Wall Street and Silicon Valley:
A Delicate Interaction*

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Abstract

Financial markets look at aggregate investment data for clues about underlying profitability. At the same time, firms’ investment depends on expected IPO value and equity prices. This generates a two-way feedback between “Wall Street” and “Silicon Valley”. In this paper we study the positive and normative implications of this interaction during episodes of intense technological change, when information about new investment opportunities is highly dispersed. Because high aggregate investment is “good news” for profitability, asset prices increase with aggregate investment. Because firms’ incentives to invest in turn increase with asset prices, an endogenous complementarity emerges in investment decisions. We show that this complementarity dampens the impact of fundamental shocks (shifts in underlying profitability) and amplifies the impact of expectational shocks (correlated errors in assessments of profitability). We next show that these effects are symptoms of inefficiency: equilibrium investment reacts too little to fundamentals and too much to noise. We finally identify policies that correct this inefficiency without requiring any informational advantage on the government’s side.

Keywords: beauty contests, heterogeneous information, complementarity, amplification, volatility, inefficiency.

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1 Introduction

In periods of intense technological change, firms make investment decisions under significant uncertainty about the profitability of the new technologies. In these circumstances, a high level of aggregate investment sends a positive signal to financial markets, as it provides a useful summary statistic of the dispersed information held by individual firms. Through this channel, high investment can contribute to high asset prices. At the same time, high asset prices can add fuel to the investment boom, by increasing the expected value of IPOs and equity issues. This generates a two-way feedback between real and financial activity (between “Silicon Valley” and “Wall Street”). In this paper, we document how this two-way interaction leads to novel positive and normative implications when information is dispersed.

We consider an environment in which a large number of “entrepreneurs” gets the option to invest in a new technology. Entrepreneurs have dispersed information about the profitability of this technology and may sell their capital in a competitive financial market before uncertainty is realized. In this environment, aggregate investment and asset prices are driven by two types of shocks: “fundamental shocks,” reflecting actual profitability, and “expectational shocks,” reflecting correlated mistakes in the entrepreneurs’ assessments of this profitability.

The positive contribution of the paper is to examine how the interaction between the real and the financial sector affects the transmission of these shocks in equilibrium. Because high aggregate investment is “good news” for profitability, asset prices increase with aggregate investment. As a result, an endogenous complementarity emerges in investment decisions. An entrepreneur anticipates that the price at which he will be able to sell his capital in the financial market will be higher the higher the aggregate level of investment. Therefore, he is more willing to invest when he expects higher investment by other entrepreneurs.

In equilibrium, this complementarity induces entrepreneurs to rely more on common sources of information regarding profitability, and less on idiosyncratic sources of information. This is because common sources of information are relatively better predictors of other entrepreneurs’ investment choices, and hence of future financial prices. For the same reason, the entrepreneurs’ choices become more anchored to the common prior, and hence less sensitive to changes in fundamentals. It follows that the interaction between “Main Street” and “Wall Street” amplifies the impact of common expectational shocks while also dampening the impact of fundamental shocks.
The normative contribution of the paper is to examine whether the reaction of the economy to different shocks is optimal from a social perspective. The mere fact that entrepreneurs care about the financial market valuation of their investment does not, on its own, imply any inefficiency. Indeed, as long as information is common across agents, equilibrium asset prices coincide with the common expectation of profitability; whether entrepreneurs try to forecast fundamentals or asset prices is then completely irrelevant for welfare and no inefficiency arises.

This is not the case, however, when information is dispersed. The fact that asset prices respond to aggregate investment induces a wedge between private and social returns to investment. To see this, consider a technology whose profitability is independent of whether the initial entrepreneurs or other agents hold the capital; nor does it depend on the aggregate investment made by other entrepreneurs (i.e. there are no investment spillovers). In this case, the social return to individual investment is independent of aggregate investment—but the private return is not. In other words, the complementarity that emerges in equilibrium due to the two-way feedback between real and financial activity is not warranted from a social perspective. By implication, the positive results discussed above are also symptoms of inefficiency: equilibrium investment reacts too little to fundamental shocks and too much to expectational shocks.

We then conclude the paper by identifying policies that can correct this inefficiency without requiring the government to have any informational advantage vis-a-vis the market. We show it suffices to make individual capital taxation contingent on realized aggregate investment. By appropriately designing the elasticity of the marginal tax rate with respect to aggregate investment, the government can manipulate the degree of strategic complementarity perceived by the entrepreneurs when making their investment choices and thereby restore efficiency in how they react to different sources of information, even if it can not directly monitor these sources of information.

**Discussion.** The US experience in the second half of the 90s has renewed interest in investment and asset-price booms driven by apparent euphoria regarding new technologies (e.g., the Internet), and on the optimal policy response to these episodes. A common view is that entrepreneurs and corporate managers are driven by noise traders and other irrational forces in financial markets, or are irrational themselves; see, e.g., Shiller (2000), Cecchetti et. al (2000), Bernanke and Gertler (2001), and Dupor (2005), along with numerous articles in the popular press. Similar concerns are currently raised for China. The presumption that the government can detect “irrational exuberance” then leads to the result sought—that the government should intervene.
While we share the view that expectational errors may play an important role in these episodes, we also recognize that these errors may originate from noise in information rather than irrational exuberance. Furthermore, we doubt that the government “knows better” than the market. Our approach is thus very different. On the positive side, we show that the interaction between real and financial activity can amplify the impact of noise and that this amplification is stronger when information is more dispersed. This helps explain, without any departure from rationality, why periods of intense technological change are likely to feature significant non-fundamental volatility. On the normative side, we show that the same mechanism is a source of inefficiency in the decentralized use of information. This opens the door to policy intervention without presuming any informational, or “intelligence”, advantage on the government’s side.

Because the source of both amplification and inefficiency in our model rests on the property that investment is largely driven by expectations about others’ choices rather than about fundamentals, our results are reminiscent of Keynes’ famous beauty-contest metaphor for financial markets:

“...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors...” Keynes (1936, p.156).

Implicit in Keynes’ argument appears to be a normative judgement: that something goes wrong when investment is driven by higher-order expectations. However, Keynes did not explain why this might be the case. More recently, Morris and Shin (2002) and Allen, Morris and Shin (2005) showed that a mechanism similar to the one articulated by Keynes increases the sensitivity of equilibrium outcomes to common sources of information. However, these papers also do not explain the source of discrepancy, if any, between the equilibrium and the socially efficient use of information. In contrast, the discrepancy emerges endogenously in our setting. In this respect, the mechanism we propose here is the first to provide a complete micro-foundation for beauty-contest-like inefficiencies in the interaction of real and financial activity.1

1Morris and Shin (2002) showed how assuming a certain discrepancy between private and social motives in the use of information can induce a negative social value for public information. In our environment, the endogeneity of the discrepancy has also different implications for the social value of information; see Section 5.
**Related literature.** The importance of the interaction between real and financial activity has long been recognized in the literature. In particular, two large strands of literature have analyzed the two channels that are crucial for our mechanism. The one strand examines the impact of news about macroeconomic activity on asset prices (e.g., Chen, Roll and Ross, 1986; Cutler, Poterba, and Summers, 1989). The other strand examines the impact of variation in asset prices on real investment (e.g., Brainard and Tobin, 1968; Tobin, 1969; Abel and Blanchard, 1986; Barro, 1990). However, the prior theoretical work has largely ignored dispersed information, which explains why the positive and normative implications that we identify in this paper are novel.

The paper also relates to the growing literature on heterogeneous information and strategic complementarities (Amato and Shin, 2006; Angeletos and Pavan, 2004, 2007a,b; Baeriswyl and Conrand, 2007; Hellwig, 2005; Lorenzoni, 2006, 2007; Morris and Shin, 2002; Roca, 2006; Mackowiak and Wiederholt, 2006; Woodford, 2002). However, unlike the complementarities that originate in monopolistic price competition, production externalities, or other payoff interdependencies, the complementarity documented in this paper originates in the dispersion of information: it emerges only because aggregate investment is a signal of the underlying fundamentals. This specific source of complementarity is the key to both the positive and the normative results of this paper.

Finally, by touching on the theme of recent “bubbly” episodes, the paper relates to at least two other lines of work in addition to the one on irrational exuberance cited earlier. One line studies rational bubbles in economies with financial frictions or asset shortages (e.g., Ventura, 2003; Caballero, 2006; Caballero, Farhi and Hammour, 2006). The mechanisms studied in these papers are, however, unrelated to information. The second line studies how heterogeneous priors can lead to speculative trading (e.g., Scheinkman and Xiong, 2003; Panageas, 2005). In these papers, however, as long as markets are complete, asset prices continue to reflect the social value of investment and no inefficiency emerges.

**Layout.** Section 2 introduces the baseline model; it also reviews the equilibrium without informational frictions, in which case there is neither complementarity nor inefficiency. Section 3 turns to equilibrium with heterogeneous information and studies the positive implications of the model. Section 4 characterizes the efficient use of information and contrasts it to equilibrium. Section 5 discusses policy implications. Section 6 considers a number of extensions. Section 7 concludes. All proofs are in the Appendix.
2 The baseline model

We consider an environment where heterogeneously informed agents choose how much to invest in a "new technology" with uncertain returns. After investment has taken place, but before uncertainty is resolved, agents trade financial claims on the returns of the installed capital. At this point, the observation of aggregate investment partially reveals the information that was dispersed in the population during the investment stage.

**Timing and actions.** There are two types of agents, each of measure 1/2, “entrepreneurs” and “traders”. We index agents by \( i \in [0,1] \), where \( i \in [0, 1/2] \) denotes entrepreneurs and \( i \in (1/2,1] \) denotes traders. There are four periods, \( t \in \{0,1,2,3\} \). The exogenous productivity of the new technology is a random variable denoted by \( \theta \).

At \( t = 0 \), nature draws \( \theta \) from a Normal distribution with mean \( \mu > 0 \) and variance \( 1/\pi_\theta \) (i.e., \( \pi_\theta \) is the precision of the prior).

At \( t = 1 \), each entrepreneur decides how much to invest in the new technology. Let \( k_i \) denote the investment of entrepreneur \( i \). The cost of this investment is \( k_i^2/2 \). Traders are not allowed to invest in period 1.

At \( t = 2 \), each entrepreneur is hit by a “liquidity shock” with probability \( \lambda \in (0,1) \). Liquidity shocks are i.i.d. across agents, so \( \lambda \) is also the fraction of entrepreneurs hit by the shock. The liquidity shock fully determines whether or not an entrepreneur sells his installed capital to the financial market. Entrepreneurs hit by the shock are forced to sell all their capital; entrepreneur not hit by the shock are not allowed to sell any of their capital. This stark assumption greatly simplifies the derivation and presentation of our results. In Section 5, we show that all results survive in a setup where entrepreneurs not hit by the shock are allowed to choose how much to trade in period 2. The financial market in period 2 is competitive and \( p \) denotes the price of one unit of installed capital. Traders participate to the financial market in period 2, where they can acquire installed capital.

Finally, at \( t = 3 \), \( \theta \) is publicly revealed and each unit of installed capital gives a payoff of \( \theta \) to its owner.

**Payoffs.** All agents are risk neutral and the discount rate is zero. Payoffs are thus given by \( u_i = c_{i1} + c_{i2} + c_{i3} \), where \( c_{it} \) denotes agent \( i \)'s consumption in period \( t \). Consider first an entrepreneur. If he is not hit by the liquidity shock his consumption is \( (c_{i1}, c_{i2}, c_{i3}) = (-k_i^2/2, 0, \theta k_i) \).
If he is hit by the shock, he sells all his capital at the price $p$ and his consumption stream is $(c_{i1}, c_{i2}, c_{i3}) = (-k_i^2/2, pk_i, 0)$. Consider next a trader, and let $q_i$ denote the units of installed capital he acquires in period 2. His consumption stream is $(c_{i1}, c_{i2}, c_{i3}) = (0, -pq_i, \theta q_i)$. Summing up, an entrepreneur’s payoff is $u_i = -k_i^2/2 + \theta k_i$ if he is not hit by the shock and $u_i = -k_i^2/2 + pk_i$ otherwise, while a trader’s payoff is $u_i = (\theta - p)q_i$.

**Information.** All agents have incomplete information about $\theta$, but the entrepreneurs are better informed than the traders. In particular, at $t = 0$, before choosing $k_i$ each entrepreneur observes a private signal $x_i = \theta + \xi_i$, where the shock $\xi_i$ is independent of $\theta$ and is normally distributed with mean 0 and variance $1/\pi_x$ (i.e., $\pi_x$ is the precision of the private signal). The shocks $\{\xi_i\}_{i \in [0,1]}$ are independent across entrepreneurs. In addition, all entrepreneurs observe a common signal $y = \theta + \varepsilon$, where $\varepsilon$ is independent of $\theta$ and of $\{\xi_i\}_{i \in [0,1]}$ and is normally distributed with mean 0 and variance $1/\pi_y$ (i.e., $\pi_y$ is the precision of the common signal). Traders never receive any exogenous information about $\theta$. However, at the beginning of $t = 2$, when they enter the financial market, they observe aggregate investment $K = \int_0^1 k_i di$ and they use this observation to update their beliefs about $\theta$.

**Remarks.** The two essential ingredients of the model are (i) that entrepreneurs have some private information, so that aggregate investment is a signal of the fundamental, and (ii) that there is some aggregate shock that is uncorrelated to the fundamental, affects aggregate investment, and is not publicly known, so that at the end of the day aggregate investment does not perfectly reveal the fundamental to all agents. The information structure used here is convenient way to obtain both (i) and (ii). In particular, the error $\varepsilon$ in the signal $y$, which is observed by the entrepreneurs and not by the traders, is a convenient way to introduce a correlated error in the entrepreneurs’ information sets, which in turn confounds the inference problem of the traders (because, in equilibrium, aggregate investment will move with both $\theta$ and $\varepsilon$). As we will see in Section 5, various other information structures are consistent with (i) and (ii) and deliver results analogous to those derived here.

Also notice that the liquidity shock in the model need not be taken too literally. Its presence captures the more general idea that the agent making the investment decision, be it a start-up entrepreneur or the manager of a more developed firm, may care about the financial market valuation of his investment at some point in the life of a project. A start-up entrepreneur may worry about

\footnote{Letting the traders observe the entire cross-sectional distribution $\{k_i\}_{i \in [0,1]}$ does not affect any of the results.}
the price at which he will be able to do a future IPO; a corporate manager may be concerned about the price at which the company will be able to issue new shares. In what follows, we interpret $\lambda$ broadly as a measure of how much firms’ investment decisions are sensitive to forecasts of future equity prices.

Finally, notice that there are no production spillovers and not direct payoff externalities of any kind: both the initial cost ($-k^2_i/2$) and the eventual return ($\theta k_i$), to an entrepreneur’s investment choice are independent of the investment decisions of other agents. The strategic complementarity that we will later document will originate purely in the dispersion of information.

**A benchmark without informational frictions.** Before we proceed, it is useful to review the case in which the dispersion of information vanishes at the time financial trades take place. Consider in particular the case that $\theta$ becomes publicly known in period 2. Clearly, the financial market then clears if and only if $p = \theta$. It follows that the expected payoff of entrepreneur $i$ in period 1 reduces to $E[u_i|x_i, y] = E[\theta|x_i, y]k_i - k^2_i/2$, and therefore his investment is given by

$$k_i = E[\theta|x_i, y] = \frac{\pi_\theta}{\pi_\theta + \pi_x + \pi_y} \mu + \frac{\pi_x}{\pi_\theta + \pi_x + \pi_y} x_i + \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y} y.$$  

The key result here is that equilibrium investment is driven solely by first-order expectations regarding the fundamental. This result does not rely per se on $\theta$ being known in period 2. Rather, it applies more generally as long as the asymmetry of information about $\theta$ vanishes in period 2. To see this, consider an arbitrary information structure; let $I_{i,t}$ denote the information of agent $i$ in period $t$ and $I_{P,t}$ the public information in period $t$; and suppose that $E[\theta|I_{i,2}] = E[\theta|I_{P,2}]$ for all $i$, which means that no agent has private information about $\theta$ in period 2. Market clearing requires $p = E[\theta|I_{P,2}]$. By the assumption, the latter coincides with $E[\theta|I_{i,2}]$ for all $i$. The law of iterated expectations then ensures that $E[p|I_{i,1}] = E[E[\theta|I_{i,2}]|I_{i,1}] = E[\theta|I_{i,1}]$. It follows that $k_i = E[\theta|I_{i,1}]$ for all $i$, no matter the details of the information structure.

We conclude that, as long as no agent has private information about $\theta$ at the time financial trades take place, equilibrium prices and investment choices are driven solely by first-order expectations regarding the fundamental, and the intensity of the entrepreneurs’ concern about financial prices is completely irrelevant. In what follows, we take this case as a point of reference and refer to it as the case without informational frictions; that is, we identify “informational frictions” with the case in which information remains dispersed at the time financial trades take place.
3 Equilibrium

The strategy of an entrepreneur is described by a function $k : \mathbb{R}^2 \rightarrow \mathbb{R}$, so that $k(x, y)$ denotes the investment made when the private signal is $x$ and the common signal is $y$. Aggregate investment is then a function of $\theta$ and $y$, and is given by

$$K(\theta, y) = \int k(x, y) \, d\Phi(x|\theta),$$

(1)

where $\Phi(x|\theta)$ denotes the c.d.f. of $x$ conditional on $\theta$. Since traders observe aggregate investment and are risk neutral, the unique market-clearing price in period 2 is $p = \mathbb{E}[\theta|K]$, where the latter denotes the expectation of $\theta$ conditional on the observed level of $K$. Since $K$ is determined by $(\theta, y)$, it follows that $p$ is also a function of $(\theta, y)$. We thus define an equilibrium as follows.

Definition 1 A (symmetric) equilibrium is an investment strategy $k : \mathbb{R}^2 \rightarrow \mathbb{R}$ and a price function $p : \mathbb{R}^2 \rightarrow \mathbb{R}$ that satisfy the following conditions:

(i) for all $(x, y)$,

$$k(x, y) \in \operatorname{arg \ max}_k \mathbb{E} \left[ (1 - \lambda) \theta k + \lambda p(\theta, y) k - k^2/2 \mid x, y \right];$$

(ii) for all $(\theta, y)$,

$$p(\theta, y) = \mathbb{E}[\theta|K(\theta, y)],$$

where $K(\theta, y) = \int k(x, y) \, d\Phi(x|\theta)$.

Condition (i) requires that the entrepreneur’s investment strategy is individually rational, taking as given the equilibrium price function. Condition (ii) requires that the equilibrium price is consistent with rational expectations and individual rationality on the side of the traders, taking as given the strategy of the entrepreneurs.

As often the case in the literature, tractability requires that we restrict attention to equilibria in which the price function is linear.

Definition 2 A linear equilibrium is an equilibrium in which $p(\theta, y)$ is linear in $(\theta, y)$.

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\(^3\)Restricting attention to symmetric strategies is not essential for any of our results.

\(^4\)Since the price is only a function of $K$ and $K$ is publicly observed, the price itself does not reveal any additional information. Therefore, we can omit conditioning on $p$. 

8
3.1 Endogenous complementarity

The optimality condition for the entrepreneurs can be written as

\[ k(x, y) = \mathbb{E} \left[ (1 - \lambda) \theta + \lambda p(\theta, y) \mid x, y \right]. \]  

(2)

The linearity of \( p(\theta, y) \) in \((\theta, y)\) and of \( \mathbb{E}[\theta \mid x, y] \) in \((x, y)\) then guarantee that the entrepreneurs’ strategy is linear in \((x, y)\); that is, there are coefficients \((\beta_0, \beta_1, \beta_2)\) such that

\[ k(x, y) = \beta_0 + \beta_1 x + \beta_2 y. \]  

(3)

By implication, aggregate investment is given by \( K(\theta, y) = \beta_0 + (\beta_1 + \beta_2)\theta + \beta_2 \varepsilon \). Observing \( K \) is thus informationally equivalent to observing a Gaussian signal \( z \) with precision \( \pi_z \), where

\[ z = \frac{K - \beta_0}{\beta_1 + \beta_2} = \theta + \frac{\beta_2}{\beta_1 + \beta_2} \varepsilon \quad \text{and} \quad \pi_z = \left( \frac{\beta_1 + \beta_2}{\beta_2} \right)^2 \pi_y. \]  

(4)

Standard Gaussian updating then gives the expectation of \( \theta \) conditional on \( K \) as a weighted average of the prior and the signal \( z \):

\[ \mathbb{E}[\theta \mid K] = \frac{\pi_\theta}{\pi_\theta + \pi_z} \mu + \frac{\pi_z}{\pi_\theta + \pi_z} z. \]

Since market clearing in period 2 requires \( p = \mathbb{E}[\theta \mid K] \), we conclude that the equilibrium price is

\[ p(\theta, y) = \gamma_0 + \gamma_1 K(\theta, y), \]  

(5)

where

\[ \gamma_0 = \frac{\pi_\theta}{\pi_\theta + \pi_z} \mu - \frac{\pi_z}{\pi_\theta + \pi_z} \frac{\beta_0}{\beta_1 + \beta_2} \quad \text{and} \quad \gamma_1 = \frac{\pi_z}{\pi_\theta + \pi_z} \frac{1}{\beta_1 + \beta_2}. \]  

(6)

These results are summarized in the following lemma.

**Lemma 1** In any linear equilibrium, there are coefficients \((\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1)\) such that

\[ k(x, y) = \beta_0 + \beta_1 x + \beta_2 y \quad \text{and} \quad p(\theta, y) = \gamma_0 + \gamma_1 K(\theta, y). \]

Moreover, \( \gamma_1 > 0 \) if and only if \( \beta_1 + \beta_2 > 0 \).
Therefore, provided that higher investment is “good news” for profitability (which is true whenever $\text{Cov}(K, \theta) > 0$, or equivalently $\beta_1 + \beta_2 > 0$), the equilibrium price increases with aggregate investment ($\gamma_1 > 0$)\footnote{Throughout the paper, we say that higher investment is good news for profitability if and only if the traders expect \( \theta \) to increase with \( K \).}. This in turn induces strategic complementarity among the entrepreneurs. Indeed, when all entrepreneurs are choosing a higher level of investment, they are sending a positive signal to the financial market, thus increasing the price \( p \) at which any given individual entrepreneur may have to sell his capital at \( t = 2 \). But then the willingness of an individual entrepreneur to invest at \( t = 1 \) is higher when the investments of other entrepreneurs are also higher, which means precisely that investment choices are strategic complements. We formalize these intuitions in the next result, which follows directly from replacing condition (5) into condition (2).

**Lemma 2** In any linear equilibrium, the investment strategy satisfies

\[
k(x, y) = E[(1 - \alpha) \kappa(\theta) + \alpha K(\theta, y) \mid x, y], \tag{7}
\]

where $\alpha \equiv \lambda \gamma_1$ and $\kappa(\theta) \equiv \frac{(1-\lambda)\theta + \lambda \gamma_1}{1-\lambda \gamma_1}$.

Condition (7) can be interpreted as the best-response condition in a coordination game among entrepreneurs: it describes the best strategy of an individual entrepreneur as a function of aggregate investment (i.e., of the strategy of other agents). The coefficient $\alpha$ then measures the degree of strategic complementarity in investment decisions: the higher $\alpha$, the higher the slope of the best response of individual investment to aggregate investment, that is, the higher the incentive of entrepreneurs to align their investment choices.

A similar best-response condition characterizes the class of linear-quadratic games examined in Angeletos and Pavan (2006), including the special case in Morris and Shin (2002). However, there is are two important difference. First, while in those games the degree of strategic complementarity is exogenously given by the payoff structure of the game, here it is endogenously determined as an integral part of the equilibrium. And second, while in those games the degree of complementarity is independent of the information structure, here it actually originates in the information structure. Indeed, the complementarity in our setup is solely due to the informational content of aggregate investment. How much information aggregate investment conveys about \( \theta \) determines the coefficient $\gamma_1$, which captures the sensitivity of prices to aggregate investment. In turn, the coefficient $\gamma_1$ pins
down the value of $\alpha$, which captures the degree of complementarity between the investment decisions of entrepreneurs. In the case of common information examined earlier, aggregate investment provided no information to the traders, prices were independent of $K$ and the complementarity in investment choices were absent. But once information is dispersed, aggregate investment becomes a signal of $\theta$, and the complementarity emerges as prices start responding to this signal. The more informative aggregate investment is about $\theta$, the stronger the complementarity.

Because the complementarity depends on the informative content of aggregate investment, which in turn depends on the strategies of the entrepreneurs, to determine the equilibrium value of $\alpha$ we need to solve a fixed point problem. Before doing so, however, we derive a central result that helps understand how the endogenous complementarity interferes with the entrepreneurs’ incentives in the use of available information.

**Lemma 3** In a linear equilibrium,\[ \frac{\beta_2}{\beta_1} = \frac{\pi_y}{\pi_x} \frac{1}{1 - \alpha}. \]

Therefore, the sensitivity of equilibrium investment to the common signal relative to the private signal is higher the higher the complementarity.

Let us provide some intuition for this result. If the strategy followed by agents is $k(x, y) = \beta_0 + \beta_1 x + \beta_2 y$, then aggregate investment is $K(\theta, y) = \beta_0 + \beta_1 \theta + \beta_2 y$ and an agent’s best predictor of aggregate investment is $E[K|x, y] = \beta_0 + \beta_1 E[\theta|x, y] + \beta_2 y$.

The private signal $x$ helps predict aggregate investment only through $E[\theta|x, y]$, while the common signal $y$ helps predict aggregate investment both through $E[\theta|x, y]$ and directly, through its effect on the term $\beta_2 y$. Therefore, relative to how much the two signals help predict the fundamentals, the common signal $y$ is a relatively better predictor of aggregate investment than the private signal $x$. But now recall that a higher $\alpha$ means a stronger incentive for the individual entrepreneur to align his investment choice with that of the others. It follows that when $\alpha$ is higher entrepreneurs find it optimal to rely more heavily on the common signal $y$ relative to the private signal $x$, for it is the former that best helps them align their choice with that of others.
3.2 Equilibrium characterization

As noted earlier, completing the equilibrium characterization requires solving for the equilibrium value of $\alpha$, which involves a fixed-point problem. On the one hand, how entrepreneurs use their available information depends on $\alpha$, the complementarity induced by the response of prices to aggregate investment. On the other hand, how sensitive prices are to aggregate investment, and hence what the value of $\alpha$ is, depends on how informative aggregate investment is about the fundamental, which in turn depends on how entrepreneurs use available information in the first place. This fixed-point problem captures the two-way feedback between the real and the financial sector of our model. Its solution is provided in the following lemma.

**Lemma 4** There exist functions $F : \mathbb{R} \times (0, 1) \times \mathbb{R}^3_+ \rightarrow \mathbb{R}$ and $G : \mathbb{R} \times (0, 1) \times \mathbb{R}^3_+ \rightarrow \mathbb{R}^5$ such that the following are true:

(i) In any linear equilibrium, $\beta_2/\beta_1$ solves

$$\frac{\beta_2}{\beta_1} = F \left( \frac{\beta_2}{\beta_1}; \lambda, \pi_\theta, \pi_x, \pi_y \right) \tag{8}$$

while $(\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1) = G \left( \beta_2/\beta_1; \lambda, \pi_\theta, \pi_x, \pi_y \right)$.

(ii) Equation (8) has at least one solution at some $\beta_2/\beta_1 > \pi_y/\pi_x$.

(iii) For any $(\pi_\theta, \pi_x, \pi_y)$ there exists a cutoff $\bar{\lambda} = \bar{\lambda}(\pi_\theta, \pi_x, \pi_y) > 0$ such that if $\lambda < \bar{\lambda}$ then (8) admits a unique solution.

(iv) There is an open set $S$ such that, if $(\lambda, \pi_\theta, \pi_x, \pi_y) \in S$, (8) admits multiple solutions.

Our fixed-point problem is set up in terms of the variable $b = \beta_2/\beta_1$, which represents the relative sensitivity of entrepreneurial investment to the two signals. Given $b$, we can determine the equilibrium value of $\gamma_1$, solving the inference problem of the traders at $t = 2$. Then, as shown in Lemma 2, the value of $\gamma_1$ determines the degree of strategic complementarity $\alpha$. Finally, as shown in Lemma 3, the value of $\alpha$ determines the relative sensitivity $\beta_2/\beta_1$, using the entrepreneurs' optimality conditions. These steps describe the mapping $F$ used in Lemma 4 and provide the intuition for part (i) of the lemma: the fixed points of $F$ identify all the linear equilibria of our economy. Parts (ii)-(iv) then characterize the fixed points of $F$, leading to the following result.

**Proposition 1** A linear equilibrium always exists and is unique if $\lambda$ is small enough.
The possibility of multiple equilibria for high values of $\lambda$ is interesting for several reasons. First, it shows that the potential strength of the two-way feedback between real and financial activity. Second, it arises solely from an informational externality rather than from any direct payoff interdependence. Finally, it can induce additional non-fundamental volatility in both real investment and financial prices if we allow for sunspots.

However, for the rest of the paper, we leave aside the possibility of multiple equilibria and focus on the case where $\lambda$ is small enough that the equilibrium is unique. The next proposition shows that in this case aggregate investment necessarily increases with $\theta$. Together with Lemma 2, this ensures that $\alpha > 0$. The proposition then further establishes that $\alpha$ is increasing in $\lambda$, which guarantees that the complementarity in investment decisions is stronger that when entrepreneurs are more concerned about financial market prices.

**Proposition 2** Whenever the equilibrium is unique, the following are true:

(i) Individual investment increases with both signals ($\beta_1, \beta_2 > 0$) and, by implication, aggregate investment is positively correlated with the fundamental ($\text{Cov}(K, \theta) > 0$).

(ii) The equilibrium price increases with aggregate investment ($\gamma_1 > 0$).

(iii) The equilibrium degree of complementarity $\alpha$ is positive and increasing in $\lambda$.

### 3.3 Sensitivity to fundamental and expectational shocks

To further appreciate the positive implications of informational frictions—and the complementarity thereof—it is useful to rewrite aggregate investment as

$$K = \beta_0 + (\beta_1 + \beta_2) \theta + \beta_2 \varepsilon.$$ 

Aggregate investment thus depends on two types of shocks: **fundamental shocks**, captured by $\theta$, and **expectational shocks**, captured by $\varepsilon$. How entrepreneurs use available information affects how investment respond to these shocks: the sensitivity to fundamentals is governed by the sum $\beta_1 + \beta_2$, while the sensitivity to expectational shocks is governed by $\beta_2$.

We showed earlier (see Lemma 3) that a stronger complementarity $\alpha$ is associated with a higher ratio $\beta_2 / \beta_1$. Because $\text{Cov}(K, \theta) > 0$ suffices for $\alpha > 0$ and hence for $\beta_2 / (\beta_1 + \beta_2) > \pi_y / (\pi_x + \pi_y)$, while $\alpha = 0$ and $\beta_2 / (\beta_1 + \beta_2) = \pi_y / (\pi_x + \pi_y)$ in the case without informational frictions, the following is immediate.
Corollary 1 (i) In any equilibrium in which high investment is “good news”, the relative impact of expectational shocks is higher than what in the absence of informational frictions. (ii) $\lambda$ small enough suffices for the equilibrium to be unique and for high investment to be “good news”.

This result is the key positive prediction of our paper: it states that informational frictions amplify non-fundamental volatility (equivalently, they reduce the R-square of a regression of aggregate investment on realized profitability). Importantly, because $\alpha$ increases with $\lambda$ (see part (iii) of Proposition 2), this amplification effect is stronger the more entrepreneurs care about asset prices. The next proposition then reinforces these findings by examining the absolute impact of the two shocks.

Proposition 3 There exists $\hat{\lambda} > 0$ such that, for all $\lambda \in (0, \hat{\lambda}]$, there is a unique linear equilibrium and the following comparative statics hold:

(i) Higher $\lambda$ reduces $\beta_1 + \beta_2$, thus dampening the impact of fundamental shocks;

(ii) Higher $\lambda$ increases $\beta_2$, thus amplifying the impact of expectational shocks.

The key intuition behind these results is again the role of the complementarity in the equilibrium use of information. To see this, suppose for a moment that $\gamma_0 = 0$ and $\gamma_1 = 1$, meaning that $p(\theta, y) = K(\theta, y)$ in all states. The entrepreneur’s best response then reduces to

$$k(x, y) = \mathbb{E}[(1 - \lambda)\theta + \lambda K(\theta, y) \mid x, y],$$

so that the degree of complementarity now coincides with $\lambda$. One can then easily show that the unique solution to the above is $k(x, y) = \beta_0 + \beta_1 x + \beta_2 y$, with

$$\beta_0 = \frac{\pi_{y}}{\pi_{\theta} + \pi_{x}(1 - \lambda) + \pi_{y}}\mu, \quad \beta_1 = \frac{\pi_{x}(1 - \lambda)}{\pi_{\theta} + \pi_{x}(1 - \lambda) + \pi_{y}}, \quad \text{and} \quad \beta_2 = \frac{\pi_{y}}{\pi_{\theta} + \pi_{x}(1 - \lambda) + \pi_{y}}.$$

It is then immediate that a higher $\lambda$ increases the sensitivity to the prior (captured by $\beta_0$) and the sensitivity to the common signal (captured by $\beta_2$), while it decreases the sensitivity to the private signal (captured by $\beta_1$). We have already given the intuition for the result that a stronger complementarity amplifies the reliance on the common signal and dampens the reliance on the private signal. That it also increases the reliance on the prior is for exactly the same reason as for the common signal: the prior is a relative good predictor of others’ investment choices.
But now note that the average return coincides with the mean of $\theta$ no matter $\lambda$; this is because the average price must equal the mean of $\theta$, for otherwise the traders would make on average non-zero profits, which would be a contradiction. This explains why the sum $\beta_0 + (\beta_1 + \beta_2) \mu$ must equal 1. But then, if it is the case that a higher $\lambda$ increases $\beta_0$, it must also be the case that a higher $\lambda$ reduces the sum $\beta_1 + \beta_2$. In other words, investment is less sensitivity to changes in the fundamentals simply because the complementarity strengthens the anchoring effect of the prior.

These intuitions are exact only if $\gamma_0 = 0$ and $\gamma_1 = 1$, so that the equilibrium price coincides with aggregate investment. In turn, this can be shown to be the case in equilibrium only when the prior is completely uninformative. More generally, the price is an increasing function of aggregate investment, but with $\gamma_0 \neq 0$ and $\gamma_1 \neq 1$. This explains why Proposition 3 has been established only for a subset of the parameter space, namely for $\lambda$ small enough. Nevertheless, extensive simulations suggest that, for all for $\lambda > 0$, any equilibrium in which $\text{Cov}(K, \theta) > 0$ features

$$\beta_1 + \beta_2 < \frac{\pi_x + \pi_y}{\pi_\theta + \pi_x + \pi_y} \quad \text{and} \quad \beta_2 > \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y},$$

which means that the impact of fundamentals is lower, and that of expectational shocks is higher, than in the case without informational frictions.

4 Constrained efficiency

The analysis so far has focused on the positive properties of the equilibrium. We now study its normative properties by examining whether there is a decentralized use of the available information that can improve upon the equilibrium.

Note that the question of interest here is whether society can do better by having agents use their available information in a different way than they actually do in equilibrium—not whether society could do better if agents had more information. We thus adopt the same constrained efficiency concept as Angeletos and Pavan (2007a,b): we consider the allocation that maximizes ex-ante utility subject to the constraint that the choice of an agent can depend only on the information available to him and not on the information that is private to other agents. In other words, we let the “planner” to dictate how agents use their available information, but not to transfer information from one agent to another. In so doing, we momentarily disregard incentive constraints; we will identify tax systems that implement the efficient allocation as an equilibrium in the next section.
Finally note that the trades in the financial market represent pure transfers between the entrepreneurs and the traders and therefore do not affect ex-ante utility. We can thus focus on the investment strategy and define the efficient allocation as follows.

**Definition 3** The efficient allocation is a strategy $k : \mathbb{R}^2 \to \mathbb{R}$ that maximizes

$$
\mathbb{E} u = \int_{y, \theta} \left\{ \int_{x} \frac{1}{2} \left[ -\frac{1}{2} k(x, y)^2 + (1 - \lambda) \theta k(x, y) \right] d\Phi(x|\theta) + \frac{1}{2} [\theta \lambda K(\theta, y)] \right\} d\Psi(\theta, y) \quad (9)
$$

with $K(\theta, y) = \int_{x} k(x, y)$. 

Here, $\mathbb{E}u$ denotes ex-ante utility and $\Psi$ denotes the c.d.f. of the joint distribution of $\theta$ and $y$. To understand (9), note the term in the first pair of brackets gives the payoff of an entrepreneur when his information $(x, y)$, while the term in the second pair of brackets gives the payoff of a trader when aggregate investment is $K(\theta, y)$; taking the expectation of these payoffs across all possible events gives (9).

Now note that the transfer of capital from the entrepreneurs that are hit by the liquidity shock to the traders does not affect the value of investment. It follows that (9) can be rewritten as

$$
\mathbb{E} u = \frac{1}{2} \mathbb{E}[V(k(x, y), \theta)] = \frac{1}{2} \mathbb{E}[\mathbb{E}[V(k(x, y), \theta)|x, y]],
$$

where

$$
V(k, \theta) \equiv \theta k - \frac{1}{2} k^2.
$$

From the society’s viewpoint, $\lambda$ is irrelevant and it is as if the entrepreneurs’ payoffs were $V(k, \theta)$. It is then obvious that a strategy $k : \mathbb{R}^2 \to \mathbb{R}$ is efficient if and only if, for almost all $x$ and $y$, $k(x, y)$ maximizes $E[V(k, \theta)|x, y]$, which gives the following result.

**Proposition 4** The efficient investment strategy is given by

$$
k(x, y) = \mathbb{E}[\theta|x, y] = \frac{\pi_\theta}{\pi_\theta + \pi_x + \pi_y} \mu + \frac{\pi_x}{\pi_\theta + \pi_x + \pi_y} x + \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y} y.
$$

Note that the efficient strategy would have been the equilibrium if it were not for informational frictions. It follows that our earlier positive results admit a normative interpretation.

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6By ex-ante, we mean before the realization of any random variable, including those that determine whether an agent will be a trader or an investor.
Corollary 2 (i) The efficient strategy is never an equilibrium in the presence of informational frictions. (ii) In any equilibrium in which high investment is “good news”, the relative impact of expectational shocks is inefficiently high.

This corollary summarizes the key normative prediction of the paper. To understand part (i), note that if all entrepreneurs $j \neq i$ were to follow the efficient strategy, then aggregate investment would be a positive signal of $\theta$ and the price would thus have to increase with $K$; but then entrepreneur $i$ would perceive a complementarity between his investment choice and those of other entrepreneurs and would best-respond by raising his reliance on the prior and common signal (and reducing his reliance on the private signal). Part (ii) then follows directly from Corollary 1 along with the property that the efficient strategy coincides with the equilibrium in the absence of informational frictions.

While part (ii) establishes the inefficiency in terms of the relative impact of expectational shocks, Proposition 3 and numerical simulations give the result in absolute terms as well: equilibrium investment reacts too little to fundamental shocks and too much to expectational shocks.

5 Policy implications

The analysis has identified a particular source of amplification and inefficiency in the sensitivity of equilibrium investment to common expectational shocks. The question that then naturally emerges is what policies can correct this inefficiency. Building on Angeletos and Pavan (2007b), we now show that this can be achieved even if the government does not have any informational advantage and can not directly monitor the different sources of information agents have access to; the key is to make individual taxation contingent on realized aggregate activity.

5.1 Policies that restore efficiency

Suppose that the government imposes taxes, or make transfers, in period 3 contingent on the information that is public in that period. This information includes $\theta$ and the capital holdings of each agent. Consider then linear tax schemes of the following type:

$$m_i = -\tau(K, \theta) s_i + T(K, \theta).$$
Here, $m_i$ denotes the net transfer made to agent $i$, $s_i$ denotes the amount of capital in the hands of agent $i$, $\tau$ the marginal tax rate on capital, and $T(K, \theta)$ a lump sum transfer. Budget balance then requires $T(K, \theta) = \tau(K, \theta) \frac{1}{2} K$. The free policy instrument is thus the marginal tax rate $\tau$, which is allowed to depend on both $\theta$ and $K$.

**Proposition 5** There exists a unique linear tax scheme that implements the efficient allocation as an equilibrium. The optimal marginal tax satisfies

$$\tau(\theta, K) = \tau_0 + \tau_1 \theta + \tau_2 K,$$

with $\tau_0 > 0$, $\tau_1 < 0$, and $\tau_2 > 0$.

The basic intuition behind this result is that the government can control the degree of strategic complementarity perceived by the agents by appropriately designing the contingency of the marginal tax rate on aggregate investment: the higher the elasticity $\tau_2$ of the marginal tax rate with respect to $K$, the lower the degree of complementarity $\alpha$ in investment choices. The optimal $\tau_2$ is then chosen to offset the informational effect of $K$ on prices and thereby to correct the inefficiency in the equilibrium sensitivity of investment to common noise relative to idiosyncratic noise, while the optimal $\tau_0$ and $\tau_1$ adjust for the mean level and the absolute sensitivities.

Clearly, allowing the government to use a richer set of transfer schemes—for example, letting the transfers depend directly on the agents private signals if these are directly observable, or otherwise on reports elicited by the agents—can not improve upon the linear tax scheme described above. This follows directly from the definition of the efficiency strategy. The only way the government could do better is if it could send information to the entrepreneurs before they make their investment choices. This, however, would endow the government with a certain informational advantage. In contrast, the tax scheme we propose above requires the government to have access only to information that is publicly available to all agents in the economy.

Nevertheless, at this point it is also interesting to consider the possibility that the government directly interferes with the information structure.

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7 Similar tax schemes implement the efficient investment strategy in all the extensions considered in Section 6.
8 The implementation of the efficient strategy considered here requires the taxes to depend on $\theta$. If $\theta$ is not publicly observed at the time taxes are collected, one can still implement the efficient strategy by making the marginal tax a decreasing function of individual investment as well as an increasing function of aggregate investment. This follows from a similar argument as in Angeletos and Pavan (2007b).
5.2 Policies that provide more information

Suppose that the government collects information about $\theta$ and contemplates whether, or to whom, to disclose this information. If the policies that restore efficiency in the equilibrium use of information are in place, welfare necessarily increases with any additional information released to either the entire population or any subset of the agents. But if these policies are absent, the details of the communication structure matter. If the information disclosed by the government becomes completely public, welfare increases, not only because real investment choices are based on better information, but also because prices react less to aggregate investment, alleviating the source of inefficiency. If, however, only a certain subset of the agents can access the information disclosed by the government, then it is possible that such disclosures decrease welfare by exacerbating the asymmetry of information and the inefficiency that emerges thereof.

A similar point applies to the release of macroeconomic data. Suppose, in particular, that the market observes $K$ with noise; the government effectively controls the level of this noise by collecting and disclosing information on macroeconomic activity. For any fixed investment strategy, reducing this noise increases the sensitivity of $p$ to $K$. This contributes towards making $p$ co-vary more with $\theta$, which by itself tends to improve investment choices; but it also increases the strategic complementarity in investment choices, possibly exacerbating inefficiency.

This discussion highlights that the social value of any additional information depends crucially on the inefficiency in the equilibrium use of such information; but keep in mind that all the caveats disappear once the tax policies that we identified in Proposition 5 are put in place.

6 Extensions

Our analysis has identified a mechanism through which the dispersion of information induces complementarity, amplification, and inefficiency, all at once. However, the baseline model made various simplifying assumptions that one may feel uncomfortable with. First, we have imposed that the demand for installed capital in the financial market is perfectly elastic; if demand were downward sloping, higher aggregate investment in period 1 could decrease the equilibrium price by increasing

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9The point that the social value of information depends on the inefficiencies of the equilibrium use of information is the theme of Angeletos and Pavan (2007a); the point that policies that restore efficiency in the use of available information are complementary to policies that communicate additional information is emphasized in Angeletos and Pavan (2007b).
the supply of the capital sold in period, which in turn would induce strategic substitutability rather than complementarity in investment decisions. Second, we have imposed that the entrepreneurs who are not hit by the liquidity shock can not trade in the financial market; if they could, the rational-expectations equilibrium price would aggregate the information that is dispersed among them, which in turn would interfere with how prices respond to aggregate investment. Third, we have modeled the common noise in aggregate investment as the noise in a signal that is commonly observed by the entrepreneurs but not by the traders; this seems too special and is probably unappealing from an applied perspective.

In this section, we relax all these assumptions. Although some interesting differences obtain, the key positive and normative predictions of the paper (Corollaries 1 and 2) remain intact: informational frictions continue to induce complementarity, amplification and inefficiency continue to emerge, much alike in the benchmark model.

6.1 Sources of strategic substitutability

In general, one expects prices to fall with supply. In our framework, the supply of the asset in the financial market is nothing else but the capital of the entrepreneurs hit by the liquidity shock. One would therefore also expect financial prices to fall with aggregate investment, thereby leading to strategic substitutability in investment choices. This “supply-side” effect was absent in the benchmark model only because the traders’ demand for the asset was perfectly elastic.

A similar source of strategic substitutability emerges when entrepreneurs compete for some input that is in fixed or less than perfectly elastic supply, such as labor or land. Higher aggregate investment then decreases the private return to investment by raising wages or property prices. We will focus here to the case of downward sloping demand, but the results we present apply equally to this alternative source of strategic substitutability.

We modify the benchmark model as follows. The net payoff of trader $j$ when he purchases (or sells) $q_j$ units of the asset at price $p$ is now given by

$$u_j = (\theta - p) q_j - \frac{1}{2\phi} q_j^2,$$

where $\phi$ is a positive scalar. The difference from the benchmark model is the term $-\frac{1}{2\phi} q_j^2$, which can be interpreted as a transaction cost, or a holding cost, for taking a position $q$ in the financial market.
The convexity of this cost then ensures a finite price elasticity for the traders’ demand for the asset. In particular, the latter is now given by \( q(p, K) = \phi \left( \mathbb{E} [\theta | K] - p \right) \), so that \( \phi \) parameterizes the price elasticity.\(^{10}\)

As in the benchmark model, in any linear equilibrium the uninformed traders’ expectation of \( \theta \) is given by \( \mathbb{E} [\theta | K] = \gamma_0 + \gamma_1 K \), for some coefficients \( \gamma_0 \) and \( \gamma_1 \). But unlike the benchmark model, the equilibrium price does not coincide with \( \mathbb{E} [\theta | K] \). Instead, market clearing (namely \( q(p, K) = \lambda K \)) now imposes that the equilibrium price satisfies

\[
p(\theta, y) = \mathbb{E} [\theta | K(\theta, y)] - \lambda K(\theta, y)/\phi = \gamma_0 + (\gamma_1 - \lambda/\phi) K(\theta, y).
\] (11)

It follows that the realized level of investment has two opposing effects on the price that clears the financial market: on the one hand, it raises the traders’ expectation of \( \theta \), thereby pushing the price up; on the other hand, it raises supply, thereby pulling the price down. Whether investment choices are strategic complements or substitutes then depends on the strength of these two effects.

**Proposition 6** (i) In any linear equilibrium, the investment strategy satisfies

\[
k(x, y) = \mathbb{E} [\left( 1 - \alpha \right) \kappa(\theta) + \alpha K(\theta, y) \mid x, y],
\] (12)

with \( \alpha = \lambda \gamma_1 - \frac{\lambda^2}{\phi} \) and \( \kappa(\theta) \equiv \frac{(1 - \lambda) \theta + \lambda \gamma_0}{1 - \lambda \gamma_1 + \lambda^2/\phi} \). (ii) \( \lambda \) small enough suffices for the equilibrium to be unique and to satisfy \( \gamma_1 > 0 \) (equivalently, \( \text{Cov}(K, \theta) > 0 \)).

The degree of complementarity or substitutability, \( \alpha \), is now the combination of two terms. The first term, \( \lambda \gamma_1 \), captures the informational effect of investment that we encountered in the benchmark model. The second term, \( \lambda^2/\phi \), captures the supply-side effect that emerges once the demand for the asset is finitely elastic. If either the information contained in aggregate investment is sufficiently poor (low \( \gamma_1 \)) or the price elasticity of demand is sufficiently low (low \( \phi \)), investment choices become strategic substitutes (\( \alpha < 0 \)). In this respect, there is an important difference from the benchmark model.

However, the question of interest is not per se whether investment choices are strategic complements or substitutes. Rather, it is how the positive and normative properties of the equilibrium are

\(^{10}\)Clearly, the benchmark model is nested as the limit where \( \phi \to \infty \).
affected by the dispersion of information. In this respect, there is actually no essential difference from the benchmark model.

Consider first the positive properties of the equilibrium. Clearly, provided that investment is a positive signal of the underlying fundamentals, $\alpha$ is higher than what it is under common information: $\text{Cov}(K, \theta) > 0 \Rightarrow \gamma_1 > 0 \Rightarrow \alpha > -\lambda^2/\phi$. It is then immediate that the dispersion of information increases the impact of common expectation shocks relative to that of fundamental shocks, even if $\alpha$ happens to be negative. Indeed, Lemma 3 directly extends from the benchmark model, guaranteeing that equilibrium investment satisfies $k_i = \beta_0 + \beta_1 x_i + \beta_2 y$ with

$$\frac{\beta_2}{\beta_1} = \frac{\pi_y}{\pi_x} \frac{1}{1 - \alpha}$$

(13)

It follows that the ratio $\beta_2/(\beta_1 + \beta_2)$, which measures the sensitivity of investment to common noise relative to fundamentals, is higher in the case with informational frictions than in the case without. In the Appendix we further saw that, as in the benchmark model, $\lambda$ small enough suffices for the equilibrium to be unique, in which case it must also satisfy $\text{Cov}(K, \theta) > 0$. We conclude that Corollary 1, the key positive prediction of the benchmark model, continues to hold here.

We next turn to the normative properties. Because of the convexity of the transaction costs, it is necessary for efficiency that all traders take the same position in the financial market: $q_j = \lambda K$ for all $j \in (1/2, 1]$. Ex-ante utility then reduces to

$$\mathbb{E}u = \int_{\theta, y} \left\{ \frac{1}{2} \int_x \left[ -\frac{1}{2} k(x, y)^2 + (1 - \lambda) \theta k(x, y) \right] d\Phi(x|\theta) + \frac{1}{2} \left[ \theta \lambda K(\theta, y) - \frac{1}{2\phi} [\lambda K(\theta, y)]^2 \right] \right\} d\Psi(\theta, y)$$

(14)

and the efficient investment strategy is simply the function $k : \mathbb{R}^2 \rightarrow \mathbb{R}$ that maximizes (14).

**Proposition 7** The efficient investment strategy is the unique linear solution to

$$k(x, y) = \mathbb{E} \left[ (1 - \alpha^*) \kappa^*(\theta) + \alpha^* K(\theta, y) \mid x, y \right],$$

(15)

where $\kappa^*(\theta) \equiv \frac{1}{1 + \lambda^2/\phi} \theta$ is the first-best level of investment and $\alpha^* \equiv -\lambda^2/\phi < 0$.

To understand this result, note that the social return to investment is given by $(1 - \lambda) \theta + \lambda (\theta - \lambda K/\phi) = \theta - \lambda^2 K/\phi$; the new term relative to the benchmark model, $-\lambda^2 K/\phi$, reflects the cost associated with transferring $\lambda K$ units of the asset from the entrepreneurs to the traders. If
information were complete, efficiency would require that each agent equates his marginal cost of investing with the social return to investment:  
k = \theta - \lambda^2 K / \phi.\] The analogue under incomplete information is that each agent equates the marginal cost with his expectation of the social return:

\[
k (x, y) = \mathbb{E} [\theta - \frac{\lambda^2}{\phi} K (\theta, y) | x, y], \tag{16}\]

Rearranging the above gives (15). Finally, the fact then that the social return to investment decreases with \( K \) explains why in the efficient allocation it is as if investment choices exhibit strategic substitutability \( (\alpha^* < 0) \).

The key finding here is that the same supply-side effect that makes the price, and hence the private return to investment, a decreasing function of \( K \) for given \( \mathbb{E} [\theta | K] \) also makes the social return to investment a decreasing function of \( K \), and indeed in a completely symmetric way. As a result, it is only the informational effect of investment that can cause any discrepancy between the equilibrium and the efficient allocation.

As in the benchmark model, this discrepancy manifests itself in the response of equilibrium to expectational and fundamental shocks. Indeed, while equilibrium investment satisfies (13), efficient investment satisfies  
k_i = \beta_0^* + \beta_1^* x_i + \beta_2^* y \text{ with}\]

\[
\frac{\beta_2^*}{\beta_1^*} = \frac{\pi_y}{\pi_x} \frac{1}{1 - \alpha^*}.
\]

It follows that the relative sensitivity of the equilibrium strategy to common noise is higher than that of the efficient strategy if and only if \( \alpha > \alpha^* \). And because the latter in turn is true if and only if \( \text{Cov}(K, \theta) > 0 \), we conclude that Corollary 2, the key normative prediction of the benchmark model, continues to hold.

### 6.2 Information aggregation through prices

The analysis so far has imposed that the entrepreneurs who are not hit by the liquidity shock can not access the financial market. Apart from being unrealistic, this assumption rules out the possibility that the price in the financial market aggregates the information that is dispersed among the entrepreneurs. In this section we thus extend the analysis by allowing entrepreneurs not hit by the liquidity shock to participate in the financial market.
To guarantee downward sloping demands, we assume that traders and entrepreneurs alike incur a transaction cost for trading in the financial market, of the same sort as in the previous section.\footnote{We assume that the entrepreneurs hit by a liquidity shock do not pay a transaction cost for the units of the asset that they have to sell in the second period; this is a completely inconsequential simplification.} Thus, the payoff of an entrepreneur $i$ who is not hit by a liquidity shock, has invested $k_i$ units in the first period, and trades an additional $q_i$ units in second period is given by

$$u_i = -k_i^2/2 - pq_i - q_i^2/(2\phi) + \theta (k_i + q_i),$$

while the payoff of a trader $j$ who takes a position $q_j$ is $u_j = -pq_i - q_i^2/(2\phi) + \theta q_i$.

Because the observation of $K$ in the second period makes $\theta$ known to every entrepreneur, their demand for the asset in the second period reduces to $q_E = \phi (\theta - p)$. The demand of the traders, on the other hand, is given by $q_T = \phi (\mathbb{E}[\theta|K,p] - p)$; note that the traders now condition their expectation of $\theta$ on the information conveyed by the rational-expectations equilibrium price.\footnote{This presumes that investors use their private information when they decide how much to invest ($\beta_1 \neq 0$), so that $K(\theta, y)$ varies with $\theta$ given $y$.} Since the aggregate demand for the asset is $\frac{1}{2} (1 - \lambda) q_E + \frac{1}{2} q_T$ and the aggregate supply is $\lambda K$, market clearing implies

$$p = \frac{1}{2\lambda} \mathbb{E}[\theta|K,p] + \frac{1 - \lambda}{2} \theta - \frac{1}{\phi (2 - \lambda)} \lambda K.$$

Clearly, for any given $K$, $p$ now perfectly reveals $\theta$. The asymmetry of information thus vanishes and the equilibrium price satisfies $p = \theta - \frac{1}{\phi (2 - \lambda)} \lambda K$. As in the previous session, this is just the social return to investment, adjusted by the fact that $\lambda K$ is now (equally) distributed among the traders and the entrepreneurs not hit by the liquidity shock. That the equilibrium price coincides with the social return to investment in turn guarantees efficiency.

This result is no different from what we already knew from the starting point of our analysis: as long as the dispersion of information vanishes at the time financial trades take place, equilibrium investment is driven merely by first-order expectations of $\theta$ and is efficient. Importantly, this result crucially depends on the absence of other sources of noise that may prevent the equilibrium price from perfectly revealing $\theta$.

To establish this, we introduce noise in prices by assuming that the entrepreneurs’ costs of holding the asset (and hence their demands) are subject to a shock that is known to them but not

\footnote{In both the benchmark model and the extension of the previous section, we did not have to consider rational-expectations equilibria because information was symmetric among all the agents who traded in the financial market; this is no more the case in the variant of this section.}
to the traders. In particular, the payoff of an entrepreneur not hit by the liquidity shock is now given by

$$u_i = -k_i^2/2 - p q_i - \omega q_i - q_i^2/(2\phi) + \theta (k_i + q_i),$$

where $\omega$ is the aforementioned shock to the cost (or value) of holding the asset; for simplicity, $\omega$ is assumed to be independent of any other random variable, with $\mathbb{E}[\omega] = 0$ and $\text{Var}[\omega] = \sigma_\omega^2 \equiv \pi^{-1}$.

In what follows, we look at linear rational-expectation equilibria, in which the investment strategy is denoted by $k(x, y)$ and the equilibrium price by $p(\theta, y, \omega)$. Since the observation of aggregate investment in the second period continues to reveal $\theta$ to the entrepreneurs (but not to the traders), asset demands can be written as $q_E = \phi(\theta - \omega - p)$ for entrepreneurs and $q_T = \phi(\mathbb{E}[\theta|K, p] - p)$ for the traders. And because market clearing requires that $\frac{1}{2}(1 - \lambda) q_E + \frac{1}{2} q_T = \lambda K$, the equilibrium price must now satisfy

$$p = \frac{1}{2 - \lambda} \mathbb{E}[\theta|K, p] + \frac{1}{2 - \lambda} (\theta - \omega) - \frac{1}{\phi(2 - \lambda)} \lambda K. \quad (17)$$

Once again, the price is a weighted average of the traders’ and the entrepreneurs’ valuations of the asset, net of transaction costs, and depends on aggregate investment through two familiar channels: the informational effect of $K$ on the traders’ expectation of $\theta$ (captured in the first term of the right-hand-side of the above condition) and the supply-side effect of $K$ (captured in the last term). But now the price also depends on the shock $\omega$, which is not known to the traders—which ensures that $\theta$ is no more perfectly revealed by the price and hence that the informational effect of $K$ is not muted as before.

It is this informational effect that once again introduces complementarity in investment choices. In any linear rational-expectations equilibrium, $\mathbb{E}[\theta|K, p]$ is the linear projection of $\theta$ on $(K, p)$ and, by (17), the price $p$ can be expressed as a linear combination of $(K, \theta, \omega)$ It follows that, for any linear equilibrium, there exist coefficients $(\gamma_0, \gamma_1, \gamma_2, \gamma_3)$ such that

$$\mathbb{E}[\theta|K, p] = \gamma_0 + \gamma_1 K + \gamma_2 \theta + \gamma_3 \omega; \quad (18)$$

this is just the projection of $\mathbb{E}[\theta|K, p]$ on $(K, \theta, \omega)$, along the equilibrium. Using the above together with (17) and the fact that the private return to investment is the expectation of $(1 - \lambda) \theta + \lambda p$, we reach the following characterization result.
Proposition 8  (i) In any linear equilibrium, the investment strategy satisfies

\[ k(x, y) = \mathbb{E}[(1 - \alpha) \kappa(\theta) + \alpha K(\theta, y) \mid x, y], \]

where \( \alpha = \frac{\lambda}{2-\lambda} \gamma_1 - \frac{\lambda^2}{\phi(2-\lambda)} \) and \( \kappa(\theta) = \kappa_0 + \kappa_1 \theta, \) with \( \kappa_0 = \frac{\lambda \gamma_0}{2-\lambda - \lambda \gamma_1 + \lambda^2 / \phi} \) and \( \kappa_1 = \frac{2(1-\lambda) + \lambda \gamma_2}{2-\lambda - \lambda \gamma_1 + \lambda^2 / \phi}. \)

(ii) \( \lambda \) small enough suffices for the equilibrium to be unique and to satisfy \( \gamma_1 > 0. \)

As in the previous section, \( \alpha \) combines an informational effect (captured by \( \frac{\lambda}{2-\lambda} \gamma_1 \)) and a supply-side effect (captured by \( -\frac{\lambda^2}{\phi(2-\lambda)} \)). The supply-side effect always contributes to strategic substitutability, while the informational effect contributes to strategic complementarity if and only if high investment is “good news” about \( \theta \) (\( \gamma_1 > 0 \) if and only if \( \beta_1 + \beta_2 > 0 \), or equivalently \( \text{Cov}(K, \theta) > 0 \)). Once again, the overall effect is ambiguous, but the role of informational frictions remains the same as before: Corollary 1 continues to hold.

We now turn to the characterization of the efficient allocation for this economy. The efficiency concept we seek to use is the same as before; but now we need to allow the planner to mimic the information aggregation that the market can achieve through prices. We thus proceed as follows.

First, we define an allocation as a collection of strategies \( k : \mathbb{R}^2 \to \mathbb{R}, q_E : \mathbb{R}^5 \to \mathbb{R}, \) and \( q_T : \mathbb{R}^2 \to \mathbb{R}, \) along with a shadow-price function \( p : \mathbb{R}^3 \to \mathbb{R}, \) to which we give the following interpretation: in the first period, an entrepreneur who has observed the exogenous signal \((x, y)\) invests \( k(x, y) \); in the second period, all agents observe get to observe the realizations of aggregate investment \( K = K(\theta, y) \) and the shadow price \( p = p(\theta, y, \omega) \); the quantity of the asset held by an entrepreneur is then given by \( q_E(x, y, K, p), \) while the quantity held by a trader is given by \( q_T(K, p). \)

Next, we say that the allocation is feasible if and only if, for all \((\theta, y, \omega)\),

\[ \lambda K(\theta, y) = (1 - \lambda) \int_x q_E(x, y, K(\theta, y), \omega, p(\theta, y, \omega)) + q_T(K(\theta, y), p(\theta, y, \omega)). \]  \(19\)

As with equilibrium, this constraint plays two roles: first, it makes sure that the second-period resource constraint is not violated; and second, it defines the technology that is used to generate the endogenous public signal (equivalently, the extent to which information can be aggregated through the shadow price).

Finally, for any given \( k : \mathbb{R}^2 \to \mathbb{R}, q_E : \mathbb{R}^5 \to \mathbb{R}, \) and \( q_T : \mathbb{R}^2 \to \mathbb{R}, \) ex ante utility can be
computed as $$
abla u = W(k, q_E, q_T),$$ where

$$W(k, q_E, q_T) = \frac{1}{2} \int_{x,y,\theta, \omega} \left\{ -\frac{1}{2}k(x, y)^2 + (1 - \lambda) \left[ \theta k(x, y) + R\left( \theta - \omega, q_E(x, y, K(\theta, y), \omega, p(\theta, y, \omega)) \right) \right] \right\},$$

$$+ \frac{1}{2} \int_{y,\theta, \omega} \left[ R\left( \theta, q_T^T(K(\theta, y), p(\theta, y, \omega)) \right) \right],$$

and where $$R(v, q) \equiv vq - q^2/(2\phi)$$ is the payoff of holding $$q$$ units of the asset for an agent whose valuation is $$v$$, net of the transaction cost. We then define an efficient allocation as follows.

**Definition 4** An efficient allocation is a collection of strategies $$k : \mathbb{R}^2 \to \mathbb{R},$$ $$q_E : \mathbb{R}^5 \to \mathbb{R},$$ and $$q_T : \mathbb{R}^2 \to \mathbb{R},$$ along with a shadow price function $$p : \mathbb{R}^3 \to \mathbb{R},$$ that jointly maximize ex-ante utility, $$\nabla u = W(k, q_E, q_T),$$ subject to the constraint (19).

Note that, because utility is transferable, the shadow price does not directly affect payoffs; its sole function is to provide an endogenous public signal upon which the allocation of the asset in the second period can be conditioned.

We start by considering the efficient trades in the financial market.

**Lemma 5** The efficient allocation in the second period is given by

$$q_E^* = \frac{\lambda K}{2 - \lambda} - \frac{\phi \omega}{2 - \lambda} \quad \text{and} \quad q_T^* = \frac{\lambda K}{2 - \lambda} + \frac{\phi(1 - \lambda)}{2 - \lambda} \omega$$

subject to $$(1 - \lambda) q_E + q_T = \lambda K.$$

To understand this result, suppose for a moment that information were complete in the second period. For any given $$K$$, efficiency in the second period would require that all entrepreneurs hold the same $$q_E$$ and that $$(q_T, q_E)$$ maximize

$$\left\{ \theta q_T - \frac{1}{2\phi} q_T^2 \right\} + (1 - \lambda) \left\{ \theta q_E - \omega q_E - \frac{1}{2\phi} q_E^2 \right\}$$

subject to $$(1 - \lambda) q_E + q_T = \lambda K.$$ Clearly, the solution to this problem is (20). In our environment, information is incomplete, but the same allocation is induced by the following shadow-price and demand functions: $$p(\theta, y, \omega) = -\frac{\phi(1 - \lambda)\omega}{2 - \lambda},$$ $$q_E(x, y, K, p, \omega) = \frac{\lambda K}{2 - \lambda} - \frac{\phi \omega}{2 - \lambda},$$ and $$q_T(K, p) = \frac{\lambda K}{2 - \lambda} - p.$$\(^{14}\)

Note that the proposed shadow price is also the unique market-clearing price given the proposed demand functions. The efficient trades can thus be implemented by inducing these demand functions through a tax system and then letting the agents trade freely in the market.
We now turn to the characterization of the efficient investment decisions. Using Lemma 5, ex ante utility reduces to

\[ \mathbb{E} u = \frac{1}{2} \mathbb{E} \left\{ -\frac{1}{2} k^2 + \theta k - \frac{1}{2\phi(2-\lambda)} (\lambda K)^2 \right\} + \frac{(1-\lambda)}{2(2-\lambda)} \phi \sigma^2 \omega. \]  

(21)

Except for two minor differences—the smaller weight in front of \((\lambda K)^2\), which adjusts the cost associated with absorbing the fixed supply \(\lambda K\) in the second period for the fact that now this quantity is split across a larger pull of agents, and the last term in (21), which captures how the volatility of \(\omega\) affects the allocation of capital across entrepreneurs and traders in the second period—ex-ante utility has the same structure as in the previous session.

**Proposition 9** The efficient investment strategy is the unique linear solution to

\[ k(x, y) = \mathbb{E} \left[ (1 - \alpha^*) \kappa^* (\theta) + \alpha^* K(\theta, y) \mid x, y \right], \]

(22)

where \(\kappa^*(\theta) \equiv \frac{1}{1-\alpha^*} \theta\) is the first-best level of investment and \(\alpha^* \equiv \frac{-\lambda^2}{\phi(2-\lambda)}\).

It follows that \(\text{Cov}(K, \theta) > 0\) suffices for \(\alpha > \alpha^*\) and that the key normative prediction, Corollary 2, continues to hold once again.

### 6.3 Alternate sources of noise

The assumption that there is a common signal that is observed by the entrepreneurs and not by the traders is unappealing from an applied perspective if taken literally. But it should not be taken literally. First, it should be interpreted as a convenient modeling device for generating common expectational errors; an alternative would be to allow for correlated errors in the entrepreneurs private information. Second, it can be replaced with other sources of noise; the key is that traders learn something, but not everything, about \(\theta\) from observing \(K\).

To illustrate, consider the following variation of the benchmark model. Entrepreneurs continue to receive private signals, \(x_i = \theta + \xi_i\) about \(\theta\), but there is no longer any common signal \(y\). Instead, the cost of investing for entrepreneur \(i\) is now \(k_i^2/2 - w_i k_i\), where \(w_i\) is a random variable. The latter is Normally distributed in the cross-section of the population with mean \(\omega\) and variance \(\sigma^2_{w_i}\), where \(\omega\) is itself a Normal random variable with mean 0 and variance \(\sigma^2_{\omega}\). Each entrepreneur knows his own \(w_i\), but does not know \(\omega\). Finally, the realizations of \(\omega, w_i - \omega, \xi_i\) and \(\theta\) are independent.
This example differs from the benchmark model in two ways. First, there is no longer any piece of information that is commonly observed by the entrepreneurs but not by the traders: entrepreneurs have only private information about either \( \theta \) or \( \omega \). Second, the problem is no longer one of pure common values: the \( w_i \) shocks introduce private values. At the same time, there is still an unobserved random variable that is not perfectly correlated with \( \theta \) and yet moves aggregate investment: whereas in the benchmark model this random variable was the noise \( \varepsilon \) in common information, here it is the common component \( \omega \) of the \( w_i \) shocks.

As in the benchmark model, the traders’ equilibrium expectations continue to satisfy \( \mathbb{E}[\theta|K] = \gamma_0 + \gamma_1 K \) for some \((\gamma_0, \gamma_1)\), with \( \gamma_1 > 0 \) if and only if investment increases with \( \theta \). As a result, the informational effect of aggregate investment once again induces a socially unwarranted complementarity, which now amplifies the sensitivity of investment to the common \( \omega \) shock.

**Proposition 10**

(i) Equilibrium investment satisfies

\[
k(x, w) = w + \mathbb{E}[(1 - \alpha) \kappa(\theta) + \alpha K(\theta, \omega)|x, w],
\]

where \( \alpha = \lambda \gamma_1 \) and \( \kappa(\theta) = \frac{(1 - \lambda)\theta + \lambda \gamma_0}{1 - \lambda \gamma_1} \). Moreover, in any equilibrium in which investment increases with \( \theta \), necessarily \( \alpha > 0 \).\(^{15}\)

(ii) The efficient investment satisfies

\[
k(x, w) = w + \mathbb{E}[\theta|x, w].
\]

(iii) In any equilibrium in which investment increases with both \( \theta \) and \( \omega \), it is necessarily the case that \( \alpha > 0 \) (\( = \alpha^* \)) and that investment exhibits inefficiently high sensitivity to \( \omega \).

### 6.4 Discussion

Throughout all the preceding extensions, we have maintained the assumption that traders can not directly invest in the new technology in the first period. Clearly, nothing changes if we relax this assumption.\(^{16}\) Hence, one could drop altogether the distinction between entrepreneurs and traders.

---

\(^{15}\) As in the benchmark model, \( \lambda \) small enough suffices for the linear equilibrium to be unique and to satisfy \( \alpha > 0 \).

\(^{16}\) For example, consider the benchmark model and suppose that each trader \( j \) chooses first-period real investment \( k_j \) at cost \( k_j^2/2 \) and then trades an additional \( q_j \) units in the second-period financial market. Neither the equilibrium price in the financial market nor the entrepreneurs’ choices in the first period are affected; all that happens is that aggregate investment now includes the investment of the traders, which is simply given by \( k_T = \mathbb{E}\theta \) and, clearly, does...
and simply talk about differentially informed agents who first make real investment choices and then trade financial claims on the installed capital.

Also, it is not necessary that some agents are better informed than others, as it was the case here with entrepreneurs vis-a-vis traders. What seems essential for our results is only that agents are heterogeneously informed and that aggregate investment is a positive signal of the underlying productivity.

Finally, the assumption that a fraction \( \lambda \) of the agents is hit by a liquidity shock and is forced to sell their capital in the financial market was a modeling device that ensured that the private return to first-period investment depends on (anticipated) second-period financial prices while ensuring tractability once coupled with the simple linear-quadratic structure of payoffs. If one were to drop the assumption of risk neutrality, or assume that the second-period transaction costs depend on gross positions, or introduce short-sale constraints in the financial market, then the profits an agent could do in the financial market would depend on how much capital he enters the market with; this in turn would ensure that private returns to first-period investment depend on expectations of future financial prices, even in the absence of liquidity shocks.

7 Conclusion

This paper examined the interaction between real and financial decisions in an economy in which information about underlying profitability is dispersed. By conveying a positive signal about profitability, higher aggregate investment stimulates higher asset prices, which in turn raise the incentives to invest. This creates an endogenous complementarity, making investment decisions sensitive to higher-order expectations, and thereby dampening the impact of fundamental shocks and amplifying the impact of common expectational shocks. Importantly, these effects are symptoms of inefficiency: equilibrium investment reacts too little to fundamental shocks and too much to expectational shocks, relative to what would be socially desirable.

These effects are likely to be stronger during periods of intense technological change, when the dispersion of information is particularly high. Our analysis therefore predicts that such periods come hand-in-hand with episodes of higher-than-usual non-fundamental volatility and comovement in investment and asset prices. At some level, this seems consistent with the recent experiences not affect the information structure in the second period.
surrounding the internet revolution or the explosion of investment opportunities in China. This is crucial. What looks like irrational exuberance could be the amplified, but rational, response to noise in information; and while both scenarios feature similar symptoms of inefficiency, they lead to very different conclusions regarding the optimal policy response.

Finally, note that the relevance of our results is clearly not limited to episodes of intense technological change. Rather, our mechanism represents also a likely source of non-fundamental volatility and inefficiency over the business cycle. Extending the analysis to richer business-cycle frameworks so as to quantify these effects is an important direction for future research.
8 Appendix

Proof of Lemma 4. The proof proceeds in several steps. We start by proving part (i). We continue with some auxiliary results regarding the function \( F \) which are used in the last steps. We conclude by establishing parts (ii), (iii) and (iv). Throughout, to simplify notation, we suppress the dependence of \( F \) and \( G \) on \((\pi_\theta, \pi_x, \pi_y, \lambda)\) and let \( \pi \equiv \pi_\theta + \pi_x + \pi_y, \delta_0 \equiv \pi_\theta / \pi, \delta_1 \equiv \pi_x / \pi, \) and \( \delta_2 \equiv \pi_y / \pi. \)

Part (i). Substituting \( K(\theta, y) = \beta_0 + \beta_1 \theta + \beta_2 y \) into \( \text{(7)} \) and using \( \mathbb{E}[\theta|x, y] = \delta_0 \mu + \delta_1 x + \delta_2 y \) gives

\[
\begin{align*}
k(x, y) &= (1 - \lambda) \mathbb{E}[\theta|x, y] + \lambda \gamma_0 + \lambda \gamma_1 (\beta_0 + \beta_1 \mathbb{E}[\theta|x, y] + \beta_2 y) \\
&= (1 - \lambda + \lambda \gamma_1 \beta_1) \delta_0 \mu + \lambda \gamma_0 + \lambda \gamma_1 \beta_0 + \lambda \gamma_1 \beta_2 y \\
&= [(1 - \lambda + \lambda \gamma_1 \beta_1) \delta_0 \mu + \lambda \gamma_0 + \lambda \gamma_1 \beta_0] + \\
&\quad + [(1 - \lambda + \lambda \gamma_1 \beta_1) \delta_1] x + [(1 - \lambda + \lambda \gamma_1 \beta_1) \delta_2 + \lambda \gamma_1 \beta_2] y
\end{align*}
\]

Because in equilibrium the above must coincide with \( \beta_0 + \beta_1 x + \beta_2 y \) for all \( x \) and \( y, \) the following conditions must hold

\[
\begin{align*}
\beta_0 &= (1 - \lambda + \lambda \gamma_1 \beta_1) \delta_0 \mu + \lambda \gamma_0 + \lambda \gamma_1 \beta_0, \\
\beta_1 &= (1 - \lambda + \lambda \gamma_1 \beta_1) \delta_1, \\
\beta_2 &= (1 - \lambda + \lambda \gamma_1 \beta_1) \delta_2 + \lambda \gamma_1 \beta_2.
\end{align*}
\]

It is immediate that any equilibrium must satisfy \( \beta_1 \neq 0. \) Then let \( b \equiv \beta_2 / \beta_1. \) From \( \text{(4)} \) and \( \text{(6)}, \)

\[
\gamma_1 \beta_1 b = h(b) \equiv \frac{\delta_2 (1 + b)}{\delta_0 b^2 + \delta_2 (1 + b)^2}
\]

while from \( \text{(26)} \) and \( \text{(27)}, \)

\[
b = \frac{\delta_2}{\delta_1} + \frac{\lambda \gamma_1 \beta_1 b}{(1 - \lambda + \lambda \gamma_1 \beta_1) \delta_1}.
\]

Substituting \( \text{(28)} \) into \( \text{(29)} \) gives \( b = F(b), \) where

\[
F(b) \equiv \frac{\delta_2}{\delta_1} \left( 1 + \frac{\lambda (1 + b) b}{(1 - \lambda)(\delta_0 + \delta_2) b^2 + (2 - \lambda) \delta_2 b + \delta_2} \right).
\]
Note that the domain of $F$ is the set of all $b \in \mathbb{R}$ such that $1 - \lambda + \lambda \gamma_1 \beta_1 \neq 0$. Using (28), the latter is given by

$$
\mathbb{B} \equiv \{ b \in \mathbb{R} : (1 - \lambda)(\delta_0 + \delta_2)b^2 + (2 - \lambda)\delta_2 b + \delta_2 \neq 0 \}.
$$

It follows that, in any linear equilibrium, $b$ is necessarily a fixed point of $F$, while the coefficients $(\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1)$ are given by the following conditions:

\begin{align*}
\beta_1 &= [1 - \lambda + \lambda h(b)]\delta_1 \\
\beta_2 &= b\beta_1 = b[1 - \lambda + \lambda h(b)]\delta_1 \\
\gamma_1 &= \frac{\gamma_1 \beta_1}{\beta_1} = \frac{h(b)}{[1 - \lambda + \lambda h(b)]\delta_1} \\
\beta_0 &= (1 - \lambda + \lambda h(b)) \delta_0 \mu + \frac{\lambda \delta_0 \mu}{\delta_0 + \delta_2 \left(\frac{1 + b}{b}\right)^2} \\
\gamma_0 &= \frac{\delta_0}{\delta_0 + \delta_2 (1 + \frac{1}{b})^2} \mu - \gamma_1 \beta_0
\end{align*}

Conditions (31)-(35) uniquely define the function $G$.

**Auxiliary results.** Let $g(b) \equiv (1 - \lambda)(\delta_0 + \delta_2)b^2 + (2 - \lambda)\delta_2 b + \delta_2$; the domain of $F$ is $\mathbb{B} = \{ b \in \mathbb{R} : g(b) \neq 0 \}$ and whose complement is $\mathbb{B}_c = \{ b \in \mathbb{R} : g(b) = 0 \}$. Note that the discriminant of $g(b)$ is $\Delta \equiv (\delta_2 \lambda)^2 - 4 \delta_0 \delta_2 (1 - \lambda)$. If $\Delta < 0$, then $\mathbb{B}_c = \varnothing$; if $\Delta = 0$, then $\mathbb{B}_c = \left\{ -\frac{(2 - \lambda)\delta_2}{2(1 - \lambda)(\delta_0 + \delta_2)} \right\}$; finally, if $\Delta > 0$, then $\mathbb{B}_c = \left\{ \frac{-(2 - \lambda)\delta_2 + \sqrt{\Delta}}{2(1 - \lambda)(\delta_0 + \delta_2)}, \frac{(2 - \lambda)\delta_2 - \sqrt{\Delta}}{2(1 - \lambda)(\delta_0 + \delta_2)} \right\}$. Because there are values for $(\delta_0, \delta_2, \lambda)$ that make $\Delta$ negative, zero, or positive, all three cases are possible in general. However, because $\Delta$ is continuous in $\lambda$ and $\Delta = -4 \delta_0 \delta_2 < 0$ when $\lambda = 0$, $\lambda$ small enough suffices for $\mathbb{B}_c = \varnothing$. Moreover, because $g(b) \geq \delta_2 > 0$ for any $b \geq 0$, $\mathbb{R}_+ \subset \mathbb{B}$ always.

The function $F$ is continuously differentiable over its entire domain, with

$$
\lim_{b \to -\infty} F(b) = \lim_{b \to +\infty} F(b) = F_\infty \equiv \frac{\delta_2}{\delta_1} \left\{ 1 + \frac{\lambda}{(1 - \lambda)(\delta_0 + \delta_2)} \right\}.
$$

$$
F(-1) = F(0) = \frac{\delta_2}{\delta_1},
$$

$$
F(\delta_2/\delta_1) > \delta_2/\delta_1,
$$

and

$$
F'(b) = \lambda \frac{\delta_2 \phi_1(b)}{\delta_1 g(b)}.
$$
with \( \phi_1 (b) \equiv [\delta_2 - (1 - \lambda) \delta_0]b^2 + 2\delta_2 b + \delta_2 \).

First, consider the case \( \delta_2 = (1 - \lambda) \delta_0 \). Then \( \phi_1 (b) = 0 \) admits a unique solution at \( b = -1/2 \). Because \( \Delta < 0 \), the function \( F \) is defined over the entire real line, it is decreasing for \( b < -1/2 \) and increasing for \( b > -1/2 \).

Next, consider the case \( \delta_2 \neq (1 - \lambda) \delta_0 \). Then \( \phi_1 (b) = 0 \) admits exactly two solutions, at \( b = b_1 \) and at \( b = b_2 \), where

\[
    b_1 \equiv \frac{-\delta_2 - \sqrt{(1 - \lambda)\delta_0}}{\delta_2 - (1 - \lambda) \delta_0} \quad \text{and} \quad b_2 \equiv \frac{-\delta_2 + \sqrt{(1 - \lambda)\delta_0\delta_2}}{\delta_2 - (1 - \lambda) \delta_0}.
\]

The function \( F \) then reaches a local maximum at \( b_1 \) and a local minimum at \( b_2 \).

**Part (ii).** By the preceding results we have that \( F \) is continuous over \( \mathbb{R}_+ \), with \( F (\delta_2/\delta_1) > \delta_2/\delta_1 \) and \( \lim_{b \to \infty} F (b) < \infty \). It follows that the equation \( F (b) = b \) admits at least one solution at \( b > \delta_2/\delta_1 \), which proves part (ii).

**Part (iii).** Fix any \((\delta_1, \delta_2) \in (0, 1)^2\). If \( \delta_2 = (1 - \lambda) \delta_0 \), where \( \delta_0 = 1 - (\delta_1 + \delta_2) \), then let \( F \equiv F (-1/2) \) and \( \bar{F} \equiv F \infty \). If, instead, \( \delta_2 \neq (1 - \lambda) \delta_0 \), then let \( F \equiv \min \{ F \infty, F (b_2) \} \) and \( \bar{F} \equiv \max \{ F \infty, F (b_1) \} \). It is easy to check that both \( F \) and \( \bar{F} \) converge to \( \delta_2/\delta_1 \) as \( \lambda \to 0 \). Since \( F \) is continuous over its entire domain, \( \mathbb{B} \), and \( \lambda \) small enough suffices for \( \mathbb{B} = \mathbb{R} \), we have that \( \lambda \) small enough also suffices for \( F \) to be bounded in \([F, \bar{F}]\). But then \( F \) has to converge uniformly to \( \delta_2/\delta_1 \) as \( \lambda \to 0 \). It follows that for any \( \varepsilon > 0 \) there exists \( \tilde{\lambda} = \tilde{\lambda} (\varepsilon) > 0 \) such that, whenever \( \lambda < \tilde{\lambda} \), \( F \) has no fixed point outside the interval \([\delta_2/\delta_1 - \varepsilon, \delta_2/\delta_1 + \varepsilon]\).

Now note that, for any given \( b, \phi_1 \) and \( g \) are continuous in \( \lambda \), with

\[
    \frac{\phi_1 (b)}{g (b)^2} \rightarrow \frac{(1 + 2b) \delta_2 + b^2 (\delta_2 - \delta_0)}{[(1 + 2b) \delta_2 + b^2 (\delta_0 + \delta_2)]^2} \equiv l (b)
\]

as \( \lambda \to 0 \). Since \( l (b) \) is continuous in \( b \) and \( l (\delta_2/\delta_1) \) is bounded, for \( \varepsilon > 0 \) small enough \( l (b) \) is bounded for all \( b \in [\delta_2/\delta_1 - \varepsilon, \delta_2/\delta_1 + \varepsilon] \). It follows that, for any \( \eta \in (0, 1) \), there exist \( \tilde{\varepsilon} = \tilde{\varepsilon} (\eta) > 0 \) and \( \tilde{\lambda} = \tilde{\lambda} (\eta) \) such that \(-1 < -\eta < F' (b) < \eta < 1 \) for all \( b \in [\delta_2/\delta_1 - \tilde{\varepsilon}, \delta_2/\delta_1 + \tilde{\varepsilon}] \) and all \( \lambda \in [0, \tilde{\lambda}] \).

Combining these results with the continuity of \( F \), we have that there exist \( \tilde{\varepsilon} > 0 \) and \( \tilde{\lambda} > 0 \) such that, for any \( \varepsilon \in [0, \tilde{\varepsilon}] \) and any \( \lambda \in [0, \tilde{\lambda}] \), the following are true: for \( b \notin [\delta_2/\delta_1 - \varepsilon, \delta_2/\delta_1 + \varepsilon] \), \( F (b) \neq b \); for \( b \in [\delta_2/\delta_1 - \varepsilon, \delta_2/\delta_1 + \varepsilon] \), \( F \) is continuous and differentiable in \( b \), with \( F' (b) < 1 \). It follows that, for \( \lambda < \tilde{\lambda} \), \( F \) has at most one fixed point. Together with the fact, from part (i), that \( F \) necessarily has at least one fixed point, this proves part (iii).
Part (iv). It is easy to check that $(\delta_1, \delta_2, \lambda) = (0.2, 1, .75)$ implies $\delta = \mathbb{R}$ (so that $F$ is continuous in over the entire real line) and $F(b_2) < b_2 < 0$. These properties, together with the properties that $F(0) > 0$ and $\lim_{b \to -\infty} F(b) > 0 > -\infty$, ensure that, in addition to a fixed point in $(\delta_2/\delta_1, + \infty)$, $F$ admits at least one fixed point in $(-\infty, b_2)$ and one in $(b_2, 0)$. Indeed, as illustrated in the left panel of Figure 1, in this example $F$ admits exactly three fixed points, which are “strict” in the sense that $F(b) - b$ changes sign around them. But since $F$ is continuous in $(b, \delta_1, \delta_2, \lambda)$ in an open neighborhood of $(\delta_1, \delta_2, \lambda) = (0.2, 1, .75)$, there necessarily exists an open set $S \subset (0,1)^3$ such that $F$ admits three fixed points whenever $(\delta_1, \delta_2, \lambda) \in S$.  

Proof of Proposition 2. Let $b(\lambda)$ denote the unique solution to $F(b; \lambda) = b$. Parts (i) and (ii) follow from conditions (31), (32) and (33) observing that $b > 0$ suffices for $h(b) > 0$. For part (iii), note that

$$
\frac{\partial F(b; \lambda)}{\partial \lambda} = \frac{\delta_2 b (1 + b) (\delta_2 (1 + 2b) + (\delta_0 + \delta_1)b^2)}{g(b)^2},
$$

so that $b \geq 0$ suffices for $F(b; \lambda)$ to increase with $\lambda$. The result then follows from this property together with the fact that $b(\lambda) > \delta_2/\delta_1$ and $\frac{\partial F(b; \lambda)}{\partial b} < 1$ at $b = b(\lambda)$.  

Proof of Proposition 3. Take any $\lambda < \bar{\lambda}$. Let $b(\lambda)$ denote the unique fixed point to $F(b; \lambda) = b$ and denote by $\beta_0(\lambda)$, $\beta_1(\lambda)$, $\beta_2(\lambda)$, $\gamma_0(\lambda)$ and $\gamma_1(\lambda)$ the corresponding equilibrium coefficients, as given by (31)-(35). Note that all these functions are continuous.

Part (i). Using conditions (31)-(35), the sensitivity of investment to the realization of $\theta$ is given by

$$
\beta_1(\lambda) + \beta_2(\lambda) = W(\lambda)(\delta_1 + \delta_2),
$$

where $W(\lambda) \equiv w(b(\lambda), \lambda)$, with $w(b, \lambda) \equiv (1 + b)(1 - \lambda + \lambda h(b))^{\frac{\delta_1}{\delta_1 + \delta_2}}$ and $h(b)$ defined as in (28). We can compute $b'(\lambda)$ and $b''(\lambda)$ using the Implicit Function Theorem on $F(b, \lambda) - b$. We can then use this to compute $W'(\lambda)$ and $W''(\lambda)$. After some tedious algebra (which is available upon request), we find that $W'(0) = 0$ and $W''(0) = -\frac{2\delta_0\delta_1\delta_2}{(\delta_1(\delta_1 + \delta_2) + \delta_2)^2} < 0$. Together with the fact that $b(0) = \delta_2/\delta_1$ and hence $W(0) = 1$, this ensures that there exists $\bar{\lambda} \in (0, \bar{\lambda})$ such that, for all $\lambda \in (0, \bar{\lambda})$, $W(\lambda) < W(0) = 1$ and $W'(\lambda) < 0$; that is, $\beta_1 + \beta_2$ is lower than $\delta_1 + \delta_2$, its value in the frictionless benchmark, and is decreasing in $\lambda$.

Part (ii). From condition (31), we have that $\beta_1(\lambda) = |1 - \lambda + \lambda h(b(\lambda))|\delta_1$ and hence $\beta_1'(\lambda) = \delta_1[-1 + h'(b(\lambda))(\lambda h(b(\lambda)))$. Since $b(0) = \delta_2/\delta_1$ and $h(\delta_2/\delta_1) = \frac{\delta_1(\delta_1 + \delta_2)}{(\delta_1 + \delta_2)^2} < 1$, we have
that $\beta_1'(0) = \delta_1 [-1 + h(\delta_2/\delta_1)] < 0$, which together with the result from part (i) that $\beta_1'(0) + \beta_2'(0) = 0$ gives $\beta_2'(0) > 0$. The result then follows from the local continuity of $\beta_2'(\lambda)$ in $\lambda$. ■

**Proof of Lemma ??**. In any linear equilibrium $E[\theta|K(\theta, y)] = \gamma_0 + \gamma_1 K$, where $\gamma_0$ and $\gamma_1$ are as in (6). Market clearing then imposes that $p(\theta, y) = E[\theta|K(\theta, y)] - \lambda K(\theta, y)/\phi = \gamma_0 + (\gamma_1 - \frac{1}{\phi})K(\theta, y)$. From (2), we then have that $k(x, y)$ must satisfy

$$k(x, y) = E\left[ (1 - \lambda) \theta + \lambda \gamma_0 + \lambda (\gamma_1 - \frac{\lambda}{\phi}) K(\theta, y) \mid x, y \right]$$

for all $(x, y)$, where $K(\theta, y) = E[\ k(x, y) \mid \theta, y \ ]$. Letting $\alpha = \lambda \gamma_1 - \frac{\lambda^2}{\phi}$ and $\kappa (\theta) \equiv \frac{(1 - \lambda) \theta + \lambda \gamma_0}{1 - \lambda \gamma_1 + \lambda^2 / \phi}$ gives condition (12). ■

**Proof of Proposition 5**. Rewrite the tax rule as

$$\tau = -\phi_0 - (\phi_1 - 1) \theta - \phi_2 K,$$

where $\phi_0 \equiv -\tau_0, \phi_1 \equiv 1 - \tau_1,$ and $\phi_2 \equiv -\tau_2$. Then, the equilibrium price is given by

$$p = E[\theta - \tau | K] = \phi_0 + \phi_1 E[\theta | K] + \phi_2 K$$

$$= \phi_0 + \phi_1 \gamma_0 + (\phi_1 \gamma_0 + \phi_2) K$$

where we have used $E[\theta | K] = \gamma_0 + \gamma_1 K$. It follows that the FOC for an entrepreneur is

$$k_i = E[(1 - \lambda) (\theta - \tau) + \lambda p | x, y] =$$

$$= E[(1 - \lambda) (\phi_0 + \phi_1 \theta + \phi_2 K) + \lambda (\phi_0 + \phi_1 \gamma_0) + (\phi_1 \gamma_1 + \phi_2) K] | x, y]$$

$$= (1 - \lambda) \phi_0 + \lambda (\phi_0 + \phi_1 \gamma_0) + (1 - \lambda) \phi_1 E[\theta | x, y] + [(1 - \lambda) \phi_2 + \lambda (\phi_1 \gamma_1 + \phi_2)] E[K | x, y]$$

For the equilibrium with the above tax rule to implement the efficient allocation, it is necessary and sufficient that the above coincides with $k_i = E[\theta | x, y]$, which gives

$$(1 - \lambda) \phi_2 + \lambda (\phi_1 \gamma_1 + \phi_2) = 0$$

$$(1 - \lambda) \phi_1 = 1$$

$$(1 - \lambda) \phi_0 + \lambda (\phi_0 + \phi_1 \gamma_0) = 0$$

36
with \((\gamma_0, \gamma_1)\) determined as in \([6]\) with \((\beta_0, \beta_1, \beta_2) = (\delta_0 \mu, \delta_1, \delta_2)\). Equivalently,
\[
\begin{align*}
\tau_0 &= -\phi_0 = \frac{\lambda}{1-\lambda} \gamma_0 > 0 \\
\tau_1 &= 1 - \phi_1 = -\frac{\lambda}{1-\lambda} < 0 \\
\tau_2 &= -\phi_2 = \frac{\lambda}{1-\lambda} \gamma_1 > 0
\end{align*}
\]
which completes the argument. 

**Proof of Proposition 6.** That any linear equilibrium satisfies condition \([12]\) follows from the analysis in the main text. Here, we prove that \(\lambda\) small enough suffices for the equilibrium to be unique and to satisfy \(\gamma_1 > 0\); in fact, we show the stronger property that \(\alpha > 0\). Substituting \([4]\) into \([6]\) gives
\[
\gamma_1 = \frac{1}{\pi_z \pi_y \beta_1 + \beta_2} \frac{\beta_1 + \beta_2}{\beta_2^2 \pi_y + (\beta_1 + \beta_2)^2 \pi_y} = \frac{\beta_1 + \beta_2}{\beta_2^2 \delta_0 + (\beta_1 + \beta_2)^2 \delta_2}.
\]
In the limit, as \(\lambda \to 0\), we have \(\beta_0 \to \delta_0\), \(\beta_1 \to \delta_1\), \(\beta_2 \to \delta_2\), and hence \(\gamma_1 \to \frac{(\delta_1 + \delta_1)\delta_2}{\delta_2^2 \delta_0 + (\delta_1 + \delta_1)^2 \delta_2} > 0\). By continuity, then, there exists \(\lambda > 0\) such that, for all \(\lambda \in (0, \lambda]\), \(\gamma_1 > \frac{\lambda}{\phi}\) and therefore \(\alpha = \lambda (\gamma_1 - \lambda/\phi) > 0\).

**Proof of Proposition 7.** Let \(V(k, K, \theta) \equiv -\frac{1}{2} k^2 + \theta k - \frac{\lambda^2}{2 \phi} K^2\). The result then follows from Proposition 3 in Angeletos and Pavan (2007a) by noting that \(\kappa^*(\theta) \equiv \arg \max_K V(K, K, \theta) = \frac{1}{1 + \lambda^2/\phi} \theta\) and
\[
\alpha^* = 1 - \frac{V_{kk} + 2 V_{kK} + V_{KK}}{V_{kk}} K_K = -\lambda^2/\phi.
\]

**Proof of Proposition 8.** From \([2]\), in any equilibrium in which \(p\) is linear in \((\theta, y, \omega)\), there are coefficients \((\beta_0, \beta_1, \beta_2)\) such that \(k(x, y) = \beta_0 + \beta_1 x + \beta_2 y\). From \([17]\) and \([18]\), the equilibrium price is then
\[
p(\theta, y, \omega) = P(K(\theta, y), \theta, \omega) \equiv \eta_0 + \eta_1 K(\theta, y) + \eta_2 \theta + \eta_3 \omega.
\]
for some \((\eta_0, \eta_1, \eta_2, \eta_3)\).

Now consider the optimality of the traders’ strategies. As in the benchmark model, the infor-
mation that $K(\theta, y)$ reveals about $\theta$ is the same as that of a signal

$$ z \equiv \frac{K(\theta, y) - \beta_0}{\beta_1 + \beta_2} = \theta + \frac{\beta_2}{\beta_1 + \beta_2} \varepsilon $$

whose precision is $\pi_z \equiv \left( \frac{\beta_1 + \beta_2}{\beta_2} \right)^2 \pi_y$, while the information that $p(\theta, y, \omega)$ reveals about $\theta$ given $K(\theta, y)$ is the same as that of a signal

$$ s = \frac{1}{\eta_2} [p(\theta, y, \omega) - \eta_0 - \eta_1 K(\theta, y)] = \theta + \frac{\eta_3 \omega}{\eta_2} $$

whose precision is $\pi_s = \left( \frac{\eta_2}{\eta_3} \right)^2 \pi_\omega = \phi^2 (1 - \lambda)^2 \pi_\omega$. A trader who observes $K$ and $p$ thus believes that $\theta$ is normally distributed with mean

$$ E[\theta \mid K(\theta, y), p(\theta, y, \omega)] = \frac{\pi_\theta}{\pi_\theta + \pi_z + \pi_s} \mu_\theta + \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s} \beta_0 \frac{1}{\beta_1 + \beta_2} + \frac{\pi_s}{\pi_\theta + \pi_z + \pi_s} s $$

where

$$ \gamma_0 = \frac{\pi_\theta}{\pi_\theta + \pi_z + \pi_s} \mu_\theta - \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s} \beta_0 \frac{1}{\beta_1 + \beta_2} $$

$$ \gamma_1 = \frac{1}{\pi_\theta + \pi_z + \pi_s} $$

$$ \gamma_2 = \frac{1}{\pi_\theta + \pi_z + \pi_s} \eta_3 $$

$$ \gamma_3 = \frac{1}{\pi_\theta + \pi_z + \pi_s} \eta_2 $$

Combining (17) with (36) we then have that

$$ \eta_0 = \frac{\gamma_0}{2 - \lambda} $$

$$ \eta_1 = \frac{1}{2 - \lambda} \left( \gamma_1 - \frac{\lambda}{\phi} \right) $$

$$ \eta_2 = \frac{1}{2 - \lambda} (\gamma_2 + 1 - \lambda) $$

$$ \eta_3 = \frac{1}{2 - \lambda} (\gamma_3 - 1 + \lambda) $$

Lastly, consider the optimality of the entrepreneurs’ investment strategies. From condition
the strategy $k(x, y) = \beta_0 + \beta_1 x + \beta_2 y = (1 - \lambda) \mathbb{E}[\theta|x, y] + \lambda \mathbb{E}[p(\theta, y, \omega)|x, y]$. That is, $(\beta_0, \beta_1, \beta_2)$ must satisfy the following conditions

\begin{align*}
\beta_0 &= [1 - \lambda + \lambda \eta_1 \beta_1 + \lambda \eta_2] \delta_0 \mu_\theta + \lambda \eta_0 + \lambda \eta_1 \beta_0 \\
\beta_1 &= (1 - \lambda + \lambda \eta_1 \beta_1 + \lambda \eta_2) \delta_1 \\
\beta_2 &= (1 - \lambda + \lambda \eta_1 \beta_1 + \lambda \eta_2) \delta_2 + \lambda \eta_1 \beta_2
\end{align*}

A linear equilibrium is a thus a solution to (37)-(47).

The existence of a linear equilibrium and its uniqueness for $\lambda$ small enough can be established following steps similar to those in the benchmark model. Here we prove that $\lambda$ small enough suffices for $\gamma_1 > 0$, and even for $\alpha > 0$.

Substituting $\pi_z \equiv (\beta_1 + \beta_2)^2 \pi_y$ and $\pi_s = \phi^2(1 - \lambda)^2 \pi_\omega$ into (38) gives

\begin{align*}
\gamma_1 &= \frac{1}{\pi_\theta + \pi_z + \pi_s \beta_1 + \beta_2} \\
&= \frac{1}{\beta_2 \delta_0 + (\beta_1 + \beta_2)^2 \pi_y + \beta_2 \phi^2(1 - \lambda)^2 \pi_\omega} \\
&= \frac{(\beta_1 + \beta_2) \delta_2}{(\beta_1 + \beta_2)^2 \delta_2 + \frac{\beta_2 \phi^2(1 - \lambda)^2 \pi_\omega}{\pi_\theta + \pi_y + \pi_z}}.
\end{align*}

In the limit, as $\lambda \to 0$, we have $\beta_0 \to \delta_0$, $\beta_1 \to \delta_1$, $\beta_2 \to \delta_2$, and $\gamma_1 \to \frac{\lambda^2}{2 \phi^2 (1 - \lambda)^2} \pi_\omega$, and therefore $\alpha = \frac{\lambda^2}{2 \phi^2 (1 - \lambda)^2} > 0$. By continuity, then, there exists $\hat{\lambda} > 0$ such that, for all $\lambda \in (0, \hat{\lambda})$, $\gamma_1 > \frac{\lambda^2}{\phi}$ and therefore $\alpha = \frac{\lambda^2}{2 \phi} \left( \gamma_1 - \frac{\lambda^2}{\phi} \right) > 0$. \hfill \blacksquare

**Proof of Proposition 9.** Let

$$V(k, K, \theta) \equiv \theta k - \frac{1}{2} k^2 - \frac{\lambda^2}{2 \phi^2 (2 - \lambda)} K^2.$$ 

The result then follows for the same argument as in the proof of Proposition 7. \hfill \blacksquare

**Proof of Proposition 10.** *Part (i).* The best-response condition is now given by

$$k(x, w) = w + \mathbb{E}[(1 - \lambda) \theta + \lambda p(\theta, \omega)|x, w].$$

(48)
It follows that, in any linear equilibrium, there are coefficients \((\beta_0, \beta_1, \beta_2)\) such that \(k(x, w) = \beta_0 + \beta_1 x + \beta_2 w\) and therefore \(K(\theta, \omega) = \beta_0 + \beta_1 \theta + \beta_2 \omega\). The equilibrium price then satisfies \(p(\theta, \omega) = \gamma_0 + \gamma_1 K(\theta, \omega),\) with \(\gamma_1 > 0\) if and only if \(\beta_1 > 0\). Using this together with (48) gives the result.

\textit{Part (ii).} This follows from the same reasoning as in the benchmark model.

\textit{Part (iii).} By (23), in any linear equilibrium,

\[
k(x, w) = w + \mathbb{E}[(1 - \alpha) \kappa(\theta) + \alpha K(\theta, \omega) | x, w] \\
= w + \mathbb{E}[(1 - \alpha) \kappa(\theta) + \alpha (\beta_0 + \beta_1 \theta + \beta_2 \omega) | x, w] \\
= w + \mathbb{E}[(1 - \alpha) \kappa(\theta) + \alpha \beta_0 + \alpha \beta_1 \theta | x, w] + \mathbb{E}[\alpha \beta_2 \omega | x, w].
\]

Since \(\mathbb{E}[\theta | x, w] = \mathbb{E}[\theta | x]\) and \(\mathbb{E}[\omega | x, w] = \mathbb{E}[\omega | w] = \eta w,\) where \(\eta \equiv \sigma_w^{-2}/(\sigma_w^{-2} + \sigma_\omega^{-2}) > 0\), we have that

\[
k(x, w) = (1 + \alpha \beta_2 \eta) w + f(x),
\]

where \(f(x) \equiv \mathbb{E}[(1 - \alpha) \kappa(\theta) + \alpha \beta_0 + \alpha \beta_1 \theta | x].\) It follows that \(\beta_2\) must equal \(1 + \alpha \beta_2 \eta,\) or equivalently \(\beta_2 = 1/(1 - \alpha \eta).\) In any equilibrium in which \(\beta_1 > 0\) and \(\beta_2 > 0,\) we have \(\alpha > 0\) and \(\eta \alpha < 1.\) But then \(\beta_2 > 1.\) In contrast, in the efficient allocation, \(k(x, w) = w + \mathbb{E}[\theta | x],\) which gives the result. ■
References


