Partial Public Ownership and Managerial Incentives

Dragan Jovanovic

Abstract

We analyze the impact of partial public ownership (PPO) on managerial incentives. A novelty of the paper is that it explicitly considers competition in the product market. We find that PPO negatively affects managerial incentives when all firms are partially owned by the government. When partially public firms compete with private firms, the effects on managerial incentives crucially depend on the degree of competitive pressure. Thereby, PPO induces either partially public firms or their private competitors to offer stronger managerial incentives. This result is essentially confirmed even if the government’s primary concern is consumer protection rather than social welfare.

JEL-Classification: D82, H32, L13, L33

Keywords: Managerial Incentives, Partial Public Ownership, Limited Control, Competition, Asymmetric Information
1 Introduction

Existing papers analyzing the impact of public ownership on firms’ productive efficiency and managerial incentives rely on the comparison of two extremes: entirely private ownership and entirely public ownership.¹ In fact, many markets are characterized by firms exhibiting mixed ownership structures. In the European Union, it is especially the public utilities sector which reflects the phenomenon of partially public firms. Despite the substantial structural reforms including privatization of the formerly governmental-owned utilities, not all of the active firms have been transferred into entirely private ownership. For instance, in the German electricity market, two of the four largest firms are partially public, while one firm is entirely public, and the other entirely private.² A further example is the telecommunications market in Germany where the incumbent firm Deutsche Telekom AG is partially public, while its main competitors are entirely private.³ This observation raises the question how partial public ownership (PPO) affects the firms’ productive efficiency and managerial incentives. The present paper addresses this issue.

Using a principal-agent setting with ex post asymmetric information, we explicitly account for product market competition by considering an oligopolistic market structure. Thereby, we specify that the principals design the contracts for their respective agents and set the price non-cooperatively in the product market where they compete à la Vickrey-Salop. Moreover, it is

¹Papers in this spirit are e.g., LaFont and Tirole (1991), and Roemer and Silvestre (1992) who explicitly account for regulation when firms are privatized. In addition, De Fraja (1993) tackles the role of “x-inefficiencies” in public firms compared with private firms. All these papers demonstrate that, in contrast to the claims by the proponents of the property rights approach (see Alchian, 1965, and Alchian and Demsetz, 1972), managerial effort is higher in public firms whose objective is social welfare rather than profits. It is worthwhile to note that Shleifer and Vishny (1994) build an exception who allow for partially public firms in their model. However, their setup can be rather classified as a political economics framework which differs form our paper in various respects.

²The four largest firms in the German electricity market are RWE, E.ON, Vattenfall and EnBW. While RWE and EnBW are partially public, E.ON is an entirely private firm. Note that the fourth competitor, Vattenfall, is entirely owned by the Swedish government. However, the present paper does not discuss the implications for the objective functions of public firms operating in foreign country.

³The German government owns directly 15% and indirectly 17% of Deutsche Telekom’s shares. Another example is the German car market where the largest company, Volkswagen, is partially owned (20%) by the local government.
assumed that each principal has private information on her firm’s marginal costs. Given that the agent accepts the contract, she can exert unobservable effort to increase her firm’s productive efficiency. Initially, we consider entirely private firms consisting of one private principal, e.g., a private investor or entrepreneur, and one agent. When analyzing partially public firms, we introduce a second principal into our model. Since we are interested in the effects of PPO on managerial incentives, we define that the second principal is a public principal, e.g., the government or a governmental institution. The public principal is assumed to be a minority shareholder whose share in firm $i$ is given by $s_i \in (0, 1/2)$. As a consequence, we postulate that the public principal has only limited control over her firms. More precisely, it is involved in the decision on the incentive scheme, but it cannot decide on prices. The pricing decision is rather exclusively made by the private principal. To motivate this assumption, one should bear in mind that the private principal represents the majority shareholder whose share always satisfies $1/2 < (1 - s_i) < 1$. Hence, we actually suppose that owning a minority share gives the principal the possibility to partially affect her firm’s personnel decisions. To give an example, one can think of the public principal choosing one or more members of the supervisory board, and thereby affecting the firm’s decision which managers to hire and how to reward them.

We analyze three cases. First, we presume that all firms are entirely private. This scenario serves as our benchmark case. Second, we suppose that the government holds identical minority shares in all firms (symmetric case). This case reflects a situation in which all firms are partially owned by the government with symmetric shares. Finally, we analyze the case in which only

4 Note that the term ‘public’ does not include private investors who are active in the (public) stock market. It rather exclusively indicates governmental ownership.

5 By assuming that $s_i < 1/2$ suffices to guarantee that $G$ is a minority shareholder, we implicitly apply a majority rule which specifies that a shareholder needs to have more than 50% of a firm’s shares in order to get full control over it. Such a rule appears to be common, and is also used by e.g., Grossman and Hart (1988).

6 Laffont and Tirole (1991) distinguish internal control and external control. It should be noted that we solely focus on internal control which comprises the design of the contract and the pricing decision. In this context, we can characterize the government as having limited internal control because it cannot decide on prices. However, external control is neglected, since we do not account for e.g., taxation or regulation.

7 Thereby, we implicitly assume that the private investor and entrepreneur, respectively, is a manager at the same time. Alternatively, one could also think of a managing director whose interests are perfectly aligned with the private shareholders.
half of the firms are partially owned by the public principal, whereas the remaining firms are entirely private. Hereby, we specify that every partially public firm exclusively competes with entirely private firms and vice versa (asymmetric case). To simplify matters, we maintain the assumption that the public principal’s shares are identical.

The substantial difference between a private investor and the government is that the former’s objective is solely to maximize her firm’s profit, while the latter additionally accounts for social welfare. In a second step, we will drop the welfare standard and presume instead that the public principal’s objective is a linear combination of her firms’ profits and consumer surplus. This modification allows us to analyze the effects of PPO given that the government’s aim is to protect consumers.

We find that managerial incentives are always larger in the benchmark case than in the symmetric case of PPO. The fact that the public principal cares relatively more about all firms’ profits in the market and, additionally, designs uniform contracts reduces managerial incentives finally given to the agents. As a consequence, firms exhibit lower productive efficiency and charge higher prices in equilibrium. Compared with the asymmetric case of PPO, our findings crucially depend on the level of competition in the market which is measured by the horizontal differentiation parameter. We demonstrate that managerial incentives either in partially public firms or in private firms can be higher than those in the benchmark case. Given that the level of competition is above a certain threshold, partially public firms offer stronger incentives whenever competition is sufficiently low. Otherwise, entirely private firms in the benchmark case give their agents stronger incentives. The opposite holds when private firms in the asymmetric case are compared with those in the benchmark case. Hence, PPO exhibits positive effects on managerial incentives when not all firms in the market are partially public, i.e., private firms compete with partially public firms. Thereby, PPO induces either the partially public firms or the private competitors to push their agents to exert more effort compared to the full private scenario. Finally, we show that, when the government adopts a consumer surplus standard rather than a social welfare standard, the effects of PPO are reversed. We take this result to propose that the government should only care about consumer protection when it holds minority shares in all firms in the market. If partially private firms compete with private firms, then there is no essential effect of PPO on managerial incentives, and thus on productive efficiency.
The remainder is organized as follows. The related literature is discussed in Section 2. We present the model in Section 3. Sections 4 presents the equilibrium for the benchmark case, i.e., all firms in the market are entirely private. The equilibria for both cases of PPO as well as the effects of PPO on managerial incentives are studied in Section 5. Section 6 analyzes the implications of a government which cares about consumer protection rather than social welfare. A discussion is provided in Section 7. Section 8 concludes the paper.

2 Related Literature

Our model is closely related to Raith (2003) whose paper is the first to explicitly model oligopolistic competition between firms following a contracting game where the principals face ex post asymmetric information. Thereby, he focuses on a comparison between exogenous and endogenous market structures with respect to their effects on managerial incentives. Raith’s basic setup with an exogenous market structure corresponds to our benchmark case. However, his paper focuses on firms which are inherently private. We are, on the contrary, interested in the effects of different ownership structures on managerial incentives where PPO with limited control is emphasized.

So far, the literature on partially public firms or, alternatively, partially private firms has not analyzed the effects on managerial incentives. As mentioned above, existing papers dealing with managerial incentives either compare private firms with entirely public firms (see e.g., De Fraja, 1993, and Corneo and Rob, 2003) or private regulated firms with entirely public firms (see e.g., Laffont and Tirole, 1991, and Roemer and Silvestre, 1992). In addition, they consider monopoly markets, and thereby do not allow for competition in the product market. In contrast, we rather focus on firms competing in an oligopolistic environment.

However, those papers on partially public firms, which allow for product market competition, do not analyze the consequences of agency issues within the firms; i.e., they suppose that firms

---

8 Other papers with a similar modelling approach are Martin (2003) and, especially, Baggs and de Bettignies (2007). The latter rather use a Hotelling model with a fixed number of firms, and they provide some empirical evidence on the effects of competition on managerial incentives. In general, this literature builds on the works by Hart (1983) and Schmidt (1997) who were among the first to formalize the relationship between managerial incentives and competitive pressure.
are entrepreneurial. Two examples are Fershtman (1990) and Matsumura (1998) who use mixed Cournot duopoly models. The former shows that a partially public firm always realizes higher profits than its private competitor, while the latter focuses on the degree to which public ownership is optimal. Thereby, Matsumura (1998) demonstrates that neither full public ownership nor full private ownership is optimal from a welfare perspective.

Finally, partially public firms are also analyzed by Cambini and Spiegel (2011) who study the strategic interactions between capital structure, investment decisions, and regulatory independence given a partially public firm which is price-regulated. They consider a regulator who is ex ante not able to fully commit to the price set at the initial stage of the game, and thus can appropriate some part of the firm’s surplus via renegotiation. Nevertheless, the authors assume that the firm is entrepreneurial and does not face any competition in the product market which clearly distinguishes their work from the present paper.

While there is empirical evidence on productive efficiency and profitability of public firms compared with private firms (see e.g., Caves and Christiansen, 1980, and Dewenter and Malatesta, 2001), there is only one empirical study by Gupta (2005) dealing with the effects of PPO on firms’ performance. Gupta (2005) finds for India that, when initially public firms are partially privatized, profitability, productivity, and investments increase, although the firms completely remain under public control. However, there is no empirical evidence on the effects of PPO where the public owner has only limited control over its firm’s (s’). The present paper attempts to fill this gap using a theoretical framework.

3 The Model

We use the Vickrey-Salop setup to model product market competition. For that purpose, we consider n firms indexed by i = 1, 2, ..., n which are equidistantly located around a circle of

9 This result holds if the “degree of nationalization”, i.e., the share of public ownership, is strictly higher than zero and strictly lower than one. Moreover, if the government’s share is below 60%, then the partially public firm’s profit is higher than the Cournot equilibrium profit with exclusively private firms.

10 It should be noted that the circle model has been already analyzed by Vickrey (1964) where he (among other things) also compares the socially optimal number of firms with the market equilibrium number (see also Vickrey / Anderson and Braid, 1999).
When entering the market, each firm has to incur fixed costs of entry denoted by $F$. To simplify matters, we focus on an exogenous market structure where we set the number of firms in the market, $n$, to be fixed.\footnote{Raith (2003) focuses on the effects of competition on managerial incentives where he distinguishes exogenous and endogenous market structures. In contrast to this work, our paper concentrates on the effects of different ownership structures on managerial incentives and assumes, for simplicity, that the market structure is exogenous.}

Consumers of mass 1 are assumed to be uniformly distributed along the unit circle. They exactly purchase one unit of the product offered either from firm $i$ or from firm $i$’s immediate neighbor firm $i-1$ and firm $i+1$, respectively. A consumer located at $x$ derives utility of

$$v_i = v - p_i - t(z_i - x)^2$$

from purchasing a product offered by firm $i$, where $v$ denotes the utility of purchasing the most preferred product, $t$ represents the horizontal product differentiation parameter, and $z_i$ is firm $i$’s location. It should be noted that due to the circle characteristic every firm $i$ competes with two competitors, i.e., firms $i-1$ and $i+1$. Using (1) and determining the marginal consumers, it is straightforward to calculate firm $i$’s demand which is given by

$$D_i = \frac{1}{n} + \frac{n[(p_{i+1}-p_i) + (p_{i-1}-p_i)]}{2t},$$

(2)

where $p_{i-1}$ and $p_{i+1}$ denote the prices of firm $i$’s immediate neighbors. Due to symmetry of the neighbors, $i-1$ and $i+1$, we can rewrite (2) as follows

$$D_i = \frac{1}{n} + \frac{n(p_j - p_i)}{t},$$

(3)

where $p_j = p_{i-1} = p_{i+1}$.

**Private firms.** We assume that each firm consists of one risk-neutral private principal and one risk-averse agent. Firm $i$’s private principal, labeled $I_i$, is assumed to maximize her (expected) profit given by

$$\pi_i = (p_i - c_i)D_i - F - w_i,$$

(4)

where $p_i$ and $D_i$ denote firm $i$’s price and demand, respectively, and $w_i$ denotes the wage. While $I_i$ sets the price, $p_i$, and designs the incentive scheme, $w_i$, her agent can exert unobservable effort to reduce marginal costs, i.e., to increase firm $i$’s productive efficiency. Marginal costs
per firm are given by $c_i = c - e_i - \theta_i$, where $c$ is a constant, $e_i$ represents agent $i$’s effort level, and $\theta_i$ denotes a normally distributed random variable with zero mean and variance $\sigma^2$, i.e., $\theta_i \sim N(0, \sigma^2)$ i.i.d. However, $I_i$ offers her agent a wage of

$$w_i = d_i + b_i (c - c_i)$$

(5)

which comprises a (fixed) salary, $d_i$, and a variable component, $b_i (c - c_i)$, depending on the extent to which productive efficiency is increased. The piece rate, $b_i$, represents the incentive the principal gives her agent to reduce marginal costs which is termed managerial incentive. Throughout the entire analysis, we assume that marginal cost reductions are verifiable, and thus can be contracted upon.

The agent can accept or reject the contract which is a take-it-or-leave-it offer. If agent $i$ rejects the contract, then she realizes her reservation utility which is normalized to zero, i.e., $\bar{u} = 0$. In contrast, if agent $i$ accepts the offer, then she receives $w_i$ and incurs costs of exerting effort which we denote by $k e_i^2/2$. For simplicity, we set $k = 1$ so that the disutility of effort can be written as $e_i^2/2$. Each agent is supposed to have a CARA utility function in the form of

$$u_i = -\exp \left( -r \left[ w_i - e_i^2/2 \right] \right),$$

(6)

where $r$ denotes the agent’s degree of risk aversion. It is straightforward that an agent only accepts the offer if $u_i \geq 0$ holds (participation constraint).

**Partially public firms.** When firms are partially owned by the government, then they consist of two principals. One principal is private and the other principal is a public principal labeled $G$. The public principal’s share in firm $i$ is given by $s_i \in (0, 1/2)$, i.e., $G$ is a minority shareholder. We postulate that $G$ is risk neutral and has only limited control over her firms. More precisely, it is involved in the decision on the incentive scheme, but it cannot decide on prices. It follows that firms are always privately managed with respect to product market decisions. Finally, given PPO, we presume that $G$ has the same information about firm $i$ as $I_i$. More specifically, $G$ cannot observe its agents’ effort levels, but it learns the marginal costs of the firms it owns.

We distinguish two cases. First, it is supposed that $G$ holds an equal share in all firms in the market which is denoted by $s^{SC}$ (symmetric case). Second, we presume that $G$ owns only half of the firms in the market, whereas the remaining firms are entirely private (asymmetric
case). Note that in this case every partially public firm exclusively competes with entirely private firms and vice versa. However, we maintain the assumption that $G$ holds an equal share in every partially owned firm which is labeled $s^{AC}$. It follows immediately that $G$ observes the marginal costs of $n$ firms and $n/2$ firms, respectively, while each $I_i$ only knows the cost of her respective firm $i$.

In contrast to the private principal, $G$’s objective function encompasses both the (expected) profits of her firms and social welfare. Let $l = SC, AC$ indicate the symmetric case ($SC$) and the asymmetric case ($AC$), respectively. Thus, $G$ maximizes

$$U^l = s^l \Pi^l + W^l,$$

where $\Pi^l$ is the sum of all the firms’ profits in the market owned by $G$, i.e.,

$$\Pi^l = \sum_{i \in O^l} \pi^l_i,$$

and $W^l$ is social welfare defined as

$$W^l = \sum_{i=1}^{n} \pi^l_i + n \left[ \int_{0}^{y^l} q' v - p^l_i - tx^2 dx + \int_{y^l}^{1/n} v - p^l_j - t (1/n - x)^2 dx \right],$$

i.e., the sum of consumer surplus and producer surplus. Note that $y^l$ indicates the marginal consumer and $O^l$ denotes the set of firms partially owned by $G$.\(^{12}\) Obviouly, both $y^l$ and $O^l$ depend on whether the symmetric case or the asymmetric case is analyzed. The public principal cares about her partially owned firms’ profitability because it benefits from their profits via e.g., dividends. Moreover, it is concerned with $W^l$. Taking social welfare into account appears to be a natural assumption, if we consider a government which cares about being reelected given that voters can be influenced by creating a higher social standard.\(^{13}\)

For a partially public firm, the wage function changes to

$$w^l_i = d^l_i + b^l_i \left( c - c^l_i \right),$$

\(^{12}\) $O^l$ contains all $n$ firms in the symmetric case, whereas it ‘only’ contains half of the firms, $n/2$, in the asymmetric case.

\(^{13}\) Maximization of social welfare is a standard assumption for a public principal’s objective (see e.g., De Fraja, 1993, Matsumura, 1998, Francois, 2000, and Corneo and Rob, 2003). Our specification of the government’s (linear) objective function is rather based on Grossman & Helpman (1994). In their setup, the government values political contributions made by (sector specific) lobby groups in addition to social welfare.
where we assume that \(d_i = s_i d_{G,i} + (1 - s_i) d_{I,i}\) and \(b_i = s_i b_{G,i} + (1 - s_i) b_{I,i}\). Hence, the incentive scheme is a weighted average of each principal’s optimal offer, where \(s_i\) and \((1 - s_i)\) represent (exogenous) measures of the principals’ bargaining power when designing the agents’ contracts. Based on the assumption that \(G\) always holds equal shares in its firms, it follows that it designs a uniform contract characterized by \((d_{G}, b_{G})\).

**Sequence of Events.** In the first stage, the principals simultaneously maximize their expected utility given in (4) ((7)) by offering their agents a contract \((d_i, b_i)\) \(((d_i, b_i))\). In doing so, the principals explicitly take their agent’s reservation utility into account (participation constraint) as well as the incentive compatibility constraint. Given that the agents accept the offer, they simultaneously choose effort levels maximizing (6). Note again that each agent’s effort level is not contractible. In the third stage, uncertainty is resolved, and each firm learns its marginal cost, \(c_i\) \((c_i)\), which is private information. Subsequently, the private principals simultaneously and non-cooperatively set prices, \(p_i\) \((p_i)\). In the last stage, prices are common knowledge and consumers make their purchasing decisions.

To ensure that each firm \(i\) only competes with its immediate neighbors, we suppose that the sufficient condition \(t < (2p_i - c_i) n^2\) holds.\(^{14}\) Thereby, the possibility of market monopolization by any firm can be neglected. Moreover, we have to make sure that a unique market-sharing equilibrium exists. Therefore, we invoke the following assumptions.

**Assumption 1a.** *In the benchmark case and in the symmetric case, \(t > n/2 (1 + r \sigma^2)\) must hold for an equilibrium to exist.*

**Assumption 1b.** *In the asymmetric case, \(t > n (4s^{AC} + 9) / 18 (1 + s^{AC}) (1 + r \sigma^2)\) must hold for an equilibrium to exist.*

To avoid too large random cost differences, we have to restrict the variance of \(\theta_i\) to be sufficiently small:\(^{15}\)

---

\(^{14}\)The sufficient condition is based on the first derivative of firm \(i\)’s profit given that one of its rival firms, say firm \(i + 1\), is not active in the market: \(\frac{\partial \pi_i}{\partial p_i} |_{p_{i+1} = 0} < 0\). Applying simple algebra, this condition can be rewritten as \(t < (2p_i - c_i) n^2\).

\(^{15}\)As in Raith (2003), a confidence interval of \([0 - 2\sqrt{3}\sigma; 0 + 2\sqrt{3}\sigma]\) is supposed which contains 99.94 per cent of all possible cost realizations, \(c_i\). Hence, the probability that \(c_i\) deviates from its mean (given by \(c - c_i\)) by more than \(2\sqrt{3}\sigma\) is below 0.1 per cent.
Assumption 2. \( \sigma^2 < t^2/3n^4 \).

We explicitly take assumptions 1a to 2 into account by allowing only for solutions if the parameters are within the feasible regions. Moreover, we focus on non-negative managerial incentives throughout the entire analysis.\(^{16}\)

The game is solved via backward induction looking for subgame perfect Bayesian-Nash equilibria. We begin our analysis with the benchmark case where firms are entirely private, i.e., \( s^i = 0 \) holds. Then, we focus on partially private firms and derive the equilibria for both cases of PPO. Finally, we examine the effects of PPO on managerial incentives.

4 Entirely Private Firms

Our benchmark case corresponds to Raith’s (2003) analysis with an exogenous market structure. Given consumer demand in (3), principals simultaneously set prices, \( p_i \), to maximize profits presented in (4). The first order condition is

\[
p_i = \frac{t}{2n^2} + \frac{c_i + E(p_j)}{2},
\]

where \( E(p_j) \) denotes the expected value of the rivals’ price. Note that, at this stage of the game, \( I_i \) does not know her rivals’ price due to private information. Making use of the symmetry specification, which is based on \( \theta_i \sim N(0, \sigma^2) \forall i \) as well as on identical objectives, we know that in equilibrium \( E(p_i) = E(p) \forall i \), where \( E(p) = t/n^2 + E(c) \) with \( E(c) \) denoting expected marginal costs in the market. Hence, equilibrium demand and equilibrium prices are given by

\[
p_i = \frac{t}{n^2} + \frac{c_i + E(c)}{2} \quad \text{and} \quad D_i = \frac{1}{n} + \frac{n}{2t} \left( E(c) - c_i \right).
\]

Both equilibrium values depend on firm \( i \)'s realized marginal costs, \( c_i \), and on the rivals’ expected costs, \( E(c) \), which are identical for all firms in the market.

In the contracting phase, uncertainty prevails so that both agents and principals rely on expectations with respect to their own marginal costs. At stage two, agents simultaneously

\(^{16}\)Whereas, by Assumption 1a, managerial incentives in the benchmark case and in the symmetric case are always positive, we need to impose additional requirements for the asymmetric case. A more detailed argumentation is offered in the Proof of Proposition 3 (see the Appendix).
choose their optimal effort levels given the incentive scheme in (5). Maximization of the certainty equivalent derived from (6) leads to the following lemma.\footnote{All proofs are provided in the Appendix.}

**Lemma 1.** Each agent’s optimal effort level is given by $e^*_i = b_i$. 

Lemma 1 demonstrates that there is a direct link between the agents’ optimal effort choice and the managerial incentive set by $I_i$. This is a standard result of moral hazard models where $e^*_i = b_i$ represents the principal’s incentive compatibility constraint.

At the initial stage of the game, principals offer their respective agents a contract, $(d_i, b_i)$, without being able to monitor their agents’ effort. In doing so, each principal faces the following optimization problem

$$
\begin{align*}
\max_{d_i, b_i} E(\pi_i) &= (p_i(e_i^*, E(c)) - E(c)) D_i(e_i^*, E(c)) - (d_i + b_i e_i^*) - F \\
\text{s.t. } e_i^* &= b_i \text{ and } u_i \geq 0,
\end{align*}
$$

where the participation constraint becomes binding, i.e., $u_i = 0$ holds. The expression in (10) says that every $I_i$ maximizes her expected profit explicitly taking into account that her agent realizes at least her reservation utility and is provided with the incentive to choose her effort level optimally. Solving (10) and imposing symmetry, i.e., $b_i = b\forall i$, leads to the following proposition.

**Proposition 1.** Given that firms are entirely private, managerial incentives are

$$
b^* = \frac{1}{n\gamma}
$$

in equilibrium, where $\gamma = (1 + r\sigma^2)$.

The equilibrium incentive does not depend on the differentiation parameter, $t$. It is rather shaped by the (exogenous) number of firms in the market, $n$, and the agents’ risk aversion reflected by $\gamma$. It is worthwhile to note that managerial incentives decrease in equilibrium when the number of firms marginally increases. With exogenous market structure, an increase in $n$ can be interpreted as a decrease in market size. One implication is that $I_i$ gives her agent stronger (weaker) incentives to reduce marginal costs when the market is declining (growing). Furthermore, it can be immediately checked that risk measured by $\sigma^2$ has a negative impact on equilibrium incentives.
In the next section, we shift our focus to firms which are partially public. Compared with our benchmark case, we ask how managerial incentives are affected when the ownership structure changes such that the government \( G \) becomes a minority shareholder with limited control.

5 Partially Public Firms

5.1 Equilibrium Analysis

Symmetric case. We begin our analysis with the symmetric case (SC) where \( G \) partially and symmetrically owns all firms in the market. Prices are continued to be set by \( I_i \) in the fourth stage of the game; i.e., the first order condition fulfills \( p_i^{SC} \left( c_i^{SC}, E(p_j^{SC}) \right) = \arg \max \pi_i^{SC} \left( p_i^{SC}, c_i^{SC}, E(p_j^{SC}) \right) \). Under symmetry, which implies \( E(p_i^{SC}) = E(p_i^{SC}) \forall i \), we get the following expression for the equilibrium prices and the equilibrium demand, respectively,

\[
p_i^{SC} = \frac{t}{n^2} + \frac{c_i^{SC} + E(c^{SC})}{2}, \quad D_i^{SC} = \frac{1}{n} + \frac{n \left( E(c^{SC}) - c_i^{SC} \right)}{2t}.
\]

(11)

As before, in the benchmark case, the equilibrium values depend on each firm's own realized marginal costs, \( c_i^{SC} \), and on the expected marginal costs in the market, \( E(c^{SC}) \). Note that \( c_i \neq c_i^{SC} \).

In the second stage, agents simultaneously choose effort levels given (8). Using the same procedure as in Lemma 1, agent \( i \)'s optimal effort choice becomes \( e_i^{SC} = b_i^{SC} \). Principal \( I_i \) faces the same problem presented in (10) when designing the contract \((b_i^{SC}, d_i^{SC})\). In contrast to that, \( G \) offers \((a_G^{SC}, b_G^{SC})\) maximizing (7) subject to both the incentive constraint and the participation constraint. In equilibrium, managerial incentives are calculated based on \( b_i^{SC} = s^{SC} b_G^{SC} + (1 - s^{SC}) b_i^{SC}_i \). Imposing symmetry, i.e., \( b_i^{SC} = b_i^{SC} \forall i \), we obtain the following result.

Proposition 2. In the symmetric case of PPO, managerial incentives are given by

\[
b^{SC} = \frac{4\gamma \left( 1 + \frac{1}{2} s^{SC} - (s^{SC})^2 \right) - s^{SC} n}{n (1 + s^{SC}) \gamma [2t \gamma - s^{SC} n]}
\]

in equilibrium, where \( \gamma = (1 + r \sigma^2) \). Furthermore, \( \partial b^{SC} / \partial s^{SC} < 0 \) always holds.

\(^{18}\) Although the distribution of the random variable \( \theta_i \) is identical for all firms, we have \( c_i \neq c_i^{SC} \) which implies \( E(c) \neq E(c^{SC}) \). The reason is that, unlike in the benchmark case, \( G \), who has a different objective than \( I_i \), appears as a second principal and, thereby, partially affects the incentive scheme.
Proposition 2 highlights that an equal governmental minority share in all \( n \) firms reduces managerial incentives when \( s^{SC} \) is increased. That is, the higher \( G \)'s share, the lower managerial incentives in equilibrium. The reason can be found in \( G \)'s objective function. Although \( G \) accounts for social welfare, \( W^{SC} \), it puts relatively more weight on all firms’ profitability which induces \( G \) to offer lower incentives than \( I_i \). In addition, the fact that \( G \) partially owns all firms in the market eliminates any strategic behavior when it decides on its individual offer, \( b_{i,G}^{SC} \). Hence, it is straightforward that the managerial incentive finally given to the agent decreases with increasing \( s^{SC} \).

**Asymmetric case.** We now derive the equilibrium incentives for the asymmetric case (\( AC \)). The first order conditions as of stage four satisfy

\[
p_i^{AC} = \frac{t}{2n^2} + \frac{c_i^{AC} + E(p_j^{AC})}{2},
\]

where symmetry cannot be imposed, since firm \( i \)'s immediate neighbors differ from \( i \) in terms of ownership structure, i.e., \( E(p_i^{AC}) \neq E(p_j^{AC}) \). Put another way, if firm \( i \) is private (partially public), then both immediate neighbors \( j \) are partially public (private). Hence, immediate competitors are asymmetric which necessitates a solution procedure accounting for asymmetric oligopolies with private information (see Basar and Ho, 1974).\(^{19}\) Solving simultaneously gives the following equilibrium prices and equilibrium demand

\[
p_i^{AC} = \frac{t}{n^2} + \frac{3c_i^{AC} + E(c_i^{AC}) + 2E(c_j^{AC})}{6} \quad \text{and} \quad D_i^{SC} = \frac{1}{n} + \frac{n \left(E(c_i^{AC}) - 3c_i^{AC} + 2E(c_j^{AC})\right)}{t}.
\]

(12)

Note that the asymmetry arises from different incentive schemes which are due to \( I_i \)'s and \( G \)'s differing objectives. All other things are kept equal. Firm \( i \)'s equilibrium values in (12) do not solely depend on its own realized marginal costs, \( c_i^{AC} \), and the rivals’ expected marginal costs, \( E(c_j^{AC}) \), but also on the expectation of its own marginal costs, \( E(c_i^{AC}) \).

Given agents’ optimal effort choices, \( e_i^{AC} = b_i^{AC} \), the incentive schemes are designed in the first stage of the game based on

\[
b_i^{AC} = s^{AC}b_{i,G}^{AC} + (1 - s^{AC})b_{i,i}^{AC} \quad \text{and} \quad b_j^{AC} \]

maximizing (10),

\(^{19}\)See also Sakai (1985) who examines the value of information in a Cournot duopoly based on the procedure proposed by Basar and Ho (1974). Thereby, the case of private information with asymmetric oligopolies is also analyzed.
respectively. Say firm $i$ is partially public, while its immediate competitors $j$ are entirely private. Then the equilibrium is presented as follows.

**Proposition 3.** In the asymmetric case of PPO, firm $i$’s and firm $j$’s equilibrium incentives are given by

$$b_i^{AC} = \frac{1}{18} \frac{243\gamma^2 t^2 \left( \frac{4}{3} + s^{AC} \right) - 180\gamma nt \left( \frac{17}{10} + s^{AC} \right) + 8n^2 \left( 9 + s^{AC} \right)}{27\gamma^2 t^2 \left( 1 + s^{AC} \right) - 18\gamma nt \left( \frac{17}{12} + s^{AC} \right) + n^2 \left( 6 + s^{AC} \right) \gamma n},$$

and

$$b_j^{AC} = \frac{1}{18} \frac{324\gamma^2 t^2 \left( 1 + s^{AC} \right) - 198\gamma nt \left( \frac{17}{11} + s^{AC} \right) + 8n^2 \left( 9 + s^{AC} \right)}{27\gamma^2 t^2 \left( 1 + s^{AC} \right) - 18\gamma nt \left( \frac{17}{12} + s^{AC} \right) + n^2 \left( 6 + s^{AC} \right) \gamma n},$$

where $\gamma = (1 + r\sigma^2)$, and $b_i^{AC} = b_j^{AC} = 2/3n\gamma$ if $s^{AC} = 0$. Firm $j$ gives her agent stronger (weaker) incentives to reduce marginal costs than firm $i$ if $t > t^H$ ($t < t_D$). Furthermore, given $s^{AC} > 1/4$, $\partial b_i^{AC}/\partial s^{AC} > 0$ holds whenever $\tilde{t} < t < \tilde{t}$, while $\partial b_i^{AC}/\partial s^{AC} < 0$ holds whenever $t < \tilde{t}$ or $t > \tilde{t}$. Given $s^{AC} < 1/4$, $\partial b_i^{AC}/\partial s^{AC} > 0$ ($\partial b_i^{AC}/\partial s^{AC} < 0$) holds whenever $t < \tilde{t}$ ($t > \tilde{t}$). The same is true for $\partial b_j^{AC}/\partial s^{AC}$.

It is shown that the private firm $j$ induces its manager to exert more effort in equilibrium than its partially public competitor $i$ if competition is sufficiently low, i.e., $t > t^H$ holds. The opposite is true for $t < t_D$. It is surprising that partially public firms impose stronger incentives on their agents than private firms when product market competition is sufficiently fierce. However, a case is found for which PPO results in stronger managerial incentives than private ownership, i.e., $b_i^{AC} > b_j^{AC}$ holds. Moreover, we demonstrate that the equilibrium incentives in the asymmetric case are not monotonically decreasing in $G$’s minority share $s^{AC}$. Given a relatively large initial public share, i.e., $s^{AC} > 1/4$, for intermediate levels of competition ($\tilde{t} < t < \tilde{t}$) both equilibrium incentives increase if $s^{AC}$ is marginally increased. If the initial public share is relatively small, i.e., $s^{AC} < 1/4$, then the level of competition has to be sufficiently high ($t < \tilde{t}$) for managerial incentives to increase when $s^{AC}$ is marginally increased. Under these conditions, expanded governmental ownership induces all firms in the market to give their managers stronger incentives to increase productive efficiency.

### 5.2 The Effects of Partial Public Ownership

In this subsection, we analyze how a change in the firms’ ownership structure affects managerial incentives. Therefore, we compare both cases of PPO with our benchmark case where all firms
are entirely private. Note again that in the asymmetric case firm \( i \) is the partially public firm, and firm \( j \) is the private firm.

The following proposition presents our results.

**Proposition 4.** Managerial Incentives are always lower in the symmetric case than in the benchmark case, i.e., \( b^* > b^{SC} \) always holds. In the asymmetric case, the effects of PPO depend on the level of competition as follows:

If \( \tilde{t} < t < t_D \), then \( b_i^{AC} > b^* > b_j^{AC} \) holds. If \( t_H < t < \tilde{t} \), then \( b_j^{AC} > b^* > b_i^{AC} \) holds. Otherwise, managerial incentives are always higher in the benchmark case than in the asymmetric case. Note that the following ordering holds \( \tilde{t} < t_D < t_H < \tilde{t} \).

Since \( G \) puts relatively more weight on its firms’ profitability than on consumer surplus, it is less tempted to give its agents strong incentives to reduce marginal costs. Thereby, the fact that \( G \) symmetrically owns all firms in the market plays a crucial role. It strictly prevents \( G \) from strategically inducing one of its managers to exert more effort because it would hurt the competitors which it also partially owns. It follows immediately that managerial incentives are always higher in the benchmark case compared with the symmetric case. The results in the asymmetric case depend on the level of competition. For relatively high levels of competition, i.e., \( t < t_D \) holds, the partially public firm offers stronger incentives than any firm in the benchmark case if \( t \) is sufficiently high. At the same time, firms in the benchmark case always give their agents stronger incentives than any private firm in the asymmetric case. If, on the contrary, the level of competition is relatively low, i.e., \( t > t_H \) holds, then the results are reversed. The firms in the benchmark case always offer higher incentives than the partially public firms in the asymmetric case. Compared with the private firms in the asymmetric case, managerial incentives are only lower in the benchmark case if \( t \) fulfills \( t_H < t < \tilde{t} \). Otherwise, \( b^* > b_j^{AC} \) always holds.

Proposition 4 highlights the idea that managerial incentives are not necessarily larger when all firms are fully private. Thereby, depending on the level of \( t \), PPO induces either the partially public firms or their private competitors to offer stronger managerial incentives than any firm in the benchmark case. We conclude that the level of competition has to be explicitly taken into account when evaluating which ownership structure is accompanied by the strongest managerial incentives. This is especially supported by the fact that in most markets, where mixed ownership
structures prevail, partially public firms compete with private firms as in e.g., the German electricity market.

Our findings in Proposition 4 can be directly transferred to the firms' (expected) productive efficiency. For that purpose, note that \( E(c^*) = c - b^* \), \( E(c^{SC}) = c - b^{SC} \) and \( E(c_{i,j}^{AC}) = c - b_{i,j}^{AC} \) hold in equilibrium. Corollary 1 summarizes our results.

**Corollary 1.** Productive efficiency is always higher in the benchmark case than in the symmetric case. In the asymmetric case, the results depend on the level of competition as follows:

Partially public firms are more efficient than any firm in the benchmark case whenever \( \bar{t} < t < t_D \). Private firms are more efficient than any firm in the benchmark case whenever \( t_H < t < \hat{t} \).

Otherwise, productive efficiency is higher in the benchmark case.

Furthermore, it is straightforward to extend our findings to the level of expected equilibrium prices, since there is a direct link from managerial incentives over productive efficiency to equilibrium prices. Therefore, it is worthwhile to recall that, in all three cases, equilibrium prices depend on the level of competition and the number of firms in the market, i.e., \( t/n^2 \), as well as on the own and the rivals' expected marginal costs. The results are correspondent to our findings on productive efficiency in Corollary 1 and are left to the reader to check.

When PPO is analyzed where the government has only limited control over its firms’ decisions, then the general claim, which associates lower productive efficiency with public ownership, does not hold true. We demonstrate that, under certain conditions, public ownership induces firms to give their managers stronger incentives to reduce marginal costs than entirely private ownership structures. The bottom line is that there is no per se rule for evaluating which ownership structure is superior in terms of managerial incentives, and thus creates higher productive efficiency. The level of competition measured by the product differentiation parameter is rather crucial, and therefore, it has to be explicitly taken into account.

6 **Consumer Protection and Partial Public Ownership**

We now consider a government which is rather concerned with consumer protection than with social welfare. For that purpose, we introduce consumer protection by simply modifying the
government’s objective function which is now given by
\[ U' = s'\Pi' + CS'. \] (13)
In contrast to the objective function used before in (7), we postulate now that \( G \) does not care about social welfare, but rather about consumer surplus, \( CS' \), in addition to its firms’ profits.

We do not derive the equilibria resulting from the modification of \( G \)’s objective function. The equilibrium analysis is rather left to the Appendix. Instead, we directly compare both cases of PPO with the benchmark case. Note again that firm \( i \) is the partially public firm, whereas firm \( j \) is the private firm. Our results are presented in the following proposition.

**Proposition 5.** Managerial Incentives are always higher in the symmetric case than in the benchmark case, i.e., \( b^{SC} > b^* \) always holds. In the asymmetric case, the effect of PPO depends on the level of \( t \) and \( s^{AC} \) as follows:

a) Given \( s^{AC} < s_1 \), the ordering \( \overline{b_i}^{AC} > b^* > \overline{b_j}^{AC} \) holds if \( t > \overline{t}_H \). Otherwise, i.e., if \( t < \overline{t}_D \), we get \( b^* > \overline{b_j}^{AC} > \overline{b_i}^{AC} \).
   
b) Given \( s^{AC} > s_2 \), the ordering \( \overline{b_i}^{AC} > b^* > \overline{b_j}^{AC} \) holds if \( t > \overline{t}_H \). If, on the contrary, \( t < \overline{t}_L \), then \( \overline{b_j}^{AC} > b^* > \overline{b_i}^{AC} \).

Note that the following ordering holds \( \overline{t}_D < \overline{t}_L < \overline{t}_H \).

When \( G \) cares about consumer surplus instead of social welfare, then the impact of PPO on managerial incentives changes. We find that managerial incentives in the symmetric case are strictly higher than in the benchmark case. This result is not surprising, since \( G \) is pushed to provide its agents with stronger incentives in order to increase consumer surplus. However, it should be noted that the difference \( b^{SC} - b^* \), though positive, is monotonically decreasing in \( s^{SC} \). Thereby, an increased public share implies that \( G \) puts more weight on its firms’ profits, and thus is less tempted to push its agents to lower prices. Compared with our analysis in the previous section, we find again that PPO may induce either firm \( i \) or firm \( j \) to give their agents stronger incentives than any firm in the benchmark case. Nevertheless, the impact of competition is reversed when \( G \)’s primary concern is consumer protection. Whereas firm \( i \) has only offered stronger incentives when competition in the product market was relatively fierce, it now offers stronger managerial incentives when the level of competition is relatively low. A similar reasoning holds for the private firm \( j \). Now, given \( s^{AC} > s_2 \), firm \( j \) offers its agent
stronger incentives only if the level of competition is relatively high. However, for a relatively low public share \((s^{AC} < s_1)\), firm \(j\) never gives its agent stronger incentives, irrespective of the level of competition.

While it appears to be rather plausible that consumer protection has a positive effect on managerial incentives in the symmetric case, it is surprising that there is no substantial effect in the asymmetric case. Though in reversed order, we still observe that the effect of PPO depends on the level of competition. We conclude that consumer protection does not have a significant effect on managerial incentives when partially public firms compete with private firms. It should be noted that this case seems to be predominant in markets where firms with mixed ownership structures compete for consumers. While some firms are partially public, their competitors are rather entirely private. Our examples, comprising the electricity market, the telecommunications market, and the car market in Germany, confirm this view. Hence, irrespective of the government’s objective, we suggest to explicitly consider the level of competition when evaluating managerial incentives in markets with mixed ownership structures.

7 Discussion and Extensions

The wage function assumed in our setup is linear and continuous in (expected) productive efficiency. Moreover, we presume that both types of principals, \(I_i\) and \(G\), use this specification for rewarding their agents. It could be claimed that especially the public principal uses some other form of incentive scheme which is closer to directly push the agent to enhance welfare or consumer surplus. Our model does not account for such instances. But it considers differences between private shareholders and the government by assuming different objectives which, finally, affect the incentive schemes. This seems to be a good compromise, although the presumed wage function remains identical for both principals. However, it should be noted that it is at least very difficult to contract upon social welfare and consumer surplus, respectively. This view in turn favors our assumption that both principals use the same wage function to incentivize their agents.

Moreover, it can be claimed that productive efficiency gains are not verifiable, and thus the principals cannot contract upon. In this case, we could make use of output measures such as profits or sales. Alternatively, we could compare different types of performance measures with
regards to their effects on managerial incentives. Such an analysis is performed by e.g., Raith (2008) who compares the effects of “input” measures and “output” measures when agents have specific knowledge of the output levels. For now, we neglect the effects of different types of incentives schemes, and leave this task for further research.

For the sake of simplicity, we assume that, provided PPO, managerial incentives are designed as the weighted sum of each principal’s individual offer, i.e., \( b^i = s^i b^i_{cL} + (1 - s^i) b^i_{I;i} \). Thereby, the respective shares, \( s^i \) and \( (1 - s^i) \), mirror the exogenous bargaining power parameters. It could be claimed that the bargaining process should have been explicitly modelled as in e.g., Shleifer and Vishny (1994), instead of treating it as exogenous. This property of our approach could be classified as a shortcoming. However, we do not focus on the process how the government and the private investor, respectively, create and exert their influence on the firm’s decision. We rather focus on the consequences of a governmental minority share on managerial incentives which can vary within the (open) interval of \((0; 1/2)\). Therefore, we believe that it is adequate to treat the governmental influence on the firms’ personnel decisions as exogenous.

Our paper does not account for regulation, although it is usually a feature of markets exhibiting mixed ownership structures (see e.g., Cambini and Spiegel, 2011). One extension could be, therefore, to introduce price regulation by an regulatory authority and examine the interplay between regulation, ownership structure, and managerial incentives.

Finally, it should be noted that our model could be extended by adopting a framework where consumers continue to make discrete choices, but all differentiated firms compete with each other, and not solely immediate neighbors (see Chen and Riordan, 2008). However, we do not account for ‘multilateral competition’ with differentiated products, and rather leave this task for further research.

8 Conclusion

In this paper, we analyze the effects of PPO on managerial incentives to increase productive efficiency. In contrast to existing works, we explicitly consider competition in the product market by introducing an oligopolistic environment à la Vickrey-Salop. Throughout the entire analysis, we assume that the government is a minority shareholder who is only able to exert limited control over her firms’, i.e., she decides on the contractual design, but has no control over the pricing
decision. We demonstrated that PPO always triggers agents to exert less effort in equilibrium when the public principal symmetrically owns all firms in the market and cares about social welfare. This negative effect of PPO is reversed if the government’s primary goal is consumer protection. The result appears to be straightforward, since the government is always tempted to offer its agents strong incentives to decrease prices, and thereby to increase consumer surplus. So far, a policy implication could be not to permit PPO if the government owns symmetric minority shares of all competitors in the market, unless it does not pursue consumer protection in the first place.

However, if the public principal only owns half of the firms in the market, so that a partially private firm always competes with a private firm and vice versa, the effect of PPO crucially depends on the level of competition. Keep in mind that we use the degree of horizontal product differentiation (product substitutability) as the measure of competition. More precisely, PPO induces either partially public firms or their private competitors to give their managers stronger incentives to reduce marginal costs than any firm in the benchmark case. Though in reverse order, this result essentially holds even if the government’s objective is to maximize consumer surplus rather than social welfare. We take this result to claim that there is no per se rule in evaluating the effects of PPO on productive efficiency. Rather, the level of competition has to be explicitly taken into account, irrespective of the government’s primary objective.
Appendix

In this Appendix we provide the omitted proofs.

**Proof of Lemma 1.** We apply the \( \mu \)-\( \sigma \)-principle for the CARA utility function with a normally and independently distributed random variable \( \theta_i \). Then agent \( i \)'s expected utility, \( E(u_i) \), can be calculated as \( E(u_i) = u_i(\mu_i - (1/2)r\sigma_i^2) \), where \( \mu_i \) and \( \sigma_i^2 \) denote the expected value of \( w_i \) and the variance of \( w_i \), respectively. This approach significantly simplifies the derivation of the certainty equivalent.

The agents simultaneously choose effort levels to maximize their expected utility which is identical with maximizing their certainty equivalent given by

\[
C_i = d_i + b_i e_i - \frac{1}{2}e_i^2 - \frac{1}{2}r b_i^2 \sigma^2,
\]

where \( (1/2)r b_i^2 \sigma^2 \) represents agent \( i \)'s risk premium. Maximizing (14) over \( e_i \) gives an optimal effort level of \( e_i^* = b_i \). It can be immediately checked that the structure of the optimal effort level holds irrespective of which of the three cases is analyzed. However, one should keep in mind that \( b_i \) differs, dependent on which ownership type is supposed.

This proves our result in Lemma 1.

**Proof of Proposition 1.** In the fourth stage of the game, principals choose prices to maximize their profits given by

\[
\pi_i = (p_i - c_i)\left(\frac{1}{n} + \frac{n (E(p_j) - p_i)}{t}\right) - w_i - F
\]

which yields equilibrium prices presented by (9). In the first stage, principals simultaneously maximize their expected profits subject to the participation constraint and incentive constraint (see (10)). Using Lemma 1, we can express principal \( i \)'s expected profit by

\[
E(\pi_i) = \left[\frac{n^2 (E(c) + b_i - c) + 2t}{n^3 t}\right]^2 + \frac{n \sigma^2}{4t} - (d_i + b_i^2) - F.
\]

Maximization yields the following first order condition

\[
b_i^* = \frac{n^2 (E(c) - c) + 2t}{n \left[2t \left(1 + r \sigma^2\right) - n\right]}
\]

Imposing symmetry, i.e., \( b_i = b_j = b^* \) for \( i \neq j \), and using \( E(c) = c - b \), we can calculate the equilibrium values, \( b^* \) and \( E(\pi^*) \), presented in Proposition 1. In addition, we ensure with
Assumption 1a that the symmetric equilibrium is unique and that it exists. However, it can be immediately checked that the first derivative of $b^*$ with respect to $n$, i.e.,

$$ \frac{\partial b^*}{\partial n} = -\frac{1}{n^2 (1 + r\sigma^2)}, $$

is strictly negative. The same is true for the marginal effect of $\sigma^2$ on $b^*$ which is given by

$$ \frac{\partial b^*}{\partial (\sigma^2)} = -\frac{r}{n (1 + r\sigma^2)^2}. $$

This completes the proof of Proposition 1.

**Proof of Proposition 2.** Since each firm’s private principal $I_i$ continues to have exclusive control over the pricing decision despite $G$’s minority share in firm $i$, prices are set by maximizing

$$ \pi_i^{SC} = (p_i^{SC} - c_i^{SC})(\frac{1}{n} + \frac{n (E(p_j^{SC}) - p_i^{SC})}{t}) - w_i^{SC} - F. $$

The first order condition is given by

$$ p_i^{SC} = \frac{t}{2n^2} + \frac{c_i^{SC} + E(p_j^{SC})}{2}. $$

Making use of symmetry gives the equilibrium values presented in (11). Based on the following optimization problem

$$ \begin{align*}
\max_{d_{I,i},b_{I,i}} & \quad E\left( \pi_i^l \right) = (p_i^l(e_i^l, E(c^l)) - E(c_i^l))D_i^l(e_i^l, E(c^l)) - (d_{I,i}^l + b_{I,i}^l e_i^l) - F \\
\text{s.t.} & \quad e_i^l = b_{I,i}^l \text{ and } u_i = 0,
\end{align*} $$

firm $i$’s private principal, $I_i$, makes her offer in the first stage of the game which is given by

$$ b_{I,i}^{SC} = \frac{2t + n^2 (E(c^{SC}) - c)}{n (2t + 2tr\sigma^2 - n)}. $$

Due to its objective, given in (7), $G$ faces a different optimization problem presented by

$$ \begin{align*}
\max_{d_{G},b_{G}} & \quad U^l = s^l \Pi^l + W^l \\
\text{s.t.} & \quad e_i^l = b_{G,i}^l \text{ and } u_i = 0,
\end{align*} $$

where expected consumer surplus as of stage 1 is given by

$$ \begin{align*}
CS^{SC} &= \left[ \int_0^{\frac{v}{y^{SC}}} v - p_i^{SC} - tx^2dx + \int_{\frac{v}{y^{SC}}}^{1/n} v - p_j^{SC} - t(1/n - x)^2dx \right] \\
&= \left[ \frac{(b^{SC} + v - c)}{n} - \frac{52t}{48n^3} - \frac{n\sigma^2}{16t} \right].
\end{align*} $$
Note again that $G$ has private information about all $n$ firms' marginal costs because it partially owns all firms in the market. Maximizing (16) leads to the following offer

$$b_G^{SC} = \frac{1}{2n(1 + s^{SC})}. $$

The equilibrium incentive can now be calculated as

$$b_i^{SC} = s^{SC} b_G^{SC} + (1 - s^{SC}) b_{I,i}^{SC}. $$

Making use of symmetry where $E(c^{SC}) = c - b^{SC}$, with $e^{SC} = b^{SC}$, we get the equilibrium expression shown in Proposition 2. Setting $s^{SC} = 0$, it can be immediately checked that $b^{SC} = b^*$. Moreover, it can be checked that the first derivative of $b^{SC}$ with respect to $s^{SC}$,

$$\frac{\partial b^{SC}}{\partial s^{SC}} = \frac{1}{2} \left[ \frac{4 \left( 2s^{SC} + \frac{1}{2} + (s^{SC})^2 \right) \gamma t + (s^{SC})^2 n}{(1 + s^{SC})^2 (n s^{SC} - 2t \gamma)^2} \right], $$

is always negative by Assumption 1a.

This completes the proof of Proposition 2.

**Proof of Proposition 3.** The pricing decisions of all firms are made by the private principals whose objective function is

$$\pi_i^{AC} = (p_i^{AC} - c_i^{AC})(1 + \frac{n \left( E(p_j^{AC}) - p_i^{AC} \right)}{t}) - w_i^{AC} - F. \tag{17} $$

Maximization of (17) yields the first order conditions given by

$$p_i^{AC} = \frac{t}{2n^2} + \frac{c_i^{AC} + E(p_j^{AC})}{2}. $$

Based on the procedure proposed by Basar and Ho (1974), we calculate the rivals' expected prices as

$$E(p_j^{AC}) = \frac{t}{2n^2} + \frac{c_j^{AC} + E(p_j^{AC})}{2}, \tag{18} $$

where firm $i$’s expected price, $E(p_i^{AC})$, is

$$E(p_i^{AC}) = \frac{t}{2n^2} + \frac{E(c_i^{AC}) + E(p_j^{AC})}{2}. \tag{19} $$

Inserting successively (18) and (19) into the first order condition, we get each firm’s equilibrium price and equilibrium demand, respectively, presented in (12).
According to Lemma 1, the agents’ optimal effort choice satisfies $e_i^{AC} = b_i^{AC}$. At the initial stage of the game, all principals simultaneously choose the incentive schemes for their respective agents. While the partially private firm’s managerial incentive is constructed based on both (16) and (15), firm $j$’s managerial incentive is solely based on (15). Thereby, expected consumer surplus used for $G$’s optimization problem presented in (16) is calculated as

$$CS^{AC} = n \left[ \int_0^{y^{AC}} v - p_i^{AC} - tx^2 dx + \int_{y^{AC}}^{1/n} v - p_j^{AC} - t (1/n - x)^2 dx \right]$$

$$= \frac{1}{36} E(c_j^{AC}) \left[ E(c_j^{AC}) - 2 \left[ \left( c - b_{G,i}^{AC} \right) + 9t \right] + n^2 \left( c - b_{G,i}^{AC} \right)^2 \right]$$

$$- \frac{1}{2} (c - b_{G,i}^{AC} - 2v) - \frac{13t}{12n^2} + \frac{n^2\sigma^2}{16t}.$$  

It is important to note that $CS^{AC} \neq CS^{SC}$ which is explained by $G$ ‘only’ knowing half of the firms’ (expected) marginal costs, but not all firms’ marginal costs as in the symmetric case.

However, the individual offers of $G$ and $I_i$ are given by

$$b_{G,i}^{AC} = \frac{1}{4} \left( 9 + 12 s^{AC} \right) - 4n^2 \left( \frac{9}{4} + s^{AC} \right) \frac{b_j^{AC}}{n} \quad \text{and}$$

$$b_{I,i}^{AC} = \frac{6t - 2n^2 b_j^{AC}}{n (9t + 9t\sigma^2 - 2n)}.$$  

Note that $E(c_j) = c - b_j^{AC}$, with $e_j^{AC} = b_j^{AC}$. Since the incentive scheme is calculated as the weighted average of each principal’s individual offer, where $s^{AC}$ is used as the weight (see (8)), the managerial incentive of firm $i$ is finally given by

$$b_i^{AC} (b_j^{AC}) = s^{AC} b_{G,i}^{AC} (b_j^{AC}) + (1 - s^{AC}) b_{I,i}^{AC} (b_j^{AC}) .$$  

The entirely private firm $j$ is exclusively controlled by one private principal, $I_j$, who offers her manager a piece rate given by

$$b_j^{AC} (b_i^{AC}) = b_{I,j}^{AC} (b_i^{AC}) = \frac{6t - 2n^2 b_i^{AC}}{n (9t + 9t\sigma^2 - 2n)}.$$  

It is easily seen that managerial incentives are strategic substitutes, i.e., $\partial b_i^{AC} (b_j^{AC}) / \partial b_j^{AC} < 0$ and $\partial b_j^{AC} (b_i^{AC}) / \partial b_i^{AC} < 0$ hold. Solving $b_i^{AC} (b_j^{AC})$ and $b_j^{AC} (b_i^{AC})$ simultaneously, we get the equilibrium values presented in Proposition 3. In addition, setting $s^{AC} = 0$, it is easily shown that both firms, $i$ and $j$, give their agents identical incentives to reduce marginal costs, i.e., $b_i^{AC} = b_j^{AC} = 2/3n\gamma$.  

25
In contrast to the previous cases, it is not guaranteed neither by definition or by Assumption 1b that managerial incentives are non-negative in equilibrium. Therefore, we need to invoke additional requirements which we explicitly take into consideration throughout the entire analysis. For the equilibrium incentive of the partially private firm $i$ the transport cost parameter, $t$, must satisfy $t \leq t_L$ or $t \geq t_H$ where $t_H > t_L$. The threshold values are given by

$$t_L = \frac{1}{36} \frac{(17 + 12s^{AC} + \sqrt{1 + 72s^{AC} + 96(s^{AC})^2})}{(1 + s^{AC})\gamma} n$$

and

$$t_H = \frac{1}{9} \frac{(17 + 10s^{AC} + \sqrt{1 + 92s^{AC} + 76(s^{AC})^2})}{(4 + 3s^{AC})\gamma} n.$$ (20)

If the transport cost parameter is such that $t \leq t_L$ or $t \geq t_H$ holds, then $b_i^{AC}$ is always non-negative in equilibrium. The conditions for the equilibrium incentive of the private firm $j$ to be non-negative are $t \leq t_D$ or $t \geq t_L$ with $t_L > t_D$. Whereas $t_D$ is given by

$$t_D = \frac{1}{36} \frac{(17 + 11s^{AC} + \sqrt{1 + 54s^{AC} + 89(s^{AC})^2})}{(1 + s^{AC})\gamma} n.$$ (22)

Hence, the transport cost parameter, $t$, must satisfy $t < t_D$ or $t > t_H$ for both managerial incentives $b_i^{AC}$ and $b_j^{AC}$ to be non-negative in equilibrium where $t_D < t_L < t_H$. For the remaining analysis, we solely consider situations in which both managerial incentives are non-negative in equilibrium.

In a next step, we compare $b_i^{AC}$ and $b_j^{AC}$ to determine which managerial incentive is larger in equilibrium. For this purpose, we define $\theta = b_j^{AC} - b_i^{AC}$ which can be calculated as

$$\theta = \frac{ts^{AC} (\frac{9}{2}t\gamma - n)}{27\gamma^2t^2 (1 + s^{AC}) - 18\gamma nt (\frac{17}{12} + s^{AC}) + n^2 (6 + s^{AC})]} n.$$  

It can be immediately checked that the numerator is always positive by Assumption 1b. Turning to the denominator simple algebra shows that it has the following two zeros

$$t_L = \frac{1}{36} \frac{(17 + 12s^{AC} + \sqrt{1 + 72s^{AC} + 96(s^{AC})^2})}{(1 + s^{AC})\gamma} n$$

and

$$t'_L = \frac{1}{36} \frac{(17 + 12s^{AC} - \sqrt{1 + 72s^{AC} + 96(s^{AC})^2})}{(1 + s^{AC})\gamma},$$

where $t'_L$ is irrelevant because it is always implied by concavity (Assumption 1b). It follows that the denominator is positive (negative) if $t > t_L$ ($t < t_L$). Non-negativity (ensuring that both
and $b_j^{AC}$ are non-negative) requires that $t \leq t_D$ or $t \geq t_H$ so that we conclude that the denominator is always positive (negative) if $t > t_H$ ($t < t_D$). Our result in Proposition 3 follows immediately.

Finally, we demonstrate that the marginal effects of $s^{AC}$ on $b_i^{AC}$ and $b_j^{AC}$ depend on both the level of $t$ and the level of $s^{AC}$. We begin with the inspection of the marginal effect of $s^{AC}$ on $b_i^{AC}$ which is given by

$$\frac{\partial b_i^{AC}}{\partial s^{AC}} = -\frac{4}{3} \frac{(n - \frac{9}{4} t\gamma) (n - \frac{9}{4} t\gamma)^2 (n - 2t\gamma)}{[27\gamma^2 n^2 (1 + s^{AC}) - 18\gamma n t (\frac{17}{12} + s^{AC}) + n^2 (6 + s^{AC})]^2 \gamma n}.$$  

While the denominator is always positive, we focus on the numerator’s sign. The following critical values can be calculated

$$\overline{t} = \frac{4n}{9\gamma} \text{ and } \underline{t} = \frac{n}{2\gamma},$$

for which $(n - 9/4 t\gamma) (n - 9/2 t\gamma)^2 (n - 2t\gamma) = 0$. Checking with concavity (Assumption 1b) it is revealed that $t > \overline{t}$ is only implied if $s^{AC} \leq 1/4$, while $t > \overline{t}$ is never implied. Thus, $\overline{t}$ is only relevant for $s^{AC} > 1/4$. Furthermore, it can be shown that the following ordering holds: $\overline{t} < t_D < t_L < t_H$. In other words, both critical values are feasible in the sense that $b_i^{AC} \geq 0$ and $b_j^{AC} \geq 0$ always hold in equilibrium. For $s^{AC} > 1/4$ both critical values are relevant and the numerator is negative resulting in $\frac{\partial b_i^{AC}}{\partial s^{AC}} > 0$ if $\overline{t} < t < \overline{t}$. If, otherwise, $t < \overline{t}$ or $t > \overline{t}$, then the numerator is always positive leading to $\frac{\partial b_i^{AC}}{\partial s^{AC}} < 0$. For $s^{AC} < 1/4$ the only relevant critical value is $\overline{t}$ where the numerator is positive (negative) if $t > \overline{t}$ ($t < \overline{t}$). The results in Proposition 3 follow immediately.

Performing the same procedure for the marginal effect of $s^{AC}$ on $b_j^{AC}$ which is given by

$$\frac{\partial b_j^{AC}}{\partial s^{AC}} = -\frac{4}{3} \frac{(n - \frac{9}{4} t\gamma) (n - \frac{9}{4} t\gamma) (n - 2t\gamma)}{[27\gamma^2 n^2 (1 + s^{AC}) - 18\gamma n t (\frac{17}{12} + s^{AC}) + n^2 (6 + s^{AC})]^2 \gamma n},$$

we find a third critical value $\underline{t} = 2n/9\gamma$ in addition to $\overline{t}$ and $\overline{t}$ for which the numerator equals zero. However, $\underline{t}$ is irrelevant because $t > \underline{t}$ always holds by Assumption 1b. Note that $\overline{t} < \overline{t} < t_D < t_L < t_H$. Thus, for $\frac{\partial b_j^{AC}}{\partial s^{AC}}$ the same results hold as for $\frac{\partial b_i^{AC}}{\partial s^{AC}}$.

This completes the proof of Proposition 3.

**Proof of Proposition 4.** It is easily checked that the equilibrium incentives in the symmetric case are equal to those in the benchmark case if $G$’s minority share is equal to zero, i.e., $b^{SC}(s^{SC} = 0) = b^* = 1/n\gamma$ holds. Moreover, we know from Proposition 2 that the managerial
incentive in the symmetric case is decreasing when the governmental minority share increases, i.e., \( \partial b^{SC}/\partial s^{SC} < 0 \) holds. This suffices to prove that \( b^* > b^{SC} \) holds for every \( s^{SC} \in (0, 1/2) \).

In a second step, we demonstrate that whether or not managerial incentives are higher in the benchmark case than in the asymmetric case depends on the level of competition in the market, \( t \), as claimed in Proposition 4. We start with the partially private firm \( i \). Let the relevant measure be \( AC_i = b^{*} - b^{AC}_i \). If \( AC_i > 0 \), then managerial incentives in the benchmark case are higher than in the asymmetric case for partially private firms. The opposite holds for \( AC_i < 0 \).

More precisely, \( AC_i \) is given by

\[
AC_i = \frac{1}{18} \frac{243\gamma^2 t^2 \left( \frac{2}{3} + s^{AC} \right) - 144\gamma nt \left( \frac{17}{18} + s^{AC} \right) + 10n^2 \left( \frac{18}{15} + s^{AC} \right)}{27\gamma^2 t^2 (1 + s^{AC}) - 18\gamma nt \left( \frac{17}{12} + s^{AC} \right) + n^2 (6 + s^{AC})} \cdot \gamma n. \tag{23}
\]

Inspection of the denominator shows that there is only one admissible critical value for which \( 27\gamma^2 t^2 (1 + s^{AC}) - 18\gamma nt \left( \frac{17}{12} + s^{AC} \right) + n^2 (6 + s^{AC}) = 0 \): it is given by \( t_L \) (see (20)). Hence, the denominator is positive (negative) if \( t > t_L \) (\( t < t_H \)). Note again that it must be that \( t < t_D \) or \( t > t_H \) to guarantee positive equilibrium incentives in the asymmetric case, i.e., \( b^*_i, b^{AC}_i \geq 0 \) always hold in equilibrium.

Turning to the numerator, we find the following two critical values

\[
\bar{t} = \frac{1}{36} n \left( 17 + 16s^{AC} + \sqrt{1 + 32s^{AC} + 136(s^{AC})^2} \right) \quad \text{and} \quad \\
\tilde{t} = \frac{1}{36} n \left( 17 + 16s^{AC} - \sqrt{1 + 32s^{AC} + 136(s^{AC})^2} \right),
\]

where \( \bar{t} \) is irrelevant because \( t > \bar{t} \) is always implied by Assumption 1a. The second zero \( \tilde{t} \) is relevant and feasible since the following ordering holds: \( \tilde{t} < t_D < t_L < t_H \), i.e., non-negative equilibrium incentives are ensured. The numerator is positive (negative) if \( t > \bar{t} \) (\( t < \tilde{t} \)). The results in Proposition 4 follow immediately.

Finally, we analyze whether or not \( b^* \) is larger than \( b^{AC}_j \). We define the relevant measure \( \phi^{AC}_j = b^* - b^{AC}_j \) which is given by

\[
\phi^{AC}_j = \frac{1}{18} \frac{162\gamma^2 t^2 (1 + s^{AC}) - 126\gamma nt \left( \frac{17}{12} + s^{AC} \right) + 10n^2 \left( \frac{18}{15} + s^{AC} \right)}{27\gamma^2 t^2 (1 + s^{AC}) - 18\gamma nt \left( \frac{17}{12} + s^{AC} \right) + n^2 (6 + s^{AC})} \cdot \gamma n.
\]

If \( \phi^{AC}_j > 0 \), then managerial incentives in the benchmark case are higher than in the asymmetric case for private firms. The opposite holds for \( \phi^{AC}_j < 0 \). We begin by examining the denominator.
It is immediately seen that the denominator is identical with (23). It follows that there also exists only one admissible zero which is given by $t_L$. The denominator is positive (negative) if $t > t_L$ ($t < t_L$). Inspection of the numerator’s sign reveals that there are two zeros given by

$$
\hat{t} = \frac{1}{36} \frac{n \left( 17 + 14s^{AC} + \sqrt{1 + 108s^{AC} + 116(s^{AC})^2} \right)}{(1 + s^{AC})\gamma} \\
\tilde{t} = \frac{1}{36} \frac{n \left( 17 + 14s^{AC} - \sqrt{1 + 108s^{AC} + 116(s^{AC})^2} \right)}{(1 + s^{AC})\gamma},
$$

where $\tilde{t}$ is irrelevant since $t > \tilde{t}$ is always implied by Assumption 1a. The second zero $\hat{t}$ is relevant and it is easily calculated that the following ordering holds: $t_D < t_L < t_H < \hat{t}$, i.e., $\hat{t}$ is feasible. The numerator is positive (negative) if $t > \hat{t}$ ($t < \hat{t}$). Our results in Proposition 4 follow immediately.

This completes the proof of Proposition 4.

**Equilibrium analysis with consumer protection.** The equilibrium incentives in the benchmark case do not change as a consequence of consumer protection, since $G$ does not own any firm, i.e., all firms are private. Hence, the equilibrium incentives remain the same and are given in Proposition 1. However, consumer protection does affect equilibrium incentives in both cases of partial public ownership. We start with the symmetric case. Recall that consumer surplus is given by

$$
CS^{SC} = n \left[ \int_0^{y^{SC}} (v - p_i^{SC} - tx^2) dx + \int_{y^{SC}}^{1/n} (v - p_j^{SC} - t(1/n - x)^2) dx \right]
$$

$$
= n \left[ \frac{(b^{SC} + v - c)}{n} - \frac{52t}{48n^3} - \frac{n\sigma^2}{16t} \right].
$$

The private principal and the public principal individually offer the following incentives

$$
b^{SC}_{I,i} = \frac{2t + n^2 (E(c^{SC}) - c)}{n(2t\gamma - n)} \quad \text{and} \quad b^{SC}_{G} = \frac{1}{2ns^{SC}\gamma}.
$$

The equilibrium incentive is derived based on $b^{SC}_{i} = s^{SC}b^{SC}_{G} + (1 - s^{SC}) b_{I,i}^{SC}$, where $E(c^{SC}) = c - b^{SC}$ with $e^{SC} = b^{SC}$. Note that due to symmetry $b^{SC}_{i} = b^{SC}$ for all $i$. Solving for $b^{SC}$ gives

$$
b^{SC} = \frac{4rt\sigma^2 \left( \frac{3}{2} - s^{SC} \right) + t(6 - 4s^{SC}) - n}{\gamma n \left( t\gamma - \frac{1}{2}s^{SC}n \right)}.
$$
in equilibrium with \( b^{SC}(s^{SC} = 0) = (6t\gamma - n) / 4nt^2 > b^* \). It can be easily checked that, by Assumption 1a, both the numerator and the denominator are strictly positive, i.e., the equilibrium incentive is always positive. The marginal effect of \( s^{SC} \) is given by

\[
\frac{\partial b^{SC}}{\partial s^{SC}} = -\frac{1}{2} \frac{(n - 2\gamma)(n - 4t\gamma)}{\gamma n (s^{SC} n - 2t\gamma)^2},
\]

where \( \frac{\partial b^{SC}}{\partial s^{SC}} < 0 \) always holds, i.e., managerial incentives are always decreasing in \( s^{SC} \) in the symmetric case.

In a next step, we turn to the asymmetric case where consumer surplus from \( G \)'s perspective is calculated as

\[
CS^{AC} = n \left[ \int_{y^{AC}}^{y^{AC} + t} v - p_i^{AC} - tx^2 dx + \int_{y^{AC}}^{y^{AC} + t} v - p_j^{AC} - t(1/n - x)^2 dx \right]
\]

\[
= \frac{1}{36} E(c_j^{AC}) \left[ E(c_j^{AC}) - 2 \left( c - b^{AC}_{G,i} \right) + 9t \right] + n^2 \left( c - b^{AC}_{G,i} \right)^2
\]

\[
- \frac{1}{2} \left( c - b^{AC}_{G,i} - 2v \right) - \frac{13t}{12n^2} + \frac{n^2 \sigma^2}{16t}.
\]

The partially public firm’s equilibrium incentive is given by

\[
b_i^{AC} = s^{AC} b_G^{AC} + (1 - s^{AC}) b_{i,i}^{AC},
\]

where

\[
b_G^{AC} = \frac{t(9 + 12s^{AC}) - b_j^{AC} n^2(1 + 4s^{AC})}{18s^{AC} t \gamma n - n^2(1 + 4s^{AC})}
\]

and

\[
b_{i,i}^{AC} = \frac{6t - 2b_j^{AC} n^2}{n(9t\gamma - 2n)}.
\]

Note that \( E(c_j) = c - b_j^{AC} \) with \( e_j^{AC} = b_j^{AC} \). The entirely private firm’s managerial incentive is given by

\[
b_j^{AC} = \frac{6t - 2b_i^{AC} n^2}{n(9t\gamma - 2n)}.
\]

Solving simultaneously, we get the following equilibrium incentives

\[
b_i^{AC} = \frac{1}{42} \frac{n^2(40s^{AC} + 8) - 324tn\gamma \left( \frac{1}{18} + s^{AC} \right) + 567s^{AC} t^2 \gamma^2}{n^2 \left( \frac{1}{18} + s^{AC} \right) - \frac{54}{17} tn\gamma + \frac{36}{7} (s^{AC} t^2 \gamma^2)}
\]

and

\[
b_j^{AC} = \frac{1}{42} \frac{n^2(40s^{AC} + 8) - 270tn\gamma \left( \frac{1}{18} + s^{AC} \right) + 324s^{AC} t^2 \gamma^2}{n^2 \left( \frac{1}{18} + s^{AC} \right) - \frac{54}{17} tn\gamma + \frac{36}{7} (s^{AC} t^2 \gamma^2)}.
\]

First, we analyze the conditions for managerial incentives to be non-negative in equilibrium.

We begin with the partially public firm’s equilibrium incentive presented in (24). Examining
the numerator first, we find the following two zeros
\[
\bar{t}_D = \frac{1}{63} n \left( 1 + 18 s^{AC} + \sqrt{1 - 20 s^{AC} + 44 (s^{AC})^2} \right) s^{AC} \gamma
\]
and
\[
\bar{t}'_D = \frac{1}{63} n \left( 1 + 18 s^{AC} - \sqrt{1 - 20 s^{AC} + 44 (s^{AC})^2} \right) s^{AC} \gamma,
\]
where \( t'_D \) is irrelevant because it is always implied by Assumption 1b. The second zero, \( t_D \), is relevant for \( s^{AC} < 5/22 - (1/22)\sqrt{11} \equiv s_1 \). Otherwise, it is implied by concavity as well. Hence, the numerator is positive (negative) if \( t > t_D \) \((t < t_D)\) given \( s^{AC} < s_1 \). If \( s^{AC} \geq 5/22 + (1/22)\sqrt{11} \equiv s_2 \) holds, then the numerator is always positive. The denominator exhibits also two zeros given by
\[
\bar{t}_L = \frac{1}{36} n \left( 1 + 15 s^{AC} + \sqrt{1 - 8 s^{AC} + 32 (s^{AC})^2} \right) s^{AC} \gamma
\]
and
\[
\bar{t}'_L = \frac{1}{36} n \left( 1 + 15 s^{AC} - \sqrt{1 - 8 s^{AC} + 32 (s^{AC})^2} \right) s^{AC} \gamma,
\]
where \( t' \) is irrelevant, i.e., \( t > t' \) always holds by concavity. The denominator is positive (negative) if \( t > \bar{t}_L \) \((t < \bar{t}_L)\). It follows that the relevant condition for \( b^{AC}_i \geq 0 \) to hold encompasses two cases: 1.) Given \( s^{AC} < \bar{s}_1 \), the equilibrium incentive is non-negative whenever \( t < \bar{t}_D \) or \( t > \bar{t}_L \); 2.) Given \( s^{AC} \geq \bar{s}_2 \), the equilibrium incentive is non-negative if \( t > \bar{t}_L \).

Now we turn to the private firm’s managerial incentive presented in (25). Setting the numerator equal to zero, i.e., \( n^2 (40 s^{AC} + 8) - 270 t n \gamma (\frac{1}{15} + s^{AC}) + 324 s^{AC} t^2 \gamma^2 = 0 \), we find the following two threshold values
\[
\bar{t}_H = \frac{1}{36} n \left( 1 + 15 s^{AC} + \sqrt{1 - 2 s^{AC} + 65 (s^{AC})^2} \right) s^{AC} \gamma
\]
and
\[
\bar{t}'_H = \frac{1}{36} n \left( 1 + 15 s^{AC} - \sqrt{1 - 2 s^{AC} + 65 (s^{AC})^2} \right) s^{AC} \gamma.
\]
The second zero \( \bar{t}'_H \) can be neglected, i.e., \( t > \bar{t}'_H \) always holds by concavity. It can be immediately seen that the numerator is positive (negative) if \( t > \bar{t}_H \) \((t < \bar{t}_H)\). Since the denominator is identical with (24), we can infer that it is positive (negative) if \( t > \bar{t}_L \) \((t < \bar{t}_L)\). Note that \( \bar{t}_D < \bar{t}_L < \bar{t}_H \). Thus, for both managerial incentives \( b^{AC}_i \) and \( b^{AC}_j \) to be non-negative the following conditions, depending on \( s^{AC} \), have to be met: 1.) Given \( s^{AC} < \bar{s}_1 \) managerial incentives are non-negative whenever \( t < \bar{t}_D \) or \( t > \bar{t}_H \); 2.) Given \( s^{AC} \geq \bar{s}_2 \) managerial incentives are
non-negative if \( t < t_L \) or \( t > t_H \). It should be noted that for the remaining analysis we solely consider cases where both managerial incentives are non-negative in equilibrium.

In a second step, we examine the marginal effects of \( s_{AC} \) on \( b_{AC}^i \) and \( b_{AC}^j \). The marginal effect of \( s_{AC} \) on \( b_{AC}^i \) is given by

\[
\frac{\partial b_{AC}^i}{\partial s_{AC}} = \frac{4}{49} \left[ \frac{81}{7} \gamma^2 t^2 s_{AC} - \frac{54}{7} \gamma nt \left( \frac{1}{12} + s_{AC} \right) + n^2 \left( \frac{2}{7} + s_{AC} \right) \right]^2 \gamma ,
\]

where the denominator is always positive. The numerator reveals one zero

\[
\tau = \frac{4n}{9\gamma}
\]

which is only relevant if \( s_{AC} > 1/4 \) holds. In this case, the marginal effect is negative (positive) if \( t > \tau \) \((t < \tau)\). In contrast, if \( s_{AC} \leq 1/4 \), then \( t > \tau \) is always implied by Assumption 1b and the marginal effect is always negative. The first derivative of \( b_{AC}^j \) with respect to \( s_{AC} \) is given by

\[
\frac{\partial b_{AC}^j}{\partial s_{AC}} = \frac{4}{49} \left[ \frac{81}{7} \gamma^2 t^2 s_{AC} - \frac{54}{7} \gamma nt \left( \frac{1}{12} + s_{AC} \right) + n^2 \left( \frac{2}{7} + s_{AC} \right) \right]^2 \gamma .
\]

Again, the denominator is always positive so that we focus on the numerator’s sign. Setting the numerator equal to zero yields the following two threshold values

\[
t = \frac{2n}{9\gamma} \text{ and } \tau = \frac{4n}{9\gamma}.
\]

The first zero is irrelevant, since \( t > t \) always holds by concavity. The second zero, \( \tau \), is only relevant for \( s_{AC} > 1/4 \). The same results hold as before when the marginal effects on \( b_{AC}^i \) were analyzed.

**Proof of Proposition 5.** It is straightforward to check that managerial incentives in the symmetric case are always larger than in the benchmark case. We already know that \( b^{SC}(s^{SC} = 0) = (6t\gamma - n) / 4nt\gamma^2 > b^* \). Moreover, we have demonstrated that \( \partial b^{SC} / \partial s^{SC} < 0 \) always holds, i.e., the marginal effect of \( s^{SC} \) on \( b^{SC} \) is strictly negative. Hence, there could possibly exist an \( s^{SC} \in (0, 1/2) \) for which \( b^* > b^{SC} \) holds. This claim can be easily rejected based on \( b^{SC}(s^{SC} = 1/2) = 1/n\gamma = b^* \), i.e., equilibrium incentives in the symmetric case are never lower than \( b^* \forall s^{SC} \in (0, 1/2) \).

Now, it is demonstrated that whether or not partially public firms offer stronger incentives than firms in the benchmark case depends on the level of competition, \( t \). We define \( \phi_{AC}^i = \)
\(b^* - b^AC\) to be our relevant measure which can be presented by
\[
\phi^AC_i = \frac{1}{42} \frac{2n^2 (2 + s^AC) - 81s^ACt^2\gamma^2 - 9nt\gamma}{\left[ \frac{81}{2} \gamma^2 t^2 s^AC - \frac{54}{7} \gamma nt \left( \frac{1}{12} + s^AC \right) + n^2 \left( \frac{7}{2} + s^AC \right) \right] \gamma n}.
\] (26)

If \(\phi^AC_i > 0\) (\(\phi^AC_i < 0\)), then managerial incentives are higher (lower) in the benchmark case. The numerator has two zeros
\[
\tilde{t}' = \frac{1}{18} \frac{n \left( \sqrt{1 + 16s^AC + 8(s^AC)^2} - 1 \right)}{s^AC \gamma}
\]
and
\[
\tilde{t} = \frac{1}{18} \frac{n \left( -\sqrt{1 + 16s^AC + 8(s^AC)^2} - 1 \right)}{s^AC \gamma},
\]
where \(\tilde{t}'\) can be ignored because it is not feasible. The second zero \(\tilde{t}\) is irrelevant, since \(t > \tilde{t}'\) always holds by concavity. It follows that the numerator is strictly negative. Turning to the denominator, we find the following two zeros
\[
\tilde{t}_L = \frac{1}{36} \frac{n \left( 1 + 12s^AC + \sqrt{1 - 8s^AC + 32(s^AC)^2} \right)}{s^AC \gamma}
\]
and
\[
\tilde{t}'_L = \frac{1}{36} \frac{n \left( 1 + 12s^AC - \sqrt{1 - 8s^AC + 32(s^AC)^2} \right)}{s^AC \gamma},
\]
where \(\tilde{t}_L\) can neglected because \(t > \tilde{t}_L\) is always implied by Assumption 1b. The denominator is positive (negative) if \(t > \tilde{t}_L\) (\(t < \tilde{t}_L\)). Accounting for non-negativity our results in Proposition 5 follow immediately.

Finally, we analyze whether or not private firms offer stronger incentives in the asymmetric case than private firms in the benchmark case. We use \(\phi^AC_j = b^* - b^AC\) as our relevant measure where
\[
\phi^AC_i = \frac{1}{42} \frac{2n^2 (2 + s^AC) + 162s^ACt^2\gamma^2 - 54n\gamma \left( \frac{1}{6} + s^AC \right)}{\left[ \frac{81}{2} \gamma^2 t^2 s^AC - \frac{54}{7} \gamma nt \left( \frac{1}{12} + s^AC \right) + n^2 \left( \frac{7}{2} + s^AC \right) \right] \gamma n}.
\]
Since the denominator is identical with the denominator in (26), the relevant threshold value is given by \(\tilde{t}_L\). The numerator has two zeros
\[
\tilde{t}' = \frac{1}{36} \frac{n \left( 1 + 6s^AC + \sqrt{1 - 20s^AC + 20(s^AC)^2} \right)}{s^AC \gamma}
\]
and
\[
\tilde{t} = \frac{1}{36} \frac{n \left( 1 + 6s^AC - \sqrt{1 - 20s^AC + 20(s^AC)^2} \right)}{s^AC \gamma},
\]
where \(\tilde{t}\) is irrelevant, i.e., \(t > \tilde{t}\) always holds by concavity. The second zero is only relevant for \(s^AC < 1/2 - (1/5)\sqrt{5} \equiv \bar{s}_1\). Otherwise, i.e., \(s^AC > \bar{s}_1\), \(t > \tilde{t}'\) always holds. Note that \(\bar{s}_1 < s_1\).
Moreover, given $s^{AC} < \bar{s}_1$, the following ordering holds: $t_D < \bar{p} < \bar{t}_L < \bar{t}_H$. Hence, $\bar{p}$ is not feasible, since it falls in the interval which leads to negative equilibrium incentives. Our results in Proposition 5 follow immediately.

This completes the proof of Proposition 5.
References


