MISINFORMATIVE ADVERTISING

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Research question

• Firms initially do not know demand for their products (e.g., how substitutable are competing products?), but they can learn about the unobservable drivers of demand based on sales made in each period.

• However, the sales made by a firm depend on variables beyond its control, such as the prices charged by competitors or their advertising effort.

• If it is too costly to (perfectly) monitor advertising effort exerted by competitors, there arises an incentive for competitors to interfere with a firm’s learning of its unobservable drivers of demand.
Setting

• Two-period duopoly model in which firms choose price and advertising in each period; at the end of the first period, price is perfectly observed by the rival, but advertising effort is not.

• Imperfectly observed advertising can be used to interfere with the competitor’s learning about an unobservable driver of demand (e.g., the degree of product substitutability).
  
  – If advertising done were observable, the unobservable driver of demand could be perfectly recovered from realized sales.

• A firm’s inferred driver of demand will depend on the advertising the rival is believed to have done and the advertising it has actually done.
Main results

• Second-period competition is affected by this inference that can be manipulated by the rival

• In a refined equilibrium, firms price statically in the first period and advertise with the aim of softening future competition (although no firm is able to fool its rival)

• Overadvertising for low marginal cost of advertising: signal-jamming decreases profits and leads to a decreasing pattern of advertising expenditures

• Underadvertising for high marginal cost of advertising: signal-jamming increases profits and leads to an increasing pattern of advertising expenditures
Related literature

• Literature on signal-jamming in oligopoly through unobservable prices
  – Riordan (1985)
  – Fudenberg and Tirole (1986)
  – Caminal and Vives (1996)

• In our paper, signal-jamming is through advertising
  – Related to the signaling of product quality through advertising and price
  – Main difference is that in our model firms are symmetrically uninformed about the relative perceived quality of their products
  – Different empirical predictions
The model: Firms

- Two-period duopoly game with discount factor $\delta$
- Firms 1 and 2 are located at the extremes of a Hotelling segment, and produce goods at a constant marginal cost normalized to zero
- At each period, firm $i$ chooses price $p_i$ and advertising effort $a_i$
- Advertising is costly: If firm $i$ exerts advertising effort $a_i$ at period $\tau = 1, 2$, then it must pay $C_\tau(a_i) = k_\tau a_i^2 / 2$
The model: Consumers

- Unit mass of unit-demand consumers is uniformly located along the Hotelling segment.
- Let $U^x_i(a_i, p_i)$ denote the (per-period) utility of a consumer who purchases firm $i$’s product at price $p_i$ given that her distance from $i$ is $x$ and that advertising effort $a_i$ has been exerted.

\[
U^x_1(a_1, p_1) = V + \tilde{v}_1 + a_1 - p_1 - tx
\]

\[
U^{1-x}_2(a_2, p_2) = V + \tilde{v}_2 + a_2 - p_2 - t(1-x)
\]
The model: Information structures and quality

- Given prices and advertising levels, all consumers know which product they like the most, but firms do not:
  - The (permanent) quality differential in favor of 1 is known by consumers, but it is random for firms.
- It is assumed that \( \tilde{\nu} = \nu_1 - \nu_2 \) has positive continuous density \( g(\nu) \) on \([\overline{\nu}, \overline{\nu}]\) where \( \overline{\nu} < 0 < \nu \).
- Expected value of \( \tilde{\nu} \) is zero (no vertical differentiation is expected), and standard deviation is \( \sigma \).
- Firm \( i \) is assumed not to observe \( \tilde{\nu}_i \), so that signaling of a product’s intrinsic quality does not play a role in the analysis.
The model: Information structures and advertising

- First-period prices are observed by the rival in the second period, but first-period advertising effort is not observable.
- Each firm faces a statistical identification problem because unobservable quality differences cannot be separated out of unobservable advertising.
  - Incentive to signal-jam the competitor’s inference about the realized quality differential.
- Advertising may be unobservable owing to private activities (direct-mail advertising, visits to customers, on-site promotions, etc).
- Key driver is the existence of an identification problem: Results are the same if advertising is observed with some (negligible) noise, which may capture media advertising done to enhance brand equity.
Solution concept for imperfect information game

- Symmetric pure-strategy Perfect Bayesian Equilibrium (PBE)
- Strategies should be sequentially rational given a system of beliefs and the belief system should be consistent with the equilibrium strategies in a Bayesian way
- Bayes’ rule is not applicable out of the equilibrium path, so there will be infinitely many PBE and we will refine the equilibrium set
- Other signal-jamming models (e.g., Riordan 1985) exhibit a unique PBE
  - In these models, there is a single strategic variable, price, which is not observed, so no deviation can be ever observed
  - In our model, price is observed, so price deviations can be observed (but advertising deviations cannot be observed)
Resolution of the model: First-period sales

- Given \( \{(p_i, a_i)\}_{i=1}^{2} \), the consumer indifferent between both firms given a realized quality differential \( v \) is located at

\[
x = \frac{t + a_1 - a_2 + p_2 - p_1 + v}{2t}
\]

- Under full market coverage, firm 1’s realized sales are

\[
q_1 = \frac{t + a_1 - a_2 + p_2 - p_1 + v}{2t}
\]

and firm 2’s realized sales are

\[
q_2 = 1 - \frac{t + a_1 - a_2 + p_2 - p_1 + v}{2t}
\]
Resolution of the model: Inference following a deviation

• Suppose that equilibrium strategies prescribe that firm $i$ charges price $\overline{p}_i$ and exerts advertising effort $\overline{a}_i$ in the first period.

• What happens if firm 2 deviates and instead charges price $p_2$ and does advertising $a_2$?
  
  – Firm 1 forms a belief about the advertising done by the rival if price $p_2$ is observed: $\hat{a}_2(p_2)$ (with $\hat{a}_2(\overline{p}_2) = \overline{a}_2$)

• Firm 1’s inference about realized quality differential given sales $q_1$:
  
  \[ \hat{v} = 2tq_1 - t - \overline{a}_1 + \hat{a}_2(p_2) - p_2 + \overline{p}_1 \]

• Since $q_1 = (t + \overline{a}_1 - a_2 + p_2 - \overline{p}_1 + v)/2t$, firm 1’s estimate is
  
  \[ \hat{v}(a_2, p_2|v) = v + \hat{a}_2(p_2) - a_2 \]
Resolution of the model: Incentive to signal-jam

- Incentive to manipulate firm 1’s inference with the aim of affecting second-period competition
- For now, assume that advertising is so costly that second-period competition is essentially driven by price choices
  - Then we know because of strategic complementarity that firm 2 would like to mislead firm 1 into believing that the realized quality differential is higher than it really is, as seen in its second-period profit:

\[
\pi_2^*(\hat{\nu}) = \frac{(6t + \hat{\nu} - 3v)^2}{72t}
\]
Resolution of the model: incentive compatibility

- Firm 2’s expected payoff for a belief system (given that the rival is following the equilibrium strategy):

\[
\Pi_2(a_2, p_2 | \hat{a}_2(p_2)) = \frac{(t + a_2 - \bar{a} + \bar{p} - p_2)p_2}{2t} + \delta \int_{\tilde{v}} \pi_2^*(\tilde{v})g(\tilde{v})d\tilde{v} - \frac{k_1a_2^2}{2}
\]

- Given a belief system \( \hat{a}_2(p_2) \), the following incentive compatibility constraint should hold: \( a_2^*(p_2 | \hat{a}_2(p_2)) \in \arg \max_{a_2} \Pi_2(a_2, p_2 | \hat{a}_2(p_2)) \)

- Hence, \( a_2^*(\bar{p} | \hat{a}_2(\bar{p})) = \bar{a} \) (since \( \hat{a}_2(\bar{p}) = \bar{a} \)), so \( \bar{a} = (18\bar{p} - 6t\delta) / 36tk_1 \)

- For a given belief system, another incentive compatibility constraint should hold:

\[
\Pi_2(\bar{a}, \bar{p} | \bar{a}) \geq \Pi_2(a_2^*(p_2 | \hat{a}_2(p_2)), p_2 | \hat{a}_2(p_2)) \text{ for all } p_2 \neq \bar{p}
\]
Existence of multiple PBE

- Focus on most favorable belief system: For fixed $p_2$, 
  \[
  \min_{\hat{a}_2} \Pi_2 (a_2^*(p_2 | \hat{a}_2 (p_2)), p_2 | \hat{a}_2 (p_2))
  \]
  leads to \( \hat{a}_2^\infty (p_2) = \max \left\{ 0, \frac{p_2}{2tk_1} - 6t \right\} \) (for \( p_2 \neq \bar{p} \))

- **Proposition 1**: There is small enough \( \varepsilon \geq 0 \) such that there always exists a symmetric equilibrium in which \( \bar{p} = t \pm \varepsilon \) and \( \bar{a} = \frac{3 - \delta}{6k_1} \pm \frac{\varepsilon}{2tk_1} \), with \( \varepsilon > 0 \) if and only if \( \delta > 0 \)
Comparative statics on advertising effort and payoffs

• Corollary 1: In the unique symmetric equilibrium in which $\bar{p} = t$ it holds that an increase in $\delta$ lowers advertising effort and increases payoff.

• Advertising decreases because firm 2 tries to mislead the rival into believing that the realized quality differential is greater than it really is and thus soften future competition.

• In equilibrium, no firm fools its rival, so future competition is not relaxed.
  – However, the rent dissipation in which the persuasive nature of advertising results is alleviated due to misinformative effect that advertising has.
Refining the equilibrium set

• When observing a price in the (closed) neighborhood of the equilibrium price, a firm has passive beliefs in that it believes that the advertising effort is unchanged
  – Passive belief restriction is usual in the literature (Hart and Tirole 1990, McAfee and Schwartz 1994, Caminal and Vives 1996): here it is applied only on a set of arbitrarily small size
  – Observing a price almost identical to $\overline{p}$ is perceived as a small but well-intended implementation error when setting prices
• Under this restriction, the unique PBE exhibits $\overline{p} = t$ and $\overline{a} = \frac{3 - \delta}{6k_1}$
Low marginal cost of advertising in the second period

- Thus far, results obtained hold for high marginal cost of advertising
- Results change dramatically if advertising is (marginally) less costly
- To study the nature of second-period competition when advertising is “cheap”, note that the optimal advertising effort simply depends on the price charged: \( a_i' = p_i' / 2tk_2 \)
- For \( 2tk_2 < 1 \), we have that \( da_i' / dp_i' > 1 \), so advertising varies in the same direction as price but the variation is always larger
- Because \( a_i' = p_i' / 2tk_2 \), we can transform the game into one of prices in which it is implicit how advertising varies when price varies
- When \( 2tk_2 < 1 \), the transformed price game displays strategic substitutability
Graphical analysis of the second-period

- Suppose that firm 1 is led to believe that the quality differential is greater than it really is. Then it believes the situation is as follows:

- But an increase in its price is associated with an even larger increase in advertising effort, which cannot be good for firm 2!
Implications of signal-jamming for second-period competition

- When $2tk_2 < 1$, firm 2 wants its rival to cut second-period advertising expenditures, so it overadvertises in the first period so as to make firm 1 believe that its quality differential is lower than it really is.

- When $2tk_2 > 1$, firm 2 wants its rival to raise its second-period price, so it underadvertises in the first period so as to make firm 1 believe that its quality differential is higher than it really is.
Empirical predictions

• Proposition 3: In the symmetric PBE in which $\bar{p} = t$ and 
  \[
  \bar{a} = \frac{1}{2k_1} - \frac{\delta(2tk_2 - 1)}{4k_1(3tk_2 - 1)}
  \]
  it holds that $\partial \bar{a} / \partial \delta < 0$ if $2tk_2 > 1$,
  whereas $\partial \bar{a} / \partial \delta > 0$ if $2tk_2 < 1 < 3tk_2$.

• Signal-jamming increases profits if and only if the marginal cost of advertising is above a certain threshold.

• Proposition 4: Let $k_1 = k_2 \equiv k$. Then price is expected to remain constant over time, whereas advertising is expected to grow if and only if $2tk > 1$. 
Conclusions

• Learning about demand in the presence of advertising is not easy
• Imperfectly observable advertising can have a signal-jamming role besides the traditional ones of signaling quality, persuading consumers or informing them about product features (e.g., price)
• Point is to manipulate the rival’s inference about unobservable drivers of demand so as to soften second-period competition
• In a refined equilibrium, price is expected not to change over time, whereas advertising grows if only if it is costly enough
Possible extensions and further work

• Insights applicable if advertising is done to inform consumers about product features or in environments with switching costs or network effects if their extent is unknown to firms.

• Horstmann and MacDonald (2003) do not find support for signaling role of advertising in CD industry.
  – Introducing learning-by-doing may explain price and advertising patterns (if advertising is costly enough).

• A firm should be able to observe a signal about the quality of its own product, even if it does not observe the rival’s product quality.
  – Interaction of signaling with signal-jamming would exacerbate incentive to soften second-period price competition, perhaps lowering first-period prices too much.