Debt Contracts in the Presence of Performance Manipulation*

Ilan Guttman
Stern School of Business
New York University

Iván Marinovic
Stanford Graduate School of Business
Stanford University

October 4, 2017

Abstract

Empirical evidence suggests that firms often manipulate reported numbers to avoid debt covenant violations. The theoretical literature on debt contracting, by and large, has ignored the borrower's ability to manipulate financial reports. Building on a standard debt financing model with continuation decisions based on reported signals, we study the effect of the borrower's ability to manipulate his report on the design of debt contracts and the resulting investment continuation and manipulation decisions. The model generates an array of novel empirical predictions regarding the covenant, the interest rate (face value), the efficiency of the continuation/liquidation decisions, and the likelihood of covenant violations. For example, the model predicts that firms with stronger corporate governance may set tighter covenants and, as a result, violate their covenants more often. It also shows that firms with stronger corporate governance may face higher interest rates. We should expect covenants to be more prevalent in environments in which the cost of manipulation is relatively high and the firm's private information is more precise.

Keywords: *Asymmetric Information, Debt Contracts, Earnings Management* JEL codes: *D82,D86, G3, M12*

*We thank Cyrus Aghamolla, Tim Baldenius (discussant), Anne Beyer, Jeremy Bertomeu, Paul Fischer (editor), Beatrice Michaeli (discussant), Ningzhong Li, Stefan Reichelstein, Stephen Ryan, Paul Povel, Andy Skrzypacz, Felipe Varas, and Jeff Zwiebel for helpful comments and seminar participants at Berkeley, New York University, Colorado Acct. Research Conference, Accounting Workshop in Basel, Stanford Summer Camp, University of Texas Dallas, UCLA.

1 Introduction

The finance, economics and accounting literature has studied debt contracts extensively, both theoretically and empirically. Unlike the empirical literature, the theoretical literature has nonetheless overlooked an important and prevalent friction: managers can, and often do, manipulate financial reports to avoid debt covenant violations.[1] In this paper, we study the design of debt contracts when managers (borrowers) can manipulate reports.

Given the vast evidence of performance manipulation to avoid covenant violations, it is important to understand its effect on the design of debt contracts, the likelihood of covenant violation, as well as on firm's investment policies. Absent a theory that considers the ability to manipulate reports, it is difficult to understand some features of debt contracts and interpret the evidence relating debt contracts to firms' information systems.[2] For example, it seems intuitive that when the firm's information system is less reliable –namely, when the manager can more easily manipulate reports– the interest rate should be higher, to compensate the lender for the expected loss of control rights caused by the manager's potential manipulation. On the other hand, one might think that a less reliable information system should lead to tighter covenants, that is, greater control rights assigned to the lender, to offset the manager's reduced cost of misreporting. While these intuitions have inspired empirical research, our model demonstrates that once the debt contract is optimally designed, none of the above hypotheses always hold. Moreover, these hypotheses cannot hold at the same time.

We study how a cash constrained firm/entrepreneur, who needs to raise debt financing to pursue a positive NPV project, optimally designs a debt contract. The main innovation in our model is that the manager has the ability to manipulate financial reports to avoid a covenant violation. Manipulating the report is assumed to be costly to the firm's manager.

We analyze how the various aspects of the optimal debt contract are affected by the firm's ability to manipulate the performance measure upon which the covenant is written.[3] These aspects include: the level of the covenant, the interest rate (face value), the efficiency of the investment termination decision, and the tightness of the covenant – or the probability of covenant violation. To gain further insight, we study how these aspects vary with firms' characteristics, including: (i) how costly it is for the manager to manipulate the report –which may capture the quality of the firm's corporate governance or the reliability of the accounting system; and (ii) the precision of the firm's private information about future cash flows, which may capture the relevance of the

[1]For empirical evidence of misreporting to avoid covenant violations, see DeFond and Jiambalvo (1994); Sweeney (1994); Dichev and Skinner (2002), Graham et al. (2008) and Dyreng et al. (2011).

[2]There is a large empirical literature in accounting documenting the impact of corporate governance and accounting quality on debt contracts. See Bharath et al. (2008); Ball et al. (2008), Costello and Wittenwerg-Moerman (2011).

[3]In this paper we restrict attention to debt financing and derive the optimal debt contract. While pure debt financing is not the optimal financing method in our setting, in additional analysis of an extended setting that includes hidden effort, we allow the firm to optimally choose the mix of debt and equity funding. Our numerical analysis demonstrates that the optimal mix includes both debt and equity. Hence the trade-off we identify in the baseline model qualitatively holds even under the optimal mix of debt and equity.

firm's private information. Our model demonstrates that the answer to these questions is often counter-intuitive.

Our setting is a simple debt contracting model with three periods. In the first period, a cash constrained manager offers a debt contract to a lender in a competitive capital market to obtain financing for the firm's investment project. The debt contract is characterized by a covenant and a face value (or, equivalently, interest rate or spread). In the second period, the firm's manager privately observes a noisy signal of the profitability of the investment project and reports it to the lender. The manager can manipulate the report in order to avoid a covenant violation. Manipulating the report is costly and the cost is increasing in the magnitude of manipulation. If the report is lower than the covenant, there is a covenant violation and the control rights are transferred to the lender, who may terminate the project. Termination of the project allows the lender to recover a fraction of the loan. If the project is continued, the firm's terminal cash flows are realized in the third period. Upon realization of the cash flow, in the third period, the lender receives the minimum of the face value and the realized cash flows, while the equity holders receive the residual cash flow.

Given the structure of a debt contract, the manager has an ex-post incentive not to liquidate the project, as the liquidation value will be given to the lender.[4] When the manager's private signal is higher than the debt covenant there is no incentive to manipulate the report. When the private signal is lower than the covenant, unless the manager manipulates the report upward, the covenant is violated and the project is liquidated. The manager manipulates the report to avoid violating the covenant if the cost of the required manipulation cost is lower than the manager's expected benefit from continuation of the project. Thus, whenever the debt contract includes a covenant, it leads to a positive expected manipulation cost.

In designing the debt contract, the firm considers the tension between investment efficiency (efficiency of the termination decision) and the expected cost of manipulation induced by the contract. This tension is typically not resolved by setting a covenant that implements the first-best termination decision. While a debt contract that implements the first-best continuation decision is feasible, it is almost always suboptimal because it induces excessive expected manipulation costs.

Naturally, the firm's investment policy is affected by the reliability of the firm's accounting system. Interestingly, the direction and magnitude of this effect vary with the level of reliability. In particular, when the cost of manipulation is high (high reliability) the optimal debt contract gives rise to over-continuation of the investment project. When the cost of manipulation is low and the precision of firm's private signal is moderate, the contract induces excessive-termination of the investment. Finally, when both the manipulation costs and the precision level of the firm's private signal are low, the cost of implementing a covenant exceeds the benefit from the real option

[4]In the equilibrium of our model the manager's payoff following liquidation is zero. However, our main results would hold even if the manager obtains positive payoff following liquidation, as long as liquidation is ex-post costly to the manager compared to continuations the project. Beneish and Press (1993) document that, in their sample, following technical covenant violation firms experience increased interest costs ranging between 0.84 and 1.63 percent of the market value of firms' equity and that costs of restructuring debt represent an average of 3.7% of the market value of equity.

to terminate, so the optimal debt contract does not include a covenant.

In addition to characterizing the optimal debt contract, the model predicts how cross sectional variation in firm characteristics affects the optimal debt contract. Cross sectional variation in manipulation costs – whether it is via real manipulation which decreases the future cash flows, or accrual manipulation which are personally costly to the manager – can arise due to differences in many firm characteristics, including the quality of corporate governance; the reliability of the accounting system; the level of scrutiny by regulators, auditors, financial intermediaries and investors; regulatory enforcement; and litigation risk.

Our model demonstrates that "intuitive" predictions that often guide empirical research do not generally hold. In particular, our model predicts that while an increase in the manipulation cost unambiguously reduces the covenant level, its effect on the frequency of covenant violation and on the face value crucially depends on the cost of manipulation. For firms with a relatively high manipulation cost (high quality of corporate governance), which in equilibrium obtain excessive-continuation of the project, a further increase in the cost of manipulation increases the likelihood of covenant violation but reduces the face value. The intuition is as follows. When manipulation cost is high, the interval of private signals following which the manager manipulates the report upwards to avoid violation is relatively small, and hence, the expected manipulation costs are relatively low. High manipulation cost decreases the importance of this friction, which in turn increases the relative importance of the efficiency of the project continuation decision. As such, the termination threshold gets closer to the first-best, i.e., the likelihood of termination increases. Since the lender benefits from greater control rights following the higher likelihood of termination, the manager can offer the lender a lower face value.

For firms with a relatively low manipulation cost (low quality of corporate governance), which in equilibrium obtain over-termination of the project, the model predictions might seem surprising. An increase in the manipulation cost decreases the likelihood of covenant violation but increases the face value. For such firms, the likelihood of manipulation is high. As a consequence, these firms have an incentive to set a high covenant to shift the manipulation interval to the right tail of the distribution of signal realizations, where the density is low, as a means of decreasing the likelihood of manipulation. This leads to over-termination of the project. However, an increase in the manipulation cost decreases the prominence of the manipulation cost and enables the firm to increase the efficiency of the termination decision by lowering the covenant and the likelihood of covenant violation. Since this change amounts to less control rights for the lender, the firm must compensate the lender via a higher face value.

While the model offers novel empirical predictions, it also highlights an important insight regarding how to measure a firm's cost of debt. The literature has widely used the interest rate as a measure of the cost of debt. This measure suffers from a severe endogeneity concern, as it ignores two important aspects that are determined in conjunction with the face value, as part of the optimal debt contract. First, this measure ignores the value of control rights implicitly transferred to the lender via covenants. Second, it ignores the extent of efficiency loss in the investment continuation decision caused by the possibility of manipulation. To address these problems, we propose a measure of the cost of debt that captures the difference between the firm's return under first best (which could be achieved in a world without manipulation) and the firm's return under the optimal debt contract in the presence of manipulation.

Our model generates a rich set of empirical predictions. We predict covenants to be more prevalent in environments in which the cost of manipulation is relatively high (which as mentioned above could be related to corporate governance, industry characteristic, country characteristic) and the precision of the firm's private information is relatively high. We predict that in industries with high cost of manipulation we should expect a positive relation between the cost of manipulation and the likelihood of debt covenant violation and a negative relation between the cost of manipulation and the face value of debt. In industries with low cost of manipulation, the above relations go in the opposite direction. These predictions can be empirically tested, for example, in an international setting (taking into account variations in regulations, enforcement, transparency, etc.), or in the US by dividing the sample to pre and post Sarbanes-Oxley, or across industries.

1.1 Related Literature

We follow the Grossman-Hart-Moore property rights program by studying the optimal assignment of control rights given contractual incompleteness (see Grossman and Hart (1986); Hart (1995); Hart and Moore (1990)). The incomplete contracting literature (see Aghion and Bolton (1992)) considers the use of financial contracts to assign control rights across different states of the world. This literature takes the information structure as exogenous: information is public but non-contractible. In our setting, by contrast, information is contractible but potentially manipulated by the firm, which sometimes results in information asymmetry. In addition, given that our model introduces the ability to manipulate the report, the likelihood and the extent of misreporting and the resulting information asymmetry depend on how control rights are assigned.

Gao (2013) is closely related. He studies optimal debt contracts, when the manager can artificially increase the probability of a positive report, and the lender can commit to verifying the report, in the spirit of Townsend (1979). The optimal debt contract prescribes verification of positive reports, consistent with a conservative accounting system. Caskey and Hughes (2012) studies the impact of fair value measures on the efficiency of project selection and continuation in a debt contracting setting, in which manipulation is not possible. They find that covenants based on a conservative fair value measure tend to perform best. The accounting literature has focused on the benefits of conservatism for debt contracting. For example Gigler et al. (2009) show that in a debt contacting setting, a liberal accounting system is more efficient than a conservative system, because the former reduces the incidence of inefficient termination. Similarly Li (2013) compares the benefits of conservative vs liberal accounting systems. Both these papers do not consider manipulation and take the information system as given, whereas we assume the information system is an indirect outcome of the contracting process. Beyer (2013) studies debt contracts and conservatism and shows that the maximum capital that can be raised by a debt contract which implements efficient post-contractual decisions is higher in a conservative than in a fair value regime. Goex and Wagenhofer (2009) follow a different approach. They study optimal impairment rules. In their setting, the information system is designed ex-ante to maximize the probability that the lender will finance the firm's project, when the firm's pledgeable assets may be insufficient to guarantee financing.

There is a large literature in accounting studying the causes and consequences of earnings manipulation. This literature has followed two strands: the first strand takes the manager incentives as exogenous and focuses on the market reaction to earnings reports (see e.g. Dye (1988); Fischer and Verrecchia (2000); Guttman et al. (2006)). The second strand studies optimal contracts when managers have discretion to manipulate the reports (see e.g., Liang (2000),Beyer et al. (2014),Dutta and Fan (2014),Stein (1989)). our model is more related to the latter strand and we restrict attention to debt contracts.

Our paper is related to the costly state verification literature started by Townsend (1979), where debt contracts are optimal because they minimize verification costs. In our setting, lenders do not monitor, and the optimal debt contract seeks to minimize expected manipulation costs while maximizing investment efficiency. Unlike in Townsend (1979) the firm can obfuscate the information available to the lender via manipulation.

Dessein (2005) studies the optimal allocation of control rights as a function of the severity of information asymmetries. Garleanu and Zwiebel (2009) also study the design and renegotiation of covenants in a setting where the lender has private information at the contracting stage. In their settings, the informed party gives up control rights to the lender to signal congruent preferences. As Garleanu and Zwiebel (2009), we focus on debt contracts, and do not address the more general security design question. The optimality of debt contracts in moral hazard settings under limited liability was first established by Innes (1990). More recently Hebert (2015) proves the optimality of debt when managers effort and risk choices are unobservable.

Cornelli and Yosha (2003) study stage financing in a settings where managers can engage only in ex-ante window dressing that shifts the distribution of signals. The signal in their setting is non-contractible and they find that the optimal contract is a convertible debt contract which results in no window dressing. We consider a very different setting where signals are "hard", privately observed by the firm, but manipulable at some cost. As such, firms' reports about the signal are contractible.

The empirical literature has provided ample evidence that managers take (costly) actions to avoid covenant violation. Some examples are DeFond and Jiambalvo (1994); Sweeney (1994) which finds that "managers of firms approaching default respond with income-increasing accounting changes and that the default costs imposed by lenders and the accounting flexibility available to managers are important determinants of managers' accounting responses."Dichev and Skinner 2002 and Dyreng et al. (2011) provide large-sample support to the debt covenant hypothesis. While companies try to avoid covenant violation, covenant violations are not rare. Dichev and Skinner document that covenant violations occur at some point to about 30 percent of the firms in their sample. Graham et al. (2005) focus on violations aimed at meeting earnings benchmarks, and find that "the bond covenants hypothesis seems to be important primarily where there are binding constraints."

Covenant violation is costly to the firm. The cost of covenant violation can vary substantially across firms in terms of the type of cost and its magnitude. For example, covenant violation costs can be due to: transfer of control rights; increased interest rate (that may lead to refinancing costs); lenders' demand for partial or full repayment (which may lead to restructuring costs and modification of operations); increased lender control and restrictions on assets sale, dividend payment and investment activities (see e.g., Beneish and Press (1993)). Note that our main results do not rely on the manager's payoff upon covenant violation being zero. As long as the manager has some personal cost from covenant violation, she will have an incentive to avoid violation of the covenant and our main results will qualitatively hold.[5]

The paper proceeds as follows. Section 2 describes the model. Section 3 presents our main results, which characterize the optimal debt contract and offer the main comparative statics. In section 3.2 we provide the intuition for the main results. In section 4 we formally derive the optimal debt contract. Sections 5 and 2.3 discuss empirical implications and theoretical underpinnings. Section 6 proposes several extension and Section 7 concludes.

2 Model

We study a debt contracting setting in which the borrower can bias his report to avoid a covenant violation.

A liquidity constrained entrepreneur/firm has access to a project that requires an initial investment of I and pays out a stochastic cash flow $\tilde{x} \sim F(x)$, if completed. In order to finance the investment opportunity, the firm needs to raise an amount of I through a debt contract. The debt contract specifies a covenant, z (as explained below) and a face value, K, which the borrower promises to pay the lender at the project's maturity.

If the lender accepts the debt contract offered by the firm, and the project is funded at $t = 1$, the sequence of events is the following. At $t = 2$, the manager privately observes the realization of a noisy signal about the project's future cash flows, denoted by $\tilde{s}$. Given the realized signal s, the manager issues a (potentially biased) report of his private signal, r. The manager is not confined to truthfully reporting his signal, however, manipulating the report is costly. We assume the manager's misreporting cost equals $c|r - s|$.[6] In the main analysis we assume, for simplicity, that the manipulation costs are personally borne by the manager (as commonly assumed in the theoretical accrual management literature). In Section 6.4, we show that similar results hold when the future cash flows are decreasing in the magnitude of the manipulation, consistent with the

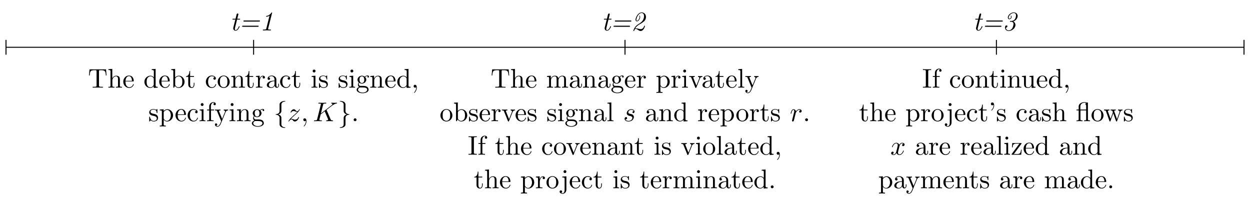
[5] In section 6.3 we study renegotiation following a covenant violation, which is an alternative way of introducing cost from covenant violation.

[6] The specific cost function does not play an important role in the analysis. All the results qualitatively go through under any strictly increasing function of the magnitude of manipulation, e.g., quadratic cost function.

notion of real earnings management rather than just accrual manipulation.

If at $t = 2$ the manager's report about his signal is lower than the contract's covenant, i.e., $r < z$, there is a covenant violation. When the covenant is violated, the lender receives the project's control rights and can terminate the project. The termination/liquidation proceeds are assumed to be L, where $L < I$. In other words, the lender can recoup part of her investment by terminating the project. If the project is not terminated, then at $t = 3$ the cash flow of the project is realized and payoffs are allocated according to the contract. Upon realization of the cash flows (at the maturity of the project at $t = 3$) the lender receives $\min(K, x)$ and the borrower retains the residual cash, that is, the manager gets $\max(x - K, 0)$. Figure 1 summarizes the timeline of the game.

Figure 1: Timeline



Both the lender and borrower are risk neutral. Cash flows are not discounted. The debt market is competitive such that the lender breaks-even. Both the manager and lender maximize their expected payoff and obtain zero payoffs when the project is not financed. The model structure is common knowledge.

2.1 Information Structure

The manager privately observes the signal, s. We assume that the distribution of the manager's signal, s, is a mixture. Specifically, with probability ρ the signal is equal to the realized cash flow x and with probability $1 - \rho$ the signal is pure noise. Thus, ρ represents the signal's precision.

Furthermore, we assume that both the cash flows x and the signal s are uniformly distributed over $[0, 1]$, i.e., $\tilde{x} \sim U\,[0, 1]$ and $\tilde{s} \sim U\,[0, 1]$. Hence, the conditional expectation of $\tilde{x}$ given the signal s is linear in s and given by

$$E(\tilde{x}|s) = \rho s + (1 - \rho)E(x).$$

We study the impact of misreporting on the design of debt contracts and abstract away from potentially confounding effects. One such effect is the variation of the density of the distributions of cash flows. To get a clearer intuition for the main economic forces determining the debt contract, we assume cash flows are uniformly distributed. In Section 6.2 we demonstrate that our main results hold under standard unbounded distributions, such as the Exponential and Log-Normal, among others.[7]

[7] We also verified that similar results hold also for truncated normal and exponential distributions, but for brevity we did not include these examples.

2.2 First Best Benchmark

Before deriving the equilibrium, we consider the first-best project termination decision. This is a useful benchmark to understand the effect of manipulation on the design of debt contracts, and quantify the efficiency loss caused by the presence of manipulation.

The first-best (labelled FB) is the continuation strategy that maximizes the expected cash flows. It is attained, in the limit, as misreporting becomes prohibitively costly ($c \to \infty$). Since the expected cash flow of the project given continuation increases in the signal s, and the payoff given termination L is independent of s, the first best continuation strategy is a threshold strategy. We denote this threshold signal by τ^{FB}. Under the first-best, for all $s < \tau^{FB}$ the project is terminated, otherwise the project is continued. τ^{FB} is the signal realization for which the expected cash flow given termination, L, equals the expected cash flow given continuation, $E\left(x|s = \tau^{FB}\right)$. Hence, the first best continuation threshold, τ^{FB}, is given by

$$\tau^{FB} = \frac{2L - (1-\rho)}{2\rho}. \tag{1}$$

The expected cash flows, which we denote by $V(\tau)$ under the first-best continuation policy are:

$$V\left(\tau^{FB}\right) = \Pr\left(s < \tau^{FB}\right) L + \Pr\left(s \geq \tau^{FB}\right) E\left(\tilde{x}|s > \tau^{FB}\right).$$

Due to the option to terminate the project, the firm value is greater than $E(x)$. Naturally, if the signal is not sufficiently informative, the the project is never terminated. A signal is sufficiently informative if there are realizations of $\tilde{s}$, such that $E(\tilde{x}|s) < L$, or equivalently if $\rho > 1 - 2L$.

2.3 Theoretical Underpinnings

A debt contract is not optimal in our stylized setting. There are multiple contacts that appropriately rewards the manager, in particular following termination, which eliminate any manipulation and induce first-best termination. Equity funding is one of those contracts. We focus on debt contracts mainly because they are highly prevalent in practice (among other reasons, due to their tax benefits, Bharath et al. (2008)). However, debt contract are optimal in slightly richer settings, where for instance, incentives are required to induce unobservable effort (see Innes (1990); Hebert (2015)) or the borrower has private information about the firm's prospects.

To focus on the impact of accounting frictions, we abstract away from alternative frictions that would render a debt contract optimal, however we argue that, under some conditions, the presence of hidden effort indeed renders a debt contract optimal (a partial analysis of this case is available upon request). To see this, consider an extended model where prior to observing his private signal the manager exerts hidden effort , a, at a convex cost, $h(a)$ (cash flows are now $x' = a + x$ and the signal becomes $s' = s + a$). When effort is infinitely costly (as in our baseline setting) the optimal contract resembles a debt contract, except that the manager gets part of the liquidation proceeds (i.e., he gets a golden parachute). Such a contract induces truth-telling as well as efficient termination.

On the other hand, when misreporting is infinitely costly, our setting is analogous to that of Innes (1990) except that the manager can exercise a real option after observing his signal, s. As such, the optimal (monotone) contract is debt. By continuity, the optimal contract should still be debt when the cost of misreporting is sufficiently large. However, as the cost of misreporting decreases, the contract should eventually include a golden parachute –thereby weakening effort incentives– as a means to mitigate expected manipulation costs. However, in general, the contract's golden parachute should be lower than that arising in the absence of effort, which suggests there should always be some misreporting in the presence of hidden effort.

A natural question in our setting –given the Revelation Principle (Myerson (1981)) – is why misreporting matters. The revelation principle assumes misreporting isn't costly to the agent. When misreporting is costly, then truth-telling isn't generally optimal; allowing for misreporting may reduce the agent's information rents when the agent has pre-contracting information, or may implement higher levels of effort (see Crocker and Slemrod (2007), Beyer et al. (2014)).

In our model, there is a potential incentive misalignment between the manager and the lender, that may trigger manipulation. We nevertheless ignore agency issues between the firm's manager and its shareholders, and simply assume the manager's objective is to maximize shareholder value net of his expected manipulation cost. In the model, the manipulation cost is borne by the manager alone. This cost may arise in practice because of litigation risk, reputation concerns, or psychic and ethical constraints. In Section 6.4, we consider the possibility that the manipulation cost is partially borne by shareholders. However, even when it is not borne by shareholders directly, the manipulation cost may be internalized by the firm's shareholders. This is trivially the case when the manager is an entrepreneur (hence, the firm's owner) and more generally when, prior to signing the debt contract, the shareholders have to hire the manager and compensate him for the expected manipulation costs he will have to bear while running the firm.

Since we focus on covenant-induced manipulation, we ignore other commonly studied frictions such as the possibility that managers expropriate the lender via empire building, cash diversion, generous dividend payout policies, asset substitution etc. These extra frictions would exacerbate the incentive misalignment between the manager and the lender.

As is usual in this strand of the contracting literature, we assume both parties have full commitment, thereby excluding the possibility of renegotiation (Section 6.3 discusses the impact of renegotiation). Our contract is not always renegotiation proof. For example, when the contract induces excessive termination, the lender could renegotiate the contract, by proposing the manager to continue the project while increasing the face value. Since the manager's default option yields zero payoffs, he should accept the lender's proposal. Conversely, when the contract prescribes inefficient continuation, the manager could propose the lender to renegotiate the contract, by terminating the project in exchange for a fraction of the liquidation proceeds. Of course the possibility of renegotiation would also distort the information transmission between the manager and the lender.

3 Equilibrium

We begin Section 3.1 by establishing some preliminary results about the reporting strategy for a any exogenously given contract. We then state the optimization program of the manager when setting the optimal debt contract and provide the main results of our paper that characterize the optimal debt contract and the main comparative statics. In Section 3.2 we provide the intuition to our main results.

3.1 The Optimal Debt Contract

A debt contract can be defined as a pair $\{K, z\}$ consisting of a face value K and a covenant z. The existence of a covenant z implies that the control rights are transferred to the lender whenever the manager's report r violates the covenant, that is, whenever $r < z$.

As an intermediate step, before deriving the optimal debt contract, we study the manager's misreporting behavior for an arbitrary contract $\{K, z\}$. Given the contract $\{K, z\}$ for any signal realization s that is higher than the covenant, that is, any $s \geq z$, the manager has no incentive to manipulate the report. When the signal realization is lower than the covenant, that is, $s < z$, the manager needs to consider the cost of manipulating the report upward to avoid covenant violation versus his expected cash flow if the project is continued (the manager's continuation value). For $s < z$, the manipulation cost to avoid covenant violation is decreasing in s where the manager's continuation value is (weakly) increasing in s. As such, any contract $\{K, z\}$ gives rise to an equilibrium termination cutoff $\tau(z, K)$, such that for lower signals, $s < \tau(z, K)$, it is too costly to manipulate the report and the manager prefers to report truthfully, violating the covenant and triggering transfer the control rights to the lender (who optimally terminates the project). The termination threshold $\tau(z, K)$ is the signal $s = \tau(z, K)$ that makes the manager indifferent between manipulating the report to meet the covenant and continue the project, versus reporting his signal truthfully to avoid the manipulation costs while letting the lender shut down the project (see Figure 2).

Formally, the manager's misreporting incentive constraint, which determines $\tau(z, K)$, as the signal value for which the manager's continuation value equals his manipulation cost, namely: [8]

$$E\left[(x-K)^{+} \,|\, s=\tau(z,K)\right] = c\,(z-\tau(z,K)), \tag{2}$$

which, given that $s \in [0, 1]$ results in the termination threshold:

$$\tau(z,K) = \max\left\{0, z - \frac{(1-\rho)(1-K)^2}{2c}\right\}, \tag{3}$$

Figure 2 depicts the manager's reporting strategy at $t = 2$ for a given contract $\{K, z\}$ and the

[8] We currently implicitly assume, and later show, that in equilibrium the lender terminates the project following a covenant violation.

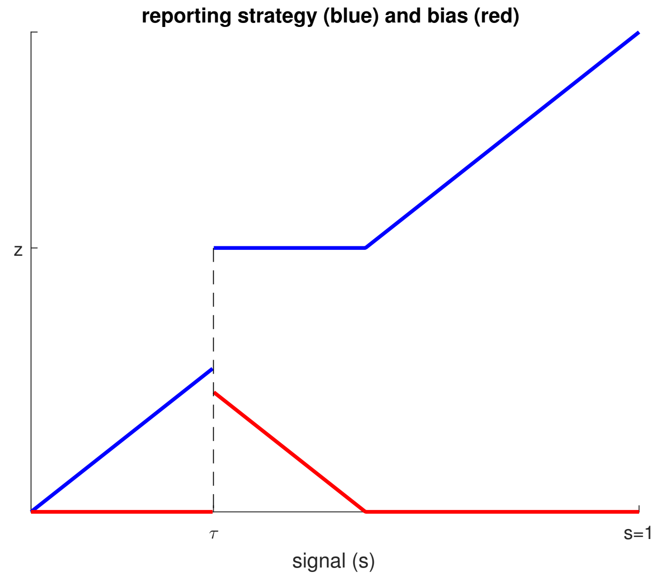


Figure 2: Reporting strategy for an arbitrary contract $\{K, z\}$ and the resulting termination threshold τ. Below τ and above z the manager reports his signal truthfully. When $s \in [\tau, z]$ the manager reports exactly z thereby over-reporting his signal s.

resulting termination threshold $\tau(z, K)$.

It is immediate from equation (3) that given all else equal, the termination threshold $\tau(z, K)$ is increasing in z, c, K and ρ. Following an increase in the contract's covenant z the manager needs to manipulate by more to avoid violating the covenant, and hence, fewer managers are willing to manipulate, thus τ increases. Following an increase in c it becomes more costly to manipulate the report to avoid covenant violation and hence, the threshold also increases in c. Following an increase in the face value, K, the manager's residual cash flow if the project is continued is lower, that is, the manager has less "skin in the game." This decreases the manager's continuation value and his willingness to manipulate, and hence, the termination threshold increases in K. Finally, an increase in the precision of the signal implies that conditional on $s = \tau(z, K)$, it is more likely that the cash flow will not be sufficient to fully pay the face value and leave a residual cash flow to the manager. Hence, the manager's continuation value is lower, which again increases the threshold τ.

The manager can always design a contract that implements efficient termination, i.e., $\tau(z, K) = \tau^{FB}$. However, any contract implementing a positive probability of termination, also induces positive expected manipulation cost. In particular, given an arbitrary contract $\{z, K\}$, the associated expected misreporting cost is given by:

$$C(z, K) \equiv \int_{\tau(z,K)}^{z} c(z - s) f(s) ds.$$

The manager can also design a contract that induces no manipulation by including no covenant ($z = 0$). However, such a contract induces over-continuation ($\tau = 0$). When designing the contract, the manager needs to optimally resolve the trade-off between efficiency of the termination decision and the magnitude of expected misreporting costs.

Formally, the optimal debt contract is given by the pair $\{K, z\}$ that solves the following program

$$\max_{\{z\geq 0, K\in[0,1]\}} \tau(z,K)\,L + \int_{\tau(z,K)}^{1} E(x|s)\,ds - C(z,K),$$

subject to the lender's participation constraint:

$$\tau(z,K)\,L + \int_{\tau(z,K)}^{1} E(\min(x,K)\,|s)\,ds \geq I.$$

The optimal debt contract maximizes expected cash flows net of manipulation costs, subject to the lender's participation constraint. As mentioned previously, when designing the debt contract the manager considers the trade-off between the expected manipulation cost and investment efficiency (the extent to which the termination threshold deviates from the first best threshold τ^{FB}). To optimally balance this trade-off, the debt contract must optimize over two levers: the lender control rights, z, and the face value, K. By controlling these aspects of the contract, the firm effectively determines not only the termination threshold, $\tau(z,K)$ but also the expected manipulation costs, $C(z,K)$.

To solve this problem, it is convenient to reformulate the above optimization program as a single variable optimization problem where the control variable is the termination threshold, τ. Before doing so, we make two observations. First, notice that in equilibrium the lender participation constraint must bind. Hence, we can define $\mathcal{K}(\tau)$ as the face value that exactly satisfies the lender's participation constraint:

$$\tau L + \int_{\tau}^{1} E(\min(x,\mathcal{K})\,|s)\,ds = I.$$

More precisely, $\mathcal{K}(\tau)$ is the face value that exactly satisfies the lender's participation constraint when the contract implements a termination threshold τ. The second observation, which we later establish formally, is that in any optimal debt contract $\mathcal{K}(\tau)$ is decreasing in the termination threshold τ, i.e., $\mathcal{K}'(\tau) < 0$. This is intuitive: If we take away control rights from the lender by reducing the termination threshold τ, we must compensate her via higher face value, so she continues to break-even. Control rights and face value are two alternative ways to reward the lender; they substitute each other.

The second definition we make to reformulate the optimization program as a single variable problem, is the covenant required to induce a termination threshold τ given face value $\mathcal{K}(\tau)$. We denote it by $\zeta(\tau)$. ζ is thus the covenant value that solves the misreporting incentive compatibility condition for a given τ when the face value is $\mathcal{K}(\tau)$, namely:

$$E\left[(x-\mathcal{K}(\tau))^{+}\,|s=\tau\right] = c(\zeta-\tau). \tag{4}$$

Armed with these observations, we can reformulate the optimal debt contract as follows:

$$\max_{\tau} \tau L + \int_{\tau}^{1} E\left(x|s\right) ds - \chi(\tau),$$

where $\chi\left(\tau\right) \equiv C\left(\zeta\left(\tau\right), \mathcal{K}(\tau)\right)$ is simply the expected manipulation cost when the contract induces termination threshold τ and the lender breaks-even.

Notice that implementing any termination threshold $\tau > 0$ induces positive manipulation costs. On the other hand, implementing a termination threshold $\tau = 0$ can be done by either not using a covenant, which induces no manipulation costs, or by setting a covenant $z = \frac{(1-\rho)(1-K)^2}{2c}$, which induces positive expected manipulation costs. Naturally, a covenant that induces no termination ($\tau = 0$) is never optimal (whenever $\tau = 0$ the manager is always better off not using a covenant, i.e., setting $z = 0$. This way, the manager implements the same termination threshold $\tau = 0$ and avoids any expected manipulation costs.)

Solving for the optimal debt contract is a two-step process. First, the manager considers the best contract with a covenant ($z > 0$), when the misreporting constraint is binding. Second, he compares the performance of such a contract versus that of a no-covenant contract, with $z = \tau = 0$. If a no-covenant contract is selected, then the manager's expected payoff is $E(x - I)$. Accordingly, the optimization problem can be rewritten as follows:

$$\Pi^* = \max\left\{\max_{\tau}\left\{V\left(\tau\right) - \chi(\tau)\right\}, E\left(x\right)\right\} - I. \tag{5}$$

where Π^* is the manager's expected payoff.

The existence of a maximum is immediate given the bounded support and continuity of the objective function. Uniqueness is not obvious, even though $V\left(\tau\right)$ is an inverse U-shape function of τ because the expected manipulation cost, $\chi(\tau)$, varies with τ in a non-linear and non-monotone way.

Next we state the paper's main result, Proposition 1, which describes the unique optimal debt contract. Then, in Proposition 2, we offer the main comparative statics describing how the contract is affected by changes in the cost of manipulation, c, and the precision of the signal, ρ. Section 3.2 discusses the intuition for the main result and only then, in Section 4, we provide the formal derivation of the optimal debt contract.

We begin by introducing two thresholds $\hat{c}$ and $\hat{\rho}$, as implicitly defined by the following equations:

$$\max_{\tau\in[0,1]}\left\{V\left(\tau|\hat{\rho}\right) - \chi\left(\tau|\hat{\rho}\right)\right\} = E\left(x\right). \tag{6}$$

and

$$\frac{2L - (1-\rho)}{2\rho} = \arg\max_{\tau\in[0,1]}\left\{V\left(\tau\right) - \chi\left(\tau|\hat{c}\right)\right\}. \tag{7}$$

$\hat{\rho}$ is the level of precision, for a given c, such that the optimal contract with a covenant results in the same expected payoff to the manager as a no-covenant contract; and $\hat{c}$ is the level of manipulation cost, given ρ, such that the optimal debt contract induces the first-best termination threshold τ^{FB}.[9] Armed with these definitions, we finally can state the paper's main result.

Proposition 1. *There exists a unique optimal debt contract characterized as follows:*

1. *If $\rho < \hat{\rho}$, the contract does not include a covenant. Hence, $z^* = 0$ and the contract induces over-continuation.*

2. *If $\rho \geq \hat{\rho}$, the contract does include a covenant (i.e., $z^* > 0$) and features one of the following two patterns:*

   (a) If $c \leq \hat{c}$, the contract entails over-termination, namely, $\tau^* \geq \tau^{FB}$,

   (b) If $c > \hat{c}$, the contract entails over-continuation, namely, $\tau^* < \tau^{FB}$.

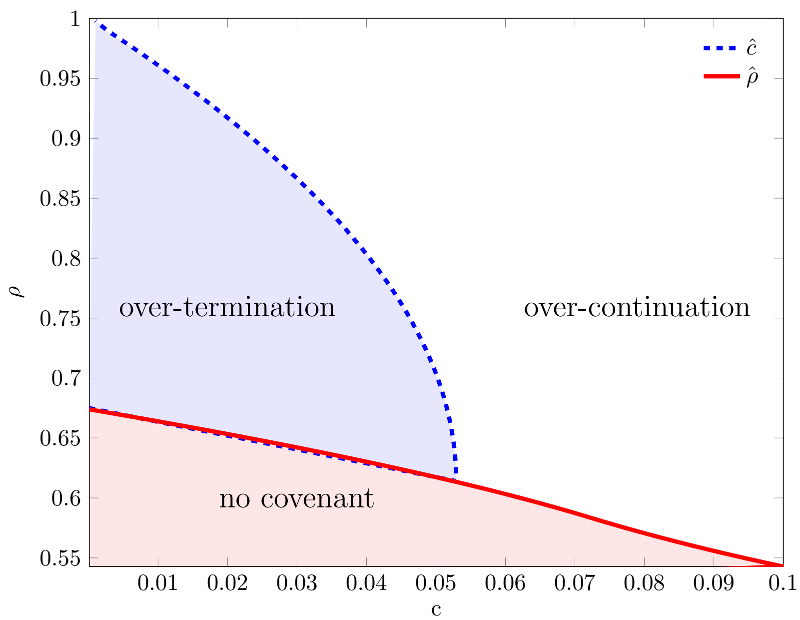


Figure 3: Accounting properties and investment efficiency. The blue curve is defined as the set of c, ρ such that the contract implements efficient termination, $\tau^* = \tau^{FB}$. The red curve is defined as the set of c, ρ such that the expected payoff of the manager, with and without covenant are the same, i.e., $\max_\tau \{V(\tau) - \chi(\tau) - E(x)\} = 0$. Parameters: $I = .45, L = .4$. Notice that the over termination region is very small. This is due to our assumption that x is uniformly distributed over $[0, 1]$. If the density of s were decreasing over its support, as in the case of Exponential distributions, over-termination tends to be more prevalent for a given c.

The above proposition reveals that both over and under-termination can arise in equilibrium in the presence of manipulation. When the manipulation friction is severe (low c) the contract induces over-termination (if $\rho > \hat{\rho}$). Covenants are set tight, leading to a high likelihood of covenant violation. This is an optimal contractual response aimed not so much at compensating the lender from the manager's future "expropriation" but instead at mitigating both the likelihood and cost

[9] Naturally, $\hat{\rho}$ depends on c and $\hat{c}$ depends on ρ. See Figure 3.

of misreporting. By contrast, when the misreporting friction is mild (high c) the more intuitive outcome of over-continuation prevails. In this case, covenants are loose and less likely to be violated than in the absence of misreporting.

Figure 3 illustrates the proposition. It shows how the properties of the accounting system (i.e., precision ρ and reliability c) affect the firm's investment choices. Broadly speaking, the accounting system not only modifies the debt contract design but, more importantly, it alters the firm's real choices, consistent with the so-called "real effects" literature in accounting. Over-termination is present when the cost of misreporting, c, is very low and precision is moderate. Observe that, for very low c, as we increase precision ρ we may transition from a contract that does not use a covenant (thus inducing over continuation) to a contract that induces over-termination (for moderate ρ) and finally to a contract that induces over-continuation again (for high ρ). Otherwise, when c is large, the effect of precision ρ on the contract is more straightforward: as ρ increases, we transition from a contract without covenant to a contract with covenant but always inducing over-continuation.

Figure 3 reveals that the firm's termination choice may be efficient (i.e., coincide with first-best τ^{FB}) even in the presence of misreporting ($c < \infty$) and noisy information ($\rho < 1$). Conversely, an increase in c or in ρ does not necessarily lead to improvements in investment efficiency but can, on the contrary, distort investment away from efficiency.

Let us turn to the main comparative statics. The debt contracting literature in accounting studies whether and how disclosure quality influences the cost of debt (see, e.g., Bharath et al. (2008), Sengupta (1998)). Next we consider the impact of two qualitative aspects of an accounting system: the precision ρ of the manager's private information, and the reliability of the accounting system, c. Specifically, the next proposition describes how the qualitative nature of the optimal debt contract varies with the main parameters of the model. The proposition focuses on the termination threshold τ, which captures the probability of covenant violation, and on the face value K of the debt contract.

Proposition 2. *Suppose $\rho \geq \hat{\rho}$ such that the optimal debt contract includes a covenant (i.e., $z^* > 0$). Then,*

1. *(effect of c) If $c \geq \hat{c}$, the likelihood of covenant violation τ^* increases in c, and the face value K^* decreases in c. If $c < \hat{c}$, the likelihood of covenant violation τ^* decreases in c, and the face value K^* increases in c.*

2. *(effect of ρ) The likelihood of covenant violation τ^* may increase or decrease in ρ. If $c > \hat{c}$, τ^* increases in ρ as $\rho \to 1$. By contrast, when both c and $I - L$ are sufficiently small, τ^* decreases in ρ as $\rho \to 1$.*

This result predicts that, once c is sufficiently high, covenant violations are more frequent among more reliable firms (with higher c). If we think of c as a proxy of the firm's corporate governance quality, then this result says that covenant violations are more likely among firms characterized by better corporate governance. To the best of our knowledge, this is a prediction that has not been tested empirically. Dichev and Skinner (2002) report "we find an unusually small number of loan/quarters with financial measures just below covenant thresholds and an unusually large number of loan/quarters with financial measures at or just above covenant thresholds' and interpret this facts as evidence of manipulation, but also recognize that they "cannot definitively rule out an explanation due to ex ante contracting– that lenders systematically set covenant thresholds just below actual values." Our study suggests precisely that the way contracts are written, and the extent to which covenants are tight, is inherently related to the firm's accounting quality. This suggests that one may extract information about accounting quality by looking at ex-post evidence of manipulation, but perhaps a more informative approach would combine ex ante information about debt contracts, along with ex post measures about the likelihood of covenant violations and evidence of misreporting.

The proposition also demonstrates that the effect of the parameters on the probability of covenant violation and the face value may qualitatively vary with the magnitude of c. In particular, when the cost of manipulation is relatively low, the likelihood of covenant violation decreases in c, and, in response to less control rights, the face value increases in c. Hence for firms with relatively low quality of corporate governance, an improvement in corporate governance will lead to a higher interest rate. More intuitively, when c is large, an increase in c increases the likelihood of violations and leads to a lower face value.

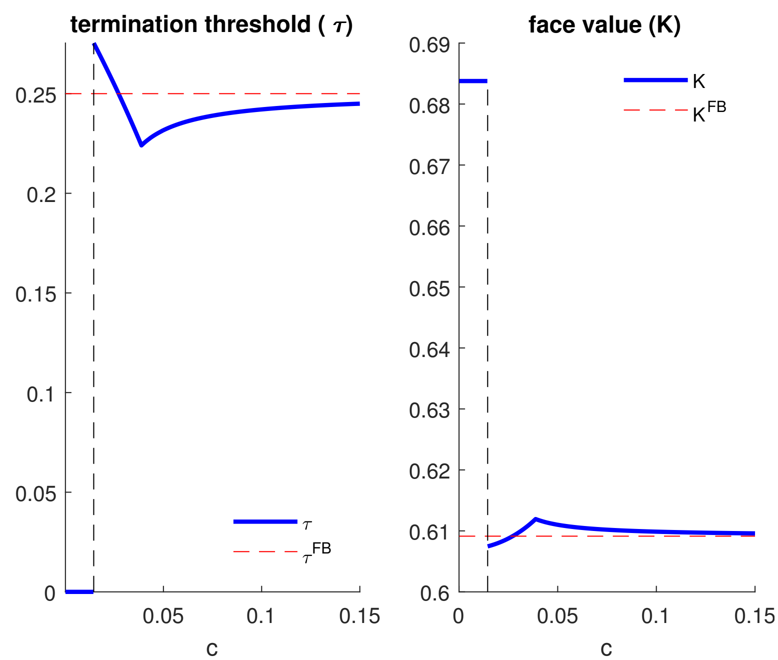


Figure 4: The effect of misreporting costs, c. Parameters: $\rho = .6, L = .35, I = .45$. The left panel shows the evolution of the termination threshold τ^* as c increases. For very low c the contract does not include a covenant and $\tau^* = 0$. Initially, as c increases the threshold jumps above the first-best level τ^{FB}, and the contract induces over-termination. As c increases further the equilibrium threshold goes down and the contract eventually induces over-continuation. Finally, as c grows large the threshold attains first-best. The evolution of the face value (left panel) is also non-monotone and mirrors that of the threshold. This is a consequence of the substitution between control rights and face value.

The likelihood of manipulation is also non-monotone in c, as Figure 5 demonstrates. In particular, for c sufficiently small such that covenant is greater than 1 we obtain over-termination, i.e., the termination threshold is relatively high. Since the probability of manipulation equals the size of the manipulation interval, which is $[\max\{1, z^*\} - \tau^*]$, the probability of manipulation is relatively low. As c increases the termination threshold decrease, however, it is still the case that $z > 1$, and hence, the probability of manipulation $[\max\{1, z^*\} - \tau^*]$ increases. This continues until $z = 1$. As c further increases, the covenant decreases and also the size of the manipulation interval and the probability of covenant violation decrease.

This non-monotonicity implies that manipulation may thus be more likely among firms featuring higher quality of corporate governance compared to lower governance quality firms, when the debt contract is endogenously determined (at least over a certain rand of corporate governance quality).

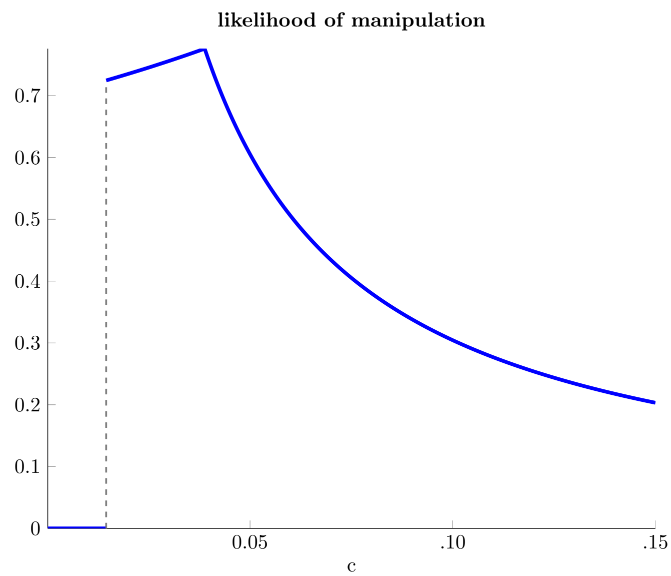


Figure 5: The likelihood of manipulation. Parameters: $\rho = .6, I = .45, L = .35$. The likelihood of manipulation is zero when c is so low that the debt contract does not include a covenant. Then it increases in c till a point where $z^* = 1$ and finally it decreases in c till it converges to 0.

The qualitative effect of the precision of the manager's signal, ρ, on the contract also depends on c. For firms with high quality of corporate governance (high c) an increase in the precision of the signal ρ increases the likelihood of covenant violation (and decreases the face value). However, for firms with low quality of corporate governance (low c) an increase in precision ρ may decrease the likelihood of covenant violation (see Figure 7). We have not been able to prove that the face value decreases in ρ (irrespective of c) but conjecture this to be true, based on extensive numerical simulations.

The non-monotone effect of the cost of misreporting, c, and precision, ρ, on the various aspects of the optimal debt contract (likelihood of covenant violation and face value), demonstrates the need for theory to better understand the empirical consequences of accounting frictions on debt contracts and the cost of capital. For example, univariate analysis and linear relations regressions may not be the adequate way to study the above aspects of debt contract.

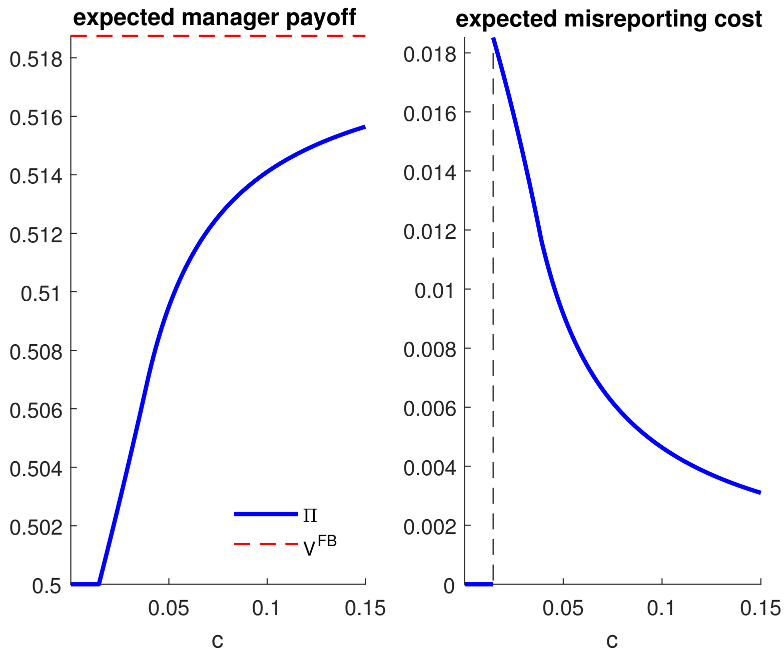


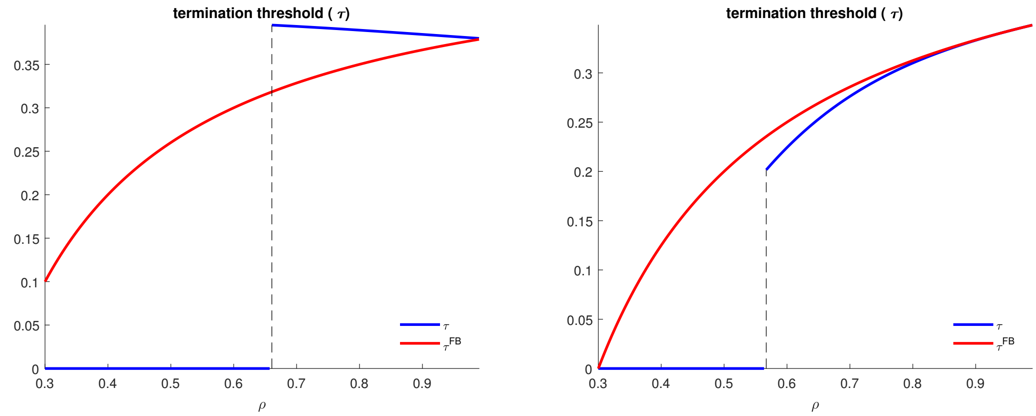
Figure 6: The effect of misreporting costs, c. Parameters: $\rho = .6, L = .35, I = .45$. The manager's expected payof Π^* increases in c eventually converging to $V^{FB} - I$ while the expected misreporting cost may initially jump up as the contract starts to incorporate a covenant but decreases in c thereafter.

To address this issue, one can think of the cost of capital, in our setting, as given by $\frac{\left(V\left(\tau^{FB}\right)-I\right)-\Pi^*}{\left(V\left(\tau^{FB}\right)-I\right)}$. This measure captures the value destroyed by the misreporting friction, which consists of two terms: i) investment distortions, measured as $V\left(\tau^{FB}\right) - V\left(\tau^*\right)$, and ii) expected misreporting costs $\chi\left(\tau^*\right)$ (relevant when the contract includes a covenant).

3.2 Intuition

The formal derivation of the equilibrium requires us to first establish some properties that must hold in any equilibrium and use these properties to solve for the optimal debt contract. Given the length of the formal derivation, we first provide economic intuition for the main results, and defer the formal derivation to the next section.

The Main Trade-off As indicated earlier, the manager can always offer a contract that implements the first-best continuation decision, τ^{FB} by adjusting the covenant z. However, implementing such a contract is typically too costly in terms of the expected manipulation cost. On the other hand, the manager can also avoid any manipulation cost by omitting a covenant, however, a no-covenant contract forgoes the value of the real option to terminate the project based on the signal s. As such, when designing the contract, the manager considers the trade-off between the efficiency of the continuation decision and the expected manipulation cost. The choice of z and K determines the termination threshold $\tau(z, K)$, which itself determines the efficiency of the investment continuation $|\tau(z, K) - \tau^{FB}|$ and the expected manipulation cost $C(z, K)$.



(a) The effect of precision ρ for small manipulation cost. $c = .001$ and $I = .4, L = .38$.

(b) The effect of precision ρ for large manipulation cost. $c = 0.1$ and $I = .4, L = .35$.

Figure 7: The figure shows the evolution of the termination threshold τ^* as precision ρ increases. For low ρ, the contract does not include a covenant and $\tau^* = 0$, so there is over-continuation. Though the first best threshold τ^{FB} increases in ρ, the equilibrium threshold may sometimes decrease in ρ, when c and $I - L$ are small (left panel).

Termination Threshold and Face Value are Substitutes Given that the capital market is efficient and the lender has no private information, the manager extracts all the rents from the lender so that the lender breaks even. That is, the lender's participation constraint is binding under the optimal debt contract. Since the manager always has an ex-post incentive to continue the project, any optimal debt contract must induce over-termination from the lender's standpoint. To see that, let us assume by contradiction that there is over-continuation from the lender's perspective, namely, that upon observing the threshold signal, the lender prefers to continue the project rather than terminate it. Given that the manager always prefers to continue the project, the manager can decrease the covenant without being required to compensate the lender for it. Hence, under the optimal debt contract, the lender strictly prefers to terminate the project when he receives the control rights (i.e., when the covenant is violated). As such, if the manager wants to increase the threshold (by increasing the covenant) the lender will require a higher face value. This implies that under the optimal debt contract, face value and covenant are substitutes; they are alternative ways of paying the lender.

Over-continuation and Over-termination As Proposition 1 indicates, the optimal debt contract can induce over-continuation or over-termination (relative to the first-best). When manipulation is sufficiently costly, i.e., c is sufficiently high, the contract induces over-continuation, whereas when c is low it either induces over-termination or else the contract includes no covenant thereby resulting in over-continuation. Note that the substitution of K and z discussed above, implies that if the firm increases the termination threshold (by increasing z) the lender will accept a lower face value. A lower face value increases the manager's continuation value and manipulation incentives.

Hence, it increases the size of the manipulation interval, i.e., increases $[z - \tau(z, K)]$. Even when the distribution is uniform, increasing the covenant, while keeping the lender's participation constraint binding, affects the manager's expected manipulation cost.

When c is high the manipulation interval $[z - \tau(z, K)]$ is relatively small. This also implies that implementing the first-best termination policy requires a covenant z lower than 1. If the manager increases the covenant, such that the termination threshold is higher than the first-best, the face value will be lower, leading to stronger manipulation incentives and a larger manipulation interval. Under a uniform distribution, this will increase the expected manipulation cost and decrease the termination efficiency. Therefore, for high c the manager will never increase the termination threshold beyond the first-best, as c increases. On the other hand, by decreasing the covenant (and the resulting termination threshold) the manager will decrease the manipulation interval and the expected manipulation cost. However, decreasing the termination threshold below the first-best will impair the efficiency of the continuation decision. However, around the first best policy τ^{FB} the investment efficiency loss is second order relative to the magnitude of expected manipulation costs. Hence, the optimal threshold is lower than than first-best. That is, for high values of c we obtain over-continuation. As c goes to infinity, the manipulation interval vanishes and the manipulation cost is no longer an issue. As a consequence, the optimal debt contract implements the first-best termination threshold.

When c is low, the manipulation friction is severe. Then, setting a tight covenant becomes an effective way of curbing manipulation. When c is small, the manipulation interval $[z - \tau(z, K)]$ is relatively large and the expected manipulation cost is high. To implement the first-best termination threshold, the manager needs to set a covenant z that is greater than 1.[10] If the manager increases the covenant beyond the one that implements first-best termination, this will also increase the termination threshold and, in response, the face value will go down. This in turn, will increase the manager's incentive to manipulate and the magnitude of the manipulation by the threshold type $s = \tau$ will be higher. However, since the support of signals is bounded by one, shifting the termination threshold to the right will decrease the size of the manipulation interval, $[1 - \tau(z, K)]$ mitigating the likelihood of manipulation and expected cost of manipulation. Notice that this result does not require that the distribution of the signal s be bounded: the result is present under unbounded distributions. The reason is that as long as the density of the signal vanishes when the signal grows large, an effective way to minimize the expected manipulation cost is to set a high covenant to push the manipulation interval toward the right tail and thereby reduce the likelihood (and expected cost) of manipulation. Here again, there is a trade-off between investment efficiency and expected manipulation costs. Around the fist best threshold, τ^{FB}, the investment efficiency loss is second order relative to the magnitude of expected manipulation costs, hence it is beneficial to increase the termination threshold and decrease investment efficiency, relative to first-best, so as to reduce expected manipulation costs. Of course, if the signal is relatively uninformative, for

[10] Allowing the manager to report something that is beyond the support of the signal, as we do here, may seem unnatural, but is not qualitatively important. All the results hold when one considers unbounded distributions.

very low values of c it may be too costly to implement a covenant and in such case the manager will sacrifice investment efficiency to eliminate any manipulation cost (the case $\rho < \hat{\rho}$).

Comparative Statics Proposition 2 demonstrates that the effect of c on the debt contract varies qualitatively based on the cost of manipulation c. Let us consider first the case of large c, for which the optimal debt contract implements over-continuation. An increase in c decreases the likelihood of manipulation and hence, decreases the expected manipulation cost (i.e., $\frac{\partial \chi(\tau)}{\partial c} < 0.$). This does not imply the threshold will go up following an increase in c. What matters is the effect of c on the marginal cost of implementing τ, i.e., $\frac{\partial \chi'(\tau)}{\partial c}$,which is negative for low values of τ and positive otherwise (i.e., $\chi(\cdot)$ is an inverted *U-shape*). For large c, the marginal cost of implementing a certain τ, in terms of manipulation costs, decreases in c (i.e., χ becomes flatter) because the manager's propensity to manipulate goes down. In contrast, the marginal benefit of τ in terms of the expected cash flow is independent of the manipulation cost ($\frac{\partial V'(\tau)}{\partial c} = 0$). As such, following an increase in c the optimal debt contract includes a higher covenant (higher termination threshold and likelihood of violation) and a lower face value. This mitigates over-continuation. Now consider the case of small c for which the optimal debt contract induces over-termination. Similar to the case of large c, an increase in c decreases the likelihood of manipulation required to induce a given termination threshold τ (the function $\chi(\tau)$ becomes more "flat"). The marginal cost of inducing τ is negative for large τ (i.e., $\chi'(\tau) < 0$) but an increase in c makes it less negative $\left(\frac{\partial \chi'(\tau)}{\partial c} > 0\right)$. The reason is that as we increase τ the face value goes down ($\mathcal{K}'(\tau) < 0$), which in turn incentivizes manipulation. However this extra manipulation incentive via the reduction in face value associated with a higher τ becomes weaker as c increases. As such, following an increase in c, the debt contract implements a higher τ^*, thereby reducing the over-termination inefficiency. In other words, a higher manipulation cost c decreases the likelihood of covenant violation and increases the face value K^*.

Consider the effect of precision ρ. Broadly speaking, a precision improvement has two effects: First, it incentivizes termination, because the signal becomes more "useful." Since the signal becomes more informative, it also becomes more attractive, from an investment perspective, to terminate the project conditional on bad news. Second, other things equal, a higher ρ mitigates manipulation incentives because the manager's option value from continuation, conditional on bad news, goes down; after all, the benefit of manipulation is predicated on the possibility that the cash flows are potentially higher than indicated by the signal. Now the qualitative effect of precision ρ on the debt contract depends on c. When c is large, such that there is over-continuation, the effect of ρ is intuitive: A higher ρ leads to a higher probability of termination and a lower face value. The intuition is as follows: Since the signal becomes more informative, from an investment point of view it's efficient to terminate the project more often (τ^{FB} goes up). Indeed, for a given face value, a marginal shift to the left in the manipulation interval $z - \tau$ reduces investment efficiency (relative to a marginal shift to the right) without reducing expected misreporting costs, for a given K. Consistent with this, the contract implements a higher probability of termination to avoid increasing the over-continuation inefficiency, namely the gap between τ^{FB} and the actual probability of termination

τ^*. In turn, since the contract provides the lender with greater control rights (and more valuable average continuation decisions) the manager can lower the face value. When c is very small, such that there is over-termination, the effect of precision ρ on τ^* is ambiguous: the probability of termination may go down as the signal becomes more precise, to mitigate the over-termination inefficiency; this is the case when liquidation proceeds L are large. Notice this does not mean the face value will increase in response to a lower likelihood of termination, because ρ also increases the average continuation value of the lender. However, we have not found examples where the face value increases in ρ.

4 Equilibrium Derivation

We begin by proving some preliminary results needed to derive the optimal debt contract. In particular, we study the lender's participation constraint and show that under the optimal debt contract the covenant and the termination threshold are substitutes; being two alternative ways to pay back the lender. We next identify the conditions under which the optimal contract does not include a covenant and turn to formally derive the optimal debt contract that includes a covenant, its properties and comparative statics (which were presented in Propositions 1 and 2).

4.1 Preliminary Results

4.1.1 Expected Manipulation Costs

As discussed in Section 3.1, any given contract $\{z, K\}$ determines a termination threshold $\tau(z, K)$ given by

$$\tau(z, K) = \max(0, z - \frac{(1-\rho)(1-K)^2}{2c}),$$

such that for $s = \tau(z, K)$ the manipulation cost incurred by the manager to meet the covenant, $c(z - \tau(z, K))$, equals the manager's expected continuation payoff, $E\left[(x-K)^+ | s = \tau(z, K)\right]$ (See equation 2). The manager manipulates the report only if the signal belongs to $s \in [\tau, z]$, and for all signals in this interval the manager reports z and the bias in the report is $z - s$. Otherwise, the manager truthfully reports his signal to the lender, even if this triggers liquidation.

We next derive the expected misreporting cost induced by any contract $\{z, K\}$. We focus on the case $\tau \leq K$ (we later show that this condition must be satisfied in equilibrium). Notice that the distance between z and τ, and the probability of manipulation is independent of τ (see the above equation for $\tau(z, K)$). This property holds for any cost function that increases in the magnitude of the manipulation. For simplicity, we assume a linear cost function.[11]

[11] Notice we do not restrict the covenant z to have the same support as the cash flows, i.e., z can be larger than one. This may seem unnatural, and is a limitation of the Uniform distribution (or more generally of distributions that have bounded support). As discussed earlier, the Uniform distribution is useful to obtain a clean intuition of the main results, but it is not strictly needed. We demonstrate the robustness of our results with respect to distributional assumptions in Section 6.2.

Given the characterization of the marginal type's misreporting cost, and the interval of types who manipulate, $s \in [\tau(z,K), \min(z,1)]$ (where the termination threshold τ is given by equation 3), we can now analyze the expected misreporting costs induced by an arbitrary contract $\{K, z\}$.

Lemma 1. *The expected misreporting cost for arbitrary $\{K, z\}$ can be written as*

$$C(z,K) \equiv \int_{\tau(z,K)}^{z} c(z-s) f(s)\, ds = \begin{cases} \frac{(1-\rho)^2}{8c}(1-K)^4 & \text{if} \quad z<1 \\ \frac{(1-\rho)^2}{8c}(1-K)^4 - \frac{c(z-1)^2}{2} & \text{if} \quad z \geq 1 \end{cases}. \tag{8}$$

Consider the effect of increasing z when $z < 1$ (and keeping K constant).[12] Increasing z induces a one-to-one increase in $\tau(z,K)$, thus shifting to the right the location of the interval of signals for which the manager manipulates the report. Given the uniform distribution, this shift does not affect the probability of manipulation. Therefore, for a given face value, the probability of misreporting and expected misreporting cost is independent of the covenant level when $z < 1$.

Now, let us consider the case of $z > 1$. Increasing z leads again to a one-to-one increase in $\tau(z,K)$. However, now the probability of manipulation — and the expected misreporting cost – decreases, because the upper bound of the interval signals for which the manager manipulates is fixed at one. Setting tighter (i.e., higher) covenants has the ability to reduce the likelihood of misreporting, *ceteris paribus*. The following corollary summarizes this result.

Corollary 1. *For a given face value K, a marginal increase in the contract's covenant z weakly reduces expected manipulation costs, i.e.,*

$$\frac{dC(z,K)}{dz} = \begin{cases} 0 & \text{if} \quad z<1 \\ <0 & \text{if} \quad z \geq 1 \end{cases}.$$

Also, for a fixed covenant z, the expected manipulation cost decreases in the face value, i.e.,

$$\frac{dC(z,K)}{dK} < 0.$$

Increasing the face value, for a given z, reduces the manager's *skin in the game*, thus weakening his manipulation incentives and lowering expected misreporting costs.

In the next section, we derive the minimal face value that the lender requires when the contract implements a termination threshold τ.

4.1.2 Lender's Participation Constraint

In addition to the expected misreporting costs, a contract $\{K, z\}$ determines the lender's expected payoff, which has two components. First, if the covenant is violated, the lender assumes control

[12] In general, the effect of z on expected misreporting costs is non-monotone. This is intuitive: the contract can eliminate misreporting costs by setting $z = 0$, i.e., no covenant. Alternatively, the contract can mitigate misreporting by setting a high covenant, so that misreporting is always too costly to the manager. (At the limit, such a contract awards all control rights to the lender.)

rights and can liquidate the project to recover L. Second, if the project is continued and generates a cash flow x, the lender receives $\min(x, K)$. The lender will accept the debt contract as long as his expected payoff is greater than the investment, I.

Since the manager can always extract all the surplus from the lender, the optimal contract should offer a face value $\mathcal{K}$ that keeps the lender's participation constraint binding, thus solving

$$\Pr(s \leq \tau(z, K)) L + \int_{\tau(z,K)}^{1} \mathbb{E}(\min(x, K) | s) \, ds = I \tag{9}$$

This equation captures the lender's participation constraint when debt markets are competitive.

Lemma 2. *The face value K that satisfies the lender's participation constraint when the contract $\{K, z\}$ induces a termination threshold τ, denoted by $\mathcal{K}(\tau)$, is given by*

$$\mathcal{K}(\tau) = 1 - \sqrt{1 - \frac{2(I - L\tau) + \rho\tau^2}{1 - \tau + \rho\tau}}. \tag{10}$$

For any $\tau \in [0, \hat{\tau}]$ the face value $\mathcal{K}(\tau)$ is a continuous U-shaped function with the following characteristics:

$$\begin{aligned} \mathcal{K}(\tau) &> I; \\ \mathcal{K}(0) &= 1 - \sqrt{1 - 2I}; \end{aligned}$$

and the unique minimum of $\mathcal{K}(\cdot)$ over the interval $[0, \hat{\tau}]$, denoted τ^+, is given by

$$\tau^+ \equiv \arg\min_{s \in [0\ \hat{\tau}]} \mathcal{K}(s),$$

where $\hat{\tau} = \frac{L + \sqrt{L^2 + 2\rho - 4\rho I}}{2\rho}$. The value of $\hat{\tau}$ is defined by $\mathcal{K}(\hat{\tau}) = 1$, and represents the termination threshold that makes the lender indifferent between accepting and not, when she has 100% rights over the firm cash flows.

The function $\mathcal{K}(\cdot)$ defines the face value that satisfies the lender's participation constraint when the project is terminated for all signals $s < \tau$, in which case the lender receives L, and continued for all $s \geq \tau$. A lender accepts a contract that implements a given cutoff τ only if $K \geq \mathcal{K}(\tau)$. However, as we discuss below, not all $\tau \in [0, \hat{\tau}]$ can be implemented, because the lender will not want to terminate the project given control rights when the signal satisfies $s > \tau^+$. For now, we will ignore this implementability issue to study the face value and termination pairs that ensure the lender breaks even regardless of whether τ can be implemented or not.

The U-shape of $\mathcal{K}(\tau)$ means that for low τ the face value and control rights are substitute ways of paying the lender: the manager can increase the lender's payoff by either increasing the face value or by increasing the covenant. However, when τ is large, face value and control rights are complements. To gain some intuition for the U-shape of $\mathcal{K}(\tau)$, note that a low value of τ (i.e., when the probability of continuation is high) means the lender is unlikely to get control rights. As such, for low values of τ the lender demands a large face value to break-even. As τ increases, the lender gets extra control rights: we add more signals under which the project is terminated – which is consistent with the lender's ex-post incentive for $s = \tau$ – and hence the lender is willing to accept a lower face value. This continues up to a certain value of τ for which the termination threshold, τ, is ex-post optimal from the lender's standpoint, that is, for $s = \tau$ the lender is indifferent between continuation and termination. As we further increase τ, the contract starts inducing excessive termination, even from the lenders's ex-post standpoint, and hence, the lender demands additional compensation in terms of face value (recall $\mathcal{K}(\cdot)$ is derived assuming the lender can commit to terminating the project when he acquires control rights). As such, the face value for which the lender breaks even is a U-shape function of τ.

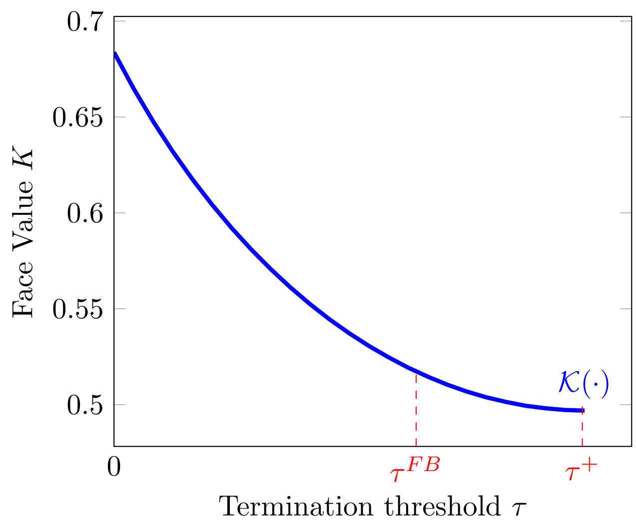


Figure 8: The lender's participation constraint. Parameters: $\rho = .95, I = .45, L = .4$. In equilibrium, the termination threshold τ^* is always below τ^+. The yellow region, above the blue curve, represents the set of feasible contracts, namely the set of contracts τ, K that satisfy the lender's participation constraint.

As mentioned above, the U-shape of $\mathcal{K}(\cdot)$ implies that, when keeping the lender's constraint binding, for values of τ such that $\tau < \tau^+$ the face value K and the termination threshold τ are substitutes. By contrast, for higher values of τ, an increase in τ requires a higher face value to satisfy the lender's participation constraint. Hence, for $\tau > \tau^+$ the termination threshold τ and the face value $\mathcal{K}(\tau)$ are complements. In the next section we demonstrate that in any equilibrium the face value and the threshold must be substitutes.

4.1.3 Covenant and Face Value are Substitutes

Given that the liquidation value L is lower than the face value, the manager always has an ex-post incentive to continue the project. For any given face value, the lender has an interior optimal continuation threshold. Lemma 2 proves that the face value that keeps the lender's participation constraint binding is U-shaped in τ. The minimum of $\mathcal{K}(\tau)$ is obtained at $\tau = \tau^+$, where the lender's ex-post optimal continuation threshold coincides with $s = \tau^+$. For any termination threshold $\tau' < \tau^+$ ($\tau'' > \tau^+$) and face value $K = \mathcal{K}(\tau')$ ($K = \mathcal{K}(\tau'')$) there is ex-post over-continuation (over-termination) from the lender's standpoint.

The following lemma shows that the equilibrium threshold τ^* is lower than τ^+. This implies that evaluated at the equilibrium threshold the face values decreases in the termination threshold, i.e., face value and termination threshold are substitutes. This also implies that for $s = \tau^*$ the lender wishes to terminate the project.

Lemma 3. *The equilibrium threshold signal, τ^*, satisfies $\left.\frac{d\mathcal{K}(\tau)}{d\tau}\right|_{\tau=\tau^*} \leq 0$.*

The reason why face value and covenant must be substitutes in equilibrium is easier to see in the case when $z^* < 1$ where for a fixed face value the expected misreporting costs are independent of the termination threshold τ. Assume by contradiction that $\left.\frac{d\mathcal{K}(\tau)}{d\tau}\right|_{\tau=\tau^*} > 0$. Then, by lowering the termination threshold while holding the face value constant the lender's participation constraint would still be satisfied but the manager would be strictly better off –in contradiction to the original contract being optimal. The case where $z^* > 1$ is more subtle, and hence the proof is in the appendix.

Thus far, we have not even considered whether a contract could implement a termination threshold above τ^+ by transferring the control right to the lender, but have demonstrated that this would never be optimal. Moreover, the lender would never want to terminate the project when given the control right for $s \geq \tau^+$. Hence, the highest termination threshold than can be implemented is indeed τ^+.

4.2 Properties of the Optimal Debt Contract

The ex-ante benefit of using a covenant is that it increases expected cash flows, by enabling some termination. However, a covenant always comes at a cost: positive expected manipulation costs. For some set of parameters, the cost exceeds the benefit, and hence the optimal contract will not include a covenant. We next identify the conditions for such contract. In the following subsection we study the optimal contract when having a covenant is optimal. We denote by τ^*, z^*, K^* the termination threshold, covenant, and face value under the optimal debt contract, and by Π^* the manager's expected payoff.

4.2.1 Optimal Debt Contract without a Covenant

The following lemma identifies a condition under which the optimal contract does not include a covenant and provides the general expression for the optimal contract, the lender's participation constraint, and the manager's incentive compatibility constraint.

Lemma 4. *If $\rho \leq \hat{\rho}$, then the debt contract does not include a covenant, the project is never terminated and there is no misreporting. Hence $\tau^* = 0$, $z^* = 0$, and the face value K^* is given by*

$$\int_0^1 E\left(\min\left(x, K^*\right)|s\right) ds = I,$$

or, equivalently,

$$K^* = 1 - \sqrt{1 - 2I}.$$

This contract generates manager's expected payoff $\Pi^* = \mathbb{E}(x) - I$ *(note that the only relevant solution is* $0 < K^* \leq 1$ *and that* $0 < I < E(x)$*).*

In general, the debt contract maximizes the firm's expected cash flows net of expected misreporting costs, subject to the lender's participation constraint, and the manager's (misreporting) incentive compatibility constraint. If ρ is small, namely the signal is not very informative, the benefit of implementing a covenant to terminate the project under bad news is very low and the misreporting cost triggered by the presence of a covenant outweighs the expected benefit. In contrast, when ρ is high, the real option to terminate the project has high value, and the benefit of implementing a covenant outweighs the associated misreporting costs.

4.2.2 Optimal Debt Contract with Covenant

We now consider the case when using a covenant is optimal, i.e., when the benefit from taking advantage of the real option from termination exceeds the cost due to manipulation. The main trade-off, investment efficiency versus manipulation costs, can be better understood from the following first order condition:

$$V'(\tau^*) = \chi'(\tau^*). \tag{11}$$

The left hand side, captures the marginal effect of increasing the probability of termination on the firm's expected cash flows. $V(\cdot)$ is a inverse U–shaped function of τ, that attains a maximum at $\tau^{FB} > 0$. The right hand side captures the marginal effect of increasing the probability of termination on the expected manipulation costs. Evaluated at the first best we have $V'\left(\tau^{FB}\right) = 0$. Hence, if $V'(\tau^*) = \chi'(\tau^*) > 0$, the equilibrium entails excessive continuation and excessive termination otherwise. We next study whether $\chi'(\tau^*)$ is positive in equilibrium.

4.2.3 Overinvestment or Underinvestment?

We first note that if a debt contract optimally implements a positive probability of termination (i.e., $\tau^* > 0$) it must necessarily induce misreporting. However, the presence of misreporting on the equilibrium path does not imply termination decisions will be distorted away from τ^{FB} namely the positive NPV rule.[13] Generically, though, (namely for almost all parameter values) the possibility of misreporting distorts termination choices away from first-best. It is however not clear whether the possibility of misreporting will induce excessive continuation or excessive termination. In order to establish this result formally, we first prove two lemmas.

Lemma 5. *Suppose* $\rho \geq \hat{\rho}$*, then there is a unique* $\tilde{c} > \hat{c}$ *such that* $z^* > 1$ *if and only if* $c < \tilde{c}$*.*[14]

[13] As we show, continuation decisions can be efficient in equilibrium ($\tau^* = \tau^{FB}$) even in the presence of equilibrium misreporting.

[14] Recall that $\hat{c}$ is defined in Proposition 1.

The above result indicates that for c sufficiently large, both misreporting and termination will take place in equilibrium. For c sufficiently small both termination and misreporting will take place in equilibrium, unless the signal is sufficiently imprecise, $\rho < \hat{\rho}$, such that not using a covenant is optimal, $\tau^* = z^* = 0$.[15]

The above lemma demonstrates that when the endogeneity of contracts is taken into account, the ability to manipulate performance could lead in equilibrium to tighter covenants compared to the no manipulation case –making termination even more likely than under first-best.

To understand whether the equilibrium entails excessive continuation or excessive termination, relative to first best, we need to study the behavior of the expected manipulation costs. Specifically, we next ask whether more termination (i.e., a higher threshold τ) increases expected misreporting costs or not. The answer depends on whether $z^* < 1$, hence it depends on the level of the cost coefficient c. Indeed, when $z^* < 1$ the expected misreporting cost depends only indirectly on τ. Inspection of equation (8) shows that

$$\chi(\tau) \propto (1 - \mathcal{K}(\tau))^4 .$$

Furthermore, we have established that in equilibrium the face value and the threshold are substitutes, namely, $\mathcal{K}'(\tau) < 0$. This means, that an increase in the continuation threshold τ allows the manager to decrease the face value without violating the lender's participation constraint. As a result, the manager retains a larger part of the realized cash flow, which increases the manager's manipulation incentive to continue the project. As such, the expected misreporting cost increases in τ for all $z \leq 1$.

When c is sufficiently high $z^* \leq 1$. Starting from the covenant that implements the first-best continuation, an increase in the termination threshold will increase the expected manipulation cost and will also decrease the efficiency of termination (moving away from first best). As such, for sufficiently high c the termination threshold is lower than the first best threshold, so the equilibrium entails over-continuation. The following Lemma formalizes this result.

Lemma 6. *If $c \geq \tilde{c}$ the equilibrium entails over-continuation. Formally $\tau^* \leq \tau^{FB}$.*

The converse is not true: a $c \leq \hat{c}$ does not necessarily lead to over-termination relative to first-best, and may even lead to efficient termination. However, if the signal is precise enough ($\rho \geq \hat{\rho}$) and reliability is sufficiently low $c \leq \hat{c} < \tilde{c}$, then we obtain over-termination.

This investment distortion is present even as $\rho \to 1$ but it vanishes as $c \to \infty$.

Corollary 2. *Suppose that $\rho \in (\hat{\rho}, 1)$ such that the optimal contract includes a covenant (i.e., $z^* > 0$), then:*

1. $\lim_{c \to 0} \tau^* = \frac{1 - \sqrt{\frac{\rho - 2(1-\rho)(I\rho - I + L)}{\rho}}}{1-\rho} > \tau^{FB}$.
2. *Suppose c is small such that $z^* > 1$. Then,* $\lim_{\rho \to 1} \tau^* = \frac{L-c}{1-c} < \tau^{FB}$.

[15] Recall that $\hat{\rho}$ is defined in Proposition 1.

3. $\lim_{c\to\infty} \tau^* = \tau^{FB}$.

Surprisingly, as $c \to 0$, instead of removing the covenant, the contract implements a very tight covenant that leads to over-termination. Interestingly, the inefficiency does not vanish as $\rho \to 1$: instead, when c is small, a very high precision leads to over-continuation.

The reason behind the excessive termination result is linked to the behavior of misreporting costs. When c is very small and ρ is large, $z^* > 1$ and the expected misreporting cost decreases in τ. The reason is that an increase in τ decreases the size of the interval of types that manipulate - $s \in [\tau, 1]$. Given the uniform distribution, the probability of misreporting decreases in τ. Note that for the same reasons discussed previously, the misreporting of the threshold type, τ, increases in τ. For sufficiently low c, the former effect always dominates, which leads to an optimal threshold that induces excessive termination. In summary, for small enough c the optimal debt contract sacrifices investment efficiency by inducing excessive termination to mitigate expected manipulation costs.

5 Discussion of the Empirical Literature

Using measures of "discretionary' accruals, DeFond and Jiambalvo (1994) find that managers use abnormal accruals to avoid debt covenant constraints. Sweeney (1994) finds that managers of firms in technical default make income-increasing accounting changes in periods before the violation, consistent with the debt covenant hypothesis. Dichev and Skinner (2002) finds an unusually small number of loan/quarters with financial measures just below covenant thresholds and an unusually large number of loan/quarters with financial measures at or just above covenant thresholds. Similar results are found by Dyreng et al. (2011).

Dichev and Skinner (2002) find strong evidence that managers take actions to avoid covenant violations but also find that covenant violations are common, occurring for about 30 percent of the observations in their sample. Garleanu and Zwiebel (2009) document that 15%–20% of outstanding loans are in violation during a typical quarter, and conditional on violating a covenant, a loan is delinquent about 40% of the time. Overall, for current-ratio covenants, about 50% of firms and 42% of loans are delinquent at some point in their lives, while for the less frequently violated net-worth covenant, about 30% of the loans are delinquent at some time.

However, the consequences of covenant violations vary across firms. Violations for financially healthy firms are typically resolved with low-cost waivers, while troubled firms face serious consequences including increased interest rates and tighter covenant restrictions. Dyreng et al. (2011) find evidence that firms engage in both real and accruals-based earnings management in order to avoid violating covenants and document that covenant violations are costly for shareholders: bank intervention following covenant violations appear to change the firm's operations in a way that is suboptimal for equity holders. Our model studies how the possibility of misreporting to avoid the consequence of covenant violations threatens the ability of covenants to induce efficient investment decisions by properly allocating decision rights between debt-holders and shareholders.

Our analysis reveals that the relation between corporate governance and debt contract design is subtle and often counter-intuitive. On some level, one would think that the possibility of misreporting should reduce the extent to which contracts will rely on financial covenants. Costello and Wittenwerg-Moerman (2011) studies the impact of reporting quality, measured by Sarbanes-Oxley internal control reports, on debt contract design and finds that when a firm experiences a material internal control weakness, lenders decrease their use of financial covenants and financial-ratio-based performance pricing provisions. On the other hand, it is natural to think that the possibility of misreporting should tighten covenants up as a means to offset the manager's tendency to exaggerate the firm's true performance in order to avoid a violation. The tension between these intuitions introduces some ambiguity as to how the accounting system's reliability affects, for example, covenant tightness (i.e., the probability of covenant violations). Our prediction is that when the accounting system is relatively reliable, further increases in the accounting system's reliability will lead to tighter covenants and a higher probability of covenant violation. In principle, one could use the probability of violation as a measure of the firm's accounting reliability, ceteris paribus. However, when the system is relatively unreliable improvements in reliability reduce covenant tightness. This non-linear relation may make it difficult to draw empirical inferences and weaken the power of empirical tests: tight covenants can in principle be a sign of a very high reliability or a very low one.

Another common intuition challenged by our theory is the relation between accounting reliability and interest rates. The evidence for the most part suggests that misreporting leads to higher interest rates. For example, Bharath et al. (2008) find that accounting quality has a significant effect on contract design but the effect differs across debt markets. In the case of private debt, both the price and non-price (i.e., maturity and collateral) terms are more stringent for poorer (accrual based) accounting quality borrowers, unlike public debt where only the price terms are more stringent. The impact of accounting quality on interest spreads of public debt is 2.5 times that of the private debt, since the price terms alone reflect the variation in accounting quality. Graham et al. (2008) study the effect of financial restatement on bank loan contracting. Compared with loans initiated before restatement, loans initiated after restatement have significantly higher spreads, shorter maturities, higher likelihood of being secured, and more covenant restrictions. These results are consistent with banks using tighter contract terms to overcome the risk and information problems arising from misreporting. Similarly Kim et al. (2011) use a sample of firms that disclosed internal control weaknesses (ICW) under Section 404 of the Sarbanes-Oxley Act, and compare the loan contracts of firms with ICW versus those without ICW. They find that the interest rate and collateral requirements are higher for ICW firms than non-ICW firms by about 28 basis points.

This idea that lower reliability leads to higher interest rates is appealing – insofar as it seems strongly supported by intuition – given that misreporting is in essence an expropriation to debtholders. To compensate for this potential expropriation –the intuition goes– debt contracts should adjust the interest rate upwards to ensure the lender is willing to provide funding (Sengupta (1998) find evidence that the interest rate is negatively associated with firms' disclosure quality as evaluated by Financial Analysts. Yu (2005) finds a similar effect on the term structure of credit spreads). Despite its intuitive appeal, this intuition misses the fact that debt contracts are multi-dimensional, and the price and non-price terms (covenants) are substitutes. The higher the interest rate given to the lender the less of a need to give him control rights via tight covenants. And though the misreporting possibility requires that we compensate the lender, it is not clear a priori what is the most efficient way to do so; by tightening covenants or by increasing the interest rate. If misreporting calls for tighter covenants as the most efficient means to compensate the lender, it will at the same time lead to lower interest rates.

A striking empirical regularity is that covenants are remarkably tight. For example, Chava and Roberts (2006) documents that at inception the average covenant threshold is only about one standard deviation away from the current value of the accounting ratio in question, so that 15%-20% of outstanding loans are in violation during a typical quarter, and conditional on violating a covenant, a loan is delinquent about 40% of the time. Similarly, Roberts and Sufi (2009) find, using a large sample of private credit agreements between U.S. publicly traded firms and financial institutions, that over 90% of long-term debt contracts are renegotiated prior to their stated maturity. Our paper offers some explanation for this observation: tight covenants can be an efficient way to resolve the consequences of misreporting if they lead to renegotiation and effectively to performance pricing. This equity aspects of the debt contract under renegotiation can sometimes mitigate the adverse consequences of misreporting.

6 Extensions

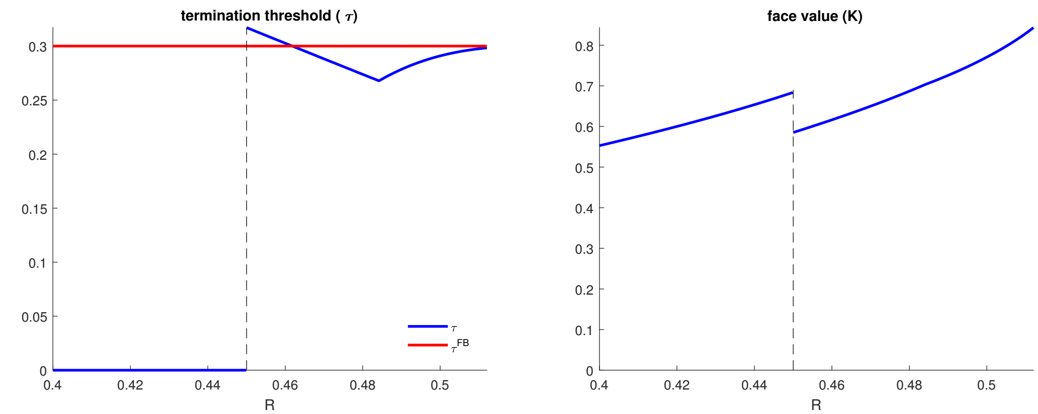
6.1 Profitability and Lending Market Competition

Thus far we have assumed that the lending market is competitive so the lender breaks-even. Next, we numerically study the effect of competition on the contract's design, investment efficiency, and misreporting. We model lending competition, in a reduced form, by assuming the lender must be given positive rents to finance the project. Formally, the lender participation constraint becomes

$$\tau L + \int_{\tau}^{1} E\left(\min\left(x, K\right)|s\right) ds \geq R$$

for $R \geq I$. R captures the lender's bargaining power. Notice that the effect of R is analogous to that of I. So increasing R is akin to lowering the profitability of the project, from the manager's perspective.

In the absence of misreporting, lending competition would only affect the face value of the contract but would not affect the contract's covenants and investment efficiency. In the presence of misreporting, the lender's bargaining power affects both misreporting and the efficiency of termination. If the contract includes a covenant, the face value increases when the lender has greater bargaining power. This in turn weakens the manager's misreporting incentives, sometimes enabling more efficient investment. Figure 9 shows the effect of R on the optimal debt contract. Intuitively, as the lender gets all the bargaining power the contract efficiency approaches first best. The reason is that as K grows, the manager misreporting incentives weaken, which in turn enables the firm to implement more efficient investment policies. As can be seen in the figure, the effect of R may be non-monotone.



(a) The effect of I on the termination threshold τ^*.

(b) The effect of I on the face value K^*.

Figure 9: Parameters: $L = .4, \rho = .5, c = .03$.

6.2 Generalizing the Distribution of Cash Flows

A distribution-free analysis is beyond the scope of this paper. In this section, we study the robustness of the over-termination result by considering the case of standard unbounded distributions such as the Exponential distribution. A similar analysis, based on the Log-Normal distribution is available from the authors upon request.

One distinct property of the Exponential distribution is that its *pdf* is monotonically decreasing. In terms of the model, this property makes over-termination more likely, since there is always an incentive to shift the manipulation interval to the right to take advantage of the decreasing density and thus reduce the likelihood of manipulation.

Assume the cash flow and signal follows an Exponential distribution, so $f(x) = e^{-x}$. The optimization program defining the optimal debt contract can be slightly adapted as follows:

$$\Pi^* = \max_{K,z,\tau} \left\{ LF(\tau) + \int_{\tau}^{\infty} E(x|s) f(s) \, ds - \chi(\tau) \right\},$$

subject to the misreporting incentive constraint:

$$c(z - \tau) = E\left[(x - K)^+ | s = \tau\right],$$

and the lender's participation constraint:

$$L \cdot F(\tau) + \int_{\tau}^{\infty} E\left[\min(x, K)\,|s\right] f(s)\, ds = I.$$

where, as before, the expected cost of manipulation is given by

$$\chi(\tau) \equiv \int_{\tau}^{z} c(z-s) f(s)\, ds.$$

We begin by studying the first-best solution. The first-best threshold maximizes the expected cash flows,

$$\begin{aligned} V(\tau) &= \left(1-e^{-\tau}\right) L + \int_{\tau}^{\infty} E(x|s) \exp(-s)\, ds \\ &= \left(1-e^{-\tau}\right) L + e^{-\tau}(1+\rho\tau). \end{aligned}$$

This yields expected cash flows of:

$$V^{FB} = L + e^{-\frac{L+\rho-1}{\rho}}\rho$$

where the threshold is

$$\tau^{FB} = \frac{L+\rho-1}{\rho}.$$

Consider the optimal contract in the presence of manipulation. Given τ, z the cost of manipulation is

$$\int_{\tau}^{z} c(z-s) f(s)\, ds = c\left(e^{-z} - e^{-\tau} + e^{-\tau}(z-\tau)\right)$$

The manager's objective function becomes

$$\Pi^* = \max_{\tau}\left\{V(\tau) - c\left(e^{-z} - e^{-\tau} + e^{-\tau}(z-\tau)\right)\right\}$$

where z is given as follows. The misreporting threshold, assuming $\tau < K$, satisfies

$$\begin{aligned} c(z-\tau) &= E\left(\max(x-K)^{+}\,|s=\tau\right) \\ &= (1-\rho)\, e^{-K}. \end{aligned}$$

On the other hand, the lender's participation constraint is

$$\begin{aligned} I &= \left(1-e^{-\tau}\right) L + \int_{\tau}^{\infty} E(\min(K,x)\,|s)\, e^{-s} ds \\ &= L\left(1-e^{-\tau}\right) + e^{-\tau}\rho\tau + e^{-\tau} - e^{-\tau-K} + e^{-\tau-K}\rho - \rho e^{-K} \end{aligned}$$

Solving for e^{-K} yields

$$e^{-K}=\frac{L\left(1-e^{-\tau}\right)+(\rho\tau+1)\,e^{-\tau}-I}{e^{-\tau}\left(1-\rho\right)+\rho} \tag{12}$$

or, equivalently,

$$K=-\ln\left(\frac{L\left(1-e^{-\tau}\right)+(\rho\tau+1)\,e^{-\tau}-I}{e^{-\tau}\left(1-\rho\right)+\rho}\right).$$

We now can rewrite the objective function as a one dimensional problem

$$\max_{\tau}\left(1-e^{-\tau}\right)L+e^{-\tau}\left(\rho\tau+1\right)-c\left(e^{-z}-e^{-\tau}+e^{-\tau}\left(z-\tau\right)\right) \tag{13}$$

subject to

$$z=\tau+\frac{1-\rho}{c}\frac{\left(1-e^{-\tau}\right)L+(\rho\tau+1)\,e^{-\tau}-I}{\rho+e^{-\tau}\left(1-\rho.\right)} \tag{14}$$

If we approximate $e^{-z}-e^{-\tau}\approx\tau-z$ the objective function reduces to

$$\max_{\tau}\left\{\left(1-e^{-\tau}\right)L+e^{-\tau}\left(\rho\tau+1\right)-c\left(\left(e^{-\tau}-1\right)\frac{1-\rho}{c}\frac{\left(1-e^{-\tau}\right)L+(\rho\tau+1)\,e^{-\tau}-I}{\rho+e^{-\tau}\left(1-\rho\right)}\right)\right\}$$

We can find a closed-form solution for this approximation. This solution is a good approximation for the original problem as $c\to 0$:

$$e^{-\tau_{approx}}=\frac{\rho\,\mathrm{LambertW}\left(\frac{1-\rho}{\rho}\exp\left(-\frac{\rho^2-\rho+L-I+I\rho}{\rho^2}\right)\right)}{1-\rho}.$$

The solution to this problem provides an upper bound for the optimal termination threshold τ. If we make ρ small, such that $\tau^{FB}\to 0$, then we get

$$\tau^*=0,$$

in which case no covenant dominates. On the other hand, if $\rho\to 1$ we get first-best termination and

$$\tau^*=L.$$

As far as comparative statics, over-termination seems more prevalent than under the Uniform distribution, given the decreasing *pdf* of the Exponential distribution. However, when ρ is very small, we may get both situations depending on the value of c (see Figure 9).

6.3 Renegotiation

Garleanu and Zwiebel (2009) argue that *given tight initial covenants, loans often fall into violation very quickly. One direct implication of this tightness is that covenants are frequently renegotiated.* The fact that violations are renegotiated does not mean they are inconsequential because they can

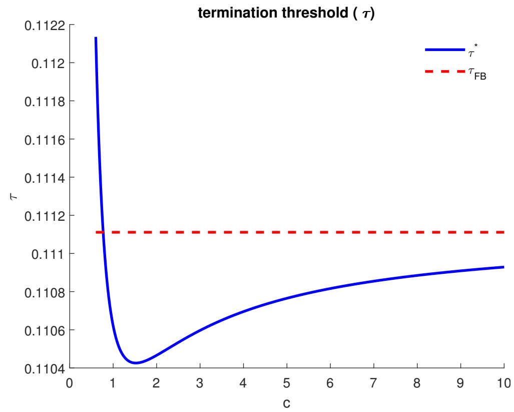


Figure 10: The effect of misreporting costs c with $x, s \sim Exponential(1)$. Parameters: $\rho = .45, L = .6, I = .65$.

affect renegotiation terms, potentially affecting the firm's misreporting choices.

Consider the case in which upon a violation of the covenant, the lender can make a take-it-or-leave-it offer (TIOLI) to the firm. In this case, renegotiation has bite only if the contract entails excessive termination (for if the contract prescribes excessive continuation, then upon a covenant violation, there are no gains from renegotiation). From the lender's standpoint, the optimal TIOLI is to offer the firm the possibility to continue the project – when the expected cash flows from continuation are higher than the liquidation proceeds – but in exchange retain full rights over the firm's continuation cash flows. The firm can accept such an offer, given that its payoff conditional on termination is zero. This renegotiation possibility and the associated efficiency gains allows the firm to reduce the face value in the original contract. If there is over-termination in the absence of renegotiation, the optimal contract, with termination, solves

$$\max_{\tau} V\left(\tau^{FB}\right) - \hat{\chi}\left(\tau\right)$$

where $\hat{\chi}$ is the expected misreporting cost under renegotiation, taking into account that the participation constraint becomes

$$\tau^{FB} L + \int_{\tau^{FB}}^{\tau} E\left(x|s\right) ds + \int_{\tau}^{1} E\left(\min\left(x, \hat{\mathcal{K}}\right) |s\right) ds = I.$$

The optimal contract seeks to minimize expected misreporting costs, subject to investment efficiency. If $\hat{\chi}\left(\cdot\right)$ is decreasing, then the threshold is such that $\hat{\mathcal{K}}\left(\tau^*\right) = 0$ (given the lender's limited liability). In other words, renegotiation leads to extremely tight covenants, as a way to minimize the probability of misreporting, but violations are waived often, when $s \in \left[\tau^{FB}, \tau^*\right]$. Notice that renegotiation has the potential to mitigate both sources of inefficiency here: by definition termination will be efficient, but also the likelihood of misreporting may go down under renegotiation. However, renegotiation won't eliminate misreporting altogether ($\tau^* < 1$) because the manager always keep a positive continuation value in equilibrium.

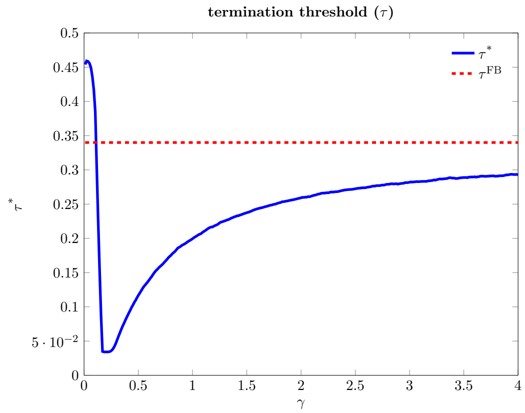


Figure 11: Optimal termination threshold with real earnings management. Parameters:$\rho = .8, c = .02, I = .4, L = .38$.

6.4 Real Earnings Management

We consider the case where manipulation not only entails personal costs to the manager but also reduces the firm cash flows.[16] Assume the cash flows are now $x' = x * \psi(m)$, where m is the amount of manipulation and $\psi(m) = e^{-\gamma m^+}$ and $\gamma > 0$. So manipulation destroys part of the firm cash flows, perhaps due to the risk of costly litigation. Notice that this model nests the previous one, when $\gamma = 0$.

The analysis parallels that in previous sections. The incentive compatibility constraint becomes now:

$$E((x * \psi(z-\tau) - K)^+|s = \tau) = c(z - \tau).$$

The lender's participation constraint is

$$F(\tau)L + \int_\tau^\infty E(\min(x * \psi(z-s), K)|s)f(s)ds = I.$$

The manager designs the optimal contract to implement a termination threshold τ that maximizes

$$F(\tau)L + \int_\tau^\infty E((x * \psi(z-s)|s)f(s)ds - \int_\tau^z c(z-s)f(s)ds.$$

subject to the above constraints. Figure 11 shows the optimal threshold τ^* as a function of γ. Notice that $\gamma = 0$ corresponds to the case without real effects. On the other hand, $\gamma = \infty$ corresponds to the case where manipulation destroys too much firm value so the manager's misreporting incentives vanish and the first best can be implemented. This example shows that in the presence of real effects we may again have both over-termination and over-continuation of projects.

[16]If misreporting did not entail personal costs to the manager, the optimal debt contract would not include a covenant.

7 Conclusion

There is ample evidence that firms can, and often do, manipulate reports in order to avoid costly covenant violation. The theoretical literature has not studies optimal design of debt contracts in the presence of performance manipulation. As has been demonstrated in other contracting settings, the ability to manipulate a report may have significant qualitative affect on the optimal contract. This paper try to feel this gap and study how debt contract are set when the manager (borrower) can manipulate his report. In particular, we study the effect of two frictions – the ability to manipulate the report and the noise in the firm's private information – on the design of debt contracts, investment efficiency, and the extent of manipulation. We consider a setting in which covenants are based on a report that can be manipulated by the manager in order to avoid a covenant violation. If truthful, the report provides useful information that determines whether a project should be continued or liquidated. In the absence of ability to manipulate the report, the covenant would transfer control rights to the lender in exactly those circumstances under which termination is efficient. However, implementing efficient termination is not necessarily optimal in the presence of manipulation. The presence of manipulation costs introduces a trade-off between expected manipulation costs and investment efficiency: a contract that aims at implementing efficient termination may result in excessive manipulation costs. Our model shows that the optimal resolution of this trade-off can implement either excessive continuation or excessive termination, depending on the parameter values (precision of the private information and the manipulation costs).

In our model, the manipulation costs, which can be proxied by the firm's corporate governance quality, affects the probability of covenant violations in a non monotone fashion. When manipulation costs are low (even vanishingly low) and the manager's private information is relatively precise, the optimal debt contract leads to excessive termination, as a means of mitigating manipulation incentives. In those cases, perhaps surprisingly, a higher manipulation cost leads to higher interest rates and looser covenants. When manipulation costs are relatively high, the optimal debt contract leads to excessive termination. We show that lender's bargaining power often mitigates misreporting and leads to more efficient investment choices. We extend the analysis to settings in which manipulation not only affect the reported number, but also has negative real effects on the firm's future cash flows. The man results are qualitatively the same in the real effect extension as well as under alternative distributions of the firm's cash flow.

Our model demonstrates that the effect of the environment in which the firm operates on the characteristics of the optimal debt contract (tightness of covenant and the face value) is complex. As such, the model can guide future empirical research that studies debt contract shed new light on the existing literature.

The complexity of the design of debt contracts, call for additional theoretical work to further our limited understanding of how to optimally design debt contracts. For example, to the best of our knowledge, there exists no theoretical guidance on how to optimally design contracts that include multiple covenants written over different performance measures.

8 Appendix

Proof of Lemma 2. Observe that

$$E\left(\min\left(x,K\right)|s\right)=\left\{\begin{array}{cc}\rho s+\left(1-\rho\right)\left(K-\frac{K^{2}}{2}\right) & s<K\\ K-\left(1-\rho\right)\frac{K^{2}}{2} & s>K\end{array}\right..$$

Using lender's participation constraint, while assuming $\tau<K$, yields

$$\tau L+\int_{\tau}^{1}E\left[\min\left(x,\mathcal{K}\right)|s\right]ds=I.$$

Solving for $\mathcal{K}$ yields

$$\mathcal{K}\left(\tau\right)=1-\sqrt{1-\frac{2\left(I-L\tau\right)+\rho\tau^{2}}{1-\tau+\rho\tau}}.$$

The function $\mathcal{K}\left(\cdot\right)$ is defined over $[0,\hat{\tau}]$ where $\hat{\tau}$ is given by the solution to

$$1-\frac{2\left(I-L\hat{\tau}\right)+\rho\hat{\tau}^{2}}{1-\hat{\tau}+\rho\hat{\tau}}=0.$$

This yields

$$1>\hat{\tau}=\frac{L+\sqrt{L^{2}+2\rho-4\rho I}}{2\rho}.$$

Furthermore, observe that $\frac{d\mathcal{K}(\tau)}{d\tau}|_{\tau=0}=\frac{-L}{\sqrt{1-2I}}<0$. Also, it's easy to verify that $\frac{d\mathcal{K}(\tau)}{d\tau}|_{\tau=\hat{\tau}}=\infty$. Further, the equation $\frac{d\mathcal{K}(\tau)}{d\tau}=0$ has two solutions. We argue by contradiction that only one of the solutions lies in the relevant range, $[0\ \hat{\tau}]$. By the Intermediate Value Theorem, at least one solution must lie in the interval, given that $\frac{d\mathcal{K}}{d\tau}|_{\tau=0}<0$ and $\frac{d\mathcal{K}}{d\tau}|_{\tau=\hat{\tau}}>0$. If both solutions lied in the interval, then the equation

$$\frac{d\mathcal{K}\left(\tau\right)}{d\tau}=0$$

would have at least three solutions, which contradicts previous results. We conclude that the function $\mathcal{K}\left(\tau\right)$ has a unique minimum over the interval $[0,\hat{\tau}]$, denoted τ^{+}, and is given by

$$\tau^{+}\equiv\arg\min_{\tau\in[0\ \hat{\tau}]}\mathcal{K}\left(\tau\right)=\frac{\rho+2\rho I-\sqrt{\rho^{2}\left(1+4I\right)+4\rho\left(I^{2}\rho-L^{2}\right)}}{2L\rho}.$$

∎

Remark 1. *In equilibrium $\tau^{*}\leq K^{*}$.*

Proof of Remark 1. In equilibrium, the lender's participation constraint is binding (Otherwise, slightly decreasing the face value, while holding constant the termination threshold, would satisfy the lender's participation constraint and increase the manager's expected payoff, despite increasing the expected manipulation cost.) Hence,

$$I = \Pr(s < \tau^*) L + \Pr(s > \tau^*) E(\min\{x, K\} | s > \tau^*).$$

For any $s > K$ the continuation value to the lender is independent of s since

$$E(\min\{x, K\} | s) = \rho K + (1-\rho) \int_0^1 \min\{x, K\} dx.$$

Suppose $\tau^* > K$. This implies that the lender's expected payoff from continuation given τ^*, which is $E(\min\{x, K\} | s = \tau^*)$, is strictly greater than his payoff upon termination, which is L. That is

$$E(\min\{x, K\} | s = \tau^*) > I > L.$$

Therefore, the lender strictly prefers to continue the project when $s = \tau^*$. The manager always prefers to continue the project. This implies there is a feasible contract that offers the lender the same face value and a lower threshold τ, and does not induce higher expected manipulation cost. Such a contract Pareto dominates the assumed contract, and hence, a contract with $\tau^* > K^*$ violates optimality. ∎

Proof of Lemma 3. We need to show that in any equilibrium $\mathcal{K}'(\tau) \leq 0$. Let $\{z^*, K^*\}$ be the equilibrium threshold and face value, then the manager's expected payoff can be written as

$$\int_{\tau^*}^1 (\mathbb{E}[\max(x - K^*, 0) | s] - c \max(z^* - s, 0)) ds, \tag{15}$$

where z^* is given by the solution to the manager's misreporting constraint

$$c(z^* - \tau^*) = \mathbb{E}\left((x - K^*)^+ | s = \tau^*\right). \tag{16}$$

Let us consider an alternative contract $\{z^o, K^*\}$ such that $z^o = z^* - \varepsilon$ for arbitrarily small $\varepsilon > 0$. Define s^o as the solution to equation (16) such that $c(z^o - s^o) = \mathbb{E}(\max(x - K^*) | s^o)$. From equation (16), we see that $z^o < z^*$ implies that $s^o < \tau^*$. If $\mathcal{K}'(\tau^*) > 0$ then the contract $\{z^o, K^*\}$ is feasible. Indeed recall that the set of feasible contracts is defined by

$$\{\{\tau, K\} : K \geq \mathcal{K}(\tau), \tau \in [0, \hat{\tau}]\},$$

where, as shown before, $\mathcal{K}(\cdot)$ is a U-shaped function over $[0, \hat{\tau}]$. We will consider two cases: $\tau^* \leq K$ and $\tau^* > K$.

When $\tau^* \leq K$ a manager with a signal $s = \tau^*$ obtains positive expected payoff only if the signal is uninformative, in which case his payoff from continuation is independent of τ^*. This implies that $z^* - \tau^*$ is independent of τ^*. In such a case, offering the contract $\{z^o, K^*\}$ is preferable to the lender, increases the likelihood of continuation (which is beneficial to the manager) and will have no effect on the manager's expected manipulation cost. As such, the contract $\{z^o, K^*\}$ is feasible and strictly dominates the contract $\{z^*, K^*\}$ for which $\mathcal{K}'(\tau^*) > 0$.

When $\tau^* > K$ a manager with a signal $s = \tau^*$ obtains positive expected payoff both when the signal is informative and when it is uninformative. In this case, the manager's expected payoff from continuation is increasing in τ^*. This implies that $z^* - \tau^*$ is also increasing in τ^*. To show that the contract $\{z^o, K^*\}$ dominates $\{z^*, K^*\}$ for which $\mathcal{K}'(\tau^*) > 0$ note that decreasing τ^* to s^0 decreases the magnitude of the manipulation for each $s \in [\tau^*, 1]$ and hence the expected payoff of these types is higher under the contract $\{z^o, K^*\}$. All types $s \in \left(s^0, \tau^*\right)$ (who didn't manipulate and terminated the project under $\{z^*, K^*\}$) prefer to manipulate and continue the project, and hence they also prefer $\{z^o, K^*\}$ over $\{z^*, K^*\}$. Since the lender also prefers the contract $\{z^o, K^*\}$, this contract is feasible and strictly dominates the contract $\{z^*, K^*\}$ for which $\mathcal{K}'(\tau^*) > 0$. ∎

Lemma 7. $z^* > 1 \rightarrow \frac{dz^*}{dc} < 0$.

Proof of Lemma 7. Let $\zeta(\tau)$ be the covenant that induces a cutoff τ when the face value is $\mathcal{K}(\tau)$, or:

$$\zeta(\tau) = \frac{(1-\rho)\left(1-\mathcal{K}(\tau)\right)^2}{2c} + \tau.$$

Now, $\frac{\partial\zeta(\tau)}{\partial c} < 0$. Also we know that $\mathcal{K}'(\tau^*) \leq 0$, which implies that $\left.\frac{d\zeta(\tau)}{d\tau}\right|_{\tau=\tau^*} > 0$ (given Lemma 3). Finally, by Lemma 4, we know that $z^* > 1 \Rightarrow \frac{d\tau^*}{dc} < 0$. Taken together these results imply that when $z^* = 1$,

$$\frac{dz^*}{dc} = \left.\frac{\partial\zeta(\tau)}{\partial c}\right|_{\tau=\tau^*} + \left.\frac{\partial\zeta(\tau)}{\partial\tau}\right|_{\tau=\tau^*}\frac{d\tau^*}{dc} < 0.$$

∎

Proof of Lemma 5. Clearly $\lim_{c\to\infty} z^* = \tau^{FB} < 1$. Also, when ρ is high enough and c is very small, we have

$$\tau^* = \arg\max_{\tau\in[0,1]}\left\{V(\tau) - \frac{(1-\tau)(1-\rho)\left(1-\frac{2(I-L\tau)+\rho\tau^2}{1-\tau+\rho\tau}\right) - c\left(\zeta(\tau)-\tau\right)^2}{2}\right\}.$$

As Proposition 1 shows, this optimization problem has a unique interior optimum. Taking limits $\lim_{c\to 0}\tau^* \frac{1-\sqrt{\frac{\rho-2(1-\rho)(I\rho-I+L)}{\rho}}}{1-\rho} > \tau^{FB}$, hence $\lim_{c\to 0} z^* = \infty$. Finally, Lemma 7 proves that $z^* = 1 \rightarrow \frac{dz^*}{dc} < 0$, so there is only one value of c, denoted $\tilde{c}$, such that $z^* = 1$, and $z^* < 1$ if and only if $c > \tilde{c}$. Now, when $z^* = 1$ the optimal threshold satisfies

$$V'(\tau^*) = -\frac{(1-\rho)^2\left(1-\mathcal{K}(\tau^*)\right)^3}{2c}\mathcal{K}'(\tau^*) > 0,$$

so at $c = \tilde{c}$ there is over-continuation, $\tau^*(\tilde{c}) < \tau^{FB}$. ∎

Proof of Lemma 6. See proof of Proposition 1. ∎

Proof of Corollary 2 . First, we show that using a covenant is optimal when ρ is sufficiently high, even as $c \to 0$. The optimal threshold, in the absence of manipulation, is

$$\tau^{FB} \equiv \arg\max_{\tau} \left\{ L\tau + \int_{\tau}^{1} E\left(x|s\right) ds \right\}.$$

By assumption $V^{FB} > \mathbb{E}\left(x\right) = \frac{1}{2}$. We argue that for ρ high enough, the debt contract includes a covenant, even as $c \to 0$. Suppose we pick τ^{FB} as the contract's threshold, perhaps sub-optimally, and set the face value accordingly at $\mathcal{K}\left(\tau^{FB}\right)$ to satisfy the lender's participation constraint. Assuming the contract leads to $z\left(\tau^{FB}\right) > 1$ (which we guarantee by making c small enough), then the expected payoff of the manager is

$$\begin{aligned}
\Pi\left(\tau^{FB}|\rho\right) + I &= V^{FB} \\
&\quad - \frac{\left(1-\tau^{FB}\right)\left(1-\rho\right)\left(1 - \frac{2\left(I - L\tau^{FB}\right) + \rho\left(\tau^{FB}\right)^2}{1-\tau^{FB}+\rho\tau^{FB}}\right) - c\left(1-\tau^{FB}\right)^2}{2} \\
&> V^{FB} - \frac{\left(1-\tau^{FB}\right)\left(1-\rho\right)\left(1 - \frac{2\left(I - L\tau^{FB}\right) + \rho\left(\tau^{FB}\right)^2}{1-\tau^{FB}+\rho\tau^{FB}}\right)}{2}
\end{aligned}$$

Now, since

$$\begin{aligned}
&\lim_{\rho \to 1} \left\{ \frac{2V^{FB} - \left(1-\rho\right)\left(1-\tau^{FB}\right)\left(1 - \frac{2\left(I - L\tau^{FB}\right) + \rho\left(\tau^{FB}\right)^2}{1-\tau^{FB}+\rho\tau^{FB}}\right)}{2} \right\} \\
&= V^{FB} > \frac{1}{2}.
\end{aligned}$$

This means we can select $\rho < 1$ large enough, denoted ρ^{+}, to ensure

$$\Pi\left(\tau^{FB}|\rho^{+}\right) + I > E(x).$$

Of course, for a fixed c, a large ρ may lead to $z\left(\tau^{FB}\right) < 1$. So to ensure $z\left(\tau^{FB}\right) > 1$, we take c small enough, say c_0, such that

$$\begin{aligned}
\Pi\left(\tau^{FB}|\rho^{+}\right) + I &= \underbrace{V^{FB} - \frac{\left(1-\tau^{FB}\right)\left(1-\rho^{+}\right)\left(1 - \frac{2\left(I - Ls^{FB}\right) + \rho^{+}\tau^{FB^2}}{1-\tau+\rho^{+}\tau^{FB}}\right)}{2}}_{>E(x)} \\
+\frac{c_0\left(1-\tau^{FB}\right)^2}{2} &> \mathbb{E}\left(x\right).
\end{aligned}$$

This proves that using a covenant is optimal for sufficiently high ρ, even as $c \to 0$. ∎

Lemma 8. $\lim_{c\to 0}\tau^* = \frac{1-\sqrt{\frac{\rho-2(1-\rho)(I\rho-I+L)}{\rho}}}{1-\rho} > \tau^{FB}$ *and* $\lim_{\rho\to 1}\tau^* = \frac{L-c}{1-c}$.

Proof of Lemma Lemma 8. To obtain the $\lim_{c\to 0}\tau^*$ we take the first order condition of the manager's optimization program with respect to τ:

$$\frac{d\left[V(\tau)-\frac{(1-\tau)(1-\rho)\left(1-\frac{2(I-L\tau)+\rho\tau^2}{1-\tau+\rho\tau}\right)-c(z-\tau)^2}{2}\right]}{d\tau}=0,$$

which leads to the first order condition

$$\frac{\left(2\rho^2+4c\rho-6c\right)\tau-2c\left(1-\rho\right)^2\tau^3-\left(1-\rho\right)\left(\rho^2+2c\rho-6c\right)\tau^2+2c+2\rho\left(I-L-I\rho\right)}{\left(1-\tau+\rho\tau\right)^2}=0.$$

Now, taking the limit as $c\to 0$ and solving for τ yields

$$\lim_{c\to 0}\tau^* = \frac{1-\sqrt{\frac{\rho-2(1-\rho)(I\rho-I+L)}{\rho}}}{1-\rho}.$$

We argue that $\lim_{c\to 0}\tau^* > \tau^{FB}$. In effect,

$$\lim_{\rho\to 1}\left[\lim_{c\to 0}\tau^*-\tau^{FB}\right]=0.$$

and

$$\lim_{\rho\to 1}\left[\frac{d\left(\lim_{c\to 0}\tau^*-\tau^{FB}\right)}{d\rho}\right]=-\frac{1}{2}L^2-\frac{1}{2}+I<0.$$

When $\rho>\hat{\rho}$, and $c\le\tilde{c}$, the optimal debt contract includes a covenant, solving

$$\max_{\tau}\left\{V(\tau)-\frac{(1-\rho)^2}{8c}\left(1-\frac{2(I-L\tau)+\rho\tau^2}{1-\tau+\rho\tau}\right)^2+\frac{c\left(\tau+\frac{(1-\rho)\left(1-\frac{2(I-L\tau)+\rho\tau^2}{1-\tau+\rho\tau}\right)}{2c}-1\right)^2}{2}\right\}$$

The first order condition of this optimization problem is

$$\begin{aligned} & \tau^2\rho^2\left(-\rho+2\tau c+1-2c\right) \\ & -2\rho\left(\rho\tau-\rho I+2\tau^3c-4\tau^2c\right) \\ -4\rho\tau c+2\rho L-2I\rho+2c\tau^3-2c\left(3\tau^2-3\tau+1\right) &= 0. \end{aligned}$$

Similarly, taking the limit as $\rho \to 1$ yields

$$\lim_{\rho \to 1} \tau^* = \frac{L-c}{1-c} < \lim_{\rho \to 1} \tau^{FB} = L.$$

∎

Corollary 3. *When $z^* \leq 1$, $\frac{\partial \tau^*}{\partial c} > 0$ and $\frac{\partial K^*}{\partial c} < 0$.*

Proof of Corollary 3. From Lemma 6, we know that $z^* \leq 1 \Rightarrow \chi'(\tau^*) > 0$. Hence $\frac{\partial \chi'(\tau^*)}{\partial c} = -\frac{1}{c}\chi'(\tau^*) < 0$. Now the first order condition is

$$\begin{aligned}
\Pi_\tau^* &= 0 \Rightarrow \frac{\partial \tau^*}{\partial c} = -\frac{\Pi_{\tau c}^*}{\Pi_{\tau\tau}^*} = -\frac{-\frac{\partial \chi'(\tau^*)}{\partial c}}{\Pi_{\tau\tau}^*} \\
&\Rightarrow \text{ sign}\left(\frac{\partial \tau^*}{\partial c}\right) = \text{sign}\left(\frac{-\partial \chi'(\tau^*)}{\partial c}\right) > 0
\end{aligned}$$

This in turn implies that the face value decreases in c, given that $\mathcal{K}'(\tau) \leq 0$ for $\tau \leq \tau^+$. ∎

Corollary 4. *When $z^* \geq 1$, $\frac{\partial \tau^*}{\partial c} < 0$ and $\frac{\partial K^*}{\partial c} > 0$.*

Proof of Corollary 4. For a given τ (such that $\zeta(\tau) \geq 1$) we can write the manager's expected payoff as

$$\Pi(\tau|c) + I \equiv V(\tau) - \frac{(1-\tau)(1-\rho)\left(1 - \frac{2(I-L\tau)+\rho\tau^2}{1-\tau+\rho\tau}\right) - c(z-\tau)^2}{2}.$$

The cross partial derivative is

$$\Pi_{\tau c} = -2(1-\tau) < 0,$$

which means that in equilibrium

$$\frac{\partial \tau^*}{\partial c} = -\frac{\Pi_{\tau c}^*}{\Pi_{\tau\tau}^*} < 0.$$

∎

Proof of Proposition 1. First we show that the contract includes a covenant if and only if precision ρ is high enough. Recall that $\hat{\rho}$ is defined by

$$\max_{\tau \in [0,\tau^+]} V(\tau|\hat{\rho}) - \chi(\tau|\hat{\rho}) = E(x).$$

$\hat{\rho}$ is the precision level such that the firm is indifferent between using a covenant and using a no-covenant contract. Next we argue that the firm will use a covenant if and only if $\rho \geq \hat{\rho}$. Suppose, by contradiction, that for some $\rho \in (\hat{\rho}, 1)$ the firm does not use a covenant. Then we have that

$$\max_\tau \{V(\tau|\rho) - \chi(\tau|\rho)\} < E(x).$$

To prove that this leads to a contradiction, we construct a feasible contract that yields payoffs above $E(x) - I$. Indeed, consider a contract implementing the same threshold $\tau^*(\hat{\rho})$ and face value $K^*(\hat{\rho})$ as the optimal debt contract under precision $\hat{\rho}$. This new contract strictly satisfies the lender's participation constraint. Furthermore,

$$\begin{aligned}\int_{\tau^*(\hat{\rho})}^{1} E^{\rho}\left[(x-K)^+|s\right] ds - \int_{\tau^*(\hat{\rho})}^{z(\hat{\rho}|\rho)} c\left(z\left(\hat{\rho}|\rho\right)-s\right) ds &> \max_{\tau\in[0,\tau^+]} V\left(\tau|\hat{\rho}\right)-\chi\left(\tau|\hat{\rho}\right) \\ &\geq E\left(x\right),\end{aligned}$$

where $z(\hat{\rho}|\rho) = \tau^*(\hat{\rho}) + \frac{(1-\rho)(1-K^*(\hat{\rho}))^2}{2c}$. Hence, the contract we have constructed yields both higher continuation cash flows and lower misreporting costs under ρ than the optimal debt contract under $\hat{\rho}$, hence it must dominate a no-covenant contract. To prove the other direction suppose $\rho < \hat{\rho}$, but the debt contract includes a covenant, so

$$\max_{\tau} V\left(\tau|\rho\right)-\chi\left(\tau|\rho\right) > E\left(x\right).$$

Then consider using $\{\tau^*(\rho), K^*(\rho)\}$ under precision $\hat{\rho}$. If this contract were feasible under ρ it must also be feasible under $\hat{\rho}$ given that $\hat{\rho} > \rho$. Clearly, this contract must give the manager a higher payoff under $\hat{\rho}$ than under ρ, hence

$$\begin{aligned}\int_{\tau^*(\rho)}^{1} E^{\hat{\rho}}\left[(x-K)^+|s\right] ds - \int_{\tau^*(\rho)}^{z(\rho|\hat{\rho})} c\left(z\left(\rho|\hat{\rho}\right)-s\right) ds &> \max_{\tau} V\left(\tau|\rho\right)-\chi\left(\tau|\rho\right) \\ &> E\left(x\right).\end{aligned}$$

which is a contradiction. Next, we show that when $\rho \geq \hat{\rho}$ there is over-continuation if and only if the cost of misreporting is high enough. Define $\tau^*(c)$ from

$$\tau^*(c) \equiv \arg\max_{\tau}\left\{V\left(\tau\right)-\chi\left(\tau|c\right)\right\}.$$

First, Corollary 2 demonstrates that $\lim_{c\downarrow 0}\tau^*(c) > \tau^{FB}$. Also, Corollary 3 and Corollary 4 prove that $\tau^*(c)$ decreases (resp. increases) in c for $c \geq \tilde{c}$ (resp. $c < \tilde{c}$) where $\tilde{c}$ is defined as $\zeta(\tau^*(\tilde{c})) = 1$ (i.e, the value of c such that the equilibrium covenant is equal to 1). Finally, $\lim_{c\rightarrow\infty}\tau^*(c) = \tau^{FB}$. Taken together these observations establish that there is a unique $\hat{c}$ defined

$$\tau^*(\hat{c}) = \tau^{FB}$$

such that if $c \leq \hat{c}$ (resp. $c > \hat{c}$) there is over-termination (resp. over-continuation). Consider uniqueness of the optimal threshold τ^*. When $z^* \geq 1$ the proof follows by contradiction. In this case, the first order condition of $V'(\tau) = \chi'(\tau)$ is a third order polynomial and has at most three real solutions, but the smallest solution is negative. The other two consecutive solutions cannot be both maxima, hence the maximum must be unique. When $z^* < 1$ the first order condition is a fourth order polynomial, so there can be (at most) two local maxima of $\Pi(\tau)$. Now, we will show that one of the maxima lies on $[\frac{1}{1-\rho},\infty)$ being outside the relevant range. Indeed,

$$\Pi(\tau) = V(\tau) - \frac{(1-\rho)^2}{8c}\left(1 - \frac{2(I - L\tau) + \rho\tau^2}{1-\tau+\rho\tau}\right)^2 - I.$$

Now note that

$$\lim_{\tau \downarrow \frac{1}{1-\rho}} \Pi(\tau) = -\infty$$

and

$$\lim_{\tau\to\infty} \Pi(\tau) = -\infty.$$

This means there is at least one local maxima in the range $[\frac{1}{1-\rho},\infty)$ which proves that the optimal threshold is unique. ∎

Proof of Proposition 2. First we prove the comparative statics with respect to c. As before, we define

$$\tau^*(c) \equiv \arg\max_{\tau\in[0,1]} \{V(\tau) - \chi(\tau|c)\}.$$

(Note that $\tau^*(c)$ is not necessarily the optimal threshold since the contract may use no covenant, in which case $z^* = \tau^* = 0$).

Lemma 7 proves that there is a unique value of c, denoted $\tilde{c}$, such that the covenant is $z^* = 1$. If $c < \tilde{c}$ (resp. $c > \tilde{c}$) the covenant is larger (smaller) than 1. On the other hand,

$$\frac{\partial\tau^*(c)}{\partial c} = -\frac{\Pi_{\tau c}}{\Pi_{\tau\tau}}.$$

Hence, $\text{sign}\left\{\frac{\partial\tau^*(c)}{\partial c}\right\} = \text{sign}\left(\frac{-\partial\chi'(\tau^*(c))}{\partial c}\right)$. Now when $c < \tilde{c}$ we have $\frac{-\partial\chi'(\tau^*(c))}{\partial c} = 2(1-\tau) < 0$ hence $\frac{\partial\tau^*(c)}{\partial c} < 0$. When $c > \tilde{c}$, we have $\frac{-\partial\chi'(\tau^*(c))}{\partial c} = -\frac{1}{c}\chi'(\tau^*(c)) < 0$, and $\chi'(\tau^*(c)) > 0$, hence $\frac{\partial\tau^*(c)}{\partial c} > 0$. Also, since $\frac{\partial K^*}{\partial c} = \mathcal{K}'(\tau^*)\frac{\partial\tau^*(c)}{\partial c}$ we have that sign $\left(-\frac{\partial K^*}{\partial c}\right) = \text{sign}\left(\frac{\partial\tau^*(c)}{\partial c}\right)$, by virtue of $\mathcal{K}'(\tau^*) < 0$.

Consider the comparative statics with respect to ρ. Consider the $z^* > 1$ case. We have:

$$\begin{aligned}
\lim_{\rho\to 1}\tau^* &= \frac{L-c}{1-c} \\
\lim_{\rho\to 1}\frac{\partial\tau^*}{\partial\rho} &= \lim_{\rho\to 1}\left(-\frac{\Pi_{\tau\rho}}{\Pi_{\tau\tau}}\right) = \frac{1}{2}\frac{3\tau^{*2} - 4\tau^*L - 4\tau^* + 2I + 2L}{1-c}.
\end{aligned}$$

For $c \approx 0$, we have $\tau^* \approx L$. Plugging $\tau^* \approx L$ above, yields $\lim_{\rho\to 1}\frac{\partial\tau^*}{\partial\rho} \approx \frac{1}{2}\frac{-L^2-2L+2I}{1-c}$ which is negative as $L \to I$. This shows that τ^* may decrease in ρ, for sufficiently high ρ and sufficiently small c. However, we can verify that even in this case, we have $\lim_{\rho\to 1, c\to 0}\frac{\partial K^*}{\partial\rho} = \frac{L\left(L+L^2-2I\right)}{2\sqrt{1-2I+L^2}} < 0$. Consider the $z^* < 1$ case. It's easy to verify that $\lim_{\rho\to 1}\frac{\partial\tau^*}{\partial\rho} = \frac{1}{2} - \tau^*$. Since $\tau^* < L < E(x)$, this implies that $\lim_{\rho\to 1}\frac{\partial\tau^*}{\partial\rho} > 0$. ∎

References

Aghion, Philippe and Patrick Bolton, "An incomplete contracts approach to financial contracting," *The review of economic Studies*, 1992, *59* (3), 473–494.

Ball, Ryan, Robert M Bushman, and Florin P Vasvari, "The debt-contracting value of accounting information and loan syndicate structure," *Journal of accounting research*, 2008, *46* (2), 247–287.

Beneish, Messod D and Eric Press, "Costs of technical violation of accounting-based debt covenants," *Accounting Review*, 1993, pp. 233–257.

Beyer, Anne, "Conservatism and aggregation: The effect on cost of equity capital and the efficiency of debt contracts," *Rock Center for Corporate Governance at Stanford University Working Paper*, 2013, (120).

_ , Ilan Guttman, and Ivan Marinovic, "Optimal contracts with performance manipulation," *Journal of Accounting Research*, 2014, *52* (4), 817–847.

Bharath, Sreedhar T, Jayanthi Sunder, and Shyam V Sunder, "Accounting quality and debt contracting," *The Accounting Review*, 2008, *83* (1), 1–28.

Caskey, Judson and John S. Hughes, "Assessing the Impact of Alternative Fair Value Measures on the Efficiency of Project Selection and Continuation," *The Accounting Review*, 2012, *87* (2), 483–512.

Chava, Sudheer and Michael R Roberts, "Is financial contracting costly? An empirical analysis of debt covenants and corporate investment," *RODNEY L WHITE CENTER FOR FINANCIAL RESEARCH-WORKING PAPERS-*, 2006, *19.*

Cornelli, Francesca and Oved Yosha, "Stage financing and the role of convertible securities," *The Review of Economic Studies*, 2003, *70* (1), 1–32.

Costello, Anna and Regina Wittenwerg-Moerman, "The impact of financial reporting quality on debt contracting: Evidence from internal control weakness reports," *Journal of Accounting Research*, 2011, *49* (1), 97–136.

Crocker, Keith and Joel Slemrod, "The Economics of Earnings Manipulation and Managerial Compensation," *The RAND Journal of Economics*, 2007, *38* (3), 698–713.

DeFond, Mark L and James Jiambalvo, "Debt covenant violation and manipulation of accruals," *Journal of accounting and economics*, 1994, *17* (1), 145–176.

Dessein, Wouter, "Information and control in ventures and alliances," *The Journal of Finance*, 2005, *60* (5), 2513–2549.

Dichev, Ilia D and Douglas J Skinner, "Large–sample evidence on the debt covenant hypothesis," *Journal of accounting research*, 2002, *40* (4), 1091–1123.

Dutta, Sunil and Qintao Fan, "Equilibrium earnings management and managerial compensation in a multiperiod agency setting," *Review of Accounting Studies*, 2014, *19* (3), 1047–1077.

Dye, Ronald, "Earnings management in an overlapping generations model," *Journal of Accounting research*, 1988, *26*, 195–235.

Dyreng, S, SA Hillegeist, and F Penalva, "Earnings management to avoid debt covenant violations and future performance," Technical Report, Working Paper, Duke University, Arizona State University 2011.

Fischer, Paul and Robert Verrecchia, "Reporting bias," *The Accounting Review*, 2000, *75* (2), 229–245.

Gao, Pingyang, "A measurement approach to conservatism and earnings management," *Journal of Accounting and Economics*, 2013, *55* (2), 251–268.

Garleanu, Nicolae and Jeffrey Zwiebel, "Design and renegotiation of debt covenants," *Review of Financial Studies*, 2009, *22* (2), 749–781.

Gigler, Frank, Chandra Kanodia, Haresh Sapra, and Raghu Venugopalan, "Accounting conservatism and the efficiency of debt contracts," *Journal of Accounting Research*, 2009, *47* (3), 767–797.

Goex, Robert F and Alfred Wagenhofer, "Optimal impairment rules," *Journal of Accounting and Economics*, 2009, *48* (1), 2–16.

Graham, John, Campbell Harvey, and Shiva Rajgopal, "The economic implications of corporate financial reporting," *Journal of Accounting and Economics*, 2005, *40* (1-3), 3–73.

Graham, John R, Si Li, and Jiaping Qiu, "Corporate misreporting and bank loan contracting," *Journal of Financial Economics*, 2008, *89* (1), 44–61.

Grossman, Sanford J and Oliver D Hart, "The costs and benefits of ownership: A theory of vertical and lateral integration," *The Journal of Political Economy*, 1986, pp. 691–719.

Guttman, Ilan, Ohad Kadan, and Eugene Kandel, "A rational expectations theory of kinks in financial reporting," *The Accounting Review*, 2006, *81* (4), 811–848.

Hart, Oliver, *Firms, contracts, and financial structure*, Clarendon Press, 1995.

_ **and John Moore**, "Property Rights and the Nature of the Firm," *Journal of political economy*, 1990, pp. 1119–1158.

Hebert, Benjamin, "Moral Hazard and the Optimality of Debt," *Available at SSRN 2185610*, 2015.

Innes, Robert D, "Limited liability and incentive contracting with ex-ante action choices," *Journal of economic theory*, 1990, *52* (1), 45–67.

Kim, Jeong-Bon, Byron Y Song, and Liandong Zhang, "Internal control weakness and bank loan contracting: Evidence from SOX Section 404 disclosures," *The Accounting Review*, 2011, *86* (4), 1157–1188.

Li, Jing, "Accounting conservatism and debt contracts: Efficient liquidation and covenant renegotiation," *Contemporary Accounting Research*, 2013, *30* (3), 1082–1098.

Liang, Pierre Jinghong, "Accounting Recognition, Moral Hazard, and Communication*," *Contemporary Accounting Research*, 2000, *17* (3), 458–490.

Myerson, Roger B, "Optimal auction design," *Mathematics of operations research*, 1981, *6* (1), 58–73.

Roberts, Michael R and Amir Sufi, "Renegotiation of financial contracts: Evidence from private credit agreements," *Journal of Financial Economics*, 2009, *93* (2), 159–184.

Sengupta, Partha, "Corporate disclosure quality and the cost of debt," *Accounting review*, 1998, pp. 459–474.

Stein, Jeremy, "Efficient capital markets, inefficient firms: A model of myopic corporate behavior," *The Quarterly Journal of Economics*, 1989, pp. 655–669.

Sweeney, Amy Patricia, "Debt-covenant violations and managers' accounting responses," *Journal of accounting and Economics*, 1994, *17* (3), 281–308.

Townsend, Robert M, "Optimal contracts and competitive markets with costly state verification," *Journal of Economic theory*, 1979, *21* (2), 265–293.

Yu, Fan, "Accounting transparency and the term structure of credit spreads," *Journal of Financial Economics*, 2005, *75* (1), 53–84.