Fear of miscoordination and the robustness of cooperation

Sylvain Chassang, MIT
Motivation

Dynamic cooperation:

- Short term conflicts of interest resolved by using promise of future cooperation to align incentives
Motivation

Dynamic cooperation:
- Short term conflicts of interest resolved by using promise of future cooperation to align incentives

Benchmark (tractable) model of cooperation:
- Common knowledge of past actions
- Common knowledge of payoffs
Motivation

Dynamic cooperation:
- Short term conflicts of interest resolved by using promise of future cooperation to align incentives

Benchmark (tractable) model of cooperation:
- Common knowledge of past actions
- Common knowledge of payoffs

This benchmark corresponds to a situation in which coordination is very easy.

Missing intuition
- In number of cases, cooperation fails not because large deviation temptation, but rather because of insufficient confidence in partner’s commitment to cooperation.
Relaxing the assumption of CK payoffs in a class of simple cooperation games

Exit games

- Infinite horizon two player games
- Each period players either Cooperate or Defect. Defection causes game to end (Exit).
Relaxing the assumption of CK payoffs in a class of simple cooperation games

Exit games
- Infinite horizon two player games
- Each period players either Cooperate or Defect. Defection causes game to end (Exit).

Private information as a model of miscoordination risk
- Payoff relevant state of the world $\omega_t$ (i.i.d.)
- Players get noisy signals $x_{i,t} = \omega_t + \sigma \varepsilon_{i,t}$
Goal for the talk: Characterize set of PBEs as $\sigma \to 0$

Step 1: show exit games not supermodular, but very structured

- Restricted monotone best response
- Existence of extreme Markovian equilibria

Step 2: Characterize impact of private information on Markovian equilibria

- Take advantage of stationarity
- Combine dynamic programming and global games
Goal for the talk: Characterize set of PBEs as $\sigma \rightarrow 0$

Step 1: show exit games not supermodular, but very structured
- Restricted monotone best response
- Existence of extreme Markovian equilibria
Goal for the talk: Characterize set of PBEs as $\sigma \to 0$

Step 1: show exit games not supermodular, but very structured
- Restricted monotone best response
- Existence of extreme Markovian equilibria

Step 2: Characterize impact of private information on Markovian equilibria
- Take advantage of stationarity
- combine dynamic programming and global games
Outline of the presentation

I. Framework and assumptions

II. Characterizing the range of perfect Bayesian equilibria

III. Further results

IV. Conclusion
Outline of the presentation

I. Framework and assumptions
II. Characterizing the range of perfect Bayesian equilibria
Outline of the presentation

I. Framework and assumptions
II. Characterizing the range of perfect Bayesian equilibria
III. Further results
Outline of the presentation

I. Framework and assumptions
II. Characterizing the range of perfect Bayesian equilibria
III. Further results
IV. Conclusion
The framework: exit games

- Two players \( i \in \{1, 2\} \), infinite horizon \( t \in \{1, \ldots, \infty\} \), discount rate \( \beta \in (0, 1) \). Two actions \( A = \{\text{Stay, Exit}\} \).
- If \((S, S)\) then \( t \to t + 1 \). If any player chooses Exit, game ends, continuation values included in flow payoffs.
- Flow payoffs (cont. in \( \omega_t \)),

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>(g^i(\omega_t))</td>
<td>(W^i_{12}(\omega_t))</td>
</tr>
<tr>
<td>E</td>
<td>(W^i_{21}(\omega_t))</td>
<td>(W^i_{22}(\omega_t))</td>
</tr>
</tbody>
</table>

- \(\omega_t \in \mathbb{R}\) is some i.i.d. state of the world, density \( f \) with convex support.
Information structure

Private noisy signals

- State $\omega_t$ unobserved at $t$
- Period $t$, player $i$ gets signal $x_{i,t} = \omega_t + \sigma \varepsilon_{i,t}$
- $\omega_t$ observed at $t + 1$.

Study game for $\sigma$ small
Example: a partnership game

Flow payoffs:

<table>
<thead>
<tr>
<th></th>
<th>$S$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$\omega_t$</td>
<td>$\omega_t - C + \beta V_E$</td>
</tr>
<tr>
<td>$E$</td>
<td>$b + V_E$</td>
<td>$V_E$</td>
</tr>
</tbody>
</table>

- $\omega_t \sim f$ with support $\mathbb{R}$, and $E|\omega_t| < \infty$
- $C > b \geq 0$
- $V_E > 0$, outside option
The partnership game under full information

- Full information Pareto optimal equilibrium characterized by $\omega$ and $\bar{V}$:

\[
\omega + \beta \bar{V} = V_E + b \\
\bar{V} = E[(\omega_t + \beta \bar{V})1_{\omega_t>\omega} + V_E1_{\omega_t<\omega}].
\]

- Parameter $C$ does not affect Pareto efficient equilibrium under full information.

<table>
<thead>
<tr>
<th></th>
<th>$S$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$\omega_t$</td>
<td>$\omega_t - C + \beta V_E$</td>
</tr>
<tr>
<td>$E$</td>
<td>$b + V_E$</td>
<td>$V_E$</td>
</tr>
</tbody>
</table>
Some notations

- **History** \( h_{i,t} \in \mathcal{H} \): \( h_{i,t} = (x_{i,1}, \ldots, x_{i,t}, \omega_1, \ldots, \omega_{t-1}) \)
- **Denote** \( V_i(h_{i,t}) \) value expected by player \( i \) at \( h_{i,t} \)
- **For mappings** \( U_i \) and \( U_{-i} : \mathbb{R} \leftrightarrow \mathbb{R} \), denote by \( G(U_i, U_{-i}, \omega_t) \) the full information one-shot game

\[
\begin{array}{c|cc}
 & S & E \\
\hline
S & g^i(\omega_t) + \beta U_i(\omega_t) & W^i_{12}(\omega_t) \\
E & W^i_{21}(\omega_t) & W^i_{22}(\omega_t)
\end{array}
\]
Important assumptions, and why they are reasonable

5 assumptions: Assumptions 2, 3, 4 standard in global games literature, Assumptions 1 and 5 specific to this dynamic setting

**Assumption 1 (boundedness).**

*In the full information game, each player $i$ has finite min-max and maximum values $m_i$ and $M_i$*

- In partnership game, $m_i \geq V_E$ and $M_i < +\infty$
Assumption 2 (increasing differences in the state of the world).

Both $g^i(\omega) - W^i_{21}(\omega)$ and $W^i_{12}(\omega) - W^i_{22}(\omega)$ increasing in $\omega$. 

<table>
<thead>
<tr>
<th></th>
<th>$S$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$g^i(\omega)$</td>
<td>$W^i_{12}(\omega)$</td>
</tr>
<tr>
<td>$E$</td>
<td>$W^i_{21}(\omega)$</td>
<td>$W^i_{22}(\omega)$</td>
</tr>
</tbody>
</table>
Assumption 3 (equilibrium symmetry).

For all $\omega$, $G(m_i, m_{-_i}, \omega)$ has a pure equilibrium. Pure equilibria are symmetric.

\[
\begin{array}{c|cc}
 & S & E \\
\hline
S & g^i(\omega) + \beta m_i & W^i_{12}(\omega) \\
E & W^i_{21}(\omega) & W^i_{22}(\omega)
\end{array}
\]
Assumption 4 (dominance regions).

There exist $\omega$ and $\bar{\omega}$ such that

$$g^i(\omega) + \beta M_i - W^i_{21}(\omega) \leq 0 \quad \text{and} \quad W^i_{12}(\bar{\omega}) - W^i_{22}(\bar{\omega}) \geq 0.$$ 

- State $\bar{\omega}$ high enough that staying is dominant
- State $\omega$ low enough that exit is dominant
Assumption 5 (staying is good).

For all $\omega \in [\underline{\omega}, \bar{\omega}]$,

\[
g^i(\omega) + \beta m_i - W^i_{12}(\omega) \geq 0 \quad (1)
\]

and

\[
W^i_{21}(\omega) - W^i_{22}(\omega) \geq 0 \quad (2)
\]

Implication: better off best facing a partner that Stays more.

<table>
<thead>
<tr>
<th></th>
<th>$S$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$g^i(\omega) + \beta m_i$</td>
<td>$W^i_{12}(\omega)$</td>
</tr>
<tr>
<td>$E$</td>
<td>$W^i_{21}(\omega)$</td>
<td>$W^i_{22}(\omega)$</td>
</tr>
</tbody>
</table>
Putting structure on Exit games

Pure strategy: \( s : \mathcal{H} \mapsto \{S, E\} \)

**Definition 1 (Markovian strategies).**

- \( s \) Markovian \( \iff \forall h_{i,t}, s(h_{i,t}) = s(x_{i,t}) \)
- \( s \) threshold-form Markovian:
  \( \exists x \in \mathbb{R} \text{ s.t. } s(h_{i,t}) = S \iff x_{i,t} > x \)
Putting structure on Exit games

Pure strategy: \( s : \mathcal{H} \mapsto \{S, E\} \)

**Definition 1 (Markovian strategies).**

\[
\begin{align*}
\triangleright \text{ } s \text{ Markovian } &\iff \forall h_i, t, s(h_i, t) = s(x_i, t) \\
\triangleright \text{ } s \text{ threshold-form Markovian: } &\exists x \in \mathbb{R} \text{ s.t. } s(h_i, t) = S \iff x_i, t > x
\end{align*}
\]

**Definition 2.**

*Partial order \( \preceq \) on strategies*

\[
s' \preceq s \iff \{ \forall h, s'(h) = S \Rightarrow s(h) = S \}.
\]

*Set of all strategies lattice wrt \( \preceq \).*
Exit games are nicely behaved

Denote $BR_{i,\sigma}$ best reply

**Lemma 1 (Restricted monotone BR).**

There exists $\bar{\sigma} > 0$ such that for all $\sigma \in (0, \bar{\sigma})$, if $s$ Markovian, then for all $s'$,

$$s' \preceq s \Rightarrow BR_{i,\sigma}(s') \preceq BR_{i,\sigma}(s) \quad \text{and} \quad s \preceq s' \Rightarrow BR_{i,\sigma}(s) \preceq BR_{i,\sigma}(s').$$
Exit games are nicely behaved

Denote $BR_{i, \sigma}$ best reply

**Lemma 1 (Restricted monotone BR).**

There exists $\sigma > 0$ such that for all $\sigma \in (0, \sigma)$, if $s$ Markovian, then for all $s'$,

$$s' \preceq s \Rightarrow BR_{i, \sigma}(s') \preceq BR_{i, \sigma}(s) \quad \text{and} \quad s \preceq s' \Rightarrow BR_{i, \sigma}(s) \preceq BR_{i, \sigma}(s').$$

**Proposition 1 (Extreme Markovian equilibria).**

(i) Set of rationalizable strategies has largest and smallest element $s^H_\sigma$ and $s^L_\sigma$

(ii) These are Markovian equilibria with thresholds $x^H_\sigma$ and $x^L_\sigma$

(iii) They are associated with highest and lowest continuation values $V^H_\sigma$ and $V^L_\sigma$
Sketch of a proof: restricted monotone BR enough for Milgrom-Roberts (1990) construction

- Always staying $S$ and always exiting $E$ are Markovian, threshold strategies
- $[E, S]$ set of all possible strategies
- $BR(E)$ and $BR(S)$ are Markovian, threshold form strategies
- Iterate forward...
- $BR^\infty(E, S) \subset BR^\infty(E), BR^\infty(S)$
Sketch of a proof: restricted monotone BR enough for Milgrom-Roberts (1990) construction

- Always staying \( S \) and always exiting \( E \) are Markovian, threshold strategies
- \([E, S]\) set of all possible strategies
- Lemma 1 implies

\[
BR([E, S]) \subset [BR(E), BR(S)]
\]
Sketch of a proof: restricted monotone BR enough for Milgrom-Roberts (1990) construction

- Always staying $S$ and always exiting $E$ are Markovian, threshold strategies
- $[E, S]$ set of all possible strategies
- Lemma 1 implies

$$BR([E, S]) \subset [BR(E), BR(S)]$$

- $BR(E)$ and $BR(S)$ are Markovian, threshold form strategies
- Iterate forward . . .

$$BR^\infty([E, S]) \subset [BR^\infty(E), BR^\infty(S)]$$
Where do we stand?

Step 1: show exit games very structured

- Restricted monotone best response
- Existence of extreme Markovian equilibria
Where do we stand?

Step 1: show exit games very structured
  ▶ Restricted monotone best response
  ▶ Existence of extreme Markovian equilibria

Step 2: characterize impact of private information on these equilibria
Apply dynamic programming to extreme equilibria

- Focus on highest Markovian equilibrium
- Markovian $\Rightarrow$ constant value $V^H_{\sigma}$
- Actions of Markovian equilibrium must be Nash equilibria of

\[
\begin{array}{c|cc}
  & S & E \\ 
  S & g^i(\omega_t) + \beta V^H_{i,\sigma} & W^i_{12}(\omega_t) \\ 
  E & W^i_{21}(\omega_t) & W^i_{22}(\omega_t) \\
\end{array}
\]

where players get signals $x_{i,t} = \omega_t + \sigma \epsilon_{i,t}$
Apply dynamic programming to extreme equilibria

- Focus on highest Markovian equilibrium
- Markovian $\Rightarrow$ constant value $V^H_{\sigma}$
- Actions of Markovian equilibrium must be Nash equilibria of

\[
\begin{array}{c|cc}
S & E \\
\hline
S & g^i(\omega_t) + \beta V^H_{i,\sigma} & W^i_{12}(\omega_t) \\
E & W^i_{21}(\omega_t) & W^i_{22}(\omega_t)
\end{array}
\]

where players get signals $x_{i,t} = \omega_t + \sigma \varepsilon_{i,t}$

This is a global game à la Carlsson and van Damme (1993)... or almost
We use uniform selection results

∀ \( V_i \in [m_i, M_i] \), consider game \( \Psi_\sigma(V) \), with info \( x_i = \omega + \sigma \varepsilon_i \)

<table>
<thead>
<tr>
<th></th>
<th>( S )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>( g^i(\omega_t) + \beta V_i )</td>
<td>( W^i_{12}(\omega_t) )</td>
</tr>
<tr>
<td>( E )</td>
<td>( W^i_{21}(\omega_t) )</td>
<td>( W^i_{22}(\omega_t) )</td>
</tr>
</tbody>
</table>

Proposition 2. Joint selection: \( \exists \sigma > 0 \) s.t. for all \( \sigma \in (0, \sigma_0) \) and \( V_i \in [m_i, M_i] \), \( \Psi_\sigma(V) \) dominance solvable. Equilibrium threshold \( x^*_\sigma(V) \).

Uniform convergence: as \( \sigma \) goes to 0, \( x^*_\sigma(V) \) converges uniformly to risk-dominant threshold \( x_{RD}(V) \) of full info game.
We use uniform selection results

∀ \( V_i \in [m_i, M_i] \), consider game \( \Psi_\sigma(V) \), with info \( x_i = \omega + \sigma \varepsilon_i \)

\[
\begin{array}{c|cc}
 & S & E \\
\hline
S & g^i(\omega_t) + \beta V_i & W_{12}^i(\omega_t) \\
E & W_{21}^i(\omega_t) & W_{22}^i(\omega_t) \\
\end{array}
\]

**Proposition 2.**

**Joint selection:** \( \exists \bar{\sigma} > 0 \) s.t. for all \( \sigma \in (0, \bar{\sigma}) \) and \( V_i \in [m_i, M_i] \), \( \Psi_\sigma(V) \) dominance solvable. Equilibrium threshold \( x^*_\sigma(V) \).
We use uniform selection results

\[ \forall V_i \in [m_i, M_i], \text{consider game } \psi_\sigma(V), \text{ with info } x_i = \omega + \sigma \varepsilon_i \]

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>( g^i(\omega_t) + \beta V_i )</td>
<td>( W_{12}^i(\omega_t) )</td>
</tr>
<tr>
<td>E</td>
<td>( W_{21}^i(\omega_t) )</td>
<td>( W_{22}^i(\omega_t) )</td>
</tr>
</tbody>
</table>

**Proposition 2.**

**Joint selection:** \( \exists \bar{\sigma} > 0 \text{ s.t. for all } \sigma \in (0, \bar{\sigma}) \text{ and } V_i \in [m_i, M_i], \psi_\sigma(V) \text{ dominance solvable. Equilibrium threshold } x^*_\sigma(V). \)**

**Uniform convergence:** \( \text{as } \sigma \text{ goes to 0, } x^*_\sigma(V) \text{ converges uniformly to risk-dominant threshold } x^{RD}(V) \text{ of full info game.} \)**
The value mapping $\phi_\sigma$

Joint selection allows to define mapping $\phi_\sigma : \mathbb{R}^2 \rightarrow \mathbb{R}^2$:

- Given $V \in [m_i, M_i] \times [m_{-i}, M_{-i}]$ consider $\Psi_\sigma(V)$

<table>
<thead>
<tr>
<th></th>
<th>$S$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$g_i^i(\omega_t) + \beta V_i$</td>
<td>$W_{12}^i(\omega_t)$</td>
</tr>
<tr>
<td>$E$</td>
<td>$W_{21}^i(\omega_t)$</td>
<td>$W_{22}^i(\omega_t)$</td>
</tr>
</tbody>
</table>
The value mapping $\phi_\sigma$

Joint selection allows to define mapping $\phi_\sigma : \mathbb{R}^2 \rightarrow \mathbb{R}^2$:

- Given $V \in [m_i, M_i] \times [m_{-i}, M_{-i}]$ consider $\Psi_\sigma(V)$
- Joint selection $\Rightarrow \Psi_\sigma(V)$ has unique equilibrium $x^*_\sigma(V)$

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>$g^i(\omega_t) + \beta V_i$</td>
<td>$W^i_{12}(\omega_t)$</td>
</tr>
<tr>
<td>E</td>
<td>$W^i_{21}(\omega_t)$</td>
<td>$W^i_{22}(\omega_t)$</td>
</tr>
</tbody>
</table>
The value mapping $\phi_\sigma$

Joint selection allows to define mapping $\phi_\sigma : \mathbb{R}^2 \rightarrow \mathbb{R}^2$:

- Given $\mathbf{V} \in [m_i, M_i] \times [m_{-i}, M_{-i}]$ consider $\Psi_\sigma(\mathbf{V})$
- Joint selection $\Rightarrow \Psi_\sigma(\mathbf{V})$ has unique equilibrium $\mathbf{x}_\sigma^*(\mathbf{V})$
- $\phi_\sigma(\mathbf{V})$ value of playing $\Psi_\sigma(\mathbf{V})$ according to its unique equilibrium

<table>
<thead>
<tr>
<th></th>
<th>$S$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$g^i(\omega_t) + \beta \mathbf{V}_i$</td>
<td>$W^i_{12}(\omega_t)$</td>
</tr>
<tr>
<td>$E$</td>
<td>$W^i_{21}(\omega_t)$</td>
<td>$W^i_{22}(\omega_t)$</td>
</tr>
</tbody>
</table>
Theorem 1.
There exists $\overline{\sigma} > 0$ such that for all $\sigma \in (0, \overline{\sigma})$

(i) $V^H_\sigma$ and $V^L_\sigma$ highest and lowest fixed points of $\phi_\sigma(\cdot)$
(ii) $V$ associated with Markovian equilibrium $\iff \phi_\sigma(V) = V$
Sketch of (ii)

- Fix $\sigma$ such that Joint Selection holds
- Markovian equilibrium $\implies$ constant value $V$
Sketch of (ii)

- Fix $\sigma$ such that Joint Selection holds
- Markovian equilibrium $\Rightarrow$ constant value $V$
- Actions must be Nash equilibria of $\psi_{\sigma}(V)$:

<table>
<thead>
<tr>
<th></th>
<th>$S$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$g^i(\omega_t) + \beta V_i$</td>
<td>$W_{12}^i(\omega_t)$</td>
</tr>
<tr>
<td>$E$</td>
<td>$W_{21}^i(\omega_t)$</td>
<td>$W_{22}^i(\omega_t)$</td>
</tr>
</tbody>
</table>
Sketch of (ii)

- Fix $\sigma$ such that Joint Selection holds
- Markovian equilibrium $\Rightarrow$ constant value $V$
- Actions must be Nash equilibria of $\Psi_\sigma(V)$:

<table>
<thead>
<tr>
<th></th>
<th>$S$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$g_i(\omega_t) + \beta V_i$</td>
<td>$W_{12}^i(\omega_t)$</td>
</tr>
<tr>
<td>$E$</td>
<td>$W_{21}^i(\omega_t)$</td>
<td>$W_{22}^i(\omega_t)$</td>
</tr>
</tbody>
</table>

- Stationarity $\Rightarrow$

  continuation value $= \text{value of playing augmented game}$
  
  $V = \phi_\sigma(V)$
Uniform selection: $x^*_\sigma(V) \to x^{RD}(V)$

**Theorem 2.**
As $\sigma$ goes to 0, $\phi_\sigma(\cdot) \to \Phi(\cdot)$ uniformly, where

$$
\Phi_i(V) = E \left[ \left( g^i + \beta V_i \right) 1_{\omega>x^{RD}(V)} + W_{i22}(W) 1_{\omega<x^{RD}(V)} \right]
$$
Characterizing Markovian equilibria

Uniform selection: \( x^*_\sigma(V) \to x^{RD}(V) \)

**Theorem 2.**

As \( \sigma \) goes to 0, \( \phi_\sigma(\cdot) \to \Phi(\cdot) \) uniformly, where

\[
\Phi_i(V) = \mathbb{E} \left[ \left( g^i + \beta V_i \right) 1_{\omega > x^{RD}(V)} + W_{22}(w) 1_{\omega < x^{RD}(V)} \right]
\]

**Note:** Characterization holds for every discount factor.
Characterizing Markovian equilibria

Uniform selection: \( x^*_\sigma(V) \to x^{RD}(V) \)

**Theorem 2.**
As \( \sigma \) goes to 0, \( \phi_\sigma(\cdot) \to \Phi(\cdot) \) uniformly, where

\[
\Phi_i(V) = \mathbb{E} \left[ \left( g^i + \beta V_i \right) 1_{\omega > x^{RD}(V)} + W^i_{22}(w) 1_{\omega < x^{RD}(V)} \right]
\]

**Note:** Characterization holds for every discount factor.

**Corollary:** \( \Phi \) has unique fixed point \( \Rightarrow \)
As \( \sigma \to 0 \), set of rationalizable strategies converges to singleton
Some questions

- Can there be multiple equilibria?
- What happens to dominance solvability?
- How to extend the usual risk-dominance criterion?
Can there be multiple equilibria?

Yes. The reason is that $V$ affects the threshold players use to decide whether to stay or exit.

$$\Phi_i(V) = \mathbb{E}\left[\left(g^i + \beta V_i\right)1_{\omega > x^{RD}(V)} + W^i_{22}(w)1_{\omega < x^{RD}(V)}\right]$$
Can there be multiple equilibria?

Yes. The reason is that $V$ affects the threshold players use to decide whether to stay or exit.

$$
\Phi_i(V) = E \left[ \left( g^i + \beta V_i \right) 1_{\omega > x^{RD}(V)} + W^i_2(w) 1_{\omega < x^{RD}(V)} \right]
$$

**Example:** partnership game

<table>
<thead>
<tr>
<th></th>
<th>$S$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$\omega_t$</td>
<td>$\omega_t - C + \beta V_E$</td>
</tr>
<tr>
<td>$E$</td>
<td>$b + V_E$</td>
<td>$V_E$</td>
</tr>
</tbody>
</table>

$\rightarrow x^{RD}(V)$ takes the form

$$
x^{RD}(V) = V_E + \frac{b + C}{2} - \beta \frac{V - V_E}{2}
$$
Simulations

Specifications: \( \omega \sim \mathcal{N}(3, \eta) \), \( V_E = 5 \), \( \beta = 0.7 \), \( C = 3 \), and \( b = 1 \)

\[
\eta = 0.2
\]
Pick $\mathbf{v}$ fixed point of $\Phi$ and $x = x^{RD}(\mathbf{v})$ the associated cooperation threshold.

Result: The stability of $\mathbf{v}$ with respect to $\Phi$ corresponds to the asymptotic stability of $s_x$ with respect to the iterated best-reply.
Pick $V$ fixed point of $\Phi$ and $x = x^{RD}(V)$ the associated cooperation threshold.

Result: The stability of $V$ with respect to $\Phi$ corresponds to the asymptotic stability of $s_x$ with respect to the iterated best-reply.

**Proposition 3.**

The following are equivalent:

(i) $V$ is a stable fixed point of $\Phi$.

(ii) there exist $x_-$ and $x_+$ so that $x \in (x_-, x_+)$ and

$$\lim_{\sigma \to 0} \lim_{k \to \infty} BR^k_\sigma([x_-, x_+]) = \{s_x\}$$
Games with approximately constant payoffs

Given distribution $f$ for $\omega$, study fixed points of $\Phi_f$. 

- Pick sequence $\{f_n\}_{n \in \mathbb{N}}$
- Point mass at $\omega_0$: $\delta_{\omega_0}$
- Define $V_H^n$ the highest fixed point of $\Phi_{f_n}$
- Define $V_{H0} = \frac{1}{1 - \beta} g(\omega_0)$

Proposition 4.

(i) If Staying RD in $G(\omega_0, V_{H0})$ then
$$\lim_{n \to \infty} V_{Hn} = V_{H0}$$

(ii) If Exiting RD in $G(\omega_0, V_{H0})$, then
$$\lim_{n \to \infty} V_{Hn} = W_{22}(\omega_0)$$
Games with approximately constant payoffs

Given distribution \( f \) for \( \omega \), study fixed points of \( \Phi_f \).

- Pick sequence \( \{f^n\}_{n \in \mathbb{N}} \rightarrow \) Point mass at \( \omega_0 : \delta_{\omega_0} \)
- Define \( V_n^H \) the highest fixed point of \( \Phi_{f_n} \)
- Define \( V_0^H = \frac{1}{1-\beta} g(\omega_0) \)
Games with approximately constant payoffs

Given distribution $f$ for $\omega$, study fixed points of $\Phi_f$.

- Pick sequence $\{f^n\}_{n \in \mathbb{N}} \rightarrow$ Point mass at $\omega_0 : \delta_{\omega_0}$
- Define $V^H_n$ the highest fixed point of $\Phi_{f_n}$
- Define $V^H_0 = \frac{1}{1-\beta}g(\omega_0)$

**Proposition 4.**

(i) If Staying RD in $G(\omega_0, V^H_0)$ then

$$\lim_{n \to \infty} V^H_n = V^H_0$$

(ii) If Exiting RD in $G(\omega_0, V^H_0)$, then

$$\lim_{n \to \infty} V^H_n = W_{22}(\omega_0)$$
Summary

Motivation

- Framework to assess impact of miscoordination fear on ability to cooperate
- First step towards extending the global games framework to repeated games
Summary

Motivation

- Framework to assess impact of miscoordination fear on ability to cooperate
- First step towards extending the global games framework to repeated games

Difficulties

- Setting with private information but not supermodular (Exit trick $\sim$ focus on trigger strategies)
- Technical tools required to combine dynamic programming (APS 90) and global games selection results (CvD 93).
Connections with literature

**Literature on dynamic global games**
- Here focus on dynamic incentives rather than on information dynamics
Connections with literature

**Literature on dynamic global games**
- Here focus on dynamic incentives rather than on information dynamics

**Literature on dynamic cooperation**
- Here, miscoordination due to private information.
- Some connexion with imperfect private monitoring (Mailath and Morris (2002, 2006))
  There, nature of the coordination problem depends on strategies used.
Further work

Applications

- How poverty affects agents’ ability to cooperate?
- Fear of miscoordination and firm collusion?
Further work

Applications
- How poverty affects agents’ ability to cooperate?
- Fear of miscoordination and firm collusion?

Theory
- Extension to fully repeated games.
- Explore connections with imperfect private monitoring