

Collusive Networks in Market Sharing Agreements in the Presence of an Antitrust Authority (Job Market Paper)

Flavia Roldán*
Universidad Carlos III de Madrid†

November, 2007

Abstract

This paper studies how the presence of an antitrust authority on market sharing agreements affects the agreements made by firms in oligopolistic markets. Market sharing agreements prevent firms from entering each other's market. The set of market sharing agreements defines a collusive network which is under suspicion by antitrust authorities. The paper shows that, from the firm's point of view, the probability of getting caught is endogenous and depends on the agreements each firm has signed. Stable collusive networks can be decomposed into a set of isolated firms and complete alliances of different sizes. While in the absence of the antitrust authority a network is stable if its alliances are large enough, when the antitrust authority is considered, the network's stability depends on the network configuration as a whole. Antitrust laws may have a pro-competitive effect as they give firms in large alliances more incentives to cut their agreements at once.

JEL Classification Numbers: D43, K21, L41

Keywords: market sharing, economic networks, antitrust authority, oligopoly

*I am grateful to Antonio Cabrales and Luis Ubeda for their advice, guidance and support. I have greatly benefited from comments and useful suggestions of Natalia Fabra and Carlos Ponce. I thank participants at the Student Workshop in Universidad Carlos III and the 2007 EARIE Meetings (Valence, Spain). The usual disclaimers apply.

†Address for Correspondence: *Flavia Roldán*: Department of Economics, Universidad Carlos III de Madrid, Calle Madrid 126, 28903 Getafe, Spain, E-mail: faroldan@eco.uc3m.es.

1 Introduction

Reciprocal market sharing agreements between firms are agreements by which firms divide up a market and agree not to enter each other's territory. These agreements are under suspicion by antitrust authorities; and if, moreover, after an investigation, the antitrust authority finds proof of market sharing agreements, the firms involved in them are penalized. It is the set of these bilateral agreements which gives rise to a collusive network among the firms.

The goal of the present article is to study how the presence of an antitrust policy on market sharing agreements affects the market sharing agreements made by firms. I examine the network structure that arises when each firm takes into account the possible punishment when a new agreement is signed.

Market sharing agreements are perennial problems in antitrust policy. As an example, in 1898, a group of iron pipe producers in the Middle West and the West in United States rigged prices on some markets and reserved some cities as exclusive domains of one of the sellers. This was the Addyston Pipes Case¹. More than one hundred years later, in January 2005, related to the case of the MCAA (Monochloroacetic Acid) chemicals cartel, the European Competition Commissioner stated "...the Commission cannot and will not tolerate price fixing and market sharing. I will not allow companies to carve up the Single Market amongst themselves and so deny customers the benefits to which they are entitled..."

Antitrust authorities have been particularly concerned about the potential harm of market sharing agreements and have spent substantial time and effort attempting to deter them. As a recent example in Europe, it is possible to mention the lifts and escalators cartel operating in Belgium, Germany, Luxembourg and the Netherlands. In February 2007, the European Commission fined the members of the cartel over €990 million. Between at least 1995 and 2004 the companies in the cartel shared markets among other anti-competitive practises.

However not only transnational competition authorities devote efforts to discourage this practice. National competition authorities also attempt to avoid it. For example, in 1999, the Irish Competition Authority issued cartel guidelines with the mission "to promote greater competition...by tackling anti-competitive practises, thereby contributing to an improvement in economic welfare". Particularly it points out two types of arrangement: 1) the price-fixing cartel and 2) the market-sharing cartel.

Additionally, as another very recent example, we can point to the saving bank cartel operating in the Basque Country and in Navarra in Spain. Last October, the Spanish Competition Authority (*Comisión Nacional de la Competencia*) fined the saving banks BBK, Kutxa, Caja Vital and Caja Navara over €24 million (the second highest fine imposed by the Spanish Competition Authority). Between 1990 and 2005, the member of the cartel had agreed to carve up markets, among other anticompetitive practises. Thus, none of the saving banks in the cartel proceeded to open any branch in the "traditional" territory of each other (while conducting a remarkable territorial expansion in other provinces, especially in the bordering).

Therefore, market-sharing cartels are a current problem in the antitrust policy. Moreover,

¹Quoted by Scherer and Ross (1990).

the antitrust authorities devote efforts to avoiding them. This point stresses the importance of understanding how the collusive agreements work and how they interact with the policy that has tried to deter them in order to promote and develop a healthy economy. The current paper goes in this direction.

Belleflamme and Bloch (2004) have analyzed the formation of market sharing agreements among firms, but they do not take into account the existence of antitrust authorities. Therefore, their results may be limited in such circumstance. The main innovation of the present article is to introduce an antitrust authority (AT), and analyze how the structure of collusive networks in market sharing agreements interact with the antitrust policy that try to deter them.

As in Belleflamme and Bloch (2004), I assume that each firm is associated to one market, i.e. its home market. In spite of this, each firm can enter and compete in all foreign markets. Market sharing agreements are modeled as bilateral agreements whereby firms commit to staying out of each other's market. The set of reciprocal agreements gives rise to a collusive network among the firms.

The antitrust authority will seek to deter collusive agreements by using two instruments. In the first of these, a fine is imposed on firms that are proved guilty of market sharing agreements. The fine is established by law and it is equal to profits of the guilty firm of collusion. On the other hand, I assume that the budget that allows the AT to audit firms is limited and costly, and hence it must select the cases to be inspected. I assume that the AT audits a given firm with some positive probability.

Furthermore, I assume that the technology of audit is such that when the AT inspects, if a market sharing agreement exists, then the AT detects it. Moreover, the AT would also identify the two firms involved in the agreement. That is, if a firm is sued for making a market sharing agreement, the AT is assumed able to detect it without error whether a market sharing agreement has occurred, and if it has occurred the AT can detect the firms that have signed that agreement. If the firms are found guilty of making market sharing agreements, both firms are penalized, and each one must pay a fine that equals the profits of each.

In the present article, I first study the actual probability of inspection in the collusive network framework. I show that from the firm's point of view, the probability of getting caught depends on the agreements each firm has signed. That is, the probability of firm i being detected depends not only on whether firm i is inspected by the antitrust authority but also on whether any firm that has formed an agreement with i , is inspected. Therefore, if a firm is inspected and a market sharing agreement exists then it is detected and the firms involved in it are penalized. But, the firm in consideration might be detected without being inspected because any firm that has an agreement with it was inspected.

I then provide a characterization of the stable network when the antitrust authority exists. I show that the pairwise stable network can be decomposed into a set of isolated firms and complete components of different sizes. Belleflamme and Bloch (2004) have shown that a network is stable if its alliances are large enough. However, under the presence of the antitrust authority, the network's stability is more complex. Alliance stability depends

not only on its size but also on the size of the other alliances. Therefore, the network's stability depends on the network configuration as a whole. To understand this, the following observation is crucial. As said above, the AT inspects with a certain constant probability. If after that it detects a market sharing agreement between two firms then the AT will punish each firm with a fine equal to the *total* profits of each firm. Notice that the *total* profit of each firm equals not only the profit that each firm makes in its own market, but also the profits that it makes in each of the other markets in which the firm participates because it does not sign a market sharing agreement. Thus, when considering whether or not to enter into a market sharing agreement, the firm must take into account that if it is detected by the AT it will lose not only the profits in its *own* market, but also the profits in those markets in which the firm is *not* colluding.

A very important goal of the paper is to explore the impact of the antitrust policy on market sharing agreements over competition.

When a pairwise stability notion is considered, it is possible to argue that the smaller components are more sensitive to the antitrust policy. Therefore, the impact of the antitrust law is perverse. However, when a strong stability criterion is taken into account the AT has a pro-competitive impact. That is, as the probability of inspection increases, firms in large components have more incentives to renege on all their agreements at once, and it might lead to complete disappearance of collusion.

Related Literature

The current article brings together elements from the literature of market sharing agreements, networks and law enforcement. I study a particular way of collusion: market sharing agreements; and these collusive agreements have a key feature: they are bilateral agreements. Therefore, I borrow from Network literature and Collusion literature.

Networks is a currently very active field of research. Prominent contributions to this literature include, among other, Jackson and Wolinsky (1996), Goyal (1993), Dutta and Mutuswami (1997) and Jackson and van den Nouweland (2005). In particular, in the first, the formation and stability of a social networks are modeled when agents choose to maintain or destroy links using the notion of pairwise stability. I follow Jackson and Wolinsky (1996) and Jackson and van den Nouweland (2004) to characterize the stable and strong stable networks. Besides these theoretical articles, there is also a growing literature that applies the theory of economic networks to models of oligopoly. Among others, Goyal and Joshi (2003), Goyal and Moraga (2001), and Belleflamme and Bloch (2004). The first two are related to the formation of bilateral agreement in order to reduce costs. The last paper is closely related to my current work.

The particular form of collusion and the stability concept used in the present paper, which bears a strong resemblance to the stability of price-fixing cartels, calls to mind Collusion literature. There is a voluminous theoretical literature on collusive behavior. After the seminal contribution of Stigler (1950) the stability of the price-fixing cartel has been extensively studied in the literature. For an excellent reference of this literature see Vives (2001).

The main goal of the current paper is to explore the impact of the antitrust authority on

the collusive network structure, and this objective is shared in some researches in the vast literature of antitrust enforcement. The following papers, among others, study the effect of antitrust policy on cartel behavior. Block et al. (1981), a classic article, is the first systematic attempt to estimate the impact of antitrust enforcement on horizontal minimum price fixing. Their model considers explicitly the effect of the antitrust enforcement on the decision of firms within an industry to fix prices collusively. They show that a cartel's optimal price is an intermediate price (between the competitive price and the cartel's price in absence of antitrust authority) and this intermediate price depends on the levels of antitrust enforcement efforts and penalties.

Besanko and Spulber (1989), and Besanko and Spulber (1990) with a different approach, use a game of incomplete information where the firms' common cost is private information and neither the antitrust authority (1989) nor the buyers (1990) observe the cartel formation. Instead, they draw inferences from the observed price and decide whether or not to pursue a case. They find that the cartel's equilibrium price is decreasing in the fines. LaCasse (1995) and Polo (1997) follow this approach.

However, the interest for studying the effect of the antitrust policy on the collusive behavior has reemerged. Harrington Jr.(2004) and Harrington Jr.(2005) explore how detection impacts cartel pricing when detection and penalties are endogenous. Firms want to raise prices but not suspicions that they are coordinating their behavior. In Harrington (2005), by assuming the probability of detection is sensitive to price changes, he shows that the steady-state price is decreasing in the damage multiple and in the probability of detection. But, he finds a long-run neutrality result with respect to fixed penalties. Harrington (2004) studies the interaction of internal cartel stability and detection avoidance. One important result that he finds is the perverse effect of the antitrust law. The risk of detection and penalties can serve to stabilize a cartel and thereby allow it to set higher prices. Additionally, Spagnolo (2000) and Motta and Polo (2003) study the effects of leniency programs on the incentives to collude when the probability of detection and penalties are both fixed. Frezal (2006) studies audit policies designed to deter explicit cartels. This article compares a standard random stationary audit strategy with a simple deterministic non stationary strategy and it shows that the latter policy is more efficient than the standard policies to deter collusion.

The outline of the paper is as follows. Section 2 presents the model of market sharing agreements. Section 3 I provide general definitions concerning networks. Section 4 characterizes the stable and strong stable collusive networks in the symmetric context. Furthermore this section studies the impact of antitrust authority over competition. Section 5 discusses the network structure when markets are asymmetric. The paper concludes in Section 6.

2 The Model

The model consists of N risk neutral and symmetric firms indexed by $i = 1, 2, \dots, N$. Each firm is associated to a market, i.e. its home market. Symmetric markets are assumed. We suppose that each firm has incentives to enter all foreign markets. However, firm i does not enter foreign market j , and viceversa, if a reciprocal market sharing agreement exists between

firm i and firm j . A reciprocal market sharing agreement is an agreement whereby two firms agree not to enter each other's territory. The existence of an agreement or link is captured by the binary variable $g_{ij} \in \{0, 1\}$ and we say that firm i has a link or forms an agreement with firm j if $g_{ij} = 1$ and $g_{ij} = 0$ otherwise².

Let n_i be the number of active firms in market i and m_i be the number of links formed by firm i . That is, $n_i = N - m_i$.

Let $\pi_j^i(\cdot)$ be the profits of firm i on market j . Firm i has two sources of profits. Firm i collects profits on its home market, $\pi_i^i(n_i)$, and on all foreign market where there does not exist a link, $\sum_{j, g_{ij}=0} \pi_j^i(n_j)$. The symmetric firm and symmetric market assumptions allow us to avoid all subscripts and superscripts on the profit functions. Therefore, the total profits of firm i can be written as follows:

$$\Pi^i = \pi(n_i) + \sum_{j, g_{ij}=0} \pi(n_j) \quad (1)$$

It is assumed that firms have a limited liability $A = \Pi^i \geq 0$, which represents an upper bound on any penalty imposed by an antitrust authority.

We define an antitrust authority (AT) as a commitment to a pair $\{\alpha, K^i\}$ where $\alpha \in [0, 1]$ is the probability that a market-sharing suit is initiated and $K^i \in [0, A]$ is the total monetary fine that a firm must pay if it is convicted of market sharing agreements.

The technology is such that when the AT inspects, if there exists a market sharing agreement, then the AT detects it. Moreover, the AT also identifies the two firms involved in the agreement. That is, if a firm is sued for making a market sharing agreement, the AT is assumed to be able to detect without error whether a market sharing agreement has occurred, and if it has occurred the AT can detect the firms that signed that agreement. If the firms are found guilty of making market sharing agreements, both firms are penalized and each one must pay K^i .

Due to the fact that resources are limited and costly, the AT will select cases to be inspected. The current paper assumes that the AT is a very simple commitment where the probability of inspection α is a constant and the penalty $K^i = \Pi^i$ ³.

The timing is as follows: given the antitrust policy $\{\alpha, K^i\}$, firms compute the incentives to form agreements, and then they decide whether to form or not. Firms form them if they yield positive profits net of expected penalties from signing market sharing agreements. If an inquiry is opened, and if a firm is proved guilty of market sharing agreement, it pays K^i .

Properties of profit functions

This paper appeals to the same properties for profit functions as Belleflamme and Bloch (2004). The profit functions must satisfy the following properties:

²It is assumed that these agreements are enforceable.

³As is well recognized in law enforcement literature, precommitment by the AT is crucial, otherwise firms might anticipate that the AT does not pursue legal action and the deterrence effect would disappear.

Property 1: Individual profits are decreasing in the number of firms active on the market, $\pi(n_i - 1) - \pi(n_i) \geq 0$.

Property 2: Individual profits are convex in the number of firms active on the market, $\pi(n_i - 1) - \pi(n_i) \geq \pi(n_i) - \pi(n_i + 1)$.

Property 3: Individual profits are log-convex in the number of firms active on the market, $\frac{\pi(n_i-1)}{\pi(n_i)} \geq \frac{\pi(n_i)}{\pi(n_i+1)}$.

It is important to note that Property 1 is satisfied in the most familiar models of oligopolies. In spite of the fact that Property 2 and Property 3 are more restrictive than Property 1, Belleflamme and Bloch (2004) provide sufficient conditions under which these properties hold.⁴

In this paper, we provide some examples which use a profit function that satisfies the above three properties. Let $P(Q) = e^{-Q}$ be the inverse demand function. Then we can compute the profit function as $\pi(n) = e^{-n}$ which satisfies properties 1,2, and 3.

Antitrust Policy

The antitrust authority is a pair $\{\alpha, K^i\}$, where $K^i = \Pi^i$ and the probability of inspection α is a constant. However, given the technology of inspection for the AT, the fact that a firm i signs a new market sharing agreement increases its probability of being inspected. That is, the probability of the firm i being caught by the AT depends not only on whether firm i is inspected but also depends on whether any firm with which firm i has a link is also inspected^{5,6}. That is, $\Pr(\text{Detection } i) = 1 - \Pr(\text{No Inspection } i)$

$$\Pr(\text{Detection } i) = 1 - \Pr\left(\text{no inspection } i \bigcap_{\substack{j \neq i \\ g_{ij}=1}} \text{no inspection } j\right)$$

$$\Pr(\text{Detection } i) = 1 - (1 - \alpha)^{m_i+1} \tag{2}$$

Therefore, from a firm point of view, the probability of a firm i being inspected is endogenous and it depends on how many links the firm i has formed. Note that as the number of links m_i increases, $\Pr(\text{Detection } i) \rightarrow 1$. And, on the other hand, as the number of links m_i goes to 0, $\Pr(\text{Detection } i) \rightarrow \alpha$.

From the AT point of view, the network structure generates an economy of scales on inspection.

⁴Let $P(Q)$ be the inverse demand function. In a symmetric Cournot oligopoly with homogeneous products, individual profits are decreasing in n if costs are increasing and convex and $E(Q) = \frac{QP'(Q)}{P(Q)} > -1$. In this context, Property 3 is satisfied if costs are linear, $E(Q) > -1$ and $E'(Q) \geq 0$.

⁵We consider only the immediate link.

⁶It is assumed that events "no inspection i " and "no inspection j " are independent.

Incentives to form agreements

An essential part of the model is the firm's incentive to form an agreement. Assume that a firm i already has $m_i \neq 0$ links but $g_{ij} = 0$, that is, the firm i does not yet form a link with some firm j . Then, by using expressions (1) and (2) we compute firm i ' expected profits

as: $\Pi^i = \left[\pi(n_i) + \sum_{k \neq j, g_{ki}=0} \pi(n_k) + \pi(n_j) \right] (1 - \alpha)^{m_i+1}$. If firm i decides to form a link

with firm j , firm i ' expected profits will be $\Pi^i = \left[\pi(n_i - 1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k) \right] (1 - \alpha)^{m_i+2}$.

Therefore, firm i 's incentive to form an agreement with firm j is:

$$\Delta\Pi = (1 - \alpha)^{m_i+1} \left[\pi(n_i - 1) - \pi(n_i) - \pi(n_j) - \alpha \left(\pi(n_i - 1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k) \right) \right] \quad (3)$$

$$\Delta\Pi \equiv J_j^i(n_i, n_j, n_k; \alpha)$$

It is worth noting that when the antitrust authority exists, firm i ' incentive to form a market sharing agreement with firm j depends not only on characteristics of markets i and j but also on characteristics of market k and on the probability of inspection α .⁷

If a firm i has an incentive to form an agreement, then $\Delta\Pi \geq 0$. And it is non negative if the bracket expression in (3) is also non negative. By focusing it we study the impact of one more link on the whole profits. One more link implies several conflicting effects. From firm i 's point of view notice that when a new link is formed, firm i agrees not to enter market j , hence firm i loses access to foreign market j and it decreases its profits by $-\pi(n_j)$. Given the reciprocal nature of this agreement, firm j does not enter market i either, then the number of active firms in market i will decrease: n_i decreases. This fact involves two opposite consequences: i) as $\pi(\cdot)$ is a convex function, when n_i decreases, expected profits increase by $(\pi(n_i - 1) - \pi(n_i))(1 - \alpha)$ and ii) as n_i decreases, the probability of inspection increases and it reduces the expected profits in all foreign markets k such that $g_{ik} = 0$. That is, firm i loses $-\alpha \sum_{k \neq j, g_{ki}=0} \pi(n_k)$.

Therefore, J_j^i is an increasing function of n_j and n_k as $\pi(\cdot)$ is a decreasing function. That is, as n_j decreases, it decreases the incentive to lose a more profitable market by forming a link. On the other hand, when firm i forms a link with j , it increases the probability of being inspected and imposes an expected cost over profits on all foreign markets k . Thus, as n_k gets smaller, the expected costs become greater and it decreases the incentive to form a link.

⁷When $m_i = 0$ and firm i is considering forming an agreement, it must evaluate: $\Delta\Pi = \pi(N - 1)(1 - \alpha)^2 - \pi(N) - \pi(n_j) + \sum_{k \neq j, g_{ki}=0} \pi(n_k)(1 - (1 - \alpha)^2)$

The relationship between J_j^i and n_i is ambiguous: 1) as n_i decreases, it increases profits in the home market as $\pi(\cdot)$ is a convex function; 2) as n_i decreases, the probability of being inspected increases, then it decreases the incentive to form a link.

In relation to the antitrust policy, when the AT increases the probability of inspection α , the incentive to form an additional agreement is reduced because it increases the expected cost of forming a link.

3 Networks and graphs: some definitions

Market sharing agreements can be interpreted as a bilateral relationship between firms giving rise to a network g in the set of firms. Hence, it is necessary to introduce some notations and definitions from graph theory.

Networks

Let $N = \{1, 2, \dots, N\}$ denote a finite set of identical firms. We assume that $N \geq 3$. For any $i, j \in N$, the pairwise relationship or link between the two firms is captured by a binary variable $g_{ij} \in \{0, 1\}$; $g_{ij} = 1$ means that a link exists between firms i and j while $g_{ij} = 0$ means that no link is formed between firms i and j . A network $g = \{(g_{ij})_{i,j \in N}\}$ is a description of the pairwise relationship between firms.

A notation for the network obtained by adding link ij to an existing network g is $g + g_{ij}$ and for the network obtained by deleting link ij from an existing network g is $g - g_{ij}$.

The *complete network*, g^c , is a network in which $g_{ij} = 1, \forall i, j \in N$. The *empty network*, g^e , is a network in which $g_{ij} = 0, \forall i, j \in N, i \neq j$. Formally, a firm is isolated if $g_{ij} = 0, \forall j \neq i$ and $\forall i, j \in N$.

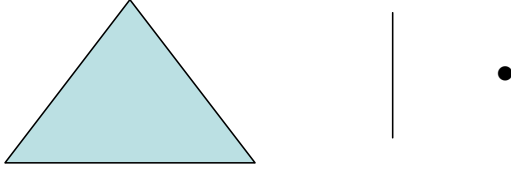
A *path* in a network g between firms i and j is a sequence of firms i_1, i_2, \dots, i_n such that $g_{i_1 i_2} = g_{i_2 i_3} = \dots = g_{i_n j} = 1$. A network is *connected* if there exists a path between any pair $i, j \in N$.

A *component* of a network g , is a nonempty subnetwork $g' \subset g$, such that: (1) if $i \in g'$ and $j \in g'$ where $i \neq j$, then there exists a path in g' between i and j . And (2) if $i \in g'$ and $i, j \in g$ then $i, j \in g'$.

Thus, a component of a network g is a maximally connected subset of g . Note that from this definition of a component, an isolated firm is not considered a component.

Let $m_i(g')$ denote the number of links that firm i has in the component g' . A component $g' \subset g$ is *complete* if $g_{ij} = 1$ for all $i, j \in g'$. For a complete component g' , $m_i(g') + 1$ denote the size of this component, that is the number of firms belonging to g' .

In the figure below there is a network with two complete components and an isolated firm.



Two complete components of sizes 3 and 2 and an isolated firm

Stable Collusive Networks Our interest is to study which networks are likely to arise. Hence we need to define a notion of stability.

Pairwise Stability Networks The following approach was taken by Jackson and Wolinsky (1996). A network g is *pairwise stable* if and only if: (i) $\forall i, j \in N$ st $g_{ij} = 1$, $\Pi^i(g) \geq \Pi^i(g - g_{ij})$ and $\Pi^j(g) \geq \Pi^j(g - g_{ij})$; and (ii) $\forall i, j \in N$ st $g_{ij} = 0$, if $\Pi^i(g + g_{ij}) > \Pi^i(g)$ then $\Pi^j(g + g_{ij}) < \Pi^j(g)$.

In terms of our model, a network g is said to be stable if and only if:

1.

$$\forall i, j \text{ s.t. } g_{ij} = 1 \left\{ \begin{array}{l} J_j^i(n_i + 1, n_j + 1, n_k; \alpha) \geq 0 \\ J_i^j(n_j + 1, n_i + 1, n_k; \alpha) \geq 0 \end{array} \right.$$

2.

$$\forall i, j \text{ s.t. } g_{ij} = 0 \left\{ \begin{array}{l} \text{if } J_j^i(n_i, n_j, n_k; \alpha) > 0 \\ \text{then } J_i^j(n_j, n_i, n_k; \alpha) < 0 \end{array} \right.$$

It is worth noting that the first part of the definition requires that no firm would want to delete a link that it serves. In other words, any firm has the discretion to *unilaterally delete* the link. This contrasts with the second part of the definition. It means that the consent of both is necessary to form a link. That is, *forming link is a bilateral decision*.

The above stability notion (in short form pws) is a relatively weak criterion in the sense that it provides broad predictions and the firm's deviations are constrained. Pairwise stability criterion only considers deviations on a single link at a time.⁸ Furthermore, pws considers only deviations by a pair of player at a time.⁹

Nevertheless, pws notion provides a test to eliminate the unstable networks and it should be seen as a necessary but not sufficient condition for a network to be stable.

⁸On the contrary, for example, it is possible that a firm would not benefit from forming a single link but would benefit from forming several links simultaneously.

⁹It could be that larger groups of player can coordinate their actions in order to all be better off.

Strong Pairwise Stability Networks In order to obtain a stronger stability concept we allow deviations by coalitions of firms. We allow firms to delete some or all market sharing agreements that they have already formed.

We say that a network is pairwise strong stable if it is immune to deviations by coalitions of two firms.

As do Belleflamme and Bloch, we consider the simultaneous linking game introduced by Myerson (1991). Each firm i chooses the set s_i of firms with which it wants to form a link. Thus, $g_{ij} = 1$ if and only if $j \in s_i$ and $i \in s_j$. Let $g(s_1, s_2, \dots, s_n)$ denote the network formed when every i chooses s_i .

We define the following. A strategy profile $\{s_1^*, s_2^*, \dots, s_n^*\}$ is a pairwise strong Nash equilibrium of the game if and only if it is a Nash equilibrium of the game and there does not exist a pair of firms i and j and strategies s_i and s_j such that $\Pi^i(g(s_i, s_j, s_{-ij}^*)) \geq \Pi^i(g(s_i^*, s_j^*, s_{-ij}^*))$ and $\Pi^j(g(s_i, s_j, s_{-ij}^*)) \geq \Pi^j(g(s_i^*, s_j^*, s_{-ij}^*))$ with a strict inequality for one of the two firms. A network g is strongly pairwise stable if and only if there exists a pairwise strong Nash equilibrium of the game $\{s_1^*, s_2^*, \dots, s_n^*\}$ such that $g = g(s_1^*, s_2^*, \dots, s_n^*)$.

It can be proved that any strongly pairwise stable network is pairwise stable¹⁰.

Thus, the strong stability notion can be thought of as sufficient condition for stability.

4 Stable Collusive Networks in the Symmetric Context

We are interested in which network structure will prevail in the symmetric context. We will characterize pairwise stable and strong pairwise stable networks. Recall that the pairwise stability notion might be thought of as a necessary but not sufficient condition for stability and the strong pairwise stable criterion provides a sufficient requirement for a network to be stable over time. Also recall that any strong pairwise stable network is pairwise stable.

The following two lemmas provide the necessary condition on pairwise stable collusive networks in a symmetric context when the AT sets a penalty $K^i = \Pi^i$ and a constant probability of inspection $\alpha \in [0, 1)$.

The following lemma shows that if two firms are connected by a market sharing agreement in an stable network, they must have the same number of competitors in their home markets.

Lemma 1 *When individual profits are decreasing in the number of active firms, if network g is stable, then $\forall i, j \in N$ such that $g_{ij} = 1$, $n_i(g) = n_j(g)$.*

Proof. See Appendix. ■

From Lemma (1), it is important to note that the firm in the market with more competitors does not have an incentive to sign an agreement with a firm in a market more profitable

¹⁰Suppose g is strong stable but not stable. If $g_{ij} = 1$ and $\Pi^i(g) < \Pi^i(g - g_{ij})$ for some i, j , then firm i would be better off if it chooses $s_i = s_i^* \setminus \{j\}$. If $g_{ij} = 0$ and $\Pi^i(g + g_{ij}) > \Pi^i(g)$ and $\Pi^j(g + g_{ij}) > \Pi^j(g)$, then i, j , would be better off if they choose $s_i = s_i^* \cup \{j\}$ and $s_j = s_j^* \cup \{i\}$. It is a contradiction with g is strong stable.

than its home market. Therefore, an isolated firm, which has N competitors in its home market, refuses to form a link with any firm j such that $n_j(g) < N$.¹¹

We can rewrite the pairwise stability condition when $g_{ij} = 1$ as:

$$\frac{\pi(n)}{\pi(n+1)} \geq \frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{hk}=0} \pi(n_k)}{(1-\alpha)\pi(n+1)} \text{ for } h = i, j \neq k \quad (4)$$

Lemma 2 *When individual profits are decreasing and log-convex in the number of firms active on the market, if the network g is stable, then any component g' of g is complete. Moreover, these complete components have different sizes.*

Proof. *See Appendix.* ■

Lemma 2 highlights two important points. One of them is that when two firms, i and j , have incentives to form an agreement ($J_j^i > 0$ and $J_i^j > 0$), the incentive is increasing in the number of agreements already signed.¹² The other relevant issue is that a necessary condition for stability is alliances of different sizes. The intuition is due to free riding. When two firms form an agreement, firms lose possibilities to free ride on the formation of market sharing agreements by the other firm. Hence, if a firm forms more agreements, the more costly it is for other firms to form market sharing agreements with it. Firms in smaller alliances free ride on firms in larger components, and the firms in the first group have no incentive to form agreements with firms in the second group.

Lemma 1 and 2 provide necessary conditions on stable collusive networks but they are not sufficient. The next lemma shows the sufficient condition on pairwise stability in the market sharing context.

Given a network g , which can be decomposed into distinct complete components, g_1, \dots, g_L , of different sizes $m(g_l) + 1 \neq m(g_{l'}) + 1, \forall l, l'$ define $m^*(g) \equiv \min \{m(g_1), \dots, m(g_L)\}$.

Lemma 3 *When individual profits are decreasing and log-convex in the number of firms active on the market, given a network g that can be decomposed into a set of isolated players and different complete components, g_1, \dots, g_L , of different sizes $m(g_l) + 1 \neq m(g_{l'}) + 1, \forall l, l'$, if firm $i \in (m^*(g) + 1)$ does not have incentives to cut a link with a player inside its component, then $j \in (m(g_l) + 1)$ does not have incentives to cut a link with a player inside its component for all $m(g_l) > m^*(g)$.*

¹¹When $n_j(g) < N$, given that profits are decreasing in n , then $\pi(N-1) \leq \pi(n_j(g))$. Therefore we can establish that $(1-\alpha)\pi(N-1) < \pi(N) + \pi(n_j(g)) + \alpha \left[\sum_{k:g_{ik}=0} \pi(n_k) \right]$.

¹²When $J_j^i > 0$ and $J_i^j > 0$ then $\left[[(1-\alpha)\pi(n_i-1) - \pi(n_i)] - \pi(n_j) - \alpha \sum_{k \neq j, g_{ki}=0} \pi(n_k) \right] > 0$ and $\left[[(1-\alpha)\pi(n_j-1) - \pi(n_j)] - \pi(n_i) - \alpha \sum_{k \neq i, g_{kj}=0} \pi(n_k) \right] > 0$

Given the properties established for profit functions, it is easy to see that these two last expressions are decreasing function in n_i (and n_j respectively). In other words, they are increasing in the number of agreements already signed by the respective firm.

Proof. See Appendix. ■

It is important to note that precedent Lemma holds for all $m^*(g) \geq 1$.

Proposition 1 *A network g is stable if and only if it can be decomposed into a set of isolated firms and distinct complete components, g_1, \dots, g_L of different sizes $m(g_1) + 1 \neq m(g_{l'}) + 1, \forall l, l'$ such that neither an isolated firm has an incentive to form a link with another isolated firm nor a firm $i \in m^*(g)$ has an incentive to cut a link with a firm inside its component.*

Proof. Lemmas 1 and 2 provide necessary conditions on stability. Let us consider the sufficient part. Consider a network g that can be decomposed into a set of isolated firms and distinct complete components, g_1, \dots, g_L of different sizes $m(g_1) + 1 \neq m(g_{l'}) + 1, \forall l, l'$. Isolated players have no incentive to create a link with another isolated player. As long as a firm $i \in m^*(g)$ does not have incentives to cut a link with a firm inside its component, then, by Lemma 3, no firm inside a component has incentives to cut a link. Additionally, given that $m(g_l) \neq m(g_{l'}), \forall l, l'$ does not exist two firms belonging to different components that have an incentive to form an agreement between themselves. ■

When $\alpha = 0$, following Belleflamme and Bloch (2004), it there is a minimal number of agreement m^* that a firm must have in order to form an additional link. This number, m^* , does not depend on g and it is a lower bound on the size of complete components in a pairwise stable network. Moreover, when $m^* = 1$, the number of isolated firms is at most 1.

However, when $\alpha \neq 0$, we are only able to define $m^*(g)$ so that it does depend on g , namely, it depends on the network structure. Additionally, $m^*(g) = 1$ is no longer an upper bound on the number of isolated firms in a pairwise stable network.

The next numerical example illustrates a pairwise stable network when $\alpha \neq 0$.

Example 1 *Network structure when $\alpha \neq 0$. Cournot competition with exponential inverse demand function $P(Q) = e^{-Q}$*

When inverse demand function is $P(Q) = e^{-Q}$, we can compute the equilibrium profits as $\pi(n) = e^{-n}$. For $N = 7$ and $\alpha = 0.025$, the following network is pairwise stable: two isolated firms and two complete components such that in one component there are two firms and in the other one there are three firms.

It is worth noting that if α were zero, the pairwise stability condition for $g_{ij} = 1$ becomes $\frac{\pi(n)}{\pi(n+1)} = e \geq 2, \forall n$. Hence, any two firms have incentives to form a link and therefore, the minimal number of agreements that are necessary to have in order to form an additional one is only 1. That is, $m^* = 1$ means that at most there is one isolated firm.

However, when $\alpha = 0.025$, the number of isolated firms is greater than $m^*(g) = 1$. It is important to stress this situation is not possible when $\alpha = 0$, because, in this example, $\alpha = 0$ implies $m^* = 1$ and it means that any two firms have an incentive to form an agreement.

By checking the sufficient conditions for pairwise stability we can show that $m^*(g)$ is the alliance of minimum size in which is profitable to maintain market sharing agreements. Thus,

for firms in the small component it is true that $\frac{\pi(n=6)}{\pi(n+1=7)} > \frac{2}{(1-\alpha)} + \frac{\alpha(3\pi(n_k=5)+2\pi(n_k=7))}{(1-\alpha)\pi(n+1=7)}$. In other words, it does not want to cut a link because it is profitable to maintain it. That is,

$$e > \frac{2}{(1-\alpha)} + \frac{\alpha(3e^{-5} + 2e^{-7})}{(1-\alpha)e^{-7}} = 2.6710$$

From Lemma 3 and given the last inequality, we know that any firm inside the bigger component does not want to cut a link. A firm in the bigger component has an incentive to maintain a link as long as $\frac{\pi(n=5)}{\pi(n+1=6)} > \frac{2}{(1-\alpha)} + \frac{\alpha(2\pi(n_k=6)+2\pi(n_k=7))}{(1-\alpha)\pi(n+1=6)}$. That is:

$$e > \frac{2}{(1-\alpha)} + \frac{2\alpha(e^{-6} + e^{-7})}{(1-\alpha)e^{-6}} = 2.1214$$

And for isolated firms, it is true that $\frac{\pi(n=6)}{\pi(n+1=7)} < \frac{2}{(1-\alpha)} + \frac{\alpha(3\pi(n_k=5)+2\pi(n_k=6))}{(1-\alpha)\pi(n+1=7)}$. That is:

$$e < \frac{2}{(1-\alpha)} + \frac{\alpha(3e^{-5} + 2e^{-6})}{(1-\alpha)e^{-7}} = 2.7591$$

The Antitrust Authority and Competition in Stable Collusive Networks In the simple setting where the antitrust authority is a commitment of a constant probability of inspection and a fine equals to profits, its presence does change the set of possible network structure that can arise. First of all, complete network is pairwise stable for α 's sufficiently low. It follows straightforward from the stability condition when $g_{ij} = 1$ and $n = 1$ that :

Proposition 2 For $\alpha \leq 1 - \frac{2\pi(n=2)}{\pi(n=1)}$ the complete network is always pairwise stable.

Proof. See Appendix. ■

Second, the empty network arises as pairwise stable for α 's sufficiently high.

Proposition 3 For $\forall N \in [3, \infty)$, $\exists \alpha_e(N)$, with $\alpha_e(N)$ strictly decreasing with N such that for $\alpha > \alpha_e(N)$ the empty network is always stable.

Proof. See Appendix. ■

Third, as we showed before, when $\alpha \neq 0$, firm i 's incentive to form an agreement with firm j depends on the characteristics of all markets, i.e., markets i, j and all markets k such that $g_{ik} = 0$. The pairwise stable network can be decomposed into a set of isolated firms and complete component of different sizes. However, when the AT exists, we can not define a unique lower bound on the size of complete components because it depends on each network structure. This is due to the fact that the free riding on the market sharing agreements made by the other firms is no longer "so free". In other words, $\alpha \neq 0$ means that there exists an additional cost of forming an agreement. If one link is formed, there exists a positive probability of finding guilty of market sharing agreement. Consequently, it exists a positive probability to lose profits not only on the market where the agreement is signed but also to lose profits on markets in which the firm is active, i.e. in markets where the firm does not

have market sharing agreements. For that reason, the smaller alliances are more sensitive to the antitrust policy. Given an $\alpha \neq 0$, a firm i inside a small alliance does not have much to gain and a lot to lose when one more link is made: i) it gains $(1 - \alpha) \pi(n_i - 1) - \pi(n_i)$ that it gets smaller as the alliance is smaller because the number of active firms is greater in small components; and ii) it loses not only the access to profits on foreign market j , $\pi(n_j)$, but it also loses, in expected terms, $\alpha \sum_{k:g_{ik}=0} \pi(n_k)$. The following Proposition formalizes this intuition.

Proposition 4 *When individual profits are decreasing in the number of active firms, given a network g that can be decomposed into a set of isolated firms and complete components of different sizes, as α increases, the incentive to form an additional market sharing agreement declines more in smaller components.*

Proof. See Appendix. ■

Therefore, as α becomes greater, the smaller components are more sensitive to the antitrust policy and, for the AT it is easier to tear down small alliances. In the limit, firms must decide to form a very large alliance (complete network) or no alliance at all (empty network).

The following example illustrates the changes that the AT imposes. (The Appendix contains all calculation details.)

Example 2 *The AT, Network Structures and Competition. Cournot competition with exponential inverse demand function $P(Q) = e^{-Q}$*

When inverse demand function is $P(Q) = e^{-Q}$, we can compute the equilibrium profits as $\pi(n) = e^{-n}$. In this case, note that $\frac{\pi(N-m)}{\pi(N-m+1)} = e, \forall m$. To construct this example we assume $N = 5$.

The stability condition when $\alpha = 0$ becomes $e \geq 2$, then any two firms have an incentive to form a link. When $\alpha \neq 0$ the pairwise stability condition for $g_{ij} = 1$ is inequality (4). This table depicts the set of pairwise stable networks for different values of the antitrust policy.

Table 1

α	Set of pairwise stable networks
$\alpha = 0$	$\{3, 2\}, \{4, 1\}, \{5\}$
$\alpha \in (0; 0.014537]$	$\{3, 2\}, \{4, 1\}, \{5\}$
$\alpha \in (0.014537; 0.040031]$	$\{3, 1, 1\}, \{4, 1\}, \{5\}$
$\alpha \in (0.040031; 0.064913]$	$\{2, 1, 1, 1\}, \{3, 1, 1\}, \{4, 1\}, \{5\}$
$\alpha \in (0.064913; 0.20795]$	$\{1, 1, 1, 1, 1\}, \{3, 1, 1\}, \{4, 1\}, \{5\}$
$\alpha \in (0.20795; 0.25171]$	$\{1, 1, 1, 1, 1\}, \{4, 1\}, \{5\}$
$\alpha \in (0.25171; 0.26424]$	$\{1, 1, 1, 1, 1\}, \{5\}$
$\alpha \in (0.26424; 1)$	$\{1, 1, 1, 1, 1\}$

From this example we can learn that:

- i. If the AT does not put in enough "effort" (it can be measured by α), for example $\alpha \leq 0.014537$, its presence does not change the results from the $\alpha = 0$ case.

- ii. As we said before, when α becomes greater, the smaller components are more sensitive to the antitrust policy. For example, when $\alpha \in (0.014537; 0.040031]$ the network structure $\{3, 2\}$ is no longer stable because firms in smaller component have incentives to break their agreement and the network graph $\{3, 1, 1\}$ becomes stable.
- iii. It is noteworthy that graphs like $\{3, 1, 1\}$ or $\{2, 1, 1, 1\}$ are not pairwise stable when $\alpha = 0$.
- iv. As α increases the set of possible pairwise stable networks becomes more polarized. For example, when $\alpha \in (0.25171; 0.26424]$ the empty or complete network are the only possible network configurations. This can be understood because the AT imposes a cost on the link formation by reducing the profitability of each link. Hence, firms must form more links, i.e. reduce the number of competitors in their home markets, in order to make it profitable to maintain each market sharing agreement given that α , which is sufficiently high, imposes a high cost on market profit k such that $g_{ik} = 0$.

The example helps us to consider the effect of the AT upon competition. When α is sufficiently low (in the Example, $\alpha \leq 0.014537$) the presence of the AT does not change the set of pairwise stable networks. It does not have an effect on competition. When α is sufficiently high (in the Example, $\alpha > 0.26424$) the empty network is the only pairwise stable network and hence all firms are active in all markets. For α between these extremes, different configurations may arise. However, we can point out that, even though the stable networks becomes more polarized when α increases, it is true that more competitive structures can be sustained through bilateral agreements for $\alpha \neq 0$.

Now, we turn our attention to strong stability criterion. Note that there will be some pairwise stable networks that will not be stable against changes in the agreements made by firms. Now, we allow firms to delete a subset of links already formed and we will study when a firm has no incentive to renege on its agreements. This point is very important in our context because a network composed of large alliances will be hard to sustain. Hence, we go into the strong pairwise stability notion. The difference between stability and strong stability arises from the firms' ability to delete multiple links in the linking game underlying strongly stable networks.

Proposition 5 *A network g is pairwise strongly stable if and only if it is pairwise stable and no firm prefers to cut all its agreements at once, that is*

$$(1 - \alpha)^m \pi(N - m + 1) \geq \pi(N) + (m - 1) \pi(N - m + 2) + \sum \pi(n_k) (1 - (1 - \alpha)^m), \quad \forall m = m(g_l) \quad (5)$$

Proof. *See Appendix.* ■

Therefore, the fact that a firm has no incentives to renege on all its links at once is a sufficient condition for strong stability.

However, a strongly stable network may fail to exist. Notwithstanding, one important advantage of strong criterion is to provide a more accurate prediction of which network

structures will prevail. The following example illustrates these two characteristics of strong stability criterion and additionally shows the effect of the AT upon competition.

Example 3 *Strong Stability Criterion: The AT and Competition. Cournot competition for exponential inverse demand function: $P(Q) = e^{-Q}$*

When the inverse demand function is $P(Q) = e^{-Q}$ we can compute the equilibrium profits as $\pi(n) = e^{-n}$. As in the last example, assume that $N = 5$. Given that a strong stable network is always stable, we only check the condition (5) for all network structures in Table 1 at different antitrust policies. (The Appendix contains all calculations details.)

Table 2

α	Set of pairwise strong stable networks
$\alpha = 0$	$\{3, 2\}$
$\alpha \in (0; 0.014537]$	$\{3, 2\}$
$\alpha \in (0.014537; 0.040031]$	it fails to exist
$\alpha \in (0.040031; 0.064913]$	$\{2, 1, 1, 1\}$
$\alpha \in (0.064913; 0.20795]$	$\{1, 1, 1, 1, 1\}$
$\alpha \in (0.20795; 0.25171]$	$\{1, 1, 1, 1, 1\}$
$\alpha \in (0.25171; 0.26424]$	$\{1, 1, 1, 1, 1\}$
$\alpha \in (0.26424; 1)$	$\{1, 1, 1, 1, 1\}$

From the example, we highlight the following:

- i. The possible set of network structure that can prevail is reduced by using the strong stability criterion.
- ii. However, in small components a firm does not have an incentive to cut all its links because it benefits from the decline of competition, $(1 - \alpha)^m \pi(N - m + 1) \geq \pi(N) + (m - 1) \pi(N - m + 2)$, this gain does not compensate the expected cost of maintaining all its m links, $\sum \pi(n_k) (1 - (1 - \alpha)^m)$. This is what happens, for example for the stable network $\{3, 2\}$ when $\alpha \in (0.014537; 0.040031]$ and therefore, it does not pass the strong stability criterion. (For details, see the *Appendix*)
- iii. Due to for $\alpha \in (0.014537; 0.040031]$ the empty network is not pairwise stable, a strong stability network fails to exist.
- iv. The incentive to free ride and delete all links is higher in larger alliances. For a very small α , for example $\alpha \in (0; 0.014537]$, the complete network and the stable network $\{4, 1\}$ they do not pass the strong stability condition.
- v. By extending the argument just presented, the empty network is the only strong stable network for $\alpha > 0.064913$.
- vi. The antitrust policy is on the side of competition as long as the empty network is the only strong stable network as α increases.

It is possible to formalize what we have learnt from the previous example.

Claim 1 $\forall \alpha > \alpha_e(N)$ the empty network is always strong pairwise stable.

For $\alpha > \alpha_e(N)$ the empty network is always pairwise stable, therefore for $\alpha > \alpha_e(N)$ it is also always strong stable.

Proposition 6 As α becomes greater, firms in large components have more incentives to delete all links at once.

Proof. See Appendix. ■

Therefore, if for some $\alpha > \alpha_e(N)$ all alliances have been taken down by the antitrust policy, the only network configuration that exists is the empty network.

5 Stable Collusive Networks in the Asymmetric Context

We extend the analysis to asymmetric situation. In particular, we assume that markets are asymmetric. By proceeding in this way, we are nearer to realistic situations but farther from providing a generalization of the results. For that reason, we use an example where three firms operate in markets with different profitability.

Example 4 *The AT and Asymmetric Markets*

Consider the inverse demand function $P_i(Q) = A_i e^{-Q}$ and assume $A_3 > A_2 > A_1 = 1$. The set of configurations of stable networks is the same for all $\alpha \geq 0$. The antitrust authority, however, imposes the necessity of "more symmetry" when any market sharing agreements are formed. Therefore, the presence of the AT implies more competition.

1. The complete network is stable if $A_3 \leq e(1 - \alpha) - 1$. And it is strong stable if $A_3 \leq \frac{(e^2(1-\alpha)^3 - 1)}{e} - A_2$.
2. The empty network is stable when (i) $A_2 > (e - 1) - \alpha(e + A_3)$ and (ii) $A_3 > A_2(e - 1) - \alpha(A_2 + 1)$
3. The three networks with two linked firms and one isolated firm are stable for different combinations of the parameter. For instance, the network with a single agreement between the firms in the larger markets is stable if and only if $A_3 \leq A_2(e - 1) - \alpha(A_2e - 1)$.
4. An incomplete collusive network is stable, for instance smallest market forms an agreement with each of the other two firms, when (i) $A_2(e - 1) - \alpha A_2 < A_3 \leq e(e - 1) - \alpha e^2$ and (ii) $A_2 \geq \frac{e + \alpha A_3}{e(1 - \alpha)^2 - 1}$. Additionally, it is strong stable if and only if $A_3 + A_2 \leq (e^2 - 1) + \alpha e^2(3\alpha - \alpha^2 - 3)$
5. Fails to exist if (i) $A_2 < (e - 1) - \alpha(e + A_3)$; (ii) $A_3 < A_2(e - 1) - \alpha(A_2 + 1)$; and $A_3 \geq e(e - 1) - \alpha e^2$.

6 Concluding Remarks

We have characterized the stable collusive network that arises when firms form agreements by which they divide up a market among themselves in a symmetric oligopolistic setting and when an antitrust authority exists. First of all, we find that the probability of inspection by the antitrust authority is endogenous and it depends on how many agreements each firm has signed. That is, the probability of firm i to be inspected depends not only on whether firm i is inspected by the antitrust authority but also on any firm j , such that firm i and j form an agreement, is inspected.

We showed that the pairwise stable network can be decomposed into a set of isolated firms and complete components of different sizes. However, when the AT exists, we can not define a unique lower bound on the size of complete components because it depends on each network configuration. This is because when the AT exists, firm i 's incentive to form an agreement with firm j depends on the characteristics of all markets, i.e., markets i, j and all markets k such that $g_{ik} = 0$. In turn, this implies that when $\alpha \neq 0$ there exists an additional cost of forming an agreement. If one link is formed, there exists a positive probability of finding guilty of market sharing agreement. Consequently, there exists a positive probability of losing profits not only in the market where the agreement is signed but also of losing profits in markets in which the firm is active, i.e. in markets where the firm does not have market sharing agreements. Antitrust laws may have a pro-competitive effect as they give firms in large alliances more incentives to cut their agreements at once. Therefore the empty network might arise as the only strong stable network.

It is important to note that we introduce a cost in forming a link by the AT. And an important implication of this is that the cost of establishing links by any two firms is not fixed and depends not only on the number of agreements these two firms have formed but also on agreements signed by other players.

The network configurations in the asymmetric context are the same as when the antitrust authority does not exist; however its presence reduces the space of profitable collusion because it imposes additional restrictions.

This is a simple setting for analyzing the effect of the antitrust policy on the collusive behavior and there are many directions that one can go from here. Although for simplicity we gain in tractability, two restrictive aspects of the article can be mentioned. One of these is that in reality market sharing agreements are more complex, and by taking complexity into account, the analysis may be enriched. In the same spirit as before, the other restrictive aspect is that the antitrust authority is pasive. Therefore, the study may be even more interesting when we move in the direction of an active antitrust authority.

7 Appendix

Necessary conditions for pairwise stability

Proof Lemma 1 Since g is stable, condition (4) is met. Hence, we must simultaneously have:

$$\begin{aligned} (1 - \alpha) \pi(n_i(g)) &\geq \pi(n_i(g) + 1) + \pi(n_j(g) + 1) + \alpha \sum_{k:g_{ik}=0} \pi(n_k(g)) \\ (1 - \alpha) \pi(n_j(g)) &\geq \pi(n_j(g) + 1) + \pi(n_i(g) + 1) + \alpha \sum_{k:g_{jk}=0} \pi(n_k(g)) \end{aligned}$$

Given that profit is a decreasing function, the following are a pair of necessary conditions that must be satisfied for the above inequalities to hold:

$$\begin{aligned} \pi(n_i(g)) &> \pi(n_j(g) + 1) \\ \pi(n_j(g)) &> \pi(n_i(g) + 1) \end{aligned}$$

From the first inequality $n_i(g) < n_j(g) + 1$ and from the second one $n_j(g) < n_i(g) + 1$ Hence:

$$n_j(g) - 1 < n_i(g) < n_j(g) + 1 \Leftrightarrow n_i(g) = n_j(g)$$

That is

$$n_i(g) = n_j(g) \equiv n(g)$$

Proof Lemma 2

Part 1: If g is stable then any component $g' \in g$ is complete. Suppose g' is not complete. Then, there exist three firms i, j, l in the component such that $g_{ij} = g_{jl} = 1$ and $g_{il} = 0$. Because g is stable, then by Lemma 1 $n_i(g) = n_j(g) \equiv n(g)$; also $n_j(g) = n_l(g) \equiv n(g)$, then $n_i(g) = n_j(g) = n_l(g) \equiv n(g)$. By stability, condition(4) holds for i and j and j and l . That is:

$$\begin{aligned} \underbrace{\frac{\pi(n)}{\pi(n+1)}}_A &\geq \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{ik}=0, i \neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n+1)}}_B \quad \text{for } i \neq k \\ \frac{\pi(n)}{\pi(n+1)} &\geq \frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{jk}=0, j \neq k} \pi^j(n_k(g))}{(1-\alpha)\pi(n+1)} \quad \text{for } j \neq k \\ \frac{\pi(n)}{\pi(n+1)} &\geq \frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{lk}=0, l \neq k} \pi^l(n_k(g))}{(1-\alpha)\pi(n+1)} \quad \text{for } l \neq k \end{aligned}$$

For i and/or l , one or both conditions hold:

$$\underbrace{\frac{\pi(n-1)}{\pi(n)}}_D < \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{ik}=0, i \neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n)}}_E \text{ for } i \neq k; \text{ and/or}$$

$$\frac{\pi(n-1)}{\pi(n)} < \frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{lk}=0, l \neq k} \pi^l(n_k(g))}{(1-\alpha)\pi(n)} \text{ for } l \neq k$$

By log-convexity, we can establish that:

$$A \leq D$$

From stability conditions:

$$B \leq A \leq D < E$$

However given that profits are decreasing functions and given that the number of terms in $\sum_{k:g_{ik}=0} \pi^i(n_k)$ in B and E are different, we can say that:

$$B > E$$

This is a contradiction. Then g' must be a complete component. ¹³

Part 2: If g is stable then the complete components must have different sizes.

Take two firms i, j in component g' and a firm l in g'' . Suppose, by contradiction, that $m(g') + 1 = m(g'') + 1$. Therefore, we have $n_i(g) = n_j(g) = n_l(g) \equiv n$. The stability of g implies condition (4) holds for i and j . That is:

$$\underbrace{\frac{\pi(n)}{\pi(n+1)}}_A \geq \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{ik}=0, i \neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n+1)}}_B \text{ for } i \neq k$$

$$\frac{\pi(n)}{\pi(n+1)} \geq \frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{jk}=0, j \neq k} \pi^j(n_k(g))}{(1-\alpha)\pi(n+1)} \text{ for } j \neq k$$

For i and/or l , one or both conditions hold:

$$\underbrace{\frac{\pi(n-1)}{\pi(n)}}_D < \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{ik}=0, i \neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n)}}_E \text{ for } i \neq k; \text{ and/or}$$

$$\frac{\pi(n-1)}{\pi(n)} < \frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{lk}=0, l \neq k} \pi^l(n_k(g))}{(1-\alpha)\pi(n)} \text{ for } l \neq k$$

¹³The same logic is for l .

By log-convexity, we can establish that:

$$A \leq D$$

From stability conditions:

$$B \leq A \leq D < E$$

However, given that profits are decreasing functions and given that the number of terms in $\sum_{k:g_{ik}=0} \pi^i(n_k)$ in B and E are different, we can say that:

$$B > E$$

But it is a contradiction with the assumption that profits are log-convex and stability of g .¹⁴

Proof Lemma 3 If $i \in (m^*(g) + 1)$ does not have incentives to cut a link with a firm inside its component, it is true that:

$$\frac{\pi(N - m^*(g))}{\pi(N - m^*(g) + 1)} > \frac{2}{(1 - \alpha)} + \frac{\alpha \left[(m(g_l) + 1) \pi(N - m(g_l)) + \sum_{k:g_{ik}=0} \pi(n_k) \right]}{(1 - \alpha) \pi(N - m^*(g) + 1)} \quad (6)$$

Assume by contradiction that $j \in (m(g_l) + 1)$ for $m(g_l) > m^*(g)$ has an incentive to cut a link with a firm inside its component. Then

$$\frac{\pi(N - m(g_l))}{\pi(N - m(g_l) + 1)} < \frac{2}{(1 - \alpha)} + \frac{\alpha \left[(m^*(g_l) + 1) \pi(N - m^*(g_l)) + \sum_{k:g_{jk}=0} \pi(n_k) \right]}{(1 - \alpha) \pi(N - m(g_l) + 1)} \quad (7)$$

When profits are decreasing in n , then $\text{RHS}(6) > \text{RHS}(7)$. By log-convexity assumption $\text{LHS}(6) < \text{LHS}(7)$. Therefore, if i does not have an incentive to cut a link with a firm inside its component, $\text{LHS}(6) > \text{RHS}(6)$, then $\text{LHS}(7) > \text{RHS}(7)$, which contradicts (7).

Proof Proposition 2 When $g_{ij} = 1$ and $n = 1$ the necessary condition for stability requires that $(1 - \alpha) \pi(n = 1) - 2\pi(n = 2) > 0$.

Proof Proposition 3 Assume that $N \geq 3$ and no firm has formed a market sharing agreement yet. A firm, in order to decide to form a link, will compare the following

$$(1 - \alpha)^2 [\pi(N - 1) + (N - 2) \pi(N)] \leq \pi(N) + \pi(N) + (N - 2) \pi(N) \quad (8)$$

¹⁴The same logic is for l .

Then, there exists an $\alpha_e(N)$ such that (8) holds with equality. That is

$$\alpha_e(N) = 1 - \left[\frac{N\pi(N)}{[\pi(N-1) + (N-2)\pi(N)]} \right]^{\frac{1}{2}} \quad (9)$$

which is strictly decreasing with N and $\forall \alpha > \alpha_e(N)$ it satisfies:

$$(1 - \alpha)^2 [\pi(N-1) + (N-2)\pi(N)] < \pi(N) + \pi(N) + (N-2)\pi(N)$$

Proof Proposition 4 Firm i 's incentive to form an agreement with firm j is $(1 - \alpha)\pi(n_i - 1) - \pi(n_i) - \pi(n_j) - \alpha \sum_{k \neq j, g_{ki}=0} \pi(n_k)$. As α increases, the expected cost that the AT imposes on firm's profits is

$$- \left[\pi(n_i - 1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k) \right]$$

We will show that the expected cost for a firm in a small component is greater than the expected cost for a firm in a large component. Assume firm $i \in m_i + 1$ and firm $j \in m_j + 1$, such that $m_i < m_j$. Then, for firm $i \in m_i + 1$, the expected cost imposed when α increases is:

$$\pi(N - m_i + 1) + (m_j + 1)\pi(N - m_j + 1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k) \quad (10)$$

And for firm $j \in m_j + 1$, the expected cost imposed when α increases is:

$$\pi(N - m_j + 1) + (m_i + 1)\pi(N - m_i + 1) + \sum_{k \neq i, g_{kj}=0} \pi(n_k) \quad (11)$$

Given that $m_i < m_j$ and profits are decreasing in the number of active firms, it is easy to see that (10) > (11).

Proof Proposition 5 \Rightarrow Consider a pairwise strong Nash equilibrium s^* . Given that any strongly pairwise stable network is pairwise stable, $g(s^*)$ can be decomposed into a set of isolated firms and complete components where no isolated firm wants to form a link with another isolated one and (6) holds. But assume, by contradiction, that some component g_l does not satisfy the condition $(1 - \alpha)^m \pi(N - m + 1) \geq \pi(N) + (m - 1)\pi(N - m + 2) + \sum \pi(n_k) (1 - (1 - \alpha)^m) \forall m = m(g_l)$. Then s^* is not a Nash equilibrium because any firm i in g_l has a profitable deviation by choosing $s'_i = \emptyset$.

\Leftarrow Assume network g can be decomposed into a set of isolated firms and complete components of different sizes, where inequality (6) holds. Also assume that $(1 - \alpha)^m \pi(N - m + 1) \geq \pi(N) + (m - 1)\pi(N - m + 2) + \sum \pi(n_k) (1 - (1 - \alpha)^m)$ holds for all $m = m(g_l)$. We will show that the following strategies form a pairwise strong Nash equilibrium. For firm $i \in g_l$ it announces $s_i^* = \{j | j \in g_l, j \neq i\}$, however, if i is isolated, it announces $s_i^* = \emptyset$. Hence,

a) No isolated firm i has an incentive to create a link with another firm j , as $i \notin s_j^*$.

b) As $(1 - \alpha)^m \pi(N - m + 1) \geq \pi(N) + (m - 1) \pi(N - m + 2) + \sum \pi(n_k) (1 - (1 - \alpha)^m)$ holds for all $m = m(g_l)$, the firm has no incentive to destroy all its m links. But we must consider the firm's incentives to cut a subset of them. Let us assume it has an incentive to delete a strict subset of its links, hence it chooses to delete h links because

$$(1 - \alpha)^h \pi(N - m + 1) < \pi(N - m + 1 + h) + h\pi(N - m + 2) + \sum \pi(n_k) \left(1 - (1 - \alpha)^h\right)$$

Given that $h \geq 1$, then

$$\pi(N - m + 1 + h) + h\pi(N - m + 2) \leq (h + 1) \pi(N - m + 2)$$

Since we are considering a strict subset of links, then $h < m - 1$ and $h + 1 < m - 1$, hence

$$(h + 1) \pi(N - m + 2) < (m - 1) \pi(N - m + 2)$$

Therefore

$$(1 - \alpha)^m \pi(N - m + 1) < (1 - \alpha)^h \pi(N - m + 1) < (m - 1) \pi(N - m + 2)$$

that contradicts our hypothesis.

c) No firm $i \in g_l$ has an incentive to create a link with firm $j \in g_{l'}$ as $i \notin s_j^*$. Moreover, as $m(g_l) \neq m(g_{l'})$ for all $l \neq l'$, no pair of firms $i \in g_l$ and $j \in g_{l'}$ has an incentive to create a new link between them.

d) As $(1 - \alpha)^m \pi(N - m + 1) \geq \pi(N) + (m - 1) \pi(N - m + 2) + \sum \pi(n_k) (1 - (1 - \alpha)^m)$ holds for all $m = m(g_l)$, when $m > 3$, no pair of firms have incentives to delete all their links nor a subsets of their agreements and to form a link between them. Let us assume, by contradiction, a pair of firms, $i \in m$ and $j \in m'$, has incentive to destroy all their m and m' links each one and form a link between them. For firm i , this is

$$\begin{aligned} & (1 - \alpha)^{m-2} \pi(N - m + 1) \\ & < \pi(N - 1) + (m - 1) \pi(N - m + 2) + (m' - 1) \pi(N - m' + 2) + \\ & + \sum_{k \neq j \neq i, g_{ik}=0} \pi(n_k) - (1 - \alpha)^{m-2} \left[\sum_{k \neq j \neq i, g_{ik}=0} \pi(n_k) + m' \pi(N - m' + 1) \right] \end{aligned} \quad (12)$$

Given that, the LHS(12) > LHS(5) and by straightforward computations we can show that RHS(5) > RHS(12), when condition(5) holds then LHS(12) > RHS(12), which contradicts (12).

Proof Proposition 6 Straightforward computations show that the incentive to delete all links at once increases, as α increases is:

$$m(1 - \alpha)^{m-1} \left[\pi(N - m + 1) + \sum \pi(n_k) \right] > 0 \quad (13)$$

Now, we must check whether the derivative of expression (13) with respect m is positive. That is, we must check if $(1 + m \ln(1 - \alpha)) (1 - \alpha)^{m-1} [\pi(N - m + 1) + \sum \pi(n_k)] + m(1 - \alpha)^{m-1} [\pi'(N - m + 1) + \sum \pi'(n_k)]$ is positive.

Simple computations allow us to show that for a sufficiently high m

$$m \left[\pi'(N - m + 1) + \sum \pi'(n_k) \right] > (1 + m \ln(1 - \alpha)) \left[\pi(N - m + 1) + \sum \pi(n_k) \right]$$

Computatios for table 1: pairwise stability

Empty Network $p(\alpha) = (1 - \alpha)^2 e^{-4} - 2e^{-5} - 3e^{-5} (1 - (1 - \alpha)^2)$
 $p(\alpha) = 0$, Solution is: 6.4913×10^{-2}

N=5, three firms isolated and two linked

For a linked firm, we must check the sign of:

$$p(\alpha) = (1 - \alpha)^2 e^{-4} - 2e^{-5} - 3e^{-5} (1 - (1 - \alpha)^2)$$

$$p(\alpha) = 0, \text{ Solution is: } 6.4913 \times 10^{-2}$$

For an isolated firm, we must check the sign of:

$$p(\alpha) = (1 - \alpha)^2 e^{-4} - 2e^{-5} - (e^{-5} + 2e^{-4}) (1 - (1 - \alpha)^2)$$

$$p(\alpha) = 0, \text{ Solution is: } 4.0031 \times 10^{-2}$$

N=5, two firms isolated and one component of three firms linked

For firms in the complete component, we must check the sign of:

$$p(\alpha) = (1 - \alpha) e^{-3} - 2e^{-4} - 2\alpha e^{-5}$$

$$p(\alpha) = 0, \text{ Solution is: } \frac{e^{-3} - 2e^{-4}}{e^{-3} + 2e^{-5}} = 0.20795$$

For an isolated firm, we must check the sign of:

$$p(\alpha) = (1 - \alpha)^2 e^{-4} - 2e^{-5} - 3e^{-3} (1 - (1 - \alpha)^2)$$

$$p(\alpha) = 0, \text{ Solution is: } 1.4537 \times 10^{-2}$$

N=5, one isolated firm and one component of four firms linked For firms in the complete component, we must check the sign of:

$$p(\alpha) = (1 - \alpha) e^{-2} - 2e^{-3} - \alpha e^{-5}$$

$$p(\alpha) = 0, \text{ Solution is: } \frac{e^{-2} - 2e^{-3}}{e^{-2} + e^{-5}} = 0.25171$$

The isolated firm does not have an incentive to form any agreement for all α .

N=5, two complete components: one of them composed of two firms and the other one of three firms.

Condition for maintaining a link in the complete component of two firms:

$$p(\alpha) = (1 - \alpha)^2 e^{-4} - 2e^{-5} - 3e^{-3} (1 - (1 - \alpha)^2)$$

$$p(\alpha) = 0, \text{ Solution is: } 1.4537 \times 10^{-2}$$

Condition for maintaining a link in the complete component of three firms:

$$p(\alpha) = (1 - \alpha) e^{-3} - 2e^{-4} - 2\alpha e^{-5}$$

$$p(\alpha) = 0, \text{ Solution is: } \frac{e^{-3} - 2e^{-4}}{e^{-3} + 2e^{-5}} = 0.20795$$

N=5, Complete Network Condition for maintaining a link in the complete component of five firms:

$$p(\alpha) = (1 - \alpha) e^{-1} - 2e^{-2}$$

$$p(\alpha) = 0, \text{ Solution is: } \frac{1}{e^{-1}} (e^{-1} - 2e^{-2}) = 0.26424$$

Computations for table 2: pairwise strong stability (ss)

Condition ss when $\alpha = 0$ $p(m) = e^{-(5-m+1)} - e^{-5} - (m-1)e^{-(5-m+2)}$
for $m = 2$ and for $m = 3$, $p(m) > 0$

Condition ss when $\alpha \neq 0$

i. $\alpha \in (0; 1.4537 \times 10^{-2}]$

for m=2

$$p(m, \alpha) = (1 - \alpha)^m e^{-(5-m+1)} - e^{-5} - (m-1)e^{-(5-m+2)} - (1 - (1 - \alpha)^m) 3e^{-3}$$

$$p(2, 1.4537 \times 10^{-2}) = 1.4253 \times 10^{-7}$$

for m=3

$$p(m, \alpha) = (1 - \alpha)^m e^{-(5-m+1)} - e^{-5} - (m-1)e^{-(5-m+2)} - (1 - (1 - \alpha)^m) 2e^{-4}$$

$$p(3, 1.4537 \times 10^{-2}) = 2.7036 \times 10^{-3}$$

for m=4

$$p(m, \alpha) = (1 - \alpha)^m e^{-(5-m+1)} - e^{-5} - (m-1)e^{-(5-m+2)} - (1 - (1 - \alpha)^m) e^{-5}$$

$$p(4, 1.4537 \times 10^{-2}) = -2.8847 \times 10^{-2}$$

for m=5

$$p(m, \alpha) = (1 - \alpha)^m e^{-(5-m+1)} - e^{-5} - (m-1)e^{-(5-m+2)}$$

$$p(5, 1.4537 \times 10^{-2}) = -0.20617$$

ii. $\alpha \in (1.4537 \times 10^{-2}; 4.0031 \times 10^{-2}]$

for m=3

$$p(m, \alpha) = (1 - \alpha)^m e^{-(5-m+1)} - e^{-5} - (m-1)e^{-(5-m+2)} - (1 - (1 - \alpha)^m) 2e^{-5}$$

$$p(3, 4.0031 \times 10^{-2}) = -8.7952 \times 10^{-4}$$

check claim ii. table 2

$$p(m, \alpha) = (1 - \alpha)^m e^{-(5-m+1)}$$

$$p(3, 4.0031 \times 10^{-2}) = 4.4044 \times 10^{-2}$$

$$p(m, \alpha) = e^{-5} + (m-1)e^{-(5-m+2)} + (1 - (1 - \alpha)^m) 2e^{-5}$$

$$p(3, 4.0031 \times 10^{-2}) = 4.4924 \times 10^{-2}$$

for m=4

$$p(m, \alpha) = (1 - \alpha)^m e^{-(5-m+1)} - e^{-5} - (m-1)e^{-(5-m+2)} - (1 - (1 - \alpha)^m) e^{-5}$$

$$p(4, 4.0031 \times 10^{-2}) = -4.2183 \times 10^{-2}$$

iii. $\alpha \in (4.0031 \times 10^{-2}; 6.4913 \times 10^{-2}]$

for $m=3$

$$p(m, \alpha) = (1 - \alpha)^m e^{-(5-m+1)} - e^{-5} - (m - 1) e^{-(5-m+2)} - (1 - (1 - \alpha)^m) 2e^{-5}$$

$$p(3, 6.491 \times 10^{-2}) = -5.119 \times 10^{-3}$$

for $m=2$

$$p(m, \alpha) = (1 - \alpha)^m e^{-(5-m+1)} - e^{-5} - (m - 1) e^{-(5-m+2)} - (1 - (1 - \alpha)^m) 3e^{-5}$$

$$p(2, 6.491 \times 10^{-2}) = 1.8438 \times 10^{-7}$$

iv. $\alpha \in (6.4913 \times 10^{-2}; 0.20795]$

for $m=3$

$$p(m, \alpha) = (1 - \alpha)^m e^{-(5-m+1)} - e^{-5} - (m - 1) e^{-(5-m+2)} - (1 - (1 - \alpha)^m) 2e^{-5}$$

$$p(3, 0.20795) = -2.5411 \times 10^{-2}$$

References

- [1] Aumann, R. and Myerson, R. (1988). 'Endogenous formation links between players and coalitions: an application of the Shapley value.' In Roth, A. *The Shapley Value*. Cambridge University Press, Cambridge, United Kingdom, pp.175-191.
- [2] Belleflamme, P. and Bloch, F. (2004). 'Market sharing agreements and collusive networks.' *International Economic Review*, vol. 45 (May), pp. 387-411.
- [3] Bensako, D. and Spulber, D. (1989) 'Antitrust enforcement under asymmetric information.' *The Economic Journal*, vol. 99 (June), pp. 408-425.
- [4] ————— and ————— (1990). 'Are Treble Damages Neutral? Sequential Equilibrium and Private Antitrust Enforcement', *American Economic Review* 80, 870-887.
- [5] Block, M. et al (1981). 'The deterrent effect of antitrust enforcement', *Journal of Political Economy*, 89, 429-445.
- [6] D'Aspremont, C et al (1983). 'On the stability of Collusive Price Leadership', *Canadian Journal of Economics* 16, 17-25.
- [7] Deneckere, R. and Whinston, M. (1990). 'Multimarket contact and collusive behavior', *Rand Journal of Economics* 21, 1-26.
- [8] Dutta, B. and Mutuswami, S. (1997). 'Stable Networks', *Journal of Economic Theory*, 76, pp. 322-344.
- [9] Frezal, S. (2006). 'On optimal cartel deterrence policies", *International Journal of Industrial Organization* 24, 1231-1240.
- [10] Goyal, S. (1993). 'Sustainable Communications Networks'. Tinbergen Institute Discussion Paper TI 93-250.
- [11] ———, and Joshi, S. (2003) 'Networks of Collaboration in Oligopoly', *Games and Economic Behavior* 43, 57-85.
- [12] ———, and Moraga, J.L. (2001) 'R&D Networks', *Rand Journal of Economics* 32, 686-707.
- [13] Harrington, Jr., J.E. (2004) 'Cartel Pricing Dynamics in the Presence of an Antitrust Authority', *Rand Journal of Economics*, 35, pp. 651-673
- [14] Harrington, Jr., J.E. (2005) 'Optimal Cartel Pricing in the Presence of an Antitrust Authority', *International Economic Review*, vol. 46, (February), pp.145-169
- [15] Jackson, M. and A. van den Nouweland (2005). 'Strongly stable networks.' *Games and Economic Behavior*, 51, pp. 420-444.

- [16] ———. and Wolinsky, A. (1996). 'A strategic model of social and economic networks.' *Journal of Economic Theory*, 71, pp. 44-74.
- [17] LaCasse (1995). 'Bid Rigging and the Threat of Government Prosecution', *Rand Journal of Economics*, 26, pp. 398-417
- [18] Lee, L.(1980). 'Some models of antitrust enforcement'. *Southern Economic Journal*, 47, 147-155.
- [19] Motta, M., *Competition Policy: Theory and Practice*. Cambridge University Press, 2004
- [20] ———and Polo (2003). 'Leniency Programs and Cartel Prosecution'. *International Journal of Industrial Organization*, 21, pp. 347-379.
- [21] Myerson, R.B. (1991). *Game theory: Analysis of conflict*. Cambridge, MA: Harvard University Press.
- [22] Mitchell Polinsky, A. and Shavell, S.(2000). 'The economic theory of public enforcement of law'. *Journal of Economic Literature*, Vol.38 (March), pp. 45-76.
- [23] Polo, M. (1997). 'The Optimal Prudential Deterrence of Price Fixing Agreements' IGER, wp. N° 120
- [24] Scherer, F.and Ross,D. *Industrial Market Structure and Economic Performance*, 3rd. ed.(Boston: Houghton-Mifflin, 1990).
- [25] Smith, W.J. and Waughan, M.B. (1986), 'Economic welfare, price and profit: the deterrent effect of alternative antitrust regimes'. *Economic Inquiry*, 24, 615-629
- [26] Spagnolo, G (2000). 'Optimal Leniency Programs". Fondazione Eni Enrico Mattei, wp. N°42/2000
- [27] Stigler, G., 'Monopoly and Oligopoly by Merger', *American Economic Review* 40 (1950), 23-34.
- [28] Vives, X., *Precios y Oligopolio*, Antoni Bosch , editor and Fundació ICO, 2001.