Vertical Integration, Collusion, and Tariffs*

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Abstract

This article presents a link between tariff rates, collusion and industry structure. We examine the role of tariffs on final-goods as to a firm’s decision to integrate and collude in the presence of competitive imports. It is shown that, under some conditions, the upstream firm has an incentive to engage in vertical integration to profitably introduce a wholesale price above the world input price while not inducing any intermediate or final good imports. Higher tariffs downstream, even with no tariff protection upstream, make this strategy more profitable, and provides a rationale for a positive relationship between tariff protection and vertical integration, that is observed in some industries.

JEL Codes: F12, L12, L42.
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1 Introduction

This paper shows that the fact that a vertically integrated firm simultaneously produces the final product and acts as an input supplier provides it with an effective instrument with which to control the downstream market, and that such control is easier to exercise the more protected this downstream market is from foreign competition. For instance, according to Lamoreaux (1985), US Steel was able for some time to control independent manufacturers of finished products by ”holding up prices on raw materials and forcing down prices on finished products”. The fact that US Steel was a vertically integrated firm meant that it was able to be an active player in the downstream market, while being a potential supplier of raw materials to competitors of its downstream divisions.

The consolidation that led to the creation of US Steel occurred amid heavy tariff protection for the iron and steel industry, following the protectionist tariff acts in the 1890’s. This protection was particularly intense in the case of finished products, see Irwin (2000) for the case of the tinplate industry. The development of the Canadian steel industry in the latter half of the 19th century saw, e.g., the subsequent integration processes of the Nova Scotia Steel and Coal Company. This company owned iron deposits, and manufactured steel as well as finished products (such as boilers, engines, axles, tools,...).1 The growth of iron and steel production in Canada is related with the 1879 protective tariffs (see Donald, 1915, where attention is drawn to the manner in which prices were fixed and competition controlled).

There are some studies point out at a positive correlation between import tariffs and vertical integration. Webb (1980) highlights the interdependence between tariffs and cartels, and reports several vertical mergers occurring in the German steel industry, arguing that they occurred mostly to circumvent high wholesale prices charged by upstream firms. Similarly, Brown (1992) documents that the German cotton industry in the 19th Century was more vertically integrated than its British counterpart, where protection was much less intense. Furthermore, protection was more intense for final products than for intermediate inputs. Mendi (2005a) studies the case of the late 19th Century Spanish steel industry. One of the main purposes of

1The recent paper by Balakrishnan et al. (2007) provides a historical overview that examines a number of factors which influenced the development in major manufacturing industries in Canada, such as iron and steel, furniture, textiles and shoes, and automobiles. The integration of production activities and protection are elements common to these industries, although there are certainty other factors that help explain the growth of manufacturing in Canada.
domestic steel producers, which were colluding throughout most of the final decades of the 19th Century, was to control the domestic producers of steel manufactures. This industry was heavily protected from foreign competition after the passing of the 1891 tariff law and vertical mergers took place in the early 20th century.

The purpose of this paper is to study the relationship between tariff protection, collusion and the degree of vertical integration in the domestic industry, which faces competitive imports both at the input and the final product levels. Initially, the domestic upstream firm may acquire some downstream firms, which behave as independent outlets. Thus, they procure the input at the world price and tacitly collude in the final product market. Then, we consider that the upstream firm can play an active role and, if profitable, offers a collusive mechanism to non-integrated producers; this consists of a price for the final product, a wholesale price, and a collusive output level for each of the non-integrated downstream firms. Downstream firms decide whether to purchase the input from the upstream firm or in the world market. The world market is the downstream firms’ alternative input source, and will effectively condition the upstream firm’s behavior. Our focus is on the effect of increasing tariffs that tax final good imports on the incentives of the upstream firm to integrate forward and sustain a more collusive outcome, when there are no tariffs protecting the input that the upstream firm sells.

Our main findings can be summarized as follows. Firstly, we find that the number of downstream firms that the upstream firm is interested in acquiring increases with tariff protection downstream. This is because prices downstream increase with the number of integrated outlets, and tariffs on the final product set an upper bound on its domestic price. This way, the higher the tariff rate, the lower the probability that it imposes an effective cap on downstream prices, and thus, the greater the likelihood that profits from integrating with more firms increase. Hence, in the presence of convex integration costs, higher tariffs downstream make integration with a larger number of downstream firms more likely, a result that is consistent with the above evidence. Secondly, under certain conditions, forward integration will allow the upstream firm to charge a wholesale price above the competitive world price for the input without inducing any imports, even in the absence of any cost ad-

\footnote{Although our paper is not on business and social networks, the survey paper by Rauch (2001) suggests that domestic networks (or market structures) can create informal barriers by facilitating collusion to restrict the market access of foreign firms.}
vantages or tariff protection on the input that it sells. Though this result might look surprising, it can be explained by noting that vertical integration provides the upstream monopolist with a powerful instrument to discipline non-integrated downstream firms. Specifically, the ability to produce the final good constitutes a credible threat that may induce non-integrated downstream firms to accept such wholesale price, and acceptance avoids retaliation by the vertical structure. Sales of the input at a wholesale price above that in the world market constitute the revenue-extracting mechanism used by the upstream monopolist. Finally, and in connection with the previous result, it is interesting to note that domestic consumers will enjoy lower final good prices when integration leads to a wholesale price above world price as compared with a setting where the input is procured at the world price; this occurs for sufficiently large values of the discount factor.

Although many studies have devoted attention to the analysis of tariffs, vertical relationships and collusion, to the best of our knowledge, no single paper seems to have combined them in a single model. There exists some research on international trade theory in the presence of vertical relationships. Seminal papers by Spencer and Jones (1991, 1992) examine the effects of trade policies on the vertical supply decision of a potentially vertically integrated foreign firm to the domestic intermediate-good market. The fact that the foreign firm controls both the exports of the intermediate and the final goods allows it to strategically react before the use of domestic trade policy towards the intermediate good industry, as in Spencer and Jones (1992), by altering its export combination. Ishikawa and Lee (1997) extend Spencer and Jones (1991, 1992) in several respects as they focus on the domestic final good market, permit entry and exit caused by trade policies, and also because both the upstream and downstream home industries are oligopolistic. These authors identify adverse effects on domestic profits when different tariff rates are imposed on final and intermediate goods sectors because the foreign vertically integrated firm may enter or exit accordingly. In contrast, we investigate the decision of a monopolist domestic supplier to integrate with a number of domestic downstream firms and consequently strategic interaction between the vertical structure and non-integrated downstream

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3Spencer and Raubitschek (1996) propose a different model where domestic firms producing the final good agree to set up a joint venture that produces part of their input requirements. These authors consider a static game, an oligopolistic market structure, and do not analyze the role of tariffs. The fact that member firms control part of input production makes the formation of a vertical production joint venture profitable, even though the domestic marginal cost of production exceeds the world input price; an idea that is present in some form in our paper.
firms may arise. We consider the extreme case of "tariff escalation"\footnote{Tari
d escalations can effectively protect the domestic final good sector by encouraging
import of intermediate products, with low or no tariffs, and discouraging
import of final products, with high or prohibitive tariffs.} since we assume
that the intermediate good industry is unprotected. By introducing tariff protection
downstream the domestic upstream firm can exercise market power by integrating
forward. The vertical structure’s strategic behavior allows it to implement a collu-
sive mechanism which offsets the costs of integration while not inducing intermediate
good imports.

A faithful description of ongoing oligopolistic rivalry in an international environ-
ment should consider a dynamic interaction setting where firms compete over several
periods. Davidson (1984) first examined the role of tariffs on the ability of home and
foreign oligopolistic firms to collude in a single market. Tariffs introduce cost asym-
metries and hence they make deviations more attractive for a home country firm
than for a foreign country firm. In such a setting, Davidson shows that when tariffs
are small the output reallocation leads to an industry structure more conducive to
collusion.\footnote{How tariffs and quotas influence cartel stability has been dealt with by
Rotemberg and Saloner (1989). An import quota restricts the foreign firm’s ability to
punish deviations and hence this trade barrier can lead to more competition. However,
predictions are sensitive to the competing assumptions. Fung (1992) considers product
differentiation. If the home firm has a smaller market share than the foreign firm, then
lowering the tariff is pro-competitive.} The fact that home and foreign firms compete in each other’s market
makes a difference as tariff reductions affect the incentives to collude in a symmetric
way. By contrast, Fung (1987) finds that a tariff may hinder collusion in a cartel of
international firms. Recently, Bond and Syropoulos (2005) have considered the case
of binding constraints and investigate how trade liberalization influences cartel prof-
itability and welfare; interestingly, these authors find that there is a non-monotonic
relation between tariff levels and the sustainability of collusion.\footnote{Firms
maximize their profits if they do not export to each other’s market and thus Pinto
(1986) finds that collusion is less likely when trade costs are low and firms compete in
quantities. His findings are consistent with Lommerud and Sorgard (2001); however,
these authors show that lower trade costs favour collusion in the form of market sharing under price competition.} Our setting differs
from the above analyses in that we concentrate on the effects of tariffs on the exercise
of market power by the domestic upstream firm and on the incentives to integrate
in a vertically related industry. It also differs in that the foreign intermediate and
the final good industries are assumed competitive. Besides, the collusive mechanism only includes the domestic industry both at the upstream and the downstream levels. These features will lead to findings in contrast with the received literature.

This paper is organized as follows: Section 2 describes the benchmark model. Section 3 analyzes the upstream firm’s optimal behavior once the number of integrated downstream firms is determined. Section 4 studies the upstream firm’s decision to integrate forward and the role played by tariffs. Section 5 discusses the implications of changes in tariffs on the outcome of the game by means of some numerical examples. Finally, Section 6 presents some conclusions.

2 The benchmark model with tacit collusion

Consider a single upstream firm and \( N > 1 \) downstream firms that operate in a given country. There are two markets in the domestic economy: one for an intermediate good, and another one for a final good produced using the former as an input. Technology allows the intermediate good to be transformed into the final good on a one-to-one basis. Additionally, there are perfectly competitive foreign producers of the intermediate good and of the final good, which are willing to supply their respective outputs at marginal cost. In all the cases considered, both the upstream firm and the foreign producers of the intermediate good have zero marginal costs. Domestic downstream firms compete in quantities. They transform the intermediate good into the final good, which is sold directly to consumers. Transformation costs are zero, and thus, the downstream firm’s marginal cost is just the price at which they purchase the intermediate good. Demand for the final product is \( p = a - bq \) where \( a, b \) are positive constants.

We will assume that there are no tariffs on the input, which implies a zero transfer price, and analyze the effect of tariffs that protect the final product. Before the game starts, the level of the per unit tariff on the final product \( T \) is announced. Given \( T \), the domestic price of the final product can be at most \( p^w + T \), where \( p^w \) is the price of the final product in the international market, which is assumed to be perfectly competitive. The price of the final product cannot exceed the world price plus the tariff because this would prompt an infinite amount of imports. Assuming that home and foreign firms bear zero marginal costs implies \( p^w = 0 \).

We propose a game with an infinite number of periods. In each period there are two stages, where, in the first one, downstream firms purchase the input and, in the second, downstream firms choose quantities. Downstream firms tacitly collude in the
production stage. Collusive outcomes will be sustained by reverting to the single-period Nash reversion outcome, as first suggested in Friedman (1971). In the Nash equilibrium, all firms produce the Cournot output with zero marginal cost, which means that downstream firms procure the input in the world market. Depending on the level of the discount factor either full collusion or partial collusion is sustained. Full collusion means that the monopoly outcome associated to this market, \( p^m = \frac{a}{T} \) and \( \pi^m = \frac{a^2}{3T} \), is realized. Partial collusion implies a level of collusion that improves upon the non-cooperative Cournot outcome, \( q^C = \frac{a}{b(N+1)} \), \( p^C = \frac{a}{N+1} \), and \( \pi^C(N) = \frac{a^2}{b(N+1)^2} \), but it falls short of full collusion. The minimum value of the discount factor that sustains the monopoly outcome, as a function of the number of firms, is given by \( \delta\hat{}(N) = \frac{(N+1)^2}{b(N+1)\pi(N+1)} \), where \( \delta\hat{}(N) \in (\frac{1}{2}, 1) \) and increases in \( N \). For realizations of the discount factor below \( \delta\hat{}(N) \), maximum per-period profits \( \pi^*(N, \delta) \) are defined as:

\[
\pi^*(N, \delta) = (1 - \delta)\pi^d(N, \delta) + \delta\pi^C(N),
\]

where \( \pi^d(N, \delta) \) is the optimal single-period deviation profits,\(^7\) given that the other \( N - 1 \) downstream firms produce a level of output that yields \( \pi^*(N, \delta) \) to each firm every period. Then, for any given \( N > 1 \) and \( \delta \in (0, \delta\hat{}(N)) \) we obtain that:

\[
\begin{align*}
   p^*(N, \delta) &= \frac{a((N+1)^2+(N-1)(1+3N)\delta)}{(N+1)((N+1)^2-(N-1)^2\delta)} \\
   q^*(N, \delta) &= \frac{a((N+1)^2-(N-1)(3+N)\delta)}{b(N+1)((N+1)^2-(N-1)(1+3N)\delta)} \\
   \pi^*(N, \delta) &= \frac{a^2((N+1)^2+(N-1)^2\delta)^2}{b(N+1)^2((N+1)^2-(N-1)(1+3N)\delta)^2}
\end{align*}
\]

It is worth commenting that \( p^*(N, \delta) \) is an increasing function of \( \delta \) and ranges from \( p^C \) to \( p^m \); similarly for \( \pi^*(N, \delta) \).

We next study how the exogenous tariff \( T \) influences the above collusive outcome. Three cases can be distinguished: first, \( T > p^m \); second, \( p^C < T < p^m \) and third, \( 0 < T < p^C \).

The first case is that of a prohibitive tariff, in the sense that the tariff rate \( T \) has no effect on the behavior of the domestic oligopoly, i.e. the level of collusion achieved depends only on \( \delta \). If \( \delta \) is below \( \delta\hat{}(N) \) then partial collusion is implemented while for \( \delta \) greater it is full collusion the market situation. In the third case, when the tariff rate is lower than the Cournot price \( p^C \), it easily follows that the equilibrium domestic price is \( T \) which obviously breaks up the above tacit collusive outcome, either full or partial, for every \( \delta \).

The interesting situation is the second case because the degree of collusion is

\(^7\)It is easy to find that \( \pi^d(N, \delta) = \frac{a^2((N+1)^2+(N-1)^2\delta)^2}{b(N+1)^2((N+1)^2-(N-1)(1+3N)\delta)^2} \). The computation of these expressions and that of \( \pi^*(N, \delta) \) appear in the Appendix.
affected depending on the value of $T$. In what follows we will concentrate on this second case. Then, for any given $T$ between $p^C$ and $p^M$, there is a threshold $\delta^*(N, T)$, which is implicitly defined by $p^*(N, \delta) = T$ i.e. $\delta^*(N, T) = \frac{(N+1)^2(T(N+1)-a)}{(N-1)[T(N^2-1)+(3N+1)a]}$. It can be verified that $\delta^*(N, T) < \delta(N)$ for $T < p^m$ and positive for $T > p^C$ and it is increasing in $T$ and in $N$. Therefore, if $\delta \in (0, \delta^*(N, T)]$ then the equilibrium price is the collusive one $p^*(N, \delta)$ while for $\delta \in (\delta^*(N, T), 1)$ then the tariff disciplines the downstream home market and the equilibrium price is $T$, regardless of the prevailing situation being either one of partial collusion or one of full collusion.

**Proposition 1** There is a direct relationship between the tariff level and the discount factor up to which the tacit collusive outcome is not modified.

Proposition 1 means that as the home country tariff is above the Cournot price that corresponds to the number $N$ of domestic home firms, the higher the tariff, the higher the discount factor level that is required for the tariff to be the equilibrium price. In other words, higher tariff levels facilitate collusion because they enlarge the set of discount factor levels for which the corresponding collusive price is not affected and also because they allow the domestic industry to achieve higher payoffs without inducing any imports.

This is in contrast with earlier findings by Davidson (1984) which emphasizes the relevance of the specific market structure assumed and cartel definition. He considers both a domestic and a foreign oligopoly without any vertical relationships to show that small tariff rates make collusion easier. This is because industry payoffs are decreasing with the tariff since the cartel includes all foreign and home firms in the industry, and also because the deviation profits earned by domestic and foreign firms are realigned so that the temptation to deviate is reduced. Our model is different in several respects. First of all, cartel membership only encompasses the domestic oligopoly. Secondly, the foreign downstream industry is competitive and therefore any tariff level keeps the domestic market effectively protected from imports. Finally, our analysis contemplates the whole range of discount factors and not just those that permit the monopoly collusive outcome.

We are now ready to provide the full solution for price, output and profits for
any $T \in (p^c, p^m)$,
\[
\bar{p}(N, T, \delta) = \begin{cases} 
p^*(N, \delta) & \text{for } 0 < \delta \leq \delta^*(N, T) \\
T & \text{for } \delta^*(N, T) < \delta < 1
\end{cases}
\]
\[
\bar{q}(N, T, \delta) = \begin{cases} 
q^*(N, \delta) & \text{for } 0 < \delta \leq \delta^*(N, T) \\
\frac{a-T}{kN} & \text{for } \delta^*(N, T) < \delta < 1
\end{cases}
\]
\[
\bar{\pi}(N, T, \delta) = \begin{cases} 
\pi^*(N, \delta) & \text{for } 0 < \delta \leq \delta^*(N, T) \\
\frac{T(a-T)}{kN} & \text{for } \delta^*(N, T) < \delta < 1
\end{cases}
\]

3 A model with vertical integration and collusion

This section contemplates the possibility that the upstream domestic firm integrates forward with a number of domestic downstream firms, $M$, where $M \leq N$. The upstream firm’s choice of how many downstream firms to acquire determines its own strategy in the following stages. The upstream firm may just acquire downstream firms without affecting the benchmark situation (that is downstream firms tacitly collude and procure the input at zero wholesale price), or might integrate with downstream firms in order to favour the implementation of an alternative collusive outcome. It is such that the non-integrated downstream firms procure the input from the home upstream firm at a positive wholesale price and the vertical structure improves upon the benchmark situation, where vertical structure denotes the upstream monopolist vertically integrated with a number of downstream firms. In any case, the upstream firm will choose how much each integrated downstream firm produces in order to maximize the vertical structure’s profits. As we will see below, this typically requires integrated outlets not to produce at all when the upstream firm is able to implement the collusive outcome with positive wholesale prices. These firms that produce nothing are part of the vertical structure for strategic reasons, specifically to induce non-integrated firms to accept a wholesale price above the world price. This revenue-collecting mechanism will make the vertical structure better off than if it were to produce the final good.

Following the announcement of the tariff $T$, the upstream firm chooses $M$ in period $t = 0$ of the game. Given $M$, the vertical structure faces organizational costs $\phi(M)$ per period, with $\phi(0) = 0$ and $\phi' > 0$, $\phi'' > 0$.\(^8\) This will prevent the equilibrium outcome from being the uninteresting case of full monopolization in all cases. Beginning in period $t = 1$, the vertical structure and the remaining

\(^8\)Convex integration costs may arise due to agency costs that are increasing with the number of integrated outlets, in a similar way to Chemla (2003).
downstream firms play an infinitely-repeated game with the following stages each period:

1. The upstream firm decides whether to service non-integrated downstream firms. If it decides to do so, it posts a collusive profile, which consists of a price for the final product, a wholesale price, $w > 0$, and an output level $\alpha > 0$ for each non-integrated downstream firm. If not, $w$ is equal to zero and tacit collusion as in the benchmark case applies.

2. Each non-integrated downstream firm decides on the amount of input to purchase and whether to acquire it from the upstream monopolist and/or from the foreign competitive industry. Payments for input purchased are realized.

3. Production takes place and firms collect revenues from sales of the final product.

Suppose that the upstream firm decides not to service non-integrated downstream firms. Since the upstream monopolist owns $M$ downstream firms, the gross profits, not accounting for organizational costs, of the vertical structure in this collusive outcome are $M\bar{\pi}(N, T, \delta)$, which increase linearly in the number of outlets integrated in the vertical structure. In this case, the upstream firm allows integrated downstream firms to behave as independent outlets. Suppose that it decides to service non-integrated downstream firms. In stage 2, the vertical structure observes its own input sales, but not whether non-integrated outlets procured any input in the world market. Non-integrated outlets do not observe other outlets’ procurement activities either. Individual output is observed by every firm. These informational assumptions are relevant for the implementation of the collusive profile specified in stage 1.

We will refer to the collusive profile specified in stage 1 above as the alternative collusive mechanism, as opposed to the tacit collusion outcome in the benchmark case. We will study under what circumstances the vertical structure will be able to successfully use such alternative collusive mechanism. This will require first that the proposal be incentive compatible both for the non-integrated downstream firms and for the vertical structure; and second that the vertical structure obtains extra profits above $M\bar{\pi}(N, T, \delta)$. The latter is an acceptance constraint which depends on the number of integrated relative to non-integrated outlets. Note that this in fact can be interpreted as the upstream firm’s decision about which collusive mechanism to implement. A remarkable consequence of such a positive wholesale price is that it will avoid imports of the input, while keeping the domestic input price above the world price. The upstream firm’s willingness to engage in such strategy and
acceptance by the downstream firms will depend on the exogenous tariff rate on final good, the discount factor and the number of acquired downstream firms. The vertical structure has to choose between a production stage where downstream firms procure the input at zero wholesale price or a production stage where the alternative collusive mechanism is at work with a positive wholesale price.9

As is usual in studying this type of games, we will proceed backwards, analyzing the final stages of the game first. In particular, we will make all our analysis conditional on the value of the tariff, and analyze how changes in this tariff affect the upstream monopolist’s decisions to integrate and/or to supply downstream firms.

Suppose the vertical structure decides to implement the alternative collusive mechanism. The collusive outcomes to be considered consist of a price for the final product, a wholesale price, and a collusive output level \( \alpha \) for each of the \( N-M \) non-integrated downstream firms. Note that there is an interval of prices for the final product that are sustainable in a collusive outcome when the upstream firm posts both a positive wholesale price and the collusive output levels. This interval is determined by the vertical structure’s incentive constraint. Among these prices, there will be an optimal price \( \hat{p} \leq p^m \) that maximizes the vertical structure’s profits, conditional on acceptance by non-integrated firms.

However, there is an upper bound on the final product price that can be sustained in the home market, given by the exogenous tariff \( T \). This way, if \( \hat{p} > T \), then the observed price will be \( T \). Let \( \hat{p} = \min \{ \hat{p}, T \} \); then the corresponding collusive aggregate output that sustains the price \( \hat{p} \) in the market is therefore \( \hat{Q}(\hat{p}) = \frac{a-\hat{p}}{b} \).

As pointed out above, collusive outcomes are sustained by Nash-reversion strategies. Any deviation in output from the collusive production profile involves reverting to the Cournot outcome with \( N \) outlets producing at zero marginal cost. In addition, the vertical structure may revert to the Nash equilibrium strategy after observing that one of the non-integrated downstream firms fails to procure input from the vertical structure. We next analyze the different constraints that the collusive mecha-

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9For a closed economy, only recently a successive oligopoly structure in a repeated game has been undertaken by Nocke and White (2007), to find that vertical integration facilitates upstream collusion. In a related article, Mendi (2005b) analyzes the ability of an upstream monopolist to achieve partial market foreclosure by means of vertical integration, in the presence of a domestic alternative supplier. Our model is different from these two in that Nocke and White (2007) assumes the existence of more than one upstream firm, while we consider an upstream monopolist in the domestic industry which faces the threat of competitive imports. On the other hand, Mendi (2005b) considers the case of integration of the upstream monopolist with a single downstream firm, while in this paper we endogenize the number of outlets to be acquired and study how tariffs influence this decision to protect the domestic market from intermediate good imports.
anism must satisfy.

**Non-Integrated Firms’ Incentive Constraint**

In the collusive equilibrium, given $\hat{p}$ and $w$, each non-integrated firm produces a quantity $\alpha$, whereas the vertical structure, which includes $M$ integrated downstream firms, produces $\frac{a-\hat{p}}{b} - (N - M)\alpha$. In this way, the price is always $\hat{p}$. Every non-integrated firm has the following incentive constraint:

$$(\hat{p} - w)\alpha \geq (1 - \delta)\pi_n^D(\hat{p}, w, \alpha, N, M) + \delta\pi^C(N),$$

(2)

where the LHS of the inequality is the per period profits a non-integrated downstream firm obtains if it agrees with the collusive mechanism proposed. The RHS are the profits when a non-integrated downstream firm deviates. It consists of $\pi_n^D(\hat{p}, w, \alpha, N, M)$ which are optimal deviation profits, and the corresponding Cournot profits after reversion to the Nash equilibrium outcome. The vertical structure must always ensure that this constraint is satisfied. Since a non-integrated downstream firm may deviate prior to purchasing the input and in the production stage, we will determine deviation profits in both cases.

First, a non-integrated downstream firm could deviate by purchasing the input in the world market. The vertical structure will, in the deviation period, respond optimally to the downstream firm’s deviation, since it observes whether non-integrated firms have purchased any amount of inputs from it. If in the collusive outcome each of the $N - M - 1$ non-integrated downstream firms produces $\alpha$, the deviating firm and each one of the integrated downstream firms optimally produce and make single-period profits of:

$$q^D_{m1}(w, \alpha, N, M) = \frac{a}{b(2 + M)} - \frac{(N - M - 1)\alpha}{2 + M}; \quad \pi^D_{m1}(w, \alpha, N, M) = b(q^D_{m1})^2,$$

(3)

which are profits for an $M + 1$ oligopoly\(^{10}\) operating with zero marginal costs, taking as given the production of the $(N - M - 1)$ non-integrated downstream firms that do not deviate. Notice that these profits are decreasing in $\alpha$.

Alternatively, a non-integrated downstream firm could acquire $\alpha$ units of the input from the upstream firm at price $w > 0$, and then purchase some additional units in the world market at zero price. Recall that these additional purchases of input remain unobserved to the vertical structure and to the rest of the non-integrated firms. If each of the $N - M$ non-integrated firms produces $\alpha$, and hence the vertical

\(^{10}\)These firms are the deviating non-integrated firm plus the $M$ downstream firms of the vertical structure.
structure, denoted by subscript vs, produces $q_{vs}(\hat{\rho}, \alpha, N, M) = \frac{a - \hat{\rho}}{b} - (N - M)\alpha$, then a non-integrated downstream firm’s optimal deviation output and profits are given by:

$$q_{n2}^D(\hat{\rho}, w, \alpha, N, M) = \frac{\hat{\rho} + b\alpha}{2b}, \pi_{n2}^D(\hat{\rho}, w, \alpha, N, M) = b \left( q_{n2}^D \right)^2 - w\alpha,$$

which means that single period deviation profits are:

$$\pi_n^D(\hat{\rho}, w, \alpha, N, M) = \max \left\{ b \left[ \frac{a}{b(2+M)} - \frac{(N - M - 1)\alpha}{2 + M} \right]^2, \frac{(\hat{\rho} + b\alpha)^2}{4b} - w\alpha \right\}. \tag{4}$$

In order to compare both deviation profits, we need to determine what is the $\alpha$ that maximizes the vertical structure’s profits. As pointed out above, deviation profits from failing to procure the input from the vertical structure are decreasing in $\alpha$, provided that the price is high enough. We show next that non-integrated firms’ profits if following the second strategy are also decreasing in $\alpha$. In order to see this, let the vertical structure choose the wholesale price $\bar{w}(\alpha)$ so as to make the non-integrated firms’ incentive constraints binding. Then, $\frac{\partial(\hat{\rho} - \bar{w}(\alpha))\alpha}{\partial \alpha} = \hat{\rho} - \bar{w}(\alpha) - \frac{\partial\bar{w}(\alpha)}{\partial \alpha}\alpha$, and given the expressions for $\bar{w}(\alpha)$ and for its partial derivative with respect to $\alpha$, we obtain:

$$\frac{\partial \left[ (\hat{\rho} - \bar{w}(\alpha))\alpha \right]}{\partial \alpha} = \left( 1 - \frac{\delta}{\delta} \right) \left( \frac{\partial (\hat{\rho} + b\alpha)^2}{\partial \alpha} - \hat{\rho} \right) = \left( 1 - \frac{\delta}{\delta} \right) \frac{b\alpha - \hat{\rho}}{2},$$

which is nonpositive when $\alpha \leq \frac{\hat{\rho}}{b}$. Hence, the vertical structure’s profits are increasing in $\alpha$ provided that $\alpha \leq \frac{\hat{\rho}}{b}$. This is because the vertical structure’s profits are total industry profits minus the amount earned by non-integrated downstream firms, that is $\hat{\rho}^{a - \hat{\rho}} - (N - M)(\hat{\rho} - \bar{w}(\alpha))\alpha$. The above result means that the vertical structure will allow non-integrated firms to produce the whole output as long as $\hat{\rho} \geq \frac{a}{N - M + 1}$. Now, if $\hat{\rho} \geq \frac{a}{N - M + 1}$, then all the aggregate output $Q(\hat{\rho})$ is produced by the $(N - M)$ non-integrated firms, which results in the following collusive output levels and corresponding wholesale price:

$$\alpha^0(\hat{\rho}) = \frac{a - \hat{\rho}}{b(N - M)}, q_{vs}(\alpha^0(\hat{\rho})) = 0,$$

$$\bar{w}(\alpha^0(\hat{\rho})) = \frac{\hat{\rho}}{\delta} - \left[ \frac{1 - \delta}{\delta} \frac{1}{4(a - \hat{\rho})(N - M)} + \frac{b(N - M)}{(a - \hat{\rho})} \pi^C(N) \right].$$

This implies that it may be optimal for the vertical structure not to produce at all. If the vertical structure were to produce, it would of course collect the full

\[\text{11In particular, this condition is satisfied when } \hat{\rho} = \frac{a}{2} \text{ and there is at least one non-integrated firm.}\]
output price $\hat{p}$. However, the cost of doing so is reduced revenues from input sales, both because total input sales are reduced and because the wholesale price is to be lowered. The previous result shows that the effect of reduced input sales dominates that of reduced output sales, and thus it is optimal for the vertical structure to let non-integrated downstream firms produce the whole output, provided that the price for the final product is high enough. We will characterize below the range of values of the discount factor for which this holds. Finally, given the values for the wholesale price and non-integrated firms’ output levels, it is shown in the Appendix that $\pi_{D2}(\hat{p}, w, \alpha, N, M)$ is always greater than $\pi_{n1}(w, \alpha, N, M)$; hence the deviation, if it occurs, will always take place in the production stage. For this reason, the non-integrated firm’s incentive constraint includes $\pi_{D2}(\hat{p}, w, \alpha, N, M)$ in its definition.

**The Vertical Structure’s Incentive Constraint**

In order for the vertical structure to introduce a positive wholesale price, it must have no incentive to deviate. If the vertical structure deviates then it will do so by expanding its output. Intuitively, the vertical structure has to refrain from increasing its output once it has received payments for its input sales. Hence, its incentive constraint is given by,

$$\hat{p}\left(\frac{a - \hat{p}}{b} - (N - M)\alpha\right) + (N - M)w\alpha \geq (1 - \delta)\pi_{vs}(w, \alpha, N, M) + \delta M\pi^C(N), \quad (6)$$

where the LHS is revenues from output sales plus input sales to non-integrated firms. The vertical structure’s deviation profits are:

$$\pi_{vs}(w, \alpha, N, M) = b\left[\frac{a}{2b} - \frac{(N - M)\alpha}{2}\right]^2 + (N - M)w\alpha, \quad (7)$$

since the vertical structure does collect revenues from the sale of its input in case it deviates from the collusive outcome; these deviation profits are decreasing in $\alpha$. Hence, given the collusive output levels chosen that maximize the vertical structure’s profits, its incentive constraint can be written as

$$(N - M)\bar{w}(\alpha^0(\hat{p})))\alpha^0(\hat{p}) \geq \frac{1 - \delta}{\delta}\left(\frac{\hat{p}^2}{4b}\right) + M\pi^C(N). \quad (8)$$

**The Vertical Structure’s Acceptance Constraint**

The vertical structure will introduce a positive wholesale price if it finds it profitable to service non-integrated downstream firms. Should the vertical structure decide not to do so, it would obtain $M\bar{\pi}(N, T, \delta)$, letting non-integrated downstream
firms acquire the input in the world market. Hence, in order for the vertical structure to introduce the alternative collusive mechanism acceptable by non-integrated firms, it must be the case that,

\[(N - M)\bar{w}(\alpha^0(\hat{p})) \geq M\bar{\pi}(N, T, \delta). \tag{9}\]

The vertical structure’s acceptance constraint it typically satisfied, except for some low realizations of the discount factor. Intuitively, if the discount factor is high enough, the vertical structure can always post in the alternative collusive mechanism \(\alpha = q^*\) together with a zero wholesale price, which yields exactly a price \(p^*\). Combining both the incentive and the acceptance constraints we have that,

\[
\bar{w}(\alpha^0(\hat{p})) \geq \max \left\{ \frac{1}{(N - M)}[(1 - \delta)\frac{\hat{p}^2}{4b} + \delta M \pi^C(N)], \frac{M\bar{\pi}(N, T, \delta)}{N - M} \right\}.
\]

Now, introducing in the vertical structure’s constraints the expression for the wholesale price and non-integrated firms’ output levels that we obtained after examining non-integrated firms’ incentive constraints, we can determine what is the optimal price \(\hat{p}\) that can be sustained as a function of the discount factor, the number of integrated firms, \(M\), and the tariff \(T\).

Provided that the vertical structure’s incentive constraint is satisfied, if the vertical structure posts \(\bar{w}(\alpha^0) > 0\) and \(\alpha^0\), then the price at which the vertical structure’s profits are maximal is the solution to

\[
\max_{p \geq 0} (N - M)\bar{w}(\alpha^0(p))\alpha^0(p) = \max_{p \geq 0} \frac{p(a - p)}{b\delta} - \frac{1 - \delta}{\delta} \frac{(a + (N - M - 1)p)^2}{4b(N - M)} - (N - M)\pi^C(N),
\]

that is,

\[
\tilde{p}(\delta, N, M) = a \frac{(N - M + 1) + \delta(N - M - 1)}{(N - M + 1)^2 - \delta(N - M - 1)^2}.
\]

It is worth commenting that \(\tilde{p}(\delta, N, M)\) is an increasing function of \(M\) for all \(0 < \delta < 1\). It is also an increasing function of \(\delta\) for all \(0 < M < N - 1\) and ranges from \(\tilde{p}(\delta = 0, N, M) = \frac{a}{N-M+1}\) to \(\tilde{p}(\delta = 1, N, M) = p^m\). Hence, for sufficiently high values of the discount factor, it is optimal for the vertical structure to let non-integrated firms produce the whole output. Furthermore, notice that \(\tilde{p} = p^m\) for \(M = N - 1\). Thus, when there is only one non-integrated outlet, the optimal outcome in the presence of a prohibitive tariff would be that the non-integrated firm produces the monopoly output, giving rise to the monopoly price.

\[12\] Note that if we had assumed that downstream firms were unable to sustain collusion the RHS would result in Cournot profits for each integrated firm which would make the acceptance constraint easier to hold.
Now, taking into account the vertical structure’s incentive constraint, \( \tilde{p} \) satisfies it as long as \( \tilde{p} \in [p^-, p^+] \) where \( p^- \) and \( p^+ \) are the roots of the following expression,

\[
(N-M) \left( \frac{\delta(a + (N-M-1)p) - (a - (N-M+1)p) + 2\pi C}{4b(N-M)} - \frac{(1-\delta)p^2}{4b} + M\pi C \right) = 0
\]

i.e. the vertical structure’s incentive constraint is verified with equality for the above \( \bar{w}(\alpha^0) \) and \( \alpha^0 \). It happens that \( \tilde{p} \in [p^-, p^+] \) for a sufficiently large discount factor, such that the equilibrium price is \( \tilde{p} \) for all \( \delta > \bar{\delta} \), where \( \bar{\delta} \) is the value that equals \( \bar{p}(\bar{\delta}) = p^+(\bar{\delta}) \), and \( \bar{\delta} \in (0, 1) \). For lower discount factor values, there is always a price that allows the vertical structure to maximize profits while satisfying its incentive constraint, although whenever \( \tilde{p} \notin [p^-, p^+] \), i.e. for \( \delta < \bar{\delta} \) it is optimal for the vertical structure to produce a positive amount of the final good, and its incentive constraint is binding. It is also true, for \( \delta < \bar{\delta} \), that

\[
\tilde{p}(\delta, N, M) < a \frac{(N-M+1) + \delta(N-M-1)}{(N-M+1)^2 - \delta(N-M-1)^2}.
\]

There is no closed-form expression for this pricing function, and the vertical structure’s optimal strategy for lower discount factor values is presented in the Appendix.

Remind that the observed price is \( \bar{p} = \min\{\tilde{p}, T\} \), so that if \( \bar{p} > T \), then the tariff imposes a binding constraint on the vertical structure’s behavior. For this reason, we next study how the exogenous tariff \( T \) influences the alternative collusive outcome, for tariff values that satisfy \( p^C < T < p^m \). Recall that the collusive price \( \bar{p}(\delta, N, M) \) is a function of \( M \) and that it belongs to \([\frac{a}{N-M+1}, p^m]\). Then, we have to distinguish two different cases. First, \( p^C < T < \frac{a}{N-M+1} \) and second, \( \frac{a}{N-M+1} < T < p^m \). It is easy to see that in the former case, since \( T < \bar{p}(\delta, N, M) \), the home industry is disciplined by the tariff for every value of the discount factor \( \delta \). For these values of the tariff rate, the vertical structure produces a positive amount of the final product in equilibrium. In the latter case, \( \frac{a}{N-M+1} < T < p^m \), there is a threshold value of the discount factor \( \bar{\delta}(N, M, T) \), which is implicitly defined by \( \bar{p}(N, M, \bar{\delta}) = T \) i.e. \( \bar{\delta}(N, M, T) = \frac{(N-M+1)(T(N-M+1)-a)}{(N-M+1)T(N-M+1)+a} \). This threshold value is increasing in \( T \) and decreasing in \( M \). Therefore, if \( \delta \in (0, \bar{\delta}(N, M, T)] \) then the equilibrium price is \( \bar{p}(N, M, \delta) \) while for \( \delta \in (\bar{\delta}(N, M, T), \bar{\delta}(N)) \) then the tariff disciplines the downstream home market and the observed price is \( T \).

Notice that when the vertical structure implements the alternative collusive mechanism, whether it is affected by the exogenous tariff might depend on the number of integrated downstream firms and therefore a relationship between vertical integration, collusion and tariffs is established. This is because the threshold value \( \bar{\delta}(N, M, T) \) decreases in the number of integrated firms \( M \).
Proposition 2 There is a direct relationship between the tariff level and the discount factor up to which the alternative collusive outcome is not modified. However, the greater the number of integrated firms the smaller the set of discount factor values for which the alternative collusive outcome is not disciplined by the tariff.

That is as the tariff level increases, the higher the discount factor that is required for the tariff to be the equilibrium price. This follows from the fact that \( \tilde{p} \) is increasing in the discount factor. Furthermore, we know that \( \tilde{p} \) is increasing in \( M \). Therefore, the threshold \( \tilde{\delta} \), for a given \( T \), is decreasing in \( M \), which explains a smaller range of discount factor values for which the tariff does not discipline the observed price. See Figure 1 below.

Before we proceed to analyze the optimal degree of integration it is interesting to provide a comparison between the impact of the exogenous tariff for each of the collusive mechanisms. A previous step is to establish the relationship between the collusive prices analyzed above. The prices \( \tilde{p} \) and \( p^* \), for given \( M < N - 1 \), intersect each other at some discount factor \( \delta' \), which is smaller than \( \tilde{\delta} \), so that for values of the discount factor below \( \delta' \) the alternative collusive mechanism produces a higher price, i.e. \( \tilde{p} > p^* \); the opposite otherwise. Furthermore the discount factor \( \delta' \) is increasing in \( M \). The relevance of the above result stands from the fact that for a each \( M < N - 1 \) there exists a set of discount factor levels such that the implementation of the alternative collusive mechanism results in a lower observed price despite the positive wholesale price.

Figure 1 is useful to understand the following result: the impact of the exogenous tariff on the different collusive mechanisms is non-monotonic since for high enough levels of \( T \) (\( T_2 \) in Figure 1) the tariff disciplines the initial collusive mechanism in more situations than those which discipline the alternative one, as \( \tilde{\delta}(N,M,T_2) > \delta^*(N,T_2) \); while for low enough levels (\( T_1 \) in Figure 1) the opposite happens \( \tilde{\delta}(N,M,T_1) < \delta^*(N,T_1) \).

This result also allows us to make the following statement. For any given \( M \), take a discount factor belonging to the interval \( [\delta', \tilde{\delta}] \). Then for all tariff levels between the Cournot price and the monopoly price the observed price of the final good is either the same for both possible collusive mechanisms or the price corresponding to the tacit collusive mechanism is higher, which is the one where downstream firms obtain the input at a zero wholesale price. Consequently, there are more tariff levels

\(^{13}\)See the Appendix for the proof.
that discipline the tacit collusive situation as compared with the alternative collusive mechanism.

4 Integration and tariffs

In this section, we analyze the upstream firm’s choice of the number of firms to acquire in the initial period of the game and how this choice is influenced by the tariff level in the downstream industry. First, we will study the upstream firm’s choice of $M$, the number of downstream firms to acquire prior to starting output competition, for a given level of the downstream tariff. The upstream firm, when choosing the number of downstream firms to acquire, will compare gross profits with organizational costs, which are convex in the number of integrated outlets. Then, we will analyze the impact of changes in the tariff that protects the product of the downstream industry on the upstream firm’s degree of forward integration.

4.1 Optimal degree of integration

The analysis in the previous section allows us to construct the reduced-form gross profit function of the vertical structure, $\pi_{\text{vs}}(M, N, T, \delta)$. This function is defined as

$$\pi_{\text{vs}}(M, N, T, \delta) = \max\{\pi_{\text{vs}}^w(M, N, T, \delta), M\bar{\pi}(N, T, \delta)\}$$

where the first term corresponds with the case of a positive wholesale price, and it typically turns out to be the receipts from input sales to non-integrated firms so that the output price is $\hat{p}$; and the second term is the vertical structure’s gross profits for a zero wholesale price. The gross profit function $\pi_{\text{vs}}^w(M, N, T, \delta)$, for $M < N$ and where $\hat{p} = \min\{T, \bar{p}\}$ is taken into account, is given by:

$$\pi_{\text{vs}}^w(M, N, T, \delta) = \begin{cases} \frac{a^2(N-M)}{b} \left[ \frac{1}{(N-M+1)^2} - \frac{1}{(N-M-1)^2} - \frac{1}{(N+1)^2} \right] & \text{for } 0 < \delta \leq \hat{\delta}(N, M, T) \\ \frac{(N-M)}{b^2} \left[ \frac{T(a-T)}{N-M} - \frac{1-\delta}{4} \left( \frac{(a-T)}{N-M} + T \right)^2 - \frac{\delta a^2}{(N+1)^2} \right] & \text{for } \hat{\delta}(N, M, T) < \delta < 1 \end{cases}$$

Obviously, for $M = N$ the gross profit equals the monopoly profit for every given value of the tariff, regardless of the value of the discount factor, i.e. $\min\left\{\frac{T(a-T)}{b}, \frac{a^2}{2T}\right\}$. Also, if there is only one non-integrated firm ($N-M = 1$) the vertical structure is able to reduce the non-integrated firm’s profits to the Cournot level. Furthermore, the vertical structure’s profits are increasing in the discount factor, and, if this parameter approaches one, then non-integrated firms profits approach the Cournot level as well.
The existence of organizational costs, an increasing and convex function of the number of integrated firms, prevents full monopolization from being the outcome in all cases. Assume that the vertical structure incurs in organizational costs \( \phi(M) \) per period. Recall that \( \phi(0) = 0 \) and \( \phi' > 0, \phi'' > 0 \). With the gross profit function at hand, the upstream firm’s decision in the initial period of the game is how many downstream firms to acquire. Also relevant in the upstream firm’s decision to integrate forward is the price paid for each outlet. Assume first that the upstream monopolist does not service non-integrated firms. If existing owners of downstream firms demand at least future expected profits, the upstream firm will integrate forward only if its profits are at least \( M\bar{\pi}(N,T,\delta) - \phi(M) \). Increasing organizational costs can be offset only if a positive wholesale price can be introduced, which allows the vertical structure to increase its own profits above \( M\bar{\pi}(N,T,\delta) \). This comparison between organizational costs and incremental profits from introducing a wholesale price above the world price is crucial in the determination of the number of integrated outlets. Hence, the optimal number of outlets is determined by the solution to this maximization problem:

\[
\max_{M \in \{0,\ldots,N\}} \pi_{vs}(M,N,T,\delta) - \phi(M) \tag{10}
\]

taking into account that \( M \) must be an integer. The functional form of \( \phi \) will determine the number of outlets that are integrated. Convexity of this function makes the upstream firm compare the benefit of integration with the increasing organizational costs of running more outlets. Since gross profits are increasing in the number of integrated downstream firms, if the \( \phi \) function is convex enough, we will obtain an interior solution.

### 4.2 Tariff protection and vertical integration

Recall that the purpose of this paper is to establish a link between tariff rates and industry structure, specifically its degree of vertical integration. As already noted, the effect of a tariff on imports of the final product is to set an upper bound on the sustainable domestic price for the final good. Consequently, the maximum domestic price is merely the tariff rate \( T \). The level of the tariff affects the upstream firm’s decision to integrate forward in the following way. If the tariff is sufficiently low, so is the maximum sustainable price in the domestic market, and the upstream firm will have little or no incentive to integrate forward, since it would not be able to make enough profits that compensate for the organizational costs. For higher values of the tariff rate, the upstream firm may choose to integrate forward, but not to service
non-integrated firms. If the tariff rate is high enough, the upstream firm chooses to integrate forward, and service non-integrated firms. The reason why there is a range of values of the tariff rate that induces the upstream firm to integrate forward, but not to sell the input to non-integrated firms is that the vertical structure should introduce too low a wholesale price to induce acceptance by non-integrated firms. For this range of values of the tariff rate, the vertical structure is best off letting its outlets behave like any other non-integrated firm.

**Proposition 3** The equilibrium number of acquired downstream firms is non-decreasing in the tariff level $T$, for $T \in (p^C, p^m)$, and for sufficiently high realizations of the discount factor.

Suppose that the discount factor is high enough so that the vertical structure’s incentive constraint is non-binding given a price $\hat{p} = \min \{\bar{p}, T\}$. Then, for any tariff $T < p^m$ we can establish an ordering of the minimum discount factors for which the tariff disciplines the behaviour of the vertical structure, in the sense that $\hat{p} = \min \{\bar{p}, T\} = T$. In particular,

$$0 = \tilde{\delta}(N, N, T) \leq \tilde{\delta}(N, N - 1, T) \leq \tilde{\delta}(N, N - 2, T) \leq \ldots \leq \tilde{\delta}(N, 1, T)$$

i.e. it is more likely that the tariff disciplines the vertical structure’s optimal behaviour the greater the number of integrated downstream firms.

Now, if the tariff increases to, say $T' > T$, for every $M$ the price -and profits- increases for $\delta > \tilde{\delta}(N, M, T)$. Given the above ordering, the range of values for which prices and profits increase after the tariff is raised decreases in $M$. In particular, for every $M$, there is an interval $[\hat{\delta}(N, M, T), \tilde{\delta}(N, M - 1, T)]$ for which profits increase if the number of integrated outlets is $M$, but not if it is $M - 1$. For $\delta > \tilde{\delta}(N, M - 1, T)$, profits increase both for $M - 1$ and $M$, but more in the latter case because $\frac{\partial \pi_{vs}}{\partial \tilde{\delta}}$ increases in $M$. To see this, notice that there are two possible expressions for $\pi_{vs}$. If the vertical structure does not produce at all, the price will be given by $\tilde{p} = T$ and each non-integrated firm will produce $\alpha = \frac{a - \tilde{p}}{b(N - M)}$. In this case, the expression for $\frac{\partial \pi_{vs}}{\partial \tilde{\delta}}$ is

$$\frac{\partial \pi_{vs}}{\partial \tilde{\delta}} = \frac{(N - M)(a - 2\tilde{\delta}) - (1 - \delta)(N - M - 1)}{b\tilde{\delta}} \frac{a + p(N - M - 1)}{2b\tilde{\delta}}$$

which is increasing in $M$. The other possibility is that the vertical structure produces a positive amount of the final good, and thus $\alpha = \frac{\bar{\delta}}{b}$, which occurs whenever $\hat{p} < \frac{a}{N - M + 1}$. For instance, this is the case if $M = N - 1$ and the tariff $T$ is less than
prohibitive. In this case, the expression for the partial derivative is

\[
\frac{\partial \pi_{vs}}{\partial \hat{p}} = \frac{(N - M)(a - 2\hat{p})}{b\delta}
\]

which turns out to be always greater than the previous expression. Hence, whenever the tariff constraints the vertical structure’s pricing strategy, a tariff raise benefits the vertical structure more the greater the number of integrated firms.

What this proposition establishes is a non-decreasing relationship between tariff protection and vertical integration, given that the realization of the discount factor is high enough. Since the optimal price increases with the number of integrated outlets, a tariff below the monopoly price is binding for more realizations of the discount factor the higher \( M \). Thus, increasing \( T \) means relaxing the constraint imposed on the vertical structure’s pricing behaviour over a range of values of the discount factor that is increasing in \( M \). For lower values of the discount factor, since there is no closed-form expression for the pricing function, the relationship cannot be proved, although in the numerical examples there is always a positive relationship between tariffs and integration.

5 Numerical examples

We resort to a set of numerical examples since the explicit solution for \( M \) cannot be neatly established. In all cases, consider \( a = 50 \), \( b = 1 \) and \( N = 3 \). All charts show the cases of vertical integration with \( M = 1, 2 \), and 3. Organizational costs are \( \phi(M) = \frac{sM^2}{2} \) with \( s = 120 \).

5.1 Wholesale and final product prices

The first set of results present optimal wholesale and final product prices for the extreme case of a prohibitive tariff downstream, i.e. a tariff that would allow downstream producers to set a price equal to the monopoly price without attracting any imports of the final good. The case of a prohibitive tariff allows us to highlight the fact that optimal prices are increasing in the number of integrated outlets, and how tariffs constraint the vertical structure’s optimal behaviour.

[insert Figures 2 and 3 here]

Figures 2 and 3 plot the posted market price and wholesale price that will be observed in equilibrium, respectively, in the case of a prohibitive tariff. Were the
vertical structure not to service non-integrated downstream firms, the price would be \( p^*(N, \delta) \), i.e. the maximum sustainable price using the first collusive mechanism. By contrast, if the vertical structure services non-integrated firms, the price will be a function of the number of integrated outlets. Specifically, given the wholesale price \( w \), each non-integrated firm purchases an amount of input \( \alpha = \min \left\{ \frac{N}{b}, \frac{a - p}{(N - M)b} \right\} \) such that the price of the final product is \( \tilde{p}(N, M, \delta) \).

Figure 2 shows that the final product price is non-decreasing both in the number of integrated downstream firms, as well as in the discount factor, if the vertical structure services non-integrated firms. In the case of the vertical structure making use of the first collusive mechanism, the price is also increasing in the discount factor, and it is greater than in the case of the alternative collusive mechanism only for \( M = 1 \). Also notice that full monopolization, i.e. \( M = 3 \), yields a price equal to monopoly price, for every value of the discount factor. By contrast, when \( M = 2 \), the monopoly price is indeed the equilibrium price only if \( \delta \geq 0.5 \), and when \( M = 1 \), which implies that there are two non-integrated firms, the monopoly price is the vertical structure’s optimal price when the discount factor is one. Figure 2 also shows that, if the alternative collusive mechanism is chosen, a tariff below the prohibitive level imposes a more restrictive constraint the greater the number of integrated downstream firms. Specifically, if \( T = 20 \), when \( M = 3 \), the constraint imposed by the tariff is binding, in the sense that the vertical structure would like to set a price greater than 20, for \( \delta \geq 0 \); if \( M = 2 \), it is binding for \( \delta \geq 0.47 \), whereas if \( M = 3 \), it is binding for \( \delta \geq 0.14 \).

Regarding the posted wholesale price, in the case of full monopolization, it will be set at the efficient level, \( w = 0 \). This is also the observed wholesale price in the case of the vertical structure not servicing non-integrated firms, since these firms acquire the intermediate good in the world market, where the price equals marginal cost. The optimal wholesale price is also non-decreasing in the number of integrated outlets and in the discount factor. Notice that, when \( M = 2 \), i.e. when there is only one non-integrated downstream firm, the wholesale price is constant at \( w = 18.75 \) for \( \delta \geq 0.5 \). For these values of the discount factor, the vertical structure is able to reduce the non-integrated downstream firm’s profits to the Nash equilibrium level. Recall that, since the non-integrated downstream firm can always refuse to procure the input from the vertical structure, it secures Nash equilibrium profits, and this is the reason why the vertical structure cannot increase the wholesale price above this level. In the case \( M = 1 \), the wholesale price asymptotically approaches the level that reduces each non-integrated firm’s profits to the Nash equilibrium level. When \( \delta = 1 \), the wholesale price \( w = 12.5 \), together with a monopoly price for the final
product, reduce the non-integrated firms’ profits to the Nash equilibrium level.

5.2 The effect of tariffs on integration

So as to analyze the impact of tariff protection on the upstream firm’s decision on whether to integrate forward and whether to service non-integrated firms, we consider three realizations of the tariff rate, namely, $T = 15, 18,$ and $25$. Demand and the functional form of the organizational costs are the same as in the previous section.

[insert Figures 4, 5, and 6 here]

First, Figure 4 compares the net profits from vertical integration as a function of the discount factor, should the firm decide to integrate with one, two, or three downstream firms when $T = 15$. This represents the upper bound on the domestic price for the final product. In this case, when $\delta \leq 0.05$ and $\delta \geq 0.12$, the upstream monopolist decides to integrate forward with one downstream firm. In the case $M = 2$, when $\delta \leq 0.05$ the vertical structure does not service the non-integrated firm, and allows them to behave as independent firms, giving rise to the first collusive mechanism described in Section 3. Both for $M = 1$ and $M = 2$, the vertical structure produces a strictly positive amount of the final good. For instance, when $\delta = 0.6$, it is optimal for the upstream firm to integrate forward with one downstream firm. The optimal price equals the tariff, 15, the wholesale price is $w = 4.583$, and each non-integrated firm produces $\alpha = 15$. Hence, the vertical structure produces 5 units of the final good. Finally, notice that in the case of full monopolization, organizational costs are so high that even if the vertical structure sets a price equal to the tariff and remains as the single producer, profits are negative.

Figure 5 plots the net profits of different degrees of integration for the case $T = 20$, which is greater than before but not yet prohibitive. In this case, it is optimal for the vertical structure to integrate with one downstream firm when $\delta \geq 0.42$, and for lower realizations of the discount factor, it is optimal for the vertical structure to acquire two outlets. Also notice that the difference in profits between the cases $M = 1$ and $M = 2$ for values of $\delta$ such that it is optimal to integrate with one firm has decreased relative to the case $T = 15$. This is an illustration of the result in Proposition 3, which establishes that profits grow more with prices the greater the number of integrated downstream firms.

Finally, Figure 6 considers the case $T = 25$, which is the minimum prohibitive tariff. Now the minimum value of the discount factor for which the upstream firm decides to integrate with only one downstream firm is 0.69, greater than before. For
all these values of the discount factor, the vertical structure produces no final output at all when $M = 1$.

Therefore the previous numerical examples disclose a positive relation between tariffs and forward integration: *higher tariff rates induce a higher degree of vertical integration.* Even absent tariff protection on the intermediate product, the upper bound on its domestic price increases with tariff protection on the final product. Since the equilibrium price increases with the number of integrated outlets, raising the tariff rate on the final product increases the likelihood that the upstream monopolist finds it optimal to integrate with a larger number of downstream firms.

6 Conclusions

This paper has analyzed the impact of tariff protection on the downstream product on the upstream monopolist’s incentive to acquire downstream firms. The acquisition of additional outlets increases the vertical structure’s profits, although it also increases its organizational costs. The effect of tariffs is to set an upper bound on domestic prices for the final good. Since it is found that collusive prices are increasing in the discount factor as well as on the number of integrated outlets, raising tariffs makes it less likely that tariffs impose a constraint on profits from integrating with more downstream firms. This is the reason why there is a direct relationship between tariffs downstream and the number of downstream firms that an upstream monopolist is interested in acquiring.

A vertical structure allows the upstream monopolist to possibly produce the final product, in addition to being a supplier to non-integrated firms. We find that the vertical structure typically finds it optimal not to produce any output and limit itself to sell the input to non-integrated outlets. This strategy minimizes non-integrated firms’ profits, and hence maximizes the vertical structure’s profits. Forward integration allows the upstream monopolist to charge a wholesale price above the world price of the input. Furthermore, both the wholesale price and the equilibrium price of the final good increase with the number of integrated outlets. Input sales thus constitute an instrument to extract rents from the downstream market in addition to possibly selling the final good by means of its integrated outlets. Higher tariffs downstream make this instrument particularly interesting, from the vertical structure’s perspective, because they increase the range of equilibrium prices of the final product.

Another remarkable finding is that, typically, the vertical structure is best off if the price of the final product is below the maximum sustainable price should
downstream outlets procure the input in the world market. This has important implications on welfare. While vertical integration has clearly an anti-competitive effect on the intermediate good, the analysis in this paper shows that there may actually be a reduction in the price of the final product, relative to a situation where all downstream firms are non-integrated. This reduction in the price for the final product has a positive effect on welfare, which must be compared with the increase in organizational costs that vertical integration generates.
References


Appendix 1. Computation of $q^*$
Suppose that $q^*$ is the maximum collusive per-period output that each firm makes when a realization of the discount factor $\delta$ is below $\delta(N)$. We now compute the output that a deviating firm will produce given that the other $N - 1$ firms do not deviate. The deviating firm chooses $q^d$ so as to solve: \[ \max_{q^d} (a - b(N - 1)q^* - bq^d)q^d \] which yields $q^d = \frac{a - b(N - 1)q^*}{2b}$ and the price is $p^d = \frac{a - b(N - 1)q^*}{2}$. Then we can use the definition of $\pi^*$ to compute $q^*$ by solving for $q^*$ in the next expression: \[ \pi^*(N, \delta) = (1 - \delta)\pi^d(N, \delta) + \delta\pi^C(N), \quad \text{or} \quad (a - bNq^*)q^* = (1 - \delta)\left(\frac{a - b(N - 1)q^*}{2b}\right)^2 + \delta\frac{a^2}{2b(N + 1)^2}. \] Then it is easy to obtain that $q^*(N, \delta) = \frac{a((N+1)^2-(N-1)^2)\delta}{b(N+1)((N+1)^2-(N-1)^2)}$, and then $p^*, q^d$ and $p^d$.

Appendix 2. Proof that $\pi^D_{n2}(p, w, \alpha, N, M) > \pi^D_{n1}(w, \alpha, N, M)$
Recall that $\alpha^0(\hat{p}) = \frac{a - \hat{p}}{b(\alpha - M)}$ and $\bar{w}(\alpha^0(\hat{p})) = \frac{b}{\alpha} - \left[\frac{1}{\delta} \frac{(a + (N+1)\hat{p})^2}{4b(\alpha - M)^2} + \frac{(a - \hat{p})\pi^C(N)}{\delta b(\alpha - M)}\right]$. Thus evaluating $\pi^D_{n1}(\bar{w}(\alpha^0(\hat{p})), \alpha^0(\hat{p}), N, M)$ simplifies to $\pi^D_{n1} = \frac{(a + (N+1)\hat{p})^2}{b(\alpha - M)^2}\sqrt{\alpha^0(\hat{p})}$ and similarly for $\pi^D_{n2}(\hat{p}, \bar{w}(\alpha^0(\hat{p})), \alpha^0(\hat{p}), N, M)$, we obtain $\pi^D_{n2} = \frac{1 + \delta}{\delta} \frac{(a + (N+1)\hat{p})^2}{4b(\alpha - M)^2} - \frac{(a - \hat{p})\pi^C(N)}{\delta b(\alpha - M)}$. Comparing both expressions it is sufficient to note that

\begin{align*}
\text{a) } & \frac{(a - \hat{p})^2}{4b(\alpha - M)^2} > \frac{(a + (N+1)\hat{p})^2}{b(\alpha - M)^2} \quad \text{for all } M > 0, \text{ and,} \\
\text{b) } & \frac{(a - \hat{p})^2}{4b(\alpha - M)^2} \text{ is always greater than } \frac{(a + (N+1)\hat{p})^2}{\delta b(\alpha - M)}, \text{ in order to reach the conclusion that } \pi^D_{n2}(\hat{p}, \bar{w}(\alpha^0(\hat{p})), \alpha^0(\hat{p}), N, M) > \pi^D_{n1}(\bar{w}(\alpha^0(\hat{p})), \alpha^0(\hat{p}), N, M). 
\end{align*}

Appendix 3. Optimal collusive price for low values of the discount factor
The vertical structure’s incentive constraint imposes lower and upper bounds on $\tilde{p}$, which read

\begin{align*}
p^+ &= a((N - M + 1) + (N - M - 1)\delta) + \sqrt{(N - M) a^2((N - M)\delta - (1 - \delta)^2) - 4bN((N - M + 1)^2 - (N - M - 1)^2\delta + (N - M)(1 - \delta))\pi^C(N)} \\
p^- &= a((N - M + 1) + (N - M - 1)\delta) - \sqrt{(N - M) a^2((N - M)\delta - (1 - \delta)^2) - 4bN((N - M + 1)^2 - (N - M - 1)^2\delta + (N - M)(1 - \delta))\pi^C(N)} \\
\end{align*}

These bounds exist as long as $\delta$ is sufficiently large, $\delta > \hat{\delta}$, where $\hat{\delta}$ is the value of the discount factor that equals the discriminant to zero, and reads

\[ \hat{\delta} = \frac{N^2(1 + 2(N - M)) - 2N(N - M)(N - M + 1) + 2(N - M) + 1}{2\sqrt{N^2 - (N - M)(N - M + 1)(1 + N - 2(N - M)(N^2 - (N - M)^2))}. \]

What happens for low deltas? The equilibrium price is $\tilde{p}$ for all $\delta > \hat{\delta}$, where $\hat{\delta}$ is the value that equals $\hat{p}(\hat{\delta}) = p^+(\hat{\delta})$, and $\hat{\delta} \in (0, 1)$. It also happens that $\hat{\delta} < \hat{\delta}$. Therefore, when $\delta < \hat{\delta}$ the vertical structure cannot set the price equal to $\tilde{p}$ since it will not satisfy its incentive constraint. In such a case the vertical structure
will set the price that implies the highest profits conditional on the fulfillment of its incentive constraint. In doing so, the vertical structure selects $\alpha$ such that the vertical structure’s incentive constraint is satisfied and then obtains the price that maximizes its profits.

We first compute the $\alpha$ that satisfies the vertical structure’s incentive constraint, given an arbitrary price $p$:

$$p(a - b(n - M)\alpha) + \delta(n - M)(\frac{(p + b\alpha)^2 - 4b\pi c}{4b} - (p - b\alpha)^2) = (1 - \delta)(\frac{a - b(n - M)\alpha}{4b} + \delta n)$$

The above expression is a quadratic convex polynomial in $\alpha$ with the following roots:

$$\alpha^{+,-} = \frac{a - p}{b(n - M + 1)} \pm \sqrt{\frac{(N - M)(1 - \delta)\delta(a - (n - M)p)^2 - (a - (n - M + 2)p)^2 - 4b(n - M + 1)\delta \pi c}{b(n - M + 1)(N - M)(1 - \delta)}}$$

Recall that the vertical structure’s profits are increasing in $\alpha$ as long as $\alpha \leq \frac{a}{N - M + 1}$. For this reason, if $\frac{p}{b} \in [\alpha^-, \alpha^+]$, it is the chosen value of $\alpha$. Otherwise, if $\frac{p}{b} < \alpha^-$, then $\alpha = \alpha^-$, and if $\frac{p}{b} > \alpha^+$, then $\alpha = \alpha^+$. However, it is true that the price will be below $\frac{a}{N - M + 1}$, which implies that the chosen share will always be below $\frac{p}{b}$. To see this, notice that even if non-integrated firms were to produce the whole output, i.e. if $\alpha = \frac{a - p}{b(n - M + 1)}$, this share is below $\frac{p}{b}$. Then, $\alpha = \alpha^+$ and the vertical structure computes the $p$ that maximizes:

$$p(a - b(n - M)\alpha^+) + (n - M)(\frac{(p + b\alpha^+)^2 - 4b\pi c}{4b\delta} - (p - b\alpha^+)^2)$$

We denote by $p'$ the argmax of the above expression which is implicitly defined by:

$$\left(1 - b\frac{\partial \alpha^+(p)}{\partial p}\right) = \frac{2\delta(a - 2p)}{(1 - \delta)(p - b\alpha^+(p))}$$

In the examples included in the text, the vertical structure’s optimal behavior for $0 < \delta < \delta'$ has been computed numerically, since no closed form solutions exist for this case. The price that maximizes the vertical structure’s profits -and that, by construction, satisfies both incentives constraints- has been obtained for every value of the discount factor $\delta$.

Appendix 4. Relationship between the collusive prices.

**Proposition 4** Consider any given $M \in (0, N - 1]$. Then, there always exists a discount factor level, denoted by $\delta'$, such that for all $0 < \delta < \delta'$ the alternative collusive mechanism is more collusive than the initial one, that is $\bar{p}(M, N, \delta) > p^*(N, \delta)$. Otherwise, the initial collusive mechanism is the most collusive one. It also happens that $\delta'$ is positive, increasing in $M$ and reaches $\delta(N)$ for $M = N - 1$. 

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The threshold $\delta'$ is implicitly defined by the equality $\tilde{p}(M, N, \delta') = p^*(N, \delta')$. Also, when $M = N - 1$ it happens that $\tilde{p}(M = N - 1, N, \delta) = p^M$ which equals $p^*(N, \delta)$ when $\delta = \delta(N)$. To prove that $\tilde{p}(M, N, \delta)$ and $p^*(N, \delta)$ always cross each other note that:

a) $\tilde{p}(M, N, \delta)$ is a continuous and increasing function in $M$ and $\delta$;
b) $p^*(N, \delta)$ is also continuous and increasing in $\delta$;
c) $\tilde{p}(M, N, \delta = 0) = \frac{a}{N-M+1}$ is greater than $p^*(N, \delta = 0) = p^C$, and
d) $\tilde{p}(M, N, \delta = \delta(N)) = \frac{a}{2} \left( \frac{N(N+1)(N+3)-M(N^2+4N+1)}{2N(N+1)^2+MN^2-N(N+1)(3N+1)} \right) < p^*(N, \delta = \delta(N)) = p^m = \frac{a}{2}$ for $M < N - 1$,

thus it is easily verified that $\tilde{p}(M, N, \delta)$ crosses $p^*(N, \delta)$ at $\delta' < \delta(N)$ when $M < N - 1$.

The precise value for $\delta'$, where $H$ stands for $N + 1$, is:

$$\delta' = \frac{2MN(N-2)H-N(N-1)H^2-(N^2-2N-1)M^2+}{(N-1)(N-M-1)(2N(N-1)-M(3N+1))} + \frac{4N^4M^4-4NH(3N^3-4N^2+N+2)M^3+H^2(13N^4-32N^3+44N^2+18N+1)M^2-H^3(N-1)(N(3N^2-6N+1)M+H^4(N-1)^2N)^{0.5}}{(N-1)(N-M-1)(2N(N-1)-M(3N+1))}$$
Figure 1: Collusive prices and Tariffs.
Figure 2. Final product prices, prohibitive tariff

Figure 3. Posted wholesale prices, prohibitive tariff