Policy with Dispersed Information*

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Abstract

This paper studies policy in a class of economies in which information about commonly-relevant fundamentals is dispersed and cannot be centralized by the government. In these economies, the equilibrium use of information can fail to be efficient either because of strategic interactions and payoff externalities, or because of informational externalities. In the first case, inefficiency manifests itself in excessive non-fundamental volatility (overreaction to common noise) or cross-sectional dispersion (overreaction to idiosyncratic noise). In the second case, inefficiency manifests itself in suboptimal quality of information contained in macroeconomic data, financial prices, or other indicators of economic activity. In either case, a novel role for policy is identified. The government can affect the way agents use information in equilibrium by making marginal taxes contingent on aggregate activity. The government may thereby improve welfare even without centralizing information or communicating information to the agents.

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“The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The economic problem of society is ... a problem of the utilization of knowledge which is not given to anyone in its totality.” (Friedrich A. Hayek, 1945)

1 Introduction

The dispersion of information is in the essence of the economic problem faced by a society. This is true not only for the idiosyncratic needs and means of households and firms but also for commonly-relevant fundamentals. Think, for example, of the arrival of a new technology, such as the PC or the Internet, or the emergence of a new market, such as India or China. Different firms deciding how much to invest in the new technology/market may have different information about the profitability of the new technology/market and may be either unwilling or unable to share it with one another. Alternatively, think of business cycles. Information about aggregate productivity and demand conditions is crucial for individual production and consumption decisions. Yet, such information is widely dispersed and it is only very slowly and imprecisely aggregated through financial prices or the release of macroeconomic data.

Because centralizing the information that is dispersed in the society is either unfeasible or prohibitively costly, society has to rely on decentralized market mechanisms for the utilization of such information. Society can then be assured that rational agents will always use their available information in the most privately-efficient way. In many situations of interest, however, the equilibrium use of information need not coincide with the one that best serves social interests. For example, because public information helps forecast the decisions of others, complementarities in investment or pricing decisions may induce firms to rely heavily on noisy public information, perhaps crowding out valuable private information or leading to excessive non-fundamental volatility. Alternatively, individuals may fail to internalize how their choices, once aggregated, affect the information contained in financial prices or other indicators of aggregate activity.

A novel role for policy then emerges: even if the government cannot either aggregate the existing dispersed information or communicate new information to the market, it may still be able to improve welfare by correcting potential inefficiencies in the decentralized use of information. Identifying policies that permit the government to do so is the goal of this paper.

**Preview.** Instead of focusing on a specific application, we conduct the analysis within a rich, yet tractable, class of games that allow for two sources of inefficiency in the decentralized use of information: payoff externalities and informational externalities. The former are short-cuts for a
variety of strategic and other external effects featured in applications, such as those originating in production spillovers, pecuniary externalities, oligopolistic competition, monopoly power, or social networking. The latter may reflect the (imperfect) aggregation of information in financial prices, the publication of data on macroeconomic activity, or other forms of social learning.

For this class of economies, we first characterize the equilibrium use of information. We then identify the best the government could achieve without centralizing information by characterizing an appropriate efficiency benchmark: the strategy that maximizes welfare (ex-ante utility) subject to the sole constraint that information cannot be transferred from one agent to another. We then identify simple tax schemes that implement this strategy as an equilibrium.

The symptoms of inefficiency that we detect by comparing the equilibrium with the aforementioned strategy depend on whether the inefficiency originates in payoff or informational externalities. In the first case, inefficiency manifests itself in excessive non-fundamental volatility (overreaction to common noise) or excessive cross-sectional dispersion (overreaction to idiosyncratic noise). In the second case, inefficiency manifests itself in too low precision of information contained in prices or other macroeconomic data.

Yet, the same policy prescription works for either case, as well as for economies that combine the two sources of inefficiency. Our key result is that the government can control how agents use information in equilibrium, and can thereby correct either type of inefficiency, by appropriately designing the contingency of marginal taxes on aggregate activity.

The logic behind this result is simple. First note that, when individuals expect marginal taxes to decrease with realized aggregate activity, they also expect the realized net-of-taxes return on their own activity to increase with aggregate activity. They thus perceive an incentive to align their choices with those of others. It follows that a negative dependence of marginal taxes on aggregate activity imputes strategic complementarity in individual choices. Symmetrically, a positive dependence imputes strategic substitutability.

Next note that public information is a better predictor of other agents’ choices than private information. It follows that the higher the incentives to align choices with one another, the higher the incentives to rely on public information. Stronger complementarity thus induces higher sensitivity of equilibrium actions to public information, whereas stronger substitutability induces higher sensitivity to private information.

Combining the aforementioned two observations, we have that the government can control the sensitivity of equilibrium actions to private and public information by appropriately designing the contingency of marginal taxes on aggregate activity. This in turn permits the government to correct any overreaction to idiosyncratic or common noise, as well as to improve the aggregation of information through prices or other indicators of aggregate activity, and therefore to improve the decentralized use of information.
Our analysis takes as exogenous the limits society faces in aggregating dispersed information; accordingly, it restricts the government, or any other social institution, from collecting and disseminating information or otherwise relaxing these limits. Investigating the foundations for these limits, and their potential implications for policy and institutional design, is a challenging topic beyond the scope of this paper. Nevertheless, our analysis offers some relevant insights. In our class of economies, equilibrium welfare may decrease with additional information because of possible inefficiencies in the equilibrium use of information. However, once these inefficiencies have been removed, more information can only improve welfare. Therefore, policies that provide the market with the right incentives for how to use available information are complements to policies, or other institutions, that provide the market with additional information.

**Related literature.** The paper deviates from the Ramsey literature on optimal taxation by introducing information heterogeneity and by avoiding exogenous restrictions on the set of available policy instruments. At the same time, it differentiates itself from the Mirrlees tradition and the “new public finance” (e.g., Kocherlakota, 2005; Golosov, Tsyvinski and Werning, 2006) in two dimensions. First, whereas that literature is primarily concerned with redistributive taxation (or social insurance), here we completely abstract from redistributive concerns and instead focus on a novel form of corrective taxation. Second, and most importantly, whereas that literature studies environments with independent private values (i.e., environments in which agents have private information only on purely idiosyncratic payoff variables such as own tastes, talent, or labor productivity), here we study environments with common values (i.e., environments in which agents have private information on commonly-relevant fundamentals such as aggregate productivity or the profitability of a new technology). With independent values, there is no gain from conditioning the transfer to one agent on other agents’ actions, because the latter convey no information about the type of the former. With common values, instead, better incentives can be provided, and hence better outcomes can be obtained, by exploiting the information contained in aggregate performance measures. This explains why the role for taxation examined here is novel to the public-finance literature.

The literature that studies economies with dispersed information goes back to Phelps (1970), Lucas (1972), and the rational-expectations revolution of the 70’s. More recently, a series of papers has raised interest on the business cycle implications of combining information heterogeneity with strategic complementarity in pricing decisions (Mankiw and Reis, 2000; Woodford, 2001; Hellwig, 2002; Amato and Shin, 2003, 2006; Lorenzoni, 2006, 2007; Mackowiak and Wiederholt, 2006). Another line of work has examined whether public information increases welfare (Morris and Shin, 2002; Heinemann and Cornand, 2004; Svensson, 2005; Hellwig, 2005; Roca, 2006; Baeriswyl and Conrad, 2007). Some of these papers have found a negative result, which has then been used to make a case against central-bank transparency; others have found the opposite result. In previous
work, we emphasized how these different results are due to different inefficiencies in the equilibrium use of available information (Angeletos and Pavan, 2007). Building on this previous work, in the current paper we identify policies that correct such inefficiencies. In doing so, these policies also guarantee that the social value of information is positive.

Finally, our analysis of economies with information externalities complements that in Vives (1993) and Amador and Weill (2007). These papers study the speed of social learning and the social value of information in a dynamic economy where agents learn from noisy observations of past aggregate activity. Our results in Section 6 identify policies that can control the speed of social learning and also guarantee that any exogenous information is socially valuable.

**Layout.** Section 2 introduces the baseline static framework: an abstract game flexible enough to nest a variety of applications. Section 3 characterizes the equilibrium use of information in the absence of policy. Section 4 characterizes the efficient use of information. Section 5 identifies tax systems that implement the latter as an equilibrium. Section 6 extends the analysis to dynamic macro-like settings. Section 7 introduces informational externalities. Section 8 discusses implications for the social value of information. Section 9 concludes. All proofs are in the Appendix.

## 2 The baseline static framework

### 2.1 Set up

**Actions and payoffs.** There is a continuum of agents of measure one, indexed by \( i \in [0, 1] \). Each agent chooses an action \( k_i \in \mathbb{R} \). The underlying common fundamental is parameterized by an exogenous random variable \( \theta \in \Theta = \mathbb{R} \). There is also a government, which imposes a tax \( \tau_i \in \mathbb{R} \) (positive or negative) on each agent \( i \), subject to the constraint that the budget is balanced.

Let \( \psi \) denote the cumulative distribution function for action \( k \) in the cross-section of the population and let \( K = \int k d\psi(k) \) and \( \sigma_k = [\int (k - K)^2 d\psi(k)]^{1/2} \) denote, respectively, the mean and the dispersion of actions. The (reduced-form) payoff of agent \( i \) is given by

\[
  u_i = U(k_i, K, \sigma_k, \theta) - \tau_i,
\]

for some \( U : \mathbb{R}^2 \times \mathbb{R}_+ \times \Theta \to \mathbb{R} \). The external and strategic effects exhibited in \( U \) may originate, not only from preferences and technologies, but also from pecuniary externalities, monopoly power, oligopolistic competition, social networks, and the like.\(^1\) What is crucial, though, is that payoffs can be reduced to the specification assumed above without missing any channels of endogenous information aggregation; the analysis of the latter is postponed till Section 7.

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\(^1\)For example, consider a two-stage Cournot game where firms choose capacity in stage 1 while facing uncertainty about demand in stage 2. The market-clearing price in stage 2 can be expressed as \( p = P(K, \theta) \), where \( K \) represents aggregate capacity, \( \theta \) is a random demand shock, and \( P \) is the inverse demand function. It follows that individual profits reduce to \( U(k, K, \theta) = P(K, \theta)k - C(k) \), where \( C \) is the cost function.
To maintain tractability, we assume that $U$ is quadratic in its arguments and that the external effect of dispersion, $U_\sigma(k, K, \sigma, \theta)$, depends only on its own level, $\sigma$. To guarantee existence and uniqueness of both the equilibrium and the efficient allocation, we assume the following: $U_{kk} < 0$, which imposes concavity at the individual’s decision problem; $-U_{kK}/U_{kk} < 1$, which ensures that the slope of the individual’s best response with respect to aggregate activity is less than one; and $U_{kk} + 2U_{kK} + U_{KK} < 0$ and $U_{kk} + U_{\sigma\sigma} < 0$, which imposes concavity at the planner’s problem. Finally, to make the analysis interesting, we assume $U_{k\theta} \neq 0$; this rules out the trivial case where the fundamental $\theta$ is irrelevant for equilibrium behavior.

**Timing and information.** There are three stages. In stage 1, the government chooses a policy rule $T$ that specifies the taxes that will be levied in stage 3. At this point, the government has no information and hence the choice of $T$ conveys no information.

In stage 2, agents simultaneously choose their actions after receiving exogenous incomplete and heterogeneous information. We model this as follows. Each agent $i$ receives a “type” $\omega_i \in \Omega$. The distribution of types in the cross-section of the population is $\phi \in \Phi$, where $\Phi$ is a family of distributions on $\Omega$. Both $\theta$ and $\phi$ are random and together constitute “the state of the world”. Their joint distribution, which is denoted by $F$, constitutes the common prior. It follows that each $\omega_i$ encodes a belief about $(\theta, \phi)$; in other words, an agent’s type summarizes his information about both the fundamentals and the information held by other agents.

Finally, in stage 3, the fundamental and the actions of all agents are publicly revealed and taxes are paid according to the rule $T$. At that point, the tax paid by agent $i$ can depend on any information that is public at that stage, including his own action, the realized distribution of actions in the population, and the realized fundamental: $\tau_i = T(k_i, \psi_i, \theta)$.

**Remark.** While most of the literature restricts attention to Gaussian information structures, here we allow for more general structures in order to show that the logic of our results is not sensitive to the details of the information structure. Nevertheless, some of the results are best illustrated in the special case of Gaussian information. In what follows, whenever we invoke this special case, we mean that the fundamental $\theta$ is drawn from $N(\mu, \sigma_\theta^2)$, while agents observe private signals $x_i = \theta + \xi_i$ and a public signal $y = \theta + \varepsilon$, where $\xi_i \sim N(0, \sigma_\xi^2)$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ are independent of one another as well as of $\theta$, and where $\xi_i$ is i.i.d. across agents. The type of agent $i$ is then simply $\omega_i = (x_i, y) \in \Omega = \mathbb{R}^2$, and similarly $\phi$ is the cross-sectional distribution of $(x_i, y)$.

Finally, note that the public signal $y$ can also be read as the result of information aggregation in society. For example, it could be the outcome of an opinion poll or of a price in an un-modeled financial market. What is crucial, though, is that information contained in this signal does not depend on the strategies of the agents; the alternative case is considered in Section 7.

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2This is with a slight abuse of terminology, because $(\theta, \phi)$ specifies only the distribution of types in the population, not the specific type received by each single agent. Such abuse is standard in games with a continuum of players.
2.2 Examples and applications

The following payoff structures are nested in our framework:

\[ u_i = Ak_i - \frac{1}{2}k_i^2, \quad \text{with } A = \theta + \alpha K \quad (\text{and } 0 < \alpha < 1); \]
\[ \pi_i = \pi^* - (p_i - p^*)^2, \quad \text{with } p^* = (1 - \alpha)\theta + \alpha P \quad (\text{and } 0 < \alpha < 1, \pi^* \in \mathbb{R}). \]

The first example can be interpreted as a simple model of production spillovers or network externalities, in which the return to investment (denoted above by \( A \)) increases with aggregate investment. The second example can be interpreted as an economy in which firms suffer a loss in their profits whenever their price \((p_i)\) deviates from some target level \((p^*)\), which itself depends both on exogenous fundamentals and the average price level \((P)\).

More generally, as it will become clear in the next section, our framework nests—at least as far as equilibrium is concerned—any model in which the agents’ strategic interaction can be summarized by a best response with the following structure:

\[ k_i = \mathbb{E}[(1 - \alpha)\kappa(\theta) + \alpha K|\omega_i] \]

for some scalar \( \alpha < 1 \) and some function \( \kappa \), and with the appropriate reinterpretation of what \( \theta \) and \( k_i \) stand for. This linear best-response structure underlies a variety of applications examined in the literature, including the following:


- The investment game in Angeletos and Pavan (2004), which is a stylized incomplete-information version of models with production or network externalities; in this context, \( \theta \) represents exogenous productivity and \( \alpha \) captures external returns to scale.

- The Cournot and Bertrand games studied in Vives (1984, 1998), Raith (1996), and various other IO papers; in this context, \( \theta \) represents a common shock to demand or costs, \( k_i \) the quantity (for Cournot) or the price (for Bertrand) set by firm \( i \), and \( \alpha \) the degree of strategic substitutability (for Cournot) or complementarity (for Bertrand) among firms decisions.

- The new-keynesian business-cycle economies in Woodford (2002), Mackowiak and Wiederholt (2006) and Baeriswyl and Conrand (2007); in this context, \( \theta \) represents exogenous nominal conditions, \( k_i \) the price set by a firm, and \( \alpha \) the degree of strategic complementarity in pricing decisions (a.k.a. “real rigidities”).

6
2.3 Common-information benchmark

Before we proceed to the core of the paper, it is useful to review the case of common information; this will help isolate the inefficiencies the emerge only under dispersed information.

Suppose first that \( \theta \) is perfectly known. Clearly, the unique equilibrium (in the absence of policy) is given by \( k_i = \kappa(\theta) \) for all \( i \), where \( \kappa(\theta) \) is the unique solution to \( U_k(\kappa, \kappa, 0, \theta) = 0 \). As for the first-best allocation, let

\[
W(K, \sigma_k, \theta) \equiv U(K, K, \sigma_k, \theta) + \frac{1}{2} U_{kk} \sigma_k^2.
\]

This function represents welfare under a utilitarian aggregator when the mean activity is \( K \), the dispersion of activity is \( \sigma_k \) and the fundamental is \( \theta \). By assumption, both \( W_{\theta\sigma}(\equiv U_{kk} + U_{\sigma\sigma}) \) and \( W_{KK}(\equiv U_{kk} + 2U_{kK} + U_{KK}) \) are negative. It follows that the first-best allocation is given \( k_i = \kappa^*(\theta) \) for all \( i \), where \( \kappa^*(\theta) \) is the unique solution to \( W_K(\kappa^*, 0, \theta) = 0 \). Finally, because of the quadratic payoffs, both \( \kappa \) and \( \kappa^* \) are linear: \( \kappa(\theta) = \kappa_0 + \kappa_1 \theta \) and \( \kappa^*(\theta) = \kappa^*_0 + \kappa^*_1 \theta \), where

\[
\kappa_0 \equiv \frac{U_k(0, 0, 0, 0)}{-U_{kk} - U_{kK}}, \quad \kappa_1 \equiv \frac{U_{k\theta}}{-U_{kk} - U_{kK}}, \quad \kappa^*_0 \equiv \frac{W_K(0, 0, 0)}{-W_{KK}}, \quad \kappa^*_1 \equiv \frac{W_{K\theta}}{-W_{KK}}.
\]

Now suppose that \( \theta \) is uncertain but let information be common: in all states of the world, all agents share the same \( \omega \). Because of the quadratic specification of payoffs, a form of certainty equivalence holds: the equilibrium and efficient actions under incomplete but common information are the best predictors of their complete-information counterparts.

**Proposition 1** Under common information, the unique equilibrium strategy is \( k(\omega) = \mathbb{E}[\kappa(\theta) | \omega] \), while the strategy that maximizes welfare is \( k^*(\omega) = \mathbb{E}[\kappa^*(\theta) | \omega] \).

Clearly, in economies where \( \kappa = \kappa^* \), the incompleteness of information does not open the door to policy intervention as long as information remains common. The core contribution of the paper will be to show that, even for these economies, inefficiency can emerge once information is dispersed. Indeed, none of our policy results require \( \kappa \neq \kappa^* \) and the reader may henceforth assume \( \kappa = \kappa^* \) if he wishes to focus on economies where inefficiency emerges only under dispersed information. Nevertheless, we allow for \( \kappa \neq \kappa^* \) to make clear that our policy conclusions are not sensitive to whether inefficiency vanishes once information is common.

3 Equilibrium use of information

In this section we study how dispersed information is used in equilibrium in the absence of policy intervention. We thus let \( T(k, \psi, \theta) = 0 \) for all \( (k, \psi, \theta) \) and define an equilibrium is the standard Bayes-Nash fashion.
Definition 1 An equilibrium is a (measurable) strategy \( k : \Omega \rightarrow \mathbb{R} \) such that, for all \( \omega \in \Omega \),

\[
k(\omega) = \arg \max_{k'} \mathbb{E}[U(k', K(\phi), \sigma_k(\phi), \theta) \mid \omega],
\]

with \( K(\phi) = \int_{\Omega} k(\omega) d\phi(\omega) \) and \( \sigma_k(\phi) = \left(\int_{\Omega} [k(\omega) - K(\phi)]^2 d\phi(\omega)\right)^{1/2} \) for all \( \phi \in \Phi \).

Define

\[
\alpha \equiv \frac{U_{kK}}{-U_{kk}}.
\]

We can then characterize the equilibrium use of information as follows.

Proposition 2 The equilibrium strategy exists, is unique, and satisfies

\[
k(\omega) = \mathbb{E}[\kappa(\theta) + \alpha \cdot (K(\phi) - \kappa(\theta)) \mid \omega]
\]

for all \( \omega \in \Omega \), with \( K(\phi) = \int_{\Omega} k(\omega) d\phi(\omega) \) for all \( \phi \in \Phi \).

Condition (6) follows directly from individual best responses and has a simple interpretation. In the case where actions are strategic independent \((\alpha = 0)\), an agent’s equilibrium action under incomplete information is simply his expectation of the action he would have taken had information been complete. Relative to this case, equilibrium behavior is tilted so as to permit more alignment of actions in the case of strategic complementarity \((\alpha > 0)\) and more differentiation in the case of strategic substitutability \((\alpha < 0)\). The coefficient \( \alpha \) then captures how much an agent tries to align his choice with aggregate activity when \( \alpha > 0 \), or how much he tries to differentiate from it when \( \alpha > 0 \). We accordingly call \( \alpha \) the equilibrium degree of coordination.

Because relying on common sources of information facilitates alignment, while relying on idiosyncratic sources inhibits alignment, strategic complementarity tends to increase the sensitivity of equilibrium actions to the former and reduce the sensitivity to the latter—and the converse is true for the case of strategic substitutability. To see this more clearly, consider the case of a Gaussian information structure. Recall that in this case the information of agent \( i \) consists of a private signal \( x_i \) and the public signal \( y \), and let \( \pi_\theta \equiv \sigma_\theta^{-2} \), \( \pi_y \equiv \sigma_y^{-2} \), and \( \pi_x \equiv \sigma_x^{-2} \) denote the precisions of, respectively, the prior, the public signal, and the private signal. The unique equilibrium is then

\[
k(\omega) = \kappa_0 + \kappa_1 \left[ \gamma_\mu \mu + \gamma_y y + \gamma_x x \right],
\]

with the coefficients \((\gamma_\mu, \gamma_y, \gamma_x)\) given as follows:

\[
\gamma_\mu = \frac{\pi_\theta}{\pi_\theta + \pi_y + (1 - \alpha) \pi_x}, \quad \gamma_y = \frac{\pi_y}{\pi_\theta + \pi_y + (1 - \alpha) \pi_x}, \quad \gamma_x = \frac{(1 - \alpha) \pi_x}{\pi_\theta + \pi_y + (1 - \alpha) \pi_x}.
\]

It follows that higher \( \alpha \) raises \( \gamma_y \), the sensitivity to the public signal, and reduces \( \gamma_x \), the sensitivity to the private signal.
4 Efficient use of information

We now turn to the following question: Suppose the government can not centralize information, or otherwise transfer information from one agent to another, but can somehow manipulate the way agents use their available information. Can the government then improve upon the equilibrium use of information?

In this section we address this question by bypassing the “details” of specific policy instruments that may permit such manipulation and instead characterizing directly the strategy that maximizes welfare under the sole restriction that information can not be centralized. We henceforth call this strategy the efficient strategy, or the efficient use of information.

Definition 2 An efficient strategy is a strategy \( k^* : \Omega \rightarrow \mathbb{R} \) that maximizes ex ante utility.

Because payoffs are linear in transfers, the latter impact welfare only through incentives. Hence, the combination of the efficient strategy with any transfer scheme that implements it defines the very best the government can do without centralizing information (that is, the best incentive-compatible direct mechanism among the ones that restrict the actions the planner recommends to an agent not to depend on the reports made by others). In the next section we will show that there, indeed, exist transfer schemes that implement the efficient strategy; here, we focus on its characterization.

Consider for a moment welfare for an arbitrary strategy \( k : \Omega \rightarrow \mathbb{R} \). Let \( \hat{K}(\theta) \equiv \mathbb{E}[k(\omega)|\theta] \) denote the component of activity that is “explained” by the fundamental. A simple Taylor expansion then gives ex ante utility as

\[
\mathbb{E}u = \mathbb{E}W(\hat{K}, 0, \theta) - \left\{ \frac{|W_{KK}|}{2} \cdot Var(K - \hat{K}) + \frac{|W_{\sigma\sigma}|}{2} \cdot Var(k - K) \right\},
\]

where \( W \) is defined as in (2). The first component of (9) captures first-order effects; the rest captures second-order effects. The term \( Var(K - \hat{K}) \) measures non-fundamental volatility, that is, variation in aggregate activity induced by common noise in information, while the term \( Var(k - K) \) measures non-fundamental dispersion, that is, variation of activity in the cross-section of the population induced by idiosyncratic noise in information. Both terms contribute towards lower welfare because of the concavity of payoffs. Their relative contribution depends on the details of the payoff structure, but is conveniently summarized in the following parameter:

\[
\alpha^* = 1 - \frac{W_{KK}}{W_{\sigma\sigma}} = 1 - \frac{U_{kk} + 2U_{kK} + U_{KK}}{U_{kk} + U_{\sigma\sigma}}.
\]

Since \( W_{KK} \) captures social aversion to volatility, while \( W_{\sigma\sigma} \) captures social aversion to dispersion, a higher \( \alpha^* \) means a higher aversion to dispersion relative to volatility.
Different strategies induce different levels of volatility and dispersion: the higher the sensitivity of actions to common sources of information relative to idiosyncratic sources, the higher the exposure to common noise relative to idiosyncratic noise, and hence the higher the non-fundamental volatility of activity relative to dispersion. At the same time, different strategies are associated with different degrees of alignment of choices across agents: the higher the sensitivity of actions to common sources of information relative to idiosyncratic sources, the higher the alignment of choices across agents. One may thus expect the degree of alignment associated with the efficient strategy to reflect social preferences over volatility and dispersion. This conjecture is verified below.

**Proposition 3** The efficient strategy exists, is unique,\(^3\) and satisfies

\[
k(\omega) = \mathbb{E}[\kappa^*(\theta) + \alpha^* (K(\phi) - \kappa^*(\theta)) \mid \omega]
\]

for almost all \(\omega\), with \(K(\phi) = \int_{\Omega} k(\omega) d\phi(\omega)\) for all \(\phi\).

In equilibrium, an agent’s equilibrium action was anchored by his expectation of \(\kappa\), the complete-information equilibrium action; but it was also adjusted on the basis of his expectation of aggregate activity, \(K\), with the weight on the latter given by \(\alpha\). A similar result holds for the efficient strategy once we replace \(\kappa\) with \(\kappa^*\) and \(\alpha\) with \(\alpha^*\). That the efficient strategy is anchored by \(\kappa^*\), the first-best action, is quite intuitive. That \(\alpha^*\) in turn is inversely related to the ratio \(W_{KK}/W_{\sigma\sigma}\) reflects our earlier discussion about volatility and dispersion: the degree of alignment associated with the efficient strategy increases with social aversion to dispersion and decreases with social aversion to volatility.

Therefore, just as \(\alpha\) summarized the private value of aligning actions across agents, \(\alpha^*\) summarizes the social value of such alignment. Furthermore, just as \(\alpha\) pinned down the relative sensitivity of equilibrium actions to different sources of information, \(\alpha^*\) pins down the corresponding relative sensitivity of efficient actions. To see this more clearly, consider again the case of Gaussian information. The efficient strategy is then given by

\[
k(x, y) = \kappa^*_0 + \kappa^*_1 \left[\gamma^*_\mu \mu + \gamma^*_y y + \gamma^*_x x\right],
\]

for almost all \((x, y)\), where the coefficients \((\gamma^*_\mu, \gamma^*_y, \gamma^*_x)\) are given by

\[
\gamma^*_\mu = \frac{\pi_\theta}{\pi_\theta + \pi_y + (1 - \alpha^*) \pi_x}, \quad \gamma^*_y = \frac{\pi_y}{\pi_\theta + \pi_y + (1 - \alpha^*) \pi_x}, \quad \gamma^*_x = \frac{(1 - \alpha^*) \pi_x}{\pi_\theta + \pi_y + (1 - \alpha^*) \pi_x}.
\]

It follows that a higher \(\alpha^*\) increases the sensitivity of the efficient strategy to public information and decreases its sensitivity to private information.

\(^3\)Hereafter, when we say “unique” we mean up to a zero-measure subset of \(\Omega\); this is a standard qualification one has to make with a continuum of types.
Comparing the equilibrium strategy with the efficient one, we conclude that the relation between $\alpha$ and $\alpha^*$ is the key to the question raised in the beginning of this section: as long as $\alpha \neq \alpha^*$, policies that manipulate the decentralized use of information can raise welfare, even in economies where there would have been no room for policy intervention had information been common.

**Corollary 1** Consider economies that are efficient under common information ($\kappa = \kappa^*$).

1. The equilibrium is efficient under dispersed information if and only if $\alpha = \alpha^*$.
2. When $\alpha > \alpha^*$ and information is Gaussian, the equilibrium exhibits overreaction to public information and excessive non-fundamental volatility.
3. When $\alpha < \alpha^*$ and information is Gaussian, the equilibrium exhibits overreaction to private information and excessive cross-sectional dispersion.

### 5 Optimal policy

The analysis so far has detected a novel objective for policy, but has not identified the instruments that may serve this objective. We complete this task in this section, first explaining how different tax schemes affect the decentralized use of information and then identifying the tax schemes that implement the efficient use of information as an equilibrium.

#### 5.1 Equilibrium with taxes

The information structure permits policies that let the tax paid by an agent in stage 3 depend, not only on as his own action and the realized fundamental, but also on the distribution of actions in the cross-section of the population. Without any loss of optimality (as it will become clear in the next subsection), we henceforth restrict attention to policies defined by

$$\tau_i = T(k_i, K, \sigma_k, \theta),$$

where the function $T : \mathbb{R}^2 \times \mathbb{R}_+ \times \Theta \rightarrow \mathbb{R}$ is quadratic in $(k, K, \theta)$ and linear in $\sigma^2_k$ and satisfies the following properties: $U_{kk} - T_{kk} < 0$, $-U_{kk} - T_{kk} < 1$, and $T(K, K, 0, \theta) + T_{kk} + T_{\sigma \sigma})\sigma^2_k/2 = 0$ for all $(K, \sigma_k, \theta)$. These properties preserve existence and uniqueness of equilibrium, while also guaranteeing budget balance state-by-state.\(^4\) We denote the class of policies that satisfy all these properties with $T$. Finally, note that this class includes linear tax schemes (in which $T_{kk} = 0$); but it also allows for either progressive or regressive tax schemes (respectively, $T_{kk} > 0$ and $T_{kk} < 0$).\(^5\)

Now, given a policy $T \in T$, let

$$\bar{U}(k, K, \sigma_k, \theta) = U(k, K, \sigma_k, \theta) - T(k, K, \sigma_k, \theta)$$

\(^4\)To see the latter, note that, for any $\psi$, $\int T(k, K, \sigma_k, \theta) d\psi(k) = T(K, K, 0, \theta) + (T_{\sigma \sigma} + T_{kk})\sigma^2_k/2$.

\(^5\)In our environment, the progressivity of the tax system will turn out to affect the decentralized use of information, but it does serve any redistributive role—this is because of the linearity of payoffs in transfers.
denote an agent’s payoff net of the tax. Next, define \((\tilde{\alpha}, \tilde{\kappa}_0, \tilde{\kappa}_1)\) as follows:

\[
\tilde{\alpha} \equiv \frac{U_{kk} - T_{kk}}{(U_{kk} - T_{kk})}, \quad \tilde{\kappa}_0 \equiv \frac{-U_{kk} (0, 0, 0, 0) + T_{kk} (0, 0, 0, 0)}{U_{kk} - T_{kk} + U_{kk} - T_{kk}}, \quad \tilde{\kappa}_1 \equiv \frac{-U_{kk} + T_{kk}}{U_{kk} - T_{kk} + U_{kk} - T_{kk}}. \tag{14}
\]

The characterization of the equilibrium strategy then follows from the results in Proposition 2, replacing \((\alpha, \kappa_0, \kappa_1)\) with \((\tilde{\alpha}, \tilde{\kappa}_0, \tilde{\kappa}_1)\).

**Proposition 4** Consider a tax scheme of the type described above, define \((\tilde{\alpha}, \tilde{\kappa}_0, \tilde{\kappa}_1)\) as in (14), and let \(\tilde{\kappa}(\theta) = \tilde{\kappa}_0 + \tilde{\kappa}_1 \theta\). The equilibrium strategy exists, is unique, and satisfies

\[
k(\omega) = \mathbb{E}[\tilde{\kappa}(\theta) + \tilde{\alpha} \cdot (K(\phi) - \tilde{\kappa}(\theta)) \mid \omega]\tag{15}
\]

for all \(\omega \in \Omega\), with \(K(\phi) = \int_\Omega k(\omega) d\phi(\omega)\) for all \(\phi \in \Phi\).

The key result here is that the government can control \(\tilde{\alpha}\), the degree of complementarity perceived by the agents, and can thereby fashion the response of equilibrium activity to different sources of information, by appropriately designing \(T_{kk}\), the sensitivity of the marginal tax to aggregate activity. The only other instrument that may help in the same direction is \(T_{kk}\), the progressivity of the tax system. However, this works only in certain cases: first, \(T_{kk}\) must exceed \(U_{kk}\), for otherwise the agent’s decision problem would fail to be convex; second, \(T_{kk}\) can affect the degree of complementarity only if there is already some (that is, only if \(U_{kk} - T_{kk} \neq 0\)). Moreover, whereas a higher \(T_{kk}\) unambiguously decreases \(\tilde{\alpha}\), the impact of \(T_{kk}\) is ambiguous: a higher \(T_{kk}\) reduces the curvature of individual payoffs, bringing \(\tilde{\alpha}\) closer to zero from either side of zero.

### 5.2 Implementation

We now turn to the implementation of the efficient strategy as an equilibrium. Clearly, if there is a policy \(T^* \in \mathcal{T}\) that implements the efficient strategy, then the very definition of the efficient strategy guarantees that there is no other transfer scheme can improve upon \(T^*\). This is true even for transfer schemes that violate budget balance and anonymity, or even if even we allow the agents to send arbitrary messages to the planner and the planner to make the transfers contingent on these messages; what is essential is only that the planner does not himself send informative messages to the agents before they make their choices. The next result then establishes the existence of such a policy and certain properties of it.

**Proposition 5** (i) There are multiple policies in \(\mathcal{T}\) that implement the efficient strategy.

(ii) All these policies have the property that, holding \(T_{kk}\) constant, the optimal \(T_{kk}\) increases with \(\alpha\) and decreases with \(\alpha^*\).

(iii) When either \(\alpha^* = 0\) or \(T_{kk} = 0\), any optimal policy satisfies

\[
T_{kk} = (-U_{kk}) (\alpha - \alpha^*).
\]
Part (i) follows directly from Propositions 3 and 4: there are multiple policies \( T \in \tilde{T} \) that induce payoffs \( \tilde{U} \) such that \( \tilde{\kappa} = \kappa^* \) and \( \tilde{\alpha} = \alpha^* \). The intuition for this multiplicity is that there are more policy instruments than policy goals. The two goals are: (a) the overall sensitivity of equilibrium activity to the fundamental \( \theta \); and (b) the extent to which agents align their choices, or, equivalently, the sensitivity of equilibrium activity to different types of information. The three instruments are: (a) \( T_{k\theta} \), the sensitivity of the marginal tax to the fundamental; (b) \( T_{kK} \), the sensitivity of the marginal tax to aggregate activity; and (c) \( T_{kk} \), the progressivity of the tax system. Since \( T_{k\theta} \) affects \( \tilde{\kappa} \) but not \( \tilde{\alpha} \), it can serve only the first policy goal. In contrast, \( T_{kk} \) and \( T_{kK} \) affect both \( \tilde{\kappa} \) and \( \tilde{\alpha} \) and can therefore serve both goals.

Parts (ii) and (iii) of the proposition then provide the key policy implications: the optimal contingency of the marginal tax on aggregate activity is dictated by the relation between \( \alpha \) and \( \alpha^* \). This becomes most clear once we restrict attention either to economies with \( \alpha^* = 0 \) (in which case \( T_{kk} \) can not help implementing the efficient degree of complementarity) or to linear tax policies (in which case \( T_{kk} \) is restricted to be zero). In either case, the marginal tax rate must increase with aggregate activity when \( \alpha > \alpha^* \), and must decrease when \( \alpha < \alpha^* \). More generally, the progressivity of the tax can also help induce the efficient use of information (subject to the constraints discussed in the previous section), but the basic intuition remains unaffected: holding \( T_{kk} \) constant, the optimal \( T_{kK} \) is higher the higher \( \alpha \) is relative to \( \alpha^* \).

To recap, when information is common, the optimal policy needs only to remove any discrepancy between \( \kappa(\theta) \) and \( \kappa^*(\theta) \); this can always be done through \( T_{k\theta} \). In contrast, when information is dispersed, the optimal policy must also remove any discrepancy between \( \alpha \) and \( \alpha^* \); for this, it is instrumental, if not essential, that the government appropriately designs \( T_{kK} \).

### 5.3 Implementations with “less information”

Although the policies considered in Proposition 5 do not require that the government have superior information than the agents at any point of time, they may require that the realized transfers be contingent on \( \theta \). Moreover, they necessarily require the agents’ actions to be perfectly revealed at the time transfers take place. In many applications, however, it seems more appealing to assume that at no time is there perfect public information about either fundamentals or actions.

To accommodate this possibility, we now consider a variant of the benchmark model, in which we modify the information available at stage 3. First, we assume that there is no public information on \( \theta \) (or simply rule out tax schedules that depend on such information). Second, we assume that actions are observed with noise. In particular, if agent \( i \) chooses \( k_i \) in stage 2, then in stage 3 the government—and all other agents—observes \( \tilde{k}_i = k_i + \eta + \nu_i \), where \( \eta \) is common noise and \( \nu_i \) is idiosyncratic noise, with respective c.d.f.’s \( f_{\eta} \) and \( f_{\nu} \) and variances \( \sigma_{\eta}^2 \) and \( \sigma_{\nu}^2 \). These noises could be interpreted as measurement errors and are assumed to be independent of \( \theta \) and of the information
that agents have in stage 2. We then let $\bar{K}$ denote the cross-sectional average of $\bar{k}$ and consider tax schedules of the form

$$\tau_i = T(\bar{k_i}, \bar{K}),$$

with $T$ satisfying the conditions described in Section 5.1. Denoting this class of policies by $\tilde{T}$, we have the following result.

**Proposition 6** There exists a policy in $\tilde{T}$ that implements the efficient strategy if and only if $\text{Cov}(\kappa(\theta), \kappa^*(\theta)) \geq 0$.

To understand this result, note first that

$$\mathbb{E}[T(\bar{k_i}, \bar{K})|\omega_i] = \mathbb{E}[T(k_i, K)|\omega_i] + \frac{1}{2} (T_{kk} + 2T_{kK} + T_{KK}) \sigma_\eta^2 + \frac{1}{2} T_{kk} \sigma_v^2.$$ 

The last two terms capture the impact of the common and idiosyncratic measurement errors on the expected transfer. Because these two terms are independent of the agent’s own action, they do not interfere with the ability of the government to control the incentives of the agents. It follows that Proposition 15 continues to hold—whether the actions of the agents are observed perfectly or with noise is irrelevant for the incentive effects of the policy.\footnote{The noise would become relevant if one introduced risk aversion over the transfers.}

Note then that, as evident in (14), the only instrument that permits a change in the sign of the overall sensitivity of equilibrium activity to $\theta$ (i.e., the sign of $\bar{k}_1$) is $T_{k\theta}$. It follows that, whenever $\text{Cov}(\kappa(\theta), \kappa^*(\theta)) < 0$ (i.e., whenever $\kappa_1$ and $\kappa_1^*$ have opposite signs), it is impossible to implement the efficient strategy with a policy that is not contingent on $\theta$ (or, at least, on a signal of $\theta$). On the other hand, whenever $\text{Cov}(\kappa(\theta), \kappa^*(\theta)) \geq 0$, the two policy instruments $T_{kK}$ and $T_{kk}$ suffice for aligning both $\alpha$ with $\alpha^*$ and $\kappa$ with $\kappa^*$,\footnote{Note that aligning $\kappa_0$ with $\kappa_0^*$ is never an issue: this can always be achieved through $T_k(0,0,0,0)$.} One can then implement the efficient strategy without conditioning the tax on realized fundamentals or on any exogenous signal about them.

### 5.4 Inefficiency only under dispersed information

As mentioned earlier, a special case of interest is economies in which inefficiency emerges only when information is dispersed, namely economies in which $\kappa = \kappa^*$ but $\alpha \neq \alpha^*$. For these economies, the condition $\text{Cov}(\kappa(\theta), \kappa^*(\theta)) \geq 0$ is trivially satisfied, and the optimal policy in $\tilde{T}$ can be characterized as follows.

**Corollary 2** Consider economies in which inefficiency emerges only under dispersed information.

(i) Conditional on $\theta$, the expected marginal tax is zero, $\mathbb{E}[T_k(\bar{k}, \bar{K})|\theta] = 0$, for every $\theta$.

(ii) When $\alpha > \alpha^*$, the optimal tax is procyclical and regressive: $T_{kK} > 0 > T_{kk}$.

(iii) When instead $\alpha < \alpha^*$, the optimal tax is countercyclical and progressive: $T_{kK} < 0 < T_{kk}$.
The intuition for this result is simple. Since \( \kappa(\theta) = \kappa^*(\theta) \) for every \( \theta \), it better be that \( \mathbb{E}[T_k(\hat{k}, \bar{K})|\theta] = 0 \) also every \( \theta \); if that were not the case, the tax would have caused a systematic deviation between the equilibrium and the efficient activity. Because we have restricted the marginal tax not to depend directly on \( \theta \) (i.e. \( T_{k|\theta} = 0 \)), this can be the case if and only if the indirect effects of \( \theta \) through individual and aggregate activity cancel each other (i.e., \( T_{kK} + T_{kk} = 0 \)).

In other words, agents must be taxed or subsidized only for deviations from aggregate activity. Finally, note that subsiding such deviations (\( T_{kK} > 0 > T_{kk} \)) tilts the agents’ incentives towards more reliance on private information and less reliance on public information, while taxing such deviations (\( T_{kK} < 0 < T_{kk} \)) has the opposite incentive effect. The result then follows from the fact that inducing the agents to rely more on private information is desirable when \( \alpha > \alpha^* \), while the latter is desirable when \( \alpha < \alpha^* \).

To recap, when inefficiency emerges only under dispersed information, not only is there a completely novel role for policy, but also the structure of the optimal tax system is sharply pinned down within \( \hat{T} \): the optimal tax is procyclical and regressive in economies with excessive non-fundamental volatility, while the converse is true in economies with excessive dispersion.

6 A dynamic economy

The analysis so far has been confined to a static game. We now show how this static game can be embedded in a dynamic setting with a more macro flavor. This serves two goals. First, it helps further appreciate how our results can be relevant for applications. Second, it accommodates the possibility that interesting dynamics in actions originate in the dynamics of information. In this section, we start by taking the dynamics of information as entirely exogenous; we turn to the analysis of endogenous information in Section 7.

6.1 Set up

There are \( T + 1 \) periods, with \( T \geq 2 \). In each period \( t = 1, \ldots, T \), each agent \( i \) chooses his level of investment in a riskless discount bond, \( b_{i,t} \), and his level of consumption, \( c_{i,t} \). The agent also chooses an action \( k_{i,t} \in \mathbb{R} \), which we henceforth interpret as capital invested in a risky technology. Investing \( k_{i,t} \) costs \( G(k_{i,t}) \) in period \( t \) and delivers \( F(k_{i,t}, \bar{K}_t, \sigma_t, A_{t+1}) \) in period \( t+1 \), where \( \bar{K}_t \) and \( \sigma_t \) are the mean and the dispersion of activities in period \( t \), \( A_{t+1} = \theta_t \) is an exogenous fundamental (which can be interpreted as productivity in period \( t+1 \) or, equivalently, the return to period-t investment), and \( G \) and \( F \) are real-valued functions. The agent’s period-t budget is thus given by

\[
c_{i,t} + G(k_{i,t}) + q_t b_{i,t} = F(k_{i,t-1}, \bar{K}_{t-1}, \sigma_{t-1}, \theta_{t-1}) + b_{i,t-1} - \tau_{i,t},
\]

where \( q_t \) denotes the period-t price of discount bonds (the reciprocal of the period-\( t \) risk-free rate) and \( \tau_{i,t} \) denotes the period-t taxes the agent pays to (or the transfers he receives from) the
government. Finally, the agent’s intertemporal preferences are given by

$$U_i = \sum_{t=1}^{T+1} \beta^{t-1} U (c_{i,t}, k_{i,t}).$$

where $U_i$ is a real-valued function.

This framework is quite flexible. For example, let $U_c(k, \sigma, \theta) = u(c)$, $G(k) = k$ and $F(k, \sigma, A) = Af(k) + (1 - \delta) k$, where $u$ is a neoclassical utility function, $f$ is a neoclassical production function, and $\delta \in [0,1]$ is the depreciation rate. With complete information, these restrictions nest a stylized version of the neoclassical growth model; with incomplete information, the model can be interpreted as an economy where a large number of entrepreneurs can freely trade a riskless bond but can not trade claims on the capital of their private businesses. Alternatively, one could interpret $k$ as individual labor supply (or effort), in which case it would be natural to let $G(k) = 0$ and $U_c(k) = u(c) - h(k)$, with $h(k)$ representing the disutility of effort.

Our framework could also allow for the private cost of investment and/or for the per-period utility to depend on the decisions of other agents in the economy, as well as on the exogenous fundamental: we could replace $G(k)$ with $G(k, K, \sigma, \theta)$ and $U_c(k)$ with $U_c(k, K, \sigma, \theta)$. This could be because of direct payoff externalities (investment spillovers, network externalities, and the like), because of externalities in leisure or, as in new-Keynesian models, because of frictions that make individual utility depend negatively on price dispersion. In certain cases, it could also be because of pecuniary externalities. Finally, that the functions $U, F$ and $G$ are stationary is clearly just a simplification—none of the subsequent results relies on such a restriction.

If agents had common information about the fundamentals in all periods, then the analysis could proceed essentially without any further restrictions on the functions $F, G$ and $U$. Here, however, we are interested in cases where agents have heterogeneous information. To keep the analysis tractable, we impose two restrictions. First, we assume that $U$ is linear in consumption and then, without further loss of generality, let $U(c, k) = c - h(k)$. Second, we let

$$V(k, K, \sigma, \theta) \equiv -[G(k) + h(k)] + \beta F(k, K, \sigma, \theta)$$

and assume that $V$ satisfies the same properties with respect to $(k, K, \sigma)$ as in the static model. The first restriction ensures that, in all periods and states, the bond market clears if and only if $q_t = \beta$ (in which case the demand for the risk-free bond is indeterminate) and that the life-time
The utility of agent $i$ (in the absence of taxes) reduces to

$$U_i = \sum_{t=1}^{T} \beta^{t-1} V(k_{i,t}, K_t, \sigma_t, \theta_t).$$

The second restriction then permits to extend our previous static analysis to the dynamic model.

The key here is to rule out informational externalities—the possibility that what an agent knows in period $t$ about $\theta_t$ depends on what actions other agents took in periods $s < t$. To ensure this, we model the dynamics of the information structure as follows. There is a sequence $\{\Omega_t, \Phi_t\}_{t=1}^{T}$, where $\Phi_t$ is a family of distributions with support $\Omega_t$. The (exogenous) information of an agent in period $t$ is represented by $\omega_t \in \Omega_t$ with (cross-sectional) distribution $\phi_t \in \Phi_t$. The joint distribution of $\{\phi_t, \theta_t\}_{t=1}^{T}$ is $F$ with support $Q$, and $F$ thus constitutes the common prior.

Finally, we assume that $(\omega_{i,t-1}, \phi_{t-1}, \theta_{t-1})$ belongs to $\omega_{i,t}$. This ensures that there is nothing to learn about $(\theta_s, \phi_s)_{s=t}^{T}$ from the observation of other agents’ (past) actions—whether such actions are observable is then irrelevant.\(^{10}\) It is then without loss of generality, for either equilibrium or efficiency, to restrict attention to strategies that depend only on $\omega_{i,t}$.

### 6.2 Equilibrium, efficiency and policy

Given that information is exogenous in all dates and states, the analysis of both the equilibrium and efficient allocations parallels that in the static benchmark. Let $\kappa(\theta)$ denote the (unique) solution to $V_k(\kappa, \kappa, 0, \theta) = 0$ and let $\kappa^*(\theta) \equiv \arg \max_{\kappa} V(\kappa, \kappa, 0, \theta)$; if information were complete, the equilibrium action in period $t$ would be $\kappa(\theta_t)$, while the first-best action would be $\kappa^*(\theta_t)$. Next, let $\alpha = -V_{kK}/V_{kk}$ and $\alpha^* = 1 - (V_{kk} + 2V_{kK} + V_{KK})/(V_{kk} + V_{\sigma\sigma})$; once again, these scalars parameterize the private and social value of aligning choices. The equilibrium and efficient allocations under incomplete information are then characterized in the following two propositions, which are direct extensions of Propositions 2 and 3.

**Proposition 7** The equilibrium strategy exists, is unique, and satisfies, for all periods $t$ and all $\omega_t \in \Omega_t$,

$$k_t(\omega_t) = \mathbb{E}[\kappa(\theta_t) + \alpha \cdot (K_t(\phi_t) - \kappa_t(\theta_t)) | \omega_t]$$

with $K_t(\phi_t) = \int_{\Omega_t} k_t(\omega) d\phi_t(\omega)$.

**Proposition 8** The efficient strategy exists, is unique, and satisfies, for all periods $t$ and almost all $\omega_t \in \Omega_t$,

$$k_t(\omega_t) = \mathbb{E}[\kappa^*(\theta_t) + \alpha^* \cdot (K_t(\phi_t) - \kappa^*(\theta_t)) | \omega_t]$$

with $K_t(\phi_t) = \int_{\Omega_t} k_t(\omega) d\phi_t(\omega)$.

\(^{10}\) An alternative that would also guarantee that agents do not learn anything about $(\theta_s, \phi_s)_{s=t}^{T}$ from the observation of past actions is to assume that for all $t > 1$, $\omega_{i,t}$ is a sufficient statistic for $(\omega_{i,t}, \phi_s, \theta_s)_{s=t}^{T}$ with respect to $(\theta_s, \phi_s)_{s=t}^{T}$. 17
The efficient strategy can be implemented as an equilibrium in a similar fashion as in Section 5. In particular, to obtain efficiency in the use of information during period $t$, it suffices to let the tax collected in period $t+1$ depend on the period-$t$ aggregate activity: $\tau_{i,t+1} = T(k_t, K_t, \theta_t)$. Once again, this policy does not require any informational advantage on the side of the government. It merely depends on the agents anticipating in period $t$ that the tax they will pay (or the transfer they will receive) in period $t+1$ will be contingent on information about realized activity and fundamentals that will become available at that time. As in Section 5, whether this information is perfect or noisy is not essential.

7 Informational externalities

A key functioning of modern economies that is missed in our preceding analysis is the aggregation of dispersed information in various indicators of aggregate activity, such as financial prices, trade volume, and aggregate employment, output and investment data. What is then crucial for our purposes is that the informational content of these indicators depends on the way agents use their available information in the first place: the more individuals rely on their private information, the more informative aggregate activity is of the underlying fundamentals. The various channels of information aggregation and social learning thus introduce informational externalities that have to be taken into account when determining the socially optimal use of information.

We study this issue, and its policy implications, within a variant of the dynamic framework introduced in the previous section. The fundamentals are no more directly revealed to the agents. Rather, the agents learn about the fundamentals from the observation of a noisy signal of past aggregate activity. This signal is a proxy for the informational role of macroeconomic data, financial prices, and other channels of information aggregation and social learning.

7.1 Set up

To simplify the exposition, we assume that the component of the fundamentals about which the agents have incomplete and heterogeneous information is constant over time. We denote this component by $\theta$ and assume that it is drawn from a Normal distribution with mean $\mu$ and variance $\sigma^2_{\theta}$. The realization of $\theta$ is never revealed. Instead, at any date $t$, agents observe a public signal $y_t = \theta + \varepsilon_t$ and private signals $x_{i,t} = \theta + \xi_{i,t}$, where $\varepsilon_t \sim N(0, \sigma^2_{y,t})$ and $\xi_{i,t} \sim N(0, \sigma^2_{x,t})$ are noises, independent of one another, independent across time, and independent of $\theta$, with $\xi_{i,t}$ also independently and identically distributed across agents. In addition, at any date $t \geq 2$, agents observe the following three random variables, which both affect payoffs and convey information about $\theta$: $\tilde{K}_{t-1} = K_{t-1} + \eta_{i,t}$, $\tilde{\sigma}_{t-1} = \sigma_{t-1} + v_t$, and $\tilde{A}_t = \theta + a_t$, where $\eta_{i,t} \sim N(0, \sigma^2_{\eta_{i,t}})$, $v_t \sim N(0, \sigma^2_{v,t})$, and $a_t \sim N(0, \sigma^2_{a,t})$ are shocks, common across agents, independent across time, and independent
of any other random variable. The period-$t$ budget of the agent is given by

$$c_{i,t} + G(k_{i,t}) + q_t b_{i,t} = F(k_{i,t-1}, \tilde{K}_{t-1}, \tilde{\sigma}_{t-1}, \tilde{A}_t) + b_{i,t-1} - \tau_{i,t}.$$  

The variable $\tilde{A}_t$ can thus be interpreted as the period-$t$ productivity, while $\theta$ is the underlying mean (trend) productivity. That the variables $\tilde{K}_{t-1}$ and $\tilde{\sigma}_{t-1}$ that enter the period-$t$ returns (through $F$) coincide with the signals about past activity is not essential. What is essential is that the observation of the period-$t$ returns do not perfectly reveal either $\theta$ or $K_{t-1}$.

The rest of the model is as in Section 6. The intertemporal payoff of an agent is given by

$$\sum_{t=1}^{T+1} \beta^{t-1} U(c_{i,t}, k_{i,t})$$

where $U(c_{i,t}, k_{i,t}) = u(c_{i,t}) - h(k_{i,t})$. Preferences are linear in consumption, ensuring once again that $q_t = \beta$ in all dates and states, that the trades of riskless bonds and the timing of consumption are indeterminate and that the intertemporal payoff of an agent reduces to

$$\sum_{t=1}^{T+1} \beta^{t-1} V(k_{i,t}, \tilde{K}_t, \tilde{\sigma}_t, \tilde{A}_{t+1}),$$

where

$$V(k, K, \sigma, A) \equiv -[G(k) + h(k)] + \beta F(k, K, \sigma, A).$$

The function $V$ is quadratic and satisfies the same restrictions as in the previous sections.

Finally, note that, unlike the case with exogenous information, here we have restricted $\theta$ and all the exogenous noises to be Gaussian. This restriction is instrumental for maintaining the analysis of the endogenous components of the information tractable.

### 7.2 Equilibrium

The only essential difference between the economy of this section and the one examined in Section 6 is the endogeneity of information: the strategy agents follow in period $t$ determines how much information is contained in $(\tilde{K}_t, \tilde{\sigma}_t)$ about $\theta$, and hence affect the information available in periods $t+1$ on. In the absence of policy, however, this informational externality is not internalized by the agents: the fact that the use of information today affects the information available tomorrow does not alter private incentives today. Thus, letting once again $\kappa(\theta)$ be the unique solution to $V_k(\kappa, \kappa, 0, \theta) = 0$ and $\alpha \equiv V_{kK}/(-V_{kk})$, we have the following result.

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11 Also, the results that follow do not depend on whether the signals about past actions are public or private. In particular, we could allow the agents to receive private signals $\tilde{K}_{i,t-1} = K_{t-1} + \eta_{i,t}$, $\tilde{\sigma}_{t-1} = \sigma_{t-1} + v_{i,t}$, and $\tilde{A}_{i,t} = \theta + a_{i,t}$, in addition to, or in substitution for, the aforementioned public signals.
Proposition 9 The equilibrium strategy exists and is unique. Let \( \{\Omega_t, \Phi_t\}_{t=1}^T \) denote the (unique) information structure generated by the equilibrium strategy. Then, for all \( t \), the strategy and the information structure jointly satisfy

\[
k_t(\omega_t) = \mathbb{E} \left[ \kappa(\theta) + \alpha \cdot (K_t(\phi_t) - \kappa(\theta)) \mid \omega_t \right]
\]

for all \( \omega_t \in \Omega_t \), with \( K_t(\phi_t) = \int_{\Omega_t} k_t(\omega) \, d\phi_t(\omega) \) for all \( \phi_t \in \Phi_t \).

This result does not require the fundamental \( \theta \) and the exogenous noises to be Gaussian. However, once we add this restriction, this result ensures that the information contained in the signals of past activity is also Gaussian. All the information—exogenous and endogenous—that is available in any given period can then be summarized in two sufficient statistics, one for the private and another for the public signals; the dynamics of these two statistics admit a simple recursive structure; and the equilibrium strategy reduces to an affine combination of the two.

Proposition 10 The equilibrium strategy is given by

\[
k_{i,t}(\omega_{i,t}) = \kappa_0 + \kappa_1 \left( \gamma_t X_{i,t} + (1 - \gamma_t) Y_t \right),
\]

with

\[
\gamma_t = \frac{(1 - \alpha) \pi_t^y}{(1 - \alpha) \pi_t^x + \pi_t^y}.
\]

The variables \( X_{i,t} \) and \( Y_t \) are sufficient statistics for all the private and public information about \( \theta \) that is available to agent \( i \) in period \( t \), while \( \pi_t^x \) and \( \pi_t^y \) are their respective precisions. The sufficient statistics are given recursively by

\[
X_{i,t} = \frac{\pi_{t-1}^x}{\pi_t^x} X_{i,t-1} + \frac{\sigma_{x,t}^2}{\pi_t^x} x_{i,t} \quad \text{and} \quad Y_t = \frac{\pi_{t-1}^y}{\pi_t^y} Y_{t-1} + \frac{\sigma_{y,t}^2}{\pi_t^y} y_t + \frac{\sigma_\alpha^2}{\pi_t^y} A_t + \frac{\gamma_t - 1}{\pi_t^x} \frac{\sigma_{\eta,t}^2}{\pi_t^y} \tilde{y}_t
\]

where

\[
\tilde{y}_t = \frac{K_{t-1} - \kappa_0 - \kappa_1 (1 - \gamma_{t-1}) Y_{t-1}}{\kappa_1 \gamma_{t-1}}
\]

is a linear transformation of the signal of past activity. Similarly, the precisions \( \pi_t^x \) and \( \pi_t^y \) are given recursively by

\[
\pi_t^x = \pi_{t-1}^x + \sigma_{x,t}^2 \quad \text{and} \quad \pi_t^y = \pi_{t-1}^y + \sigma_{y,t}^2 + \sigma_\alpha^2 + \kappa_1 \gamma_{t-1} \sigma_{\eta,t}^2.
\]

Finally, the initial conditions are \( \gamma_0 = 0 \), \( X_{i,0} = 0 \), \( y_{i,0} = \mu_0 \), \( \pi_0^x = 0 \) and \( \pi_0^y = \sigma_\theta^{-2} \).

The logic behind condition (17) is the same as the one we encountered in the static benchmark: a higher \( \alpha \) reflects a higher private value of aligning choices across agents and tilts the equilibrium use of information away from private information and towards public information. The evolution
of the information, on the other hand, can be understood as follow. Note first that, since $X_{i,t+1}$ equals $\theta$ plus idiosyncratic noise, aggregate activity in period $t - 1$ is given by

$$K_{t-1} = \kappa_0 + \kappa_1 \left( \gamma_{t-1} \theta + (1 - \gamma_{t-1}) Y_{t-1} \right).$$

Since $Y_{t-1}$ is publicly known (and so are the coefficients $\kappa_0$, $\kappa_1$, and $\gamma_{t-1}$), observing $K_{t-1} = K_{t-1} + \eta_t$ in period $t$ is informational equivalent to observing the variable $\tilde{y}_t$ defined in (19). But now note that

$$\tilde{y}_t = \theta + \frac{1}{\kappa_1 \gamma_{t-1}} \eta_t,$$

which is simply a Gaussian signal with precision $\tilde{\pi}_t = \gamma_{t-1}^2 \kappa_1^2 \sigma_{\eta t}^{-2}$. It follows that all the signals can be combined in two sufficient statistics, one for the private signals ($X_{i,t}$) and another for the public signals ($Y_t$). Condition (18) then states that these statistics are simply weighted averages of all the available signals, with the weights dictated by the respective precisions of these signals, while condition (20) states that the precisions of these statistics are simply the sums of the precisions of the component signals.

The key property to take notice is that the precision of information available in one period depends on the strategy followed in previous periods. In particular, for all $t$, $\tilde{\pi}_t$ and thereby $\pi^y_t$ is increasing in $\gamma_{t-1}$. This is because the informative content of the signals of aggregate activity is higher the more sensitive the strategies of the agents are to their private information. This is an important informational externality that the equilibrium fails to internalize in the absence of policy intervention.

### 7.3 Efficiency and policy

We now study the policy implications of the aforementioned informational externality by characterizing the strategy that maximizes ex-ante utility taking into account this externality. However, unlike the cases of exogenous information, we now have to restrict attention to strategies that are linear in the available private signals. Without this restriction, the endogenous signals are not more Gaussian and the analysis becomes intractable.

Suppose, for a moment, that the government fails to recognize period-$t$ strategy affects the information available in subsequent periods. Suppose further that the period-$t$ private and public information were summarized in sufficient statistics $X_{i,t}$ and $Y_t$ with respective precisions $\pi^x_t$ and $\pi^y_t$, so that

$$E[\theta | \omega_{i,t}] = \frac{\pi^x_t}{\pi^x_t + \pi^y_t} X_{i,t} + \frac{\pi^y_t}{\pi^x_t + \pi^y_t} Y_t,$$

12 When the common component of productivity changes over time, $\pi_t$ should be interpreted as the period-$t$ precision of the agents’ posterior about $\theta_{t+1}$; in this case, $\pi_t$ need not be monotonic over time, but it remains an increasing function of $\gamma_t$ for $\tau < t$.

13 A similar property typically holds in rational-expectation-equilibria models: the information contained in the price increases with the sensitivity of individual asset demands to private information.
and let $\kappa^*(\theta) \equiv \arg \max_\kappa V(\kappa, \kappa, 0, \theta) = \kappa_0^* + \kappa_1^*(\theta)$ and $\alpha^* \equiv 1-(V_{kk} + 2V_{kK} + V_{KK}) / (V_{kk} + V_{o\sigma})$. Proposition 8 would then imply that the efficient strategy is given by

$$k_{i,t}(\omega_{i,t}) = \kappa_0^* + \kappa_1^*(\gamma_t^* X_{i,t} + (1-\gamma_t^*) Y_t),$$

with

$$\gamma_t^* = \frac{(1-\alpha^*) \pi_t^\pi}{(1-\alpha^*) \pi_t^\pi + \pi_t^\beta}.$$ 

Furthermore, as we will further explain in Section 8, this strategy would guarantee that, in any given period, welfare is increasing in the precision of available information.

Now take into account the endogeneity of the information. As long as welfare is increasing in the precision of available information, it should be desirable to adjust the current use of information so as to induce more learning in subsequent periods. Because more learning is achieved only by the aggregation of private information, this suggests that the informational externality raises the sensitivity of efficient strategies to private information. The following result verifies this intuition.

**Proposition 11** The linear strategy that maximizes ex-ante utility is given by

$$k_{i,t}(\omega_{i,t}) = \kappa_0^* + \kappa_1^*(\gamma_t^{**} X_{i,t} + (1-\gamma_t^{**}) Y_t),$$

where

$$\gamma_t^{**} = \frac{(1-\alpha^*) \pi_t^\pi}{(1-\alpha^*) \pi_t^\pi + \pi_t^\beta - \beta (1-\alpha^*) (1-\gamma_{t+1}^{**})^2 \pi_t^\pi \pi_t^\beta (\pi_t^\beta)^{-2} (\kappa_t^*)^2 \sigma_{t+1}^{-2}$$

for all $t < T$, while $\gamma_T^{**} = (1-\alpha^*) \pi_T^\pi / [(1-\alpha^*) \pi_T^\pi + \pi_T^\beta]$, and where $X_{i,t}$ and $Y_t$ are sufficient statistics for all the private and public information about $\theta$ available to agent $i$ in period $t$, while $\pi_t^\pi$ and $\pi_t^\beta$ are their respective precisions. The sufficient statistics are obtained recursively using (18)-(20), replacing $(\gamma_t, \kappa_0, \kappa_1)$ with $(\gamma_t^{**}, \kappa_0^*, \kappa_1^*)$.

The key result here is that, holding constant the current precisions of private and public information, the optimal weight on private information is higher than what it would have been if information in the subsequent periods were exogenous:

$$\gamma_t^{**} > \frac{(1-\alpha^*) \pi_t^\pi}{(1-\alpha^*) \pi_t^\pi + \pi_t^\beta}.$$ 

As anticipated, this follows directly from the internalization of the informational externality: by raising the reliance on private information in one period, society achieves higher precision of information and hence higher welfare in subsequent periods. (Of course, this informational externality is absent in the very last period, which explains why the result does not hold at $t = T$.)

The following alternative representation of the optimal strategy helps translate the result here in terms of the degree of complementarity that the policy must induce in equilibrium.
Proposition 12 Consider the efficient linear strategy and let \( \{\Omega_t, \Phi_t\}_{t=1}^T \) be the associated information structure. There exists a unique sequence \( \{\alpha_t^{**}\}_{t=1}^T \), with \( \alpha_t^{**} < \alpha^* \) for all \( t < T \) and \( \alpha_T^{**} = \alpha^* \), such that the strategy and the information structure jointly satisfy, for all \( t \),

\[
k_t^* (\omega_t) = \mathbb{E}[\kappa^* (\theta) + \alpha_t^{**} \cdot (K_t (\phi_t) - \kappa^* (\theta_t)) \mid \omega_t]
\]

for almost all \( \omega_t \in \Omega_t \), with \( K_t (\phi_t) = \int_{\Omega} k_t^* (\omega) d\phi_t (\omega) \) for all \( \phi_t \in \Phi_t \).

As in the case without informational externalities, the weight \( \alpha_t^{**} \) in condition (21) summarizes how much society would like the agents to factor their expectations of others’ agents choices in their own choices. But unlike the case without informational externalities, these weights are now endogenous: they depend on the information structure and, in particular, they are lower the higher the social value of information. Nevertheless, condition (21) remains a valid and insightful representation of the optimal strategy: the result that \( \alpha_t^{**} < \alpha^* \) highlights that having the agents internalize the informational externality is isomorphic to having them perceive a lower complementarity in their actions than the one they should have perceived if information were exogenous. This in turn guides policy analysis: the optimal linear strategy can be implemented with similar tax schemes as in the benchmark model, but now the sensitivity \( T_k K \) of the marginal tax to aggregate activity must be higher than what it would have been had information been exogenous.

Corollary 3 Informational externalities unambiguously contribute to a higher optimal sensitivity of the marginal tax to aggregate activity.

This is true no matter what are the payoff interdependencies, or whether the equilibrium would have been efficient had information been exogenous. It relies merely on two properties: (i) that a higher \( T_k K \) induces more learning by increasing the sensitivity of actions to private information; and (ii) that the social value of such learning is guaranteed to be positive as long as the optimal policy is in place. We further discuss the importance of this last point in the next section.

8 Implications for the social value of information

Throughout the analysis, we have ruled out policies that, directly or indirectly, convey information to the agents. If the government processes information that is not directly available to the agents, as it is the case for macroeconomic statistics, then it certainly has the option to communicate this information to the market. Whether it should do so, however, is not obvious.

Indeed, as long the equilibrium use of available information is inefficient, an increase in the precision of available information can have a detrimental effect on welfare. For example, if \( \alpha > \alpha^* \), agents may overreact to any additional public information, exacerbating the already excessive non-fundamental volatility and leading to lower welfare.
However, this can not be the case if the equilibrium use of information is efficient. This is because the equilibrium strategy then coincides with the solution to a planning problem where the planner directly controls how agents use available information. An argument analogous to Blackwell’s theorem then guarantees that additional information can not hurt welfare: if the opposite were true, the planner would have preferred the agents to “ignore” the additional information when making their choices, which would contradict the assumption that the equilibrium is efficient.

A direct implication, then, is that policies that guarantee efficiency in the decentralized use of information also guarantee a positive social value for any additional information disseminated by policy makers or other institutions.

**Corollary 4** In general, more precise information can reduce welfare. However, once the optimal policy restores efficiency in the equilibrium use of information, welfare necessarily increases with the precision of available information.

In an influential paper, Morris and Shin (2002) used an elegant example to illustrate the possibility that more precise public information reduce welfare when strategic complementarities that are not warranted from a social perspective cause overreaction to public news.  

14 This example has lead to a renewed debate on the merits of transparency in central bank communications.  

Whereas this question has been studied largely in isolation from other aspects of policy making, our results indicate that the optimal communication policy of the central bank is far from orthogonal to the corrective roles of monetary and fiscal policies.

In a related but different line of reasoning, Amador and Weill (2007) argue that, by crowding out private information, an increase in the precision of exogenous public information can reduce the precision of the endogenous information contained in prices and other indicators of economic activity, and can thereby slow down social learning.  

A similar theme is explored in Morris and Shin (2005) and Amato and Shin (2006). Our results imply that the government can improve the informational content of prices, can raise the speed of social learning, and can guarantee that any public information it disseminates is welfare improving, once it sets in place policies that correct the underlying inefficiency in the decentralized use of information.

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14 Their example is nested in our baseline static framework with $\kappa = \kappa^*$ and $\alpha > \alpha^*$; that is, it is an economy where inefficiency emerges only under dispersed information and manifests itself in excessive non-fundamental volatility.


16 An important exception is Baeriswyl and Conrand (2007), which though focuses on the signaling effects of monetary policy.

17 Amador and Weill (2007) extend Vives (1993) to situations with both private and public learning. Both models are nested in our analysis of Section 7 as special cases that rule out payoff externalities.
9 Concluding remarks

This paper examined economies in which information about commonly-relevant fundamentals is dispersed in society and can not be perfectly aggregated by the government or any other social institution. In these economies, inefficiency can emerge in the decentralized use of information either because of payoff interdependencies and strategic interactions, or because of informational externalities.

The key contribution was to identify a novel role for policy: by making marginal taxes contingent on aggregate activity, the government can manipulate the decentralized use of information and can thereby improve welfare even if it cannot collect and disseminate information, or create new channels through which information is aggregated in society.

The details of the optimal contingency depend on the details of the application under examination: if the key inefficiencies are overreaction to public news, excessive non-fundamental volatility, or informational externalities, then the theory predicts that marginal taxes must increase with aggregate activity; the converse is true in economies exhibiting overreaction to private information and excessive cross-sectional dispersion. Nevertheless, the key principle—the optimality of marginal taxes that are contingent on aggregate activity—remains valid for any economy featuring dispersed information about commonly-relevant fundamentals.

In order to highlight the novel role for policy identified in this paper, we deliberately abstracted from two other, familiar policy objectives: (i) redistribution; and (ii) collection and dissemination of information. Interesting interactions emerge when the different roles are combined. First, consider redistribution achieved through progressive income taxation; such progressivity may discourage agents from relying on their private information and hence may conflict with the desire to reduce excessive non-fundamental volatility, or to improve the quality of information contained in prices and macroeconomic data, as documented here. Second, suppose that the government has the possibility to collect and disseminate additional information; whether it will find it optimal to do so depends on whether the government also has the instruments to correct possible inefficiencies in the decentralized use of information. Further investigating these interactions remains a fruitful direction for future research.

Finally, our analysis focused on the implications of dispersed information for optimal taxation. However, our insights are relevant for monetary policy as well: through its response to realized inflation, aggregate employment data, or other indicators of realized aggregate activity, monetary policy can also affect incentives in the decentralized use of information. Further exploring how the type of policy objectives we have studied in this paper filter in the optimal design of monetary policy is another interesting direction for future research.\textsuperscript{18}

\textsuperscript{18}For some work in this direction, see Lorenzoni (2007).
Proof of Proposition 2. Step 1 proves that condition (6) characterizes any equilibrium; this result follows from adapting Proposition 1 of Angeletos and Pavan (2007) to the more general information structures of this paper. Step 2 then proves existence and uniqueness.

Step 1. Take any strategy $k : \Omega \to \mathbb{R}$ and let $K(\phi) = \int k(\omega) d\phi(\omega)$ and $\sigma_k(\phi) = \int [k(\omega) - K(\phi)]^2 d\phi(\omega)$. A best-response is a strategy $k' : \Omega \to \mathbb{R}$ such that, for all $\omega$, $k'(\omega)$ solves the first-order condition

$$\mathbb{E}[U_k(k', K(\phi), \sigma_k(\phi), \theta) | \omega] = 0. \quad (22)$$

Using the fact that $U_k(k', K, \sigma_k, \theta) = U_k(\kappa, \kappa, 0, \theta) + U_{kk} \cdot (k' - \kappa) + U_{kK} \cdot (K - \kappa)$, where $\kappa$ stands for the complete-information equilibrium allocation and the fact that $\kappa$ solves $U_k(\kappa, \kappa, 0, \theta) = 0$ for all $\theta$, (22) reduces to

$$\mathbb{E}[U_{kk} \cdot (k' - \kappa(\theta)) + U_{kK} \cdot (K(\phi) - \kappa(\theta)) | \omega] = 0,$$

or equivalently $k'(\omega) = \mathbb{E}[\kappa(\theta) + \alpha(\kappa(\phi) - \kappa(\theta)) | \omega]$. In equilibrium, $k'(\omega) = k(\omega)$ for all $\omega$, which gives (6).

Step 2. What remains to prove is that the equilibrium exists and is unique; this can be done with the help of Proposition 3, which concerns the efficient use of information (and is proved below). Let $\mathcal{U}$ denote the class of payoff functions $U$ that satisfy the properties specified in Section 2. An economy is given by the collection $e = (U, \Theta, \Omega, \Phi, \mathcal{F})$. By comparing conditions (6) and (11), it is immediate that the set of equilibrium strategies for economy $e = (U, \Theta, \Omega, \Phi, \mathcal{F})$ coincides with the set of efficient strategies for an economy $e' = (U', \Theta, \Omega, \Phi, \mathcal{F})$ such that the optimal $\kappa^*$ and $\alpha^*$ corresponding to $e'$ coincide with the equilibrium $\kappa$ and $\alpha$ corresponding to $e$. Moreover, $U' \in \mathcal{U}$ as long as $U \in \mathcal{U}$. But, as shown in Proposition 3, the efficient strategy for any economy $e'$ with $U' \in \mathcal{U}$ exists and is uniquely determined for all but a measure-zero set of $\omega$. It follows that $e$ must have a unique equilibrium.

Proof of Proposition 3. This result follows from adapting Proposition 2 of Angeletos and Pavan (2007) to the more general information structures of this paper.

By definition, a strategy is efficient if and only if it maximizes

$$\mathbb{E}u = \int_{\Theta \times \Phi} \int_{\Omega} U(k(\omega), K(\phi), \sigma_k(\phi), \theta) d\phi(\omega) d\mathcal{F}(\theta, \phi),$$

with $K(\phi) = \int_{\Omega} k(\omega) d\phi(\omega)$ and $\sigma_k(\phi) = [\int_{\Omega} [k(\omega) - K(\phi)]^2 d\phi(\omega)]^{1/2}$. The strict concavity and the quadratic specification of $U$ ensures that a solution to this problem exists and is unique for almost all $\omega$. Let $G(\phi)$ be the marginal distribution of $\phi$ and $H(\theta|\phi)$ the distribution of $\theta$ conditional on...
\( \phi \), as implied by their joint distribution, \( \mathcal{F} \). The Lagrangian for this problem can be written as

\[
\Lambda = \int_\Phi \int_\Theta \int_\Omega U(k(\omega), K(\phi), \sigma_k(\phi), \theta) d\phi(\omega) dH(\theta|\phi) dG(\phi)
+ \int_\Phi \lambda(\phi) \left[ K(\phi) - \int_\Omega k(\omega) d\phi(\omega) \right] dG(\phi)
+ \int_\Phi \eta(\phi) \left[ \int_\Omega [k(\Omega) - K(\phi)]^2 d\phi(\omega) \right] dG(\phi)
\]

Therefore, the first order conditions with respect to \( K(\phi), \sigma_k(\phi), \) and \( k(\omega) \), which are necessary and sufficient for optimality, are given by the following:

\[
\int_\Theta \int_\Omega [U_K(k(\omega), K(\phi), \sigma_k(\phi), \theta) + \lambda(\phi) + 2\eta(\phi)(k(\omega) - K(\phi))] d\phi(\omega) dH(\theta|\phi) = 0
\tag{23}
\]

for almost all \( \phi \)

\[
\int_\Theta \left[ \int_\Omega U_\sigma(k(\omega), K(\phi), \sigma_k(\phi), \theta) d\phi(\omega) + 2\eta(\phi)\sigma_k(\phi) \right] dH(\theta|\phi) = 0
\tag{24}
\]

for almost all \( \phi \)

\[
\int_\Theta \times_\Phi [U_k(k(\omega), K(\phi), \sigma_k(\phi), \theta) - \lambda(\phi) - 2\eta(\phi)(k(\omega) - K(\phi))] dP(\theta, \phi|\omega) = 0
\tag{25}
\]

for almost all \( \omega \)

where \( P(\theta, \phi|\omega) \) denotes the cumulative distribution function of \( (\theta, \phi) \) conditional on \( \omega \).

Using the facts that \( U_K(k, K, \sigma_k, \theta) \) is linear in its arguments, that \( K(\phi) = \int_\Omega k(\omega) d\phi(\omega) \), and that \( U_\sigma(k, K, \sigma_k, \theta) = U_{\sigma\sigma}\sigma_k \), conditions (23) and (24) reduce to

\[
\lambda(\phi) = -\int_\Theta U_K(K(\phi), K(\phi), \sigma_k(\phi), \theta) dH(\theta|\phi)
\]

\[
\eta(\phi) = -\frac{1}{2} U_{\sigma\sigma}.
\]

Substituting the above into (25), we conclude that an allocation \( k : \Omega \rightarrow \mathbb{R} \) is efficient if and only if it satisfies

\[
\mathbb{E}[ U_k(k(\omega), K(\phi), \sigma_k(\phi), \theta) + U_K(K(\phi), K(\phi), \sigma_k(\phi), \theta) + U_{\sigma\sigma}[k(\phi) - K(\phi)] \mid \omega ] = 0,
\tag{26}
\]

for almost all \( \omega \). Next, because \( U \) is quadratic in \( (k, K, \theta) \) and linear in \( \sigma_k^2 \), condition (26) can be rewritten as

\[
\mathbb{E}[ U_k(\kappa^*, \kappa^*, 0, \theta) + U_{kk} \cdot (k - \kappa^*) + U_{KK} \cdot (K - \kappa^*) + U_K(\kappa^*, \kappa^*, 0, \theta) + (U_{kk} + U_{KK}) \cdot (K - \kappa^*) + U_{\sigma\sigma}(k - K) \mid \omega ] = 0,
\]

where we have suppressed the dependence of \( \kappa^*, k, \) and \( K \) on \( \theta, \omega, \) and \( \phi \), respectively. Using \( U_k(\kappa^*, \kappa^*, 0, \theta) + U_K(\kappa^*, \kappa^*, 0, \theta) = 0 \), by definition of first best, the above reduces to

\[
\mathbb{E}[ U_{kk}(k - \kappa^*) + (2U_{kk} + U_{KK})(K - \kappa^*) + U_{\sigma\sigma}(k - K) \mid \omega ] = 0,
\]

and rearranging gives (11).

**Proof of Proposition 4.** The result follows directly from Proposition 2, replacing \((\alpha, \kappa_0, \kappa_1)\) with \((\tilde{\alpha}, \tilde{\kappa}_0, \tilde{\kappa}_1)\).  

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Proof of Proposition 5. For the equilibrium with policy to coincide with the efficient strategy, it must be that $U_{kk} - T_{kk} < 0$ and
\[ \tilde{\alpha} = \alpha^*, \quad \tilde{\epsilon}_0 = \epsilon_0^*, \quad \text{and} \quad \tilde{\epsilon}_1 = \epsilon_1^* \]
where
\[ \tilde{\alpha} \equiv -\frac{U_{kK} - T_{kk}}{U_{kk} - T_{kk}}, \quad \tilde{\epsilon}_0 \equiv -\frac{U_k (0, 0, 0, 0) + T_k (0, 0, 0, 0)}{U_{kk} - T_{kk} + U_{kK} - T_{kK}} \quad \text{and} \quad \tilde{\epsilon}_1 \equiv \frac{-U_{k\theta} + T_{k\theta}}{U_{kk} - T_{kk} + U_{kK} - T_{kK}} \]
Combining and using the definition of $\alpha$, gives the result. ■

Proof of Proposition 6. The argument in the main text implies that we can ignore the noise in the observation of actions. Thus take any policy $T(k, K)$. From (10), we have that, for any $T_{kk}$, with $U_{kk} - T_{kk} < 0$, there exists a unique $T_{kK}$ such that $\tilde{\epsilon}_0 = \epsilon_0^*$ and hence also a unique $T_k (0, 0)$ such that $\tilde{\epsilon}_1 = \epsilon_1^*$. But then
\[ \tilde{\epsilon}_1 = \frac{U_{k\theta}}{(U_{kk} - T_{kk})(1 - \alpha^*)}, \]
while $\epsilon_1^* = -W_{K\theta}/W_{KK}$. Therefore, a $T_{kk}$ that ensures $\tilde{\epsilon}_1 = \epsilon_1^*$ and $U_{kk} - T_{kk} < 0$ exists if and only if $U_{k\theta}$ and $W_{k\theta}$ (equivalently, $\epsilon_1$ and $\epsilon_1^*$) have the same sign, i.e. if and only if $\text{Cov}(\epsilon(\theta), \epsilon^*(\theta)) > 0$. Provided this is the case, the optimal $T_{kk}$ is given by
\[ (U_{kk} - T_{kk}) = -\frac{U_{k\theta}}{\epsilon_1^* (1 - \alpha^*)} = \frac{U_{k\theta}W_{KK}}{W_{K\theta} (1 - \alpha^*)} = \frac{U_{k\theta}W_{\sigma\sigma}}{W_{K\theta}} \]
or equivalently
\[ T_{kk} = (-U_{kk}) \left( \frac{U_{k\theta}/W_{\sigma\sigma}}{U_{kk}/W_{K\theta}} - 1 \right). \]
The optimal $T_{kK}$ is then given by
\[ \left( \frac{T_{kK}}{-U_{kk}} \right) = \alpha - \alpha^* \left( 1 + \frac{T_{kk}}{-U_{kk}} \right) = \alpha - \alpha^* \left( \frac{U_{k\theta}/W_{\sigma\sigma}}{U_{kk}/W_{K\theta}} \right), \]
which is clearly increasing in $\alpha$ and decreasing in $\alpha^*$. All other coefficients of the tax policy $T$ are then set to balance the budget. ■

Proof of Corollary 2.
Recall that a policy implements the efficient strategy if and only if $(\tilde{\alpha}, \tilde{\epsilon}_0, \tilde{\epsilon}_1) = (\alpha^*, \epsilon_0^*, \epsilon_1^*)$, with $(\tilde{\alpha}, \tilde{\epsilon}_0, \tilde{\epsilon}_1)$ as in (14). By assumption, $(\epsilon_0^*, \epsilon_1^*) = (\epsilon_0, \epsilon_1)$ and $T_{k\theta} = 0$. The conditions $\tilde{\epsilon}_0 = \epsilon_0^*$ and $\tilde{\epsilon}_1 = \epsilon_1^*$ then reduces to
\[ -\frac{U_k (0, 0, 0, 0) + T_k (0, 0)}{U_{kk} - T_{kk} + U_{kK} - T_{kK}} = -\frac{U_k (0, 0, 0, 0)}{U_{kk} + U_{kK}} \quad \text{and} \quad \frac{-U_{k\theta}}{U_{kk} - T_{kk} + U_{kK} - T_{kK}} = \frac{-U_{k\theta}}{U_{kk} + U_{kK}}, \]

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which can be true if and only if \( T_k(0,0) = 0 \) and \( T_{kk} + T_{KK} = 0 \). Hence, \( T_k = T_{kK} (K - k) \), for some coefficient \( T_{kK} \). Since \( \mathbb{E}[K(\phi) | \theta] = \mathbb{E}[k(\omega) | \theta] \), part (i) is immediate. Parts (ii) and (iii) then follow from the fact that

\[
\bar{\alpha} \equiv \frac{U_{kK} - T_{kK}}{- (U_{kk} - T_{kk})} = \frac{U_{kk} + T_{kk}}{- (U_{kk} - T_{kk})}
\]

is increasing in \( T_{kk} \) and equal to \( \alpha \) at \( T_{kk} = 0 \). ■

**Proof of Propositions 7 and 8.** These are direct extensions of Propositions 2 and 3. ■

**Proof of Proposition 9.** Start with \( t = 1 \). Because information is exogenous in the first period, that the equilibrium strategy at \( t = 1 \) is unique and solves (16) follows directly from Proposition 2. Now consider \( t = 2 \). The information structure is now endogenous but uniquely determined by the unique equilibrium strategy for \( t = 1 \). That the equilibrium strategy at \( t = 2 \) is unique and solves (16) thus follows again from Proposition 2. Repeating the same argument for all \( t > 2 \) establishes the result. ■

**Proof of Proposition 10.** Start with \( t = 1 \). In the first period, information is exogenous with \( \omega_{i,1} = (x_{i,1}, y_{1}, A_1) \). Standard Gaussian updating then gives

\[
\mathbb{E}[\theta | \omega_{i,1}] = \frac{\pi^x}{\pi^x + \pi^y} X_{i,1} + \frac{\pi^y}{\pi^x + \pi^y} Y_1,
\]

where \( X_{i,1} = x_{i,1} \), \( \pi^x_1 = \sigma_{x,1}^{-2} \), \( Y_1 = \frac{\sigma_{\theta}^2}{\pi^y_1} \mu_0 + \frac{\sigma_{y}^2}{\pi^y_1} y_1 + \frac{\sigma_{a}^2}{\pi^y_1} A_1 \) and \( \pi^y_1 = \sigma_{\theta}^{-2} + \sigma_{y,1}^{-2} + \sigma_{a,1}^{-2} \). It is then easy to show that the unique solution to (16) is given by

\[
k_1(\omega_{i,1}) = \kappa_0 + \kappa_1 (\gamma_1 X_{i,1} + (1 - \gamma_1) Y_1),
\]

with \( \gamma_1 \equiv [(1 - \alpha) \pi^x_1] / [(1 - \alpha) \pi^x + \pi^y_1] \). (To see this, start by guessing that the equilibrium strategy satisfies (28) holds for some coefficient \( \gamma_1 \); next, use this guess to compute aggregate activity as \( K_1(\phi_1) = \kappa_0 + \kappa_1 (\gamma_1 \theta + (1 - \gamma_1) Y_1) \); finally, use the latter along with (16) and (27) to derive the best response to our initial guess. It is then immediate to check that the best response coincides with our initial guess—and hence that our initial guess is correct—if and only if \( \gamma_1 \) takes the aforementioned value.)

Next, consider \( t = 2 \). In the second period, \( \omega_{i,2} = \omega_{i,1} \cup (x_{i,2}, y_2, A_2, \tilde{K}_1, \tilde{\sigma}_1) \). The endogenous signal is given by

\[
\tilde{K}_1 = \kappa_0 + \kappa_1 (\gamma_1 \theta + (1 - \gamma_1) Y_1) + \eta_2
\]

Clearly, the information about \( \theta \) contained in \( \tilde{K}_1 \) is the same as that contained in

\[
\tilde{y}_2 \equiv \frac{\tilde{K}_1 - \kappa_0 - \kappa_1 (1 - \gamma_1) Y_1}{\kappa_1 \gamma_1} = \theta + \tilde{\eta}_2.
\]
where \( \tilde{\eta}_2 = \eta_2 / (\kappa_1 \gamma_1) \) is Gaussian noise with variance \( \sigma_{\eta,2}^2 = \sigma_{\eta,2}^2 / \kappa_1^2 \gamma_1^2 \). The signal \( \tilde{\sigma}_1 \), on the other hand, conveys no information about \( \theta \), because (28) implies that \( \sigma_1 = \kappa_1^2 \gamma_1^2 \sigma_{x,1}^2 \), which is common knowledge. It follows that the period-2 public information about \( \theta \) can be summarized in a sufficient statistic \( Y_2 \) such that the posterior about \( \theta \) conditional on \((y_1, A_1, \tilde{K}_1, \tilde{\sigma}_1, y_2, A_2)\) is Gaussian with mean

\[
Y_2 = \frac{\pi_1^y}{\pi_2^y} Y_1 + \frac{\sigma_{y,2}^2}{\pi_2^y} y_2 + \frac{\sigma_{a,2}^2}{\pi_2^y} A_2 + \frac{\gamma_1^2 \kappa_1^2 \sigma_{\eta,2}^2}{\pi_2^y} \]

and precision \( \pi_2^y = \pi_1^y + \sigma_{y,2}^2 + \sigma_{a,2}^2 + \gamma_1^2 \kappa_1^2 \sigma_{\eta,2}^2 \). Similarly, the private information can be summarized in the sufficient statistic \( X_{i,2} \) such that the posterior about \( \theta \) conditional on \((x_{i,1}, x_{i,2})\) is Gaussian with mean

\[
X_{i,2} = \frac{\pi_1^x}{\pi_2^x} X_{i,1} + \frac{\sigma_{x,1}^2}{\pi_2^x} x_{i,2}
\]

and precision \( \pi_2^x = \pi_1^x + \sigma_{x,2}^2 \). The unique solution to (16) is then given by

\[
k_2(\omega_{i,2}) = \kappa_0 + \kappa_1 (\gamma_2 X_{i,2} + (1 - \gamma_2) Y_2),
\]

with \( \gamma_2 \equiv [(1 - \alpha) \pi_2^x] / [(1 - \alpha) \pi_2^x + \pi_2^y] \).

It is immediate that the construction for \( t = 2 \) applies also to any \( t \geq 3 \) with the statistics \( X_{i,t} \) and \( Y_t \) defined recursively as in the proposition. We conclude that the unique equilibrium strategy is

\[
k_{i,t}(\omega_{i,t}) = \kappa_0 + \kappa_1 (\gamma_t X_{i,t} + (1 - \gamma_t) Y_t),
\]

with \( \gamma_t \equiv [(1 - \alpha) \pi_t^x] / [(1 - \alpha) \pi_t^x + \pi_t^y] \).

**Proof of Proposition 11.** We prove the result in two steps. Part (i) characterizes the efficient linear strategy in the absence of payoff externalities; this helps isolate the role of informational externalities. Part (ii) then extends the result to general payoff structures.

**Part (i).** Suppose \( V(k, K, \sigma, \theta) \) does not depend on \((K, \sigma)\) and, without any further loss of generality, let

\[
V(k, K, \sigma, \theta) = -(k - \theta)^2.
\]

Let \( h_t = \{y_1, A_1, \tilde{K}_1, ..., y_{t-1}, A_{t-1}, \tilde{K}_{t-1}, y_t, A_t\} \) denote the public history in period \( t \) and suppose agents follow a strategy \( k = \{k_t\}_{t=1}^T \) such that

\[
k_t(\omega_{i,t}) = P_t(h_t) + \sum_{\tau=1}^t Q_{i,\tau} x_{i,\tau},
\]

where \( P_t(h_t) \) is a deterministic function of \( h_t \) and \( Q_{t,\tau} \) are deterministic coefficients. It follows that

\[
k_{i,t} = P_i + \gamma_t \theta + \sum_{\tau=1}^t Q_{i,\tau} \xi_{i,\tau},
\]

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and hence $\tilde{K}_t = P_t + \gamma_t \theta + \eta_{t+1}$, where $P_t$ is a shortcut for $P_t(h_t)$ and $\gamma_t$ is defined as

$$\gamma_t \equiv \sum_{\tau=1}^{t} Q_{t,\tau}.$$ 

Next consider welfare. Given any linear strategy, ex-ante utility is $\mathbb{E}u = \sum_{t=1}^{T} w_t$, where $w_t \equiv \mathbb{E}[v(k_{i,t}, A_{t+1})] = \mathbb{E}[v(k_{i,t}, \theta)] - \sigma_a^2 e_{i,t+1}$ and where

$$\mathbb{E}[v(k_{i,t}, \theta)] = \mathbb{E}\left[\mathbb{E}\left[-\left\{P_t + \gamma_t \theta + \sum_{\tau=1}^{t} Q_{t,\tau} \xi_{i,\tau}\right\} - \theta\right] \mid \theta, h_t\right]$$

$$= \mathbb{E}\left[-(P_t + \gamma_t \theta - \theta)^2 - \sum_{\tau=1}^{t} Q_{t,\tau}^2 \sigma_{\xi,\tau}^2\right]$$

Now consider a strategy $\hat{k} = \{\hat{k}_t\}_{t=1}^{T}$ that is a variation of the initial strategy $k = \{k_t\}_{t=1}^{T}$ constructed as follows. First, pick an arbitrary $t$ and let $\hat{k}_{i,s}(\omega_{i,s}) = k_{i,s}(\omega_{i,s})$ for all $s < t$. Next, in period $t$, pick an arbitrary function $\hat{P}_t$ and any coefficients $\hat{Q}_{t,\tau}$ such that $\sum_{\tau=1}^{t} \hat{Q}_{t,\tau} = \gamma_t$, and let

$$\hat{k}_t(\omega_{i,t}) = \hat{P}_t(h_t) + \sum_{\tau=1}^{t} \hat{Q}_{t,\tau} x_{i,\tau}.$$ 

Finally, for all $s > t$, let

$$\hat{k}_s(\omega_{i,s}) = \hat{P}_s(h_s) + \sum_{\tau=1}^{s} Q_{s,\tau} x_{i,\tau},$$

where the functions $\hat{P}_s$ are such that

$$\hat{P}_s \left(\ldots, \hat{K}_t, \ldots\right) = \hat{P}_s \left(\ldots, \hat{K}_t - \hat{P}_t(h_t) + P_t(h_t), \ldots\right).$$

By construction, at any period $s \neq t$, the strategy $\hat{k}$ induces the same outcomes, and by implication the same per-period welfare level $w_t$, as the initial strategy $k$. It follows that a necessary condition for the strategy $k$ to be efficient is that, for all $t$ and all $h_t$,

$$(P_t, (Q_{t,\tau})_{\tau=1}^{t}) \in \mathop{\arg\max}_{\hat{P}_t, \hat{Q}_{t,\tau}} \mathbb{E}\left[-(\hat{P}_t + \gamma_t \theta - \theta)^2 - \sum_{\tau=1}^{t} \hat{Q}_{t,\tau}^2 \sigma_{\xi,\tau}^2 \right] \mid h_t$$

s.t. $\sum_{\tau=1}^{t} \hat{Q}_{t,\tau} = \gamma_t$

This in turn is the case if and only if, for all $t$ and all $h_t$

$$P_t(h_t) = (1 - \gamma_t) \mathbb{E}[\theta | h_t] \quad \text{and} \quad Q_{t,\tau} = \gamma_t \frac{\sigma_{\xi,\tau}^2}{\sum_{j=1}^{\tau} \sigma_{\xi,j}^2} \forall \tau.$$ (29)

Next note that, because $P_t$ is public information, the observation in period $t + 1$ of $\tilde{K}_t = K_t + \eta_t = P_t + \gamma_t \theta + \eta_t$ is informationally equivalent to the observation of a signal

$$\tilde{y}_{t+1} \equiv \frac{\tilde{K}_t - P_t}{\gamma_t} = \theta + \tilde{\eta}_{t+1}.$$ (30)
where $\eta_{t+1} = \eta_{t+1}/\gamma_t$ is Gaussian noise with precision $\sigma_{\eta_{t+1}}^{-2} = \gamma_t^2 \sigma_{\eta_{t+1}}^{-2}$. It follows that, given any linear strategy, the common posterior about $\theta$ in period $t$ is Gaussian with mean $\mathbb{E}[\theta|\eta_t] = Y_t$ and precision $\pi_t^y$, where $Y_t$ and $\pi_t^y$ are defined recursively by

$$Y_t = \frac{\pi_{t-1}^x}{\pi_t^x} X_{t-1} + \frac{\sigma_{x,t}^{-2}}{\pi_t^x} x_t + \frac{\sigma_{\eta,t}^{-2}}{\pi_t^x} \eta_t \quad \text{and} \quad \pi_t^x = \pi_{t-1}^x + \frac{\sigma_{x,t}^{-2}}{\pi_t^x},$$

$$\pi_t^y = \frac{\pi_{t-1}^y}{\pi_t^y} Y_{t-1} + \frac{\sigma_{y,t}^{-2}}{\pi_t^y} y_t + \frac{\sigma_{\eta,t}^{-2}}{\pi_t^y} \eta_t \quad \text{and} \quad \pi_t^y = \pi_{t-1}^y + \frac{\sigma_{y,t}^{-2}}{\pi_t^y} + \gamma_{t-1}^2 \sigma_{\eta,t},$$

with initial conditions $Y_1 = \mu_0$ and $\pi_1^y = \sigma_0^{-2}$. Similarly, the private posteriors are Gaussian with mean

$$\mathbb{E}[\theta|\omega_{i,t}] = \frac{\pi_t^x}{\pi_t^x + \pi_t^y} X_{i,t} + \frac{\pi_t^y}{\pi_t^x + \pi_t^y} Y_t,$$

and precision $\pi_t = \pi_t^x + \pi_t^y$, where

$$X_{i,t} = \frac{\pi_{t-1}^x}{\pi_t^x} X_{i,t-1} + \frac{\sigma_{x,t}^{-2}}{\pi_t^x} x_{i,t} \quad \text{and} \quad \pi_t^x = \pi_{t-1}^x + \frac{\sigma_{x,t}^{-2}}{\pi_t^x},$$

with initial conditions $X_{i,1} = x_{i,1}$ and $\pi_1^x = \sigma_{x,1}^{-2}$.

Now note that

$$X_{i,t} = \sum_{\tau=1}^s \sum_{j=1}^{l_t} \sigma_{x,\tau}^{-2} x_{i,\tau},$$

which together with (29) gives

$$\sum_{\tau=1}^s Q_{t,\tau} x_{i,\tau} = \gamma_t X_{i,t}.$$

We conclude that a linear strategy $k$ maximizes ex-ante utility only if, for all $t$ and all $\omega_{i,t}$,

$$k_{i,t}(\omega_{i,t}) = (1 - \gamma_t) Y_t + \gamma_t X_{i,t},$$

for some coefficient $\gamma_t \in \mathbb{R}$.

To determine the optimal $\gamma$'s, note that, when agents follow a strategy as in (31),

$$\mathbb{E}[v(k_{i,t}, \theta)] = \mathbb{E}[v(k_{i,t}, \theta)] = -\left\{ \gamma_t^2 (\pi_t^x)^{-1} + (1 - \gamma_t)^2 (\pi_t^y)^{-1} \right\}$$

and hence

$$\mathbb{E}u = W_{FB} - \sum_{t=1}^T \beta^{t-1} \left\{ \gamma_t^2 (\pi_t^x)^{-1} + (1 - \gamma_t)^2 (\pi_t^y)^{-1} \right\},$$

where $W_{FB}$ is the first-best level of welfare (the one obtained under complete information about $\theta$). Because the evolution of $\pi_t^y$ does not depend on $\gamma_t$, the choice of the optimal linear strategy reduces to the following problem:

$$\min_{\{\gamma_t\}} \sum_{t=1}^T \beta^{t-1} \left\{ \gamma_t^2 (\pi_t^x)^{-1} + (1 - \gamma_t)^2 (\pi_t^y)^{-1} \right\}$$

s.t. $\pi_{t+1}^y = \pi_t^y + \Delta_t + \sigma_{\eta,t}^{-2} \gamma_t^2 \forall t$.
with initial condition $\pi^y_1 = \sigma^2_\theta$, where $\Delta_t \equiv \sigma^2_{\epsilon,t} + \sigma^2_{\alpha,t}$ is the exogenous change in the precision of public information.

Consider the value functions $L_t : \mathbb{R}^2_+ \to \mathbb{R}_+$ defined by

$$L_t(\pi^x_t, \pi^y_t) \equiv \min_{\{\gamma_s\}_{s=t}} \sum_{s=t}^T \beta^{s-t} \left\{ \gamma^2_t (\pi^x_s)^{-1} + (1 - \gamma_t)^2 (\pi^y_s)^{-1} \right\}$$

s.t. $\pi^y_{s+1} = \pi^y_s + \Delta_s + \sigma^2_{\eta,s} \gamma_s \forall s \geq t$

For all $t \leq T$, $L_t(\pi^x_t, \pi^y_t)$ must satisfy

$$L_t(\pi^x_t, \pi^y_t) = \min_{\gamma^*_t} \left\{ \gamma^2_t (\pi^x_t)^{-1} + (1 - \gamma_t)^2 (\pi^y_t)^{-1} + \beta L_{t+1} (\pi^x_{t+1}, \pi^y_{t+1}) \right\}$$

s.t. $\pi^y_{t+1} = \pi^y_t + \Delta_t + \gamma^2_t$

and hence the optimal $\gamma_t$ is the solution to the following FOC

$$\gamma_t (\pi^x_t)^{-1} - (1 - \gamma_t) (\pi^y_t)^{-1} + \frac{1}{2} \frac{\partial L_{t+1}}{\partial \pi^y_{t+1}} \frac{\partial \pi^y_{t+1}}{\partial \gamma_t} = 0.$$

>From the envelope theorem,

$$\frac{\partial L_{t+1}}{\partial \pi^y_{t+1}} = - (1 - \gamma_{t+1})^2 (\pi^y_{t+1})^{-2}.$$

Finally, from the low of motion for $\pi^y_t$,

$$\frac{\partial \pi^y_{t+1}}{\partial \gamma_t} = 2\sigma^{-2}_{\eta,t+1} \gamma_t.$$

It follows that, for all $t \leq T - 1$, the optimal $\gamma_t$ satisfies

$$\gamma^{**}_t = \frac{\pi^x_t}{\pi^y_t + \pi^x_t} - \beta (1 - \gamma^{**}_{t+1})^2 \pi^x_t \pi^y_t (\pi^y_{t+1})^{-2} \sigma^{-2}_{\eta,t+1} > \frac{\pi^x_t}{\pi^y_t + \pi^x_t}.$$

Finally, for $t = T$, $\gamma^*_T = \pi^x_T / (\pi^x_T + \pi^y_T)$, simply because this is the last period and hence there is no more an informational externality.

Part (ii). Consider now the more general payoffs $V$ and let $\kappa^*(\theta) = \arg \max_{\kappa} V(\kappa, \kappa, 0, \theta)$. A similar argument as in part (i) ensures that the efficient linear strategy must satisfy

$$k_t(\omega_t) = \kappa^*_0 + \kappa^*_1 (\gamma_t X_t + (1 - \gamma_t) Y_t),$$

for some $\gamma_t$, with $X_t$ and $Y_t$ are the relevant sufficient statistics of available private and public information. Ex ante utility is then given by

$$\mathbb{E}u = W_{FB} + \sum_{t=1}^T \beta^{t-1} \kappa^* \left\{ \frac{W_{\alpha\sigma}}{2} (\gamma_t)^2 (\pi^x_t)^{-1} + \frac{W_{KK}}{2} (1 - \gamma_t)^2 (\pi^y_t)^{-1} \right\},$$

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where $W_{FB} \equiv \sum_{t=1}^{T} \beta^{t-1} W(\kappa^*(\theta), 0, \theta)$ is the first-best level of welfare. Using the fact that $W_{\sigma\sigma} < 0$, $W_{KK} < 0$, and $W_{KK}/W_{\sigma\sigma} = 1 - \alpha^*$, we conclude that the optimal $\gamma$’s must solve the following problem:

$$
\min_{\{\gamma_t\}} \sum_{t=1}^{T} \beta^{t-1} \left\{ \gamma_t^2 (\pi_t^y)^{-1} + (1 - \alpha^*) (1 - \gamma_t)^2 (\pi_t^y)^{-1} \right\}
$$

s.t. $\pi_{t+1}^y = \pi_t^y + \Delta_t + (\kappa_t^*)^2 \sigma_{\eta_t}^{-2} \gamma_t^2 \forall t$

Letting $L_t(\pi_t^x, \pi_t^y)$ denote the associated value function in period $t$, we have

$$
L_t(\pi_t^x, \pi_t^y) = \min_{\gamma_t} \left\{ \gamma_t^2 (\pi_t^y)^{-1} + (1 - \alpha^*) (1 - \gamma_t)^2 (\pi_t^y)^{-1} + \beta L_{t+1}(\pi_{t+1}^x, \pi_{t+1}^y) \right\}
$$

s.t. $\pi_{t+1}^y = \pi_t^y + \Delta_t + (\kappa_t^*)^2 \sigma_{\eta_{t+1}}^{-2} \gamma_t^2$

The FOC for $\gamma_t$ gives

$$
\gamma_t (\pi_t^y)^{-1} - (1 - \alpha^*) (1 - \gamma_t) (\pi_t^y)^{-1} + \frac{1}{2} \beta \frac{\partial L_{t+1}}{\partial \pi_{t+1}^y} \frac{\partial \pi_{t+1}^y}{\partial \gamma_t} = 0.
$$

The envelope condition for $\pi_{t+1}^y$ gives

$$
\frac{\partial L_t}{\partial \pi_{t+1}^y} = -(1 - \alpha^*) (1 - \gamma_{t+1})^2 (\pi_{t+1}^y)^{-2},
$$

while the law of motion for $\pi_{t+1}^y$ gives

$$
\frac{\partial \pi_{t+1}^y}{\partial \gamma_t} = 2 (\kappa_{t+1}^*)^2 \sigma_{\eta_{t+1}}^{-2} \gamma_t.
$$

It follows that the optimal $\gamma$’s satisfy

$$
\gamma_t^{**} = \frac{(1 - \alpha^*) \pi_t^y}{\pi_t^x + (1 - \alpha^*) \pi_t^x - \beta (1 - \alpha^*) (1 - \gamma_{t+1}^{**})^2 \pi_t^x \pi_{t+1}^y (\pi_{t+1}^y)^{-2} (\kappa_{t+1}^*)^2 \sigma_{\eta_{t+1}}^{-2}},
$$

which completes the proof.

**Proof of Proposition 12.** Let $\{\gamma_t^{**}\}$ be the coefficients that characterize the efficient linear strategy as in Proposition 11 and let $\{\pi_t^x, \pi_t^y\}$ be the corresponding precisions of private and public information generated by the efficient linear strategy. The result then follows from letting $\alpha_t^{**}$ be the unique solution to

$$
\frac{(1 - \alpha_t^{**}) \pi_t^y}{(1 - \alpha_t^{**}) \pi_t^x + \pi_t^y} = \gamma_t^{**}.
$$

In fact, it is then and only then the unique solution to (21) coincides with the strategy obtained in Proposition 11.

**Proof of Corollary 4.** Consider the environments with both exogenous and endogenous Gaussian signals studied in Section 7. The result follows directly from the proof of Proposition 34.
12, where it is shown that, for all periods $t$, the present-value welfare losses $L_t$ obtained along the efficient linear strategy are decreasing functions of $\pi_t^x$ and $\pi_t^y$, the precisions of private and public information available in the beginning of period $t$. (Putting aside informational externalities, the result can also be established for environments with non-Gaussian signals using a Blackwell-like argument for the planner’s problem that characterizes the efficient strategy; the details of this argument are available upon request.) ■
References


