Supply signals, complementarities, and multiplicity in asset prices and information acquisition*

Jayant V Ganguli and Liyan Yang†

May 24, 2007

Abstract

Allowing speculation using private information on a stock's payoff and supply generates (i) complementarity in information acquisition and (ii) multiple equilibria in *the financial market and the information market* independently in a standard CARA-normal model. Information can be a complement irrespective of the financial market equilibrium coordinated upon and generates multiplicity in the information market. Multiplicity in the financial market exists irrespective of the information market equilibrium and is generated as traders seek to profit from information about the payoffs and the supply of the asset. The multiplicity and complementarity provides insights and explanations for market frenzies and other phenomenon. This extension also allows for general equilibrium analysis of the financial market suggesting that multiplicity is inherent to the CARA-normal framework.

*We thank several colleagues and audiences from Cornell, Penn State, and the 2006 NBER General Equilibrium and Fall 2006 Midwest Economic Theory meetings for comments and suggestions. All errors are our responsibility and we welcome any comments. **Keywords** CARA-normal, supply signals, complementarities, multiplicity. **JEL codes** D82, D83, G14.

†The authors are at the Department of Economics, Cornell University, Ithaca, NY 14850. Their e-mail addresses are <jvg6@cornell.edu> and <ly52@cornell.edu>, respectively.

1 Introduction

Do investors seek information that other investors have? Does the stock price always reveal more information when more investors with private information trade? Is this price uniquely predicted?

Noisy REE (rational expectations equilibria) models are commonly used to analyze trading in stocks by investors based on differences in information. The CARA-normal (constant-absolute-risk-aversion-normal) framework is possibly the most widely used framework for noisy REE modeling since it permits tractable analysis and closed-form solutions.

The answers to the above questions in most existing CARA-normal REE models have been no, yes, and yes respectively. We analyze a natural extension of a standard CARA-normal model (Grossman, 1976; Grossman and Stiglitz, 1980; Diamond and Verrecchia, 1981; and Admati, 1985) and find that the answers to the first question can in fact be positive and the answers to the last two negative.

In the CARA-normal framework, investors live for two periods, trading in risky and riskless assets in a financial market in the second period. In the first period, called the information market, the traders choose whether or not to acquire (costly) information about the stocks. In previous models, only information about the payoffs per unit invested in the stocks can be purchased.

Stock prices are affected by factors unrelated to the payoffs and traders often seek or possess information about these aspects, such as the (net) supply of stocks in the market in any trading period.[1] The (net) supply of the stocks is determined by the trading decisions of a variety of traders including those who possess information about the payoffs of the stocks and those who trade for reasons, such as liquidity, unrelated to the payoffs.

Traders who provide liquidity in the market (and may be informed about the payoffs) seek information about the liquidity demanded, typically to avoid trades

[1] This is distinguished from the shares outstanding for a firm, which is fixed and publicly known.

with anyone better informed than they are. Examples of information about the supply of stocks include the NYSE Open Book, information obtained by sentiment-oriented technical traders and frontrunners, information on the float maintained by investment banks, or information about the demand from liquidity traders for stock initial public offerings (IPO) that can be obtained from the book-building activity of investment banks, or information obtained by dealers or market-makers with access to the order book. Finally, information about the aggregate supply can also be naturally obtained by a trader from her endowment of the risky assets, as in a general equilibrium model of trade in assets. Hence, it is of interest to assess the effects of traders who possess information about supply of the stocks.

We allow each trader to independently buy information about the supply of the stocks, thus permitting private information about the supply. We term this the *supply signal* and term the signal about the payoff the *payoff signal*. We call the traders who purchase the signals *informed traders*.

We find that strategic complementarity in information acquisition can exist leading to multiple equilibria in the information market. When information is a strategic complement then a trader will want to acquire this information even when other traders have it which can lead to the multiplicity. We also find the financial market has two (linear) partially revealing financial market REE price functions, which have opposing properties with regard to information content. In fact, our extension allows us to uncover an equilibrium map of partially revealing REE prices which indicates that the uniqueness of equilibrium in previous analyses is not robust.

Our results are in stark contrast to most previous analyses of the CARA-normal model in the partial equilibrium framework by, inter alia, Grossman (1976), Grossman and Stiglitz (1980), and Admati (1985), and in the general equilibrium framework by Diamond and Verrecchia (1981) where information acquisition exhibits strategic substitutability and there is a unique (linear) partially revealing REE price in the financial market. If information is a strategic substitute then a trader has no incentive to acquire the information when other traders already possess it and as a consequence the information market also has a unique equilibrium. Also, in the financial market, the (unique) REE price always reveals more of the private information when more traders with private information are present.

The multiplicity of equilibria in the financial market in our model is driven by the presence of supply signals *and* payoff signals, which allow the informed traders to use their information about the supply or the payoff of stocks. The absence of either payoff signals or supply signals will lead to a unique (linear) partially revealing REE price.

The financial market equilibria in our model are closely related to the financial market equilibrium in the models by Grossman (1976), Grossman and Stiglitz (1980), and Diamond and Verrecchia (1981). In one of the equilibria, the price of the stock reveals more information about the payoff as the number of informed traders in the financial market increases similarly to the unique equilibrium in the previous studies. Due to this and other similarities that we elaborate on in section 3, we refer to this equilibrium as the GS-REE (Grossman-Stiglitz REE). We refer to the other financial market equilibrium as the NGS-REE (non-Grossman-Stiglitz REE).[2] The information about the payoff revealed by the NGS-REE price decreases as the number of informed traders increases in contrast to the unique equilibrium price in the previous studies. In fact, the NGS-REE price reveals more information about the supply of the stock as the number of informed traders increases.

Since the financial market equilibrium in the previous analyses of the standard CARA-normal model has been widely used to analyze a variety of phenomena, it is of interest to assess whether information can be a strategic complement if the traders coordinate on the GS-REE in our model.

We indeed find that even when the traders coordinate on the GS-REE, information acquisition can still exhibit strategic complementarities. Whether information is a

[2] While we use the NGS label in a very specific sense, we realize that it may seem vague and uninformative and welcome any suggestions for a better label.

strategic complement or not depends on the quality of the signals about the payoff and the supply of the stock. We show that if the supply signal is 'relatively of better quality' than the payoff signal, then information is a strategic complement (proposition 3).

Since the NGS-REE price reveals less information about the payoff as the number of informed traders increases, if the traders coordinate on the NGS equilibrium, then information acquisition may be a strategic complement. However, the conditions under which information is a strategic complement in this scenario are the opposite of those when the traders coordinate on the GS-REE. This is a direct consequence of the fact that the two REE prices have opposite properties regarding information revelation as the number of informed traders increases.

The multiplicity of financial market equilibria and complementarity in information acquisition provide insights into and explanations for market frenzies, the cost of capital, and price crashes and excess volatility (sections 3 and 4.4). In particular, we find that in contrast to the existing analysis by Easley and O'Hara (2004) the cost of capital to a firm can in fact increase if the private information about the payoff is more precise or the number of informed traders increases.

Since complementarity exists for both financial market equilibria, leading to multiple equilibria in the information market, there can be a large number of overall equilibria in the model. In particular, prices can crash as traders switch from one information market equilibrium to another without switching between financial market equilibria or as traders switch between financial market equilibria while staying at the same information market equilibrium. The switches between equilibria can also lead to price volatility that is excessive compared to those of the payoff- or supply-fundamentals.

The multiplicity of equilibria also provides an insight into market frenzies, or 'price surges' that can be thought of as occuring due to changes in information about the stock, as in Veldkamp (2006a). A new result is that a price surge may occur with a rise in information about the supply of the stock, suggesting that supply-based speculation has a role in explaining large market fluctuations.

Related literature The consequences of private information about the supply of stocks are also examined by Gennotte and Leland (1991) to analyze price crashes. Palomino (2001) studies the informational efficiency of imperfectly competitive markets where traders use supply information to speculate, extending the analyses of Bhattacharya and Spiegel (1991) and Madhavan (1992) who study the possibility of market breakdowns in REE models.

The financial market equilibrium in the CARA-normal framework has been widely applied to study, for example, price crashes (Yuan, 2005 and Gennotte and Leland, 1991), contagion (Kodres and Pritsker, 2002 and Yuan, 2005), and uniqueness of equilibria in coordination games (Angeletos and Werning, forthcoming).

In Grossman and Stiglitz (1980) information acquisition exhibits strategic substitutability. As the number of informed traders increases, the (REE) price of the stock reveals more information about the payoff to the uninformed traders leading to a unique equilibrium in the information market. Veldkamp (2006a and 2006b) introduces an information production sector that supplies the payoff information at an endogenous price into the Grossman-Stiglitz model in a dynamic setting to generate complementarity. She uses the complementarity to explain the occurrence of media 'frenzies' in investment markets (2006a) and the excess covariance of asset prices relative to that of their fundamentals (2006b). We discuss her model in more detail later (section 4) and just note here that we are able to generate similar results to Veldkamp's (2006a) dynamic model in a static setting.

Hellwig and Veldkamp (2006) conclude that when actions chosen are strategically complementary (substitutable), then information acquisition is complementary (substitutable) as well, thus explaining the substitutability in Grossman-Stiglitz (1980) since "[i]nvestment is a strategic substitute: investors prefer purchasing assets that others don't want, because these assets have low prices" (p 4). While they note that allowing freely observable aggregate variables, like prices, may change their result, they present the Grossman-Stiglitz model as an example where it does not. Our results show that the payoff signal being identical is crucial to this result, since in our model, although investment is still a strategic substitute in the financial market, the information choice of the traders can exhibit complementarity.

Other models also generate complementarity in information acquisition outside the CARA-normal framework (Barlevy and Veronesi, 2000 and Chamley, 2005).[3] Complementarities occur in a large number of economic interactions, such as bank runs; regime switches; and currency, debt, and financial crises resulting in, inter alia, synchronization of actions, multiplicity of equilibria, and amplification of volatility. Our results suggest that the analytically tractable CARA-normal REE models can be used to analyze situations where the information is complementary across agents and multiplicity is potentially useful to understand the movement of economic data and possibly puzzling economic phenomena.

The rest of the paper is organized as follows. We present the model in section 2. We discuss the equilibria in the financial market with its applications in section 3 and we discuss the general equilibrium analysis of the CARA-normal framework in sub-section 3.1. Section 4 describes the equilibria in the information market and its applications. The two main results are presented in propositions 1 and 3. All the results are proved in the appendix and section 5 concludes the paper.

2 The model

There is a continuum of traders, indexed by $[0, 1]$, who are identical ex ante and live for 2 periods. There are 2 assets - one riskless (money) and one risky (a stock) - traded in a financial market that opens in period 2. The payoff to each unit invested

[3] However, see the discussion just before proposition 3.

in money is normalized to one and each trader is endowed with $\bar{N}$ units of money and zero units of the stock at the beginning of period 1.[4]

The payoff to each unit invested in the stock is denoted by v, and v is normally distributed with mean $\bar{v}$ and precision $\rho_v > 0$, i.e.,

$$v \sim N\left(\bar{v}, 1/\rho_v\right) \text{ with } \rho_v > 0.$$

We assume that the aggregate supply (x) of the stock is normally distributed, i.e.,

$$x \sim N\left(\bar{x}, 1/\rho_x\right) \text{ with } \rho_x > 0.$$

We call the pair (v, x) the fundamentals in this financial market.

At the beginning of period 1, the information market opens. In this market, at a cost $\kappa \in \left(0, \bar{N}\right)$, trader i can independently purchase a two-dimensional private information signal about the fundamentals in the financial market,[5] $S_i = (y_i, x_i)$, where y_i and x_i are mutually independent and

$$\begin{aligned} y_i &= v + \varepsilon_i, \text{with } \varepsilon_i \sim N\left(0, 1/\rho_\varepsilon\right), \rho_\varepsilon > 0, \text{ and} \\ x_i &= x + \eta_i, \text{ with } \eta_i \sim N\left(0, 1/\rho_\eta\right), \rho_\eta > 0. \end{aligned}$$

The noise $(\varepsilon_i)_{i\in[0,1]}$ and $(\eta_i)_{i\in[0,1]}$ in the signals are i.i.d. across the traders.[6] Trader

[4]The assumption about zero endowment of the stock is not important for our results. We obtain the same results, if for example, we assume that the endowment of the stock is random across the traders and is independent of the aggregate supply of the stock. See also section 3.1

[5]Alternatively, we can assume that the traders pay $\kappa_x \geq 0$ to buy a supply signal, $\kappa_y > 0$ to buy a payoff signal, and $\kappa = \kappa_x + \kappa_y \in \left(0, \bar{N}\right)$ to buy both. Then there will be four groups of traders in the financial market: supply-informed, payoff-informed, supply-payoff-informed, and uninformed. This generalization is straightforward and the main results of this paper still hold under this extension. For the sake of simplicity, we adopt the current version.

[6]Here and in what follows, when we are dealing with a continuum of i.i.d. random variables $(Y_i)_{i\in[0,1]}$ with mean $\bar{Y} < \infty$ and variance $(\rho_Y)^{-1} < \infty$, we adopt the Bewley integral so that $\int_0^1 Y_i di = \bar{Y}$ with probability one. The interested reader is refered to Bewley (1986), Judd (1985), Laffont (1985), or Admati (1985) for details. Sun (2006) provides a resolution to the issue by extending the usual product probability space. Our main results do not depend on the assumption of a continuum as we discuss later.

i is called *informed* if she chooses to purchase the signal S_i and *uninformed* otherwise. Once the traders have made their information acquisition decision, period 1 ends and financial market opens in period 2. We relabel the traders so that the set of traders who are informed is $[0, \lambda]$, i.e., the fraction of informed traders is λ.

Note that unlike the model of Grossman and Stiglitz (1980), the private information is diverse. While, this assumption is not uncommon in the literature, see for example Grossman (1976), Hellwig (1981), Diamond and Verrecchia (1981), Admati (1985), and Verrecchia (2001), the results that we derive are new. Simply introducing diverse supply signals in a CARA-normal REE model with diverse payoff signals is what drives these new findings.

Each trader only cares about wealth W at the end of period 2 and has the (von-Neumann-Morgenstern) utility function u with CARA parameter $\gamma > 0$,

$$u(W) = -e^{-\gamma W}.$$

We normalize the price of money to 1 and denote by P the price of the stock. The initial wealth of trader i is $W_1^i = \bar{N}$. We denote trader i's demand for the stock by $D^i(P)$ and for the bond by $B^i(P)$. Then trader i's wealth at the end of period 2 is $W_2^i = B^i(P) + vD^i(P)$.

Since we use the concept of an REE, the price, P, of the stock is a function, $P(v, x)$, of the fundamentals (v, x) of the economy.[7] Denoting by $D_I^i(P)$ (respectively, $D_U(P)$) the demand function of an informed (respectively, uninformed) trader i in the financial market, an *overall equilibrium* in the model is a tuple

$$\left(\lambda^*, P, \left(D_I^i\right)_{i\in[0,\lambda^*]}, \left(D_U^i\right)_{i\in(\lambda^*,1]}\right) \tag{1}$$

[7] To be precise, in an REE, the price P is a function of the information of all agents in the economy, i.e., $P \equiv P\left((y_i, x_i)_{i\in[0,\lambda]}\right)$. However, in equilibrium the price will aggregate all the diverse information and hence will be a function of only the aggregates v and x, i.e., $P = P(v, x)$.

such that the tuple $\left(P, \left(D_I^i(P)\right)_{i\in[0,\lambda^*]}, \left(D_U^i(P)\right)_{i\in(\lambda^*,1]}\right)$ constitutes an REE in the financial market and λ^* is the equilibrium fraction of informed traders given that the traders coordinate on $\left(P, \left(D_I^i(P)\right)_{i\in[0,\lambda^*]}, \left(D_U^i(P)\right)_{i\in(\lambda^*,1]}\right)$ in the financial market. This is formalized in definitions 1 and 2 where $E[V_I(\lambda)]$ denotes the expected (indirect) utility of an informed trader and $E[V_U(\lambda)]$ denotes the expected (indirect) utility of an uninformed trader, for any $\lambda \in [0,1]$, and $R(\lambda) = E[V_I(\lambda)]/E[V_U(\lambda)]$. Note that the expected (indirect) utility of being informed (uninformed) is identical across traders, since they are identical ex-ante.

Definition 1 *(Financial market equilibrium) Given a fraction* (λ) *of informed traders in the market, a price function* $P(x,v)$ *and demand functions* $\left(D_I^i(P)\right)_{i\in[0,\lambda]}$ *and* $\left(D_U^i(P)\right)_{i\in(\lambda,1]}$ *constitute an REE if (i)* $D_I^i(P)$ *(respectively* $D_U^i(P)$*) maximizes the expected utility of informed (respectively, uniformed) trader* i *conditional on her information, including that provided by the prices, given the price* P *and (ii) the markets for the stock and the bond clear for each realization of* (v,x)*.*

Noting that we have defined utility to be negative, an equilibrium $\lambda^* \in [0,1]$ in the information market is given by the following.

Definition 2 *(Information market equilibrium)*

$$\lambda^* = 0 \;\; \textit{if } R(0) > 1, \;\; \lambda^* = 1 \;\; \textit{if } R(1) < 1, \;\; \textit{or } \lambda^* \in [0,1] \;\; \textit{if } R(\lambda^*) = 1. \qquad (2)$$

Hence, if a trader does not benefit from becoming informed when no other trader is informed, i.e., $R(0) > 1$, then it is an equilibrium in the information market for no one to buy the information, i.e., $\lambda^* = 0$. On the other hand, if a trader is strictly better off from being informed when all other traders are also informed, i.e., $R(1) < 1$, then in equilibrium all traders in the market will be informed, i.e., $\lambda^* = 1$. In general, for a given fraction of informed traders (λ) if a trader is indifferent between becoming informed and staying uninformed, then that fraction λ is an information market equilibrium.

3 Equilibria in the financial market

We now establish the existence of two REE in the financial market. We are interested in REE that have price $P(v, x)$ as a linear function of the fundamentals. So, suppose the traders conjecture the price function as

$$P = a + bv - cx, \text{ with } b \geq 0 \text{ and } c \geq 0. \quad (3)$$

Note that this price is always measurable with respect to the information of both types of traders since it is a function of the prior beliefs of the traders. Then, when $b > 0$ and $c > 0$ the information contained in the price can be expressed by the public signal

$$s = \frac{P - a + c\bar{x}}{b} = v - \frac{c}{b}(x - \bar{x}).$$

Let $\phi = -\frac{c}{b}(x - \bar{x})$, then ϕ is normally distributed with mean 0 and precision given by

$$\rho_\phi = \left(\frac{b}{c}\right)^2 \rho_x. \quad (4)$$

So, informed trader i, uses the sufficient private signal[8]

$$z_i = v - \frac{c}{b}(x - E[x|x_i]) = v - \frac{c}{b}\left[\frac{\rho_x}{\rho_x + \rho_\eta}(x - \bar{x}) - \frac{\rho_\eta}{\rho_x + \rho_\eta}\eta_i\right]. \quad (5)$$

Let $\theta_i = -\frac{c}{b}(x - E[x|x_i])$. So, conditional on x_i, θ_i is normally distributed with mean 0 and precision given by

$$\rho_\theta = \left(\frac{b}{c}\right)^2 (\rho_x + \rho_\eta). \quad (6)$$

Equation (6) captures the benefit to informed trader i from the supply signal x_i in the form of the additional precision $(b/c)^2 \rho_\eta$ in the information she tries to extract

[8] Throughout the paper, we use $A|\{B_1, B_2, ..., B_n\}$ to mean the random variable A conditioned on the random variables $\{B_1, B_2, ..., B_n\}$.

from the price. This additional precision depends on (b/c), which in turn will be determined by the value of λ in the (overall) equilibrium. This provides the channel through which the benefit from acquisition of supply information depends on the fraction of informed traders, which underlies the complementarity or substitutability of information as formalized in section 4. Note that (b/c) can be used as a measure of the *informativeness* of the price about the payoff. When (b/c) is larger, price changes relatively more in response to changes in v than changes in x. Similarly, (c/b) is a measure of the informativeness of price about x. This is consistent with the measure used by Grossman and Stiglitz (1980).

Informed traders An informed trader i has information $\{x_i, y_i, P\}$ and uses $\{x_i, y_i, z_i\}$ to update her beliefs. $v|\{x_i, y_i, z_i\}$ is normally distributed with mean μ_I^i and precision ρ_I given by

$$\mu_I^i = \frac{\rho_v \bar{v} + \rho_\varepsilon y_i + \rho_\theta z_i}{\rho_I}, \rho_I = \rho_v + \rho_\varepsilon + \rho_\theta. \tag{7}$$

Given her posterior beliefs about the stock (7), the demand function of informed trader i is

$$D_I^i(P) = \frac{\mu_I^i - P}{\gamma \rho_I^{-1}}. \tag{8}$$

Note that each informed trader has two sources of information to possibly profit from – the information about payoff v, which constitutes direct speculation, and the information about supply x.

Uninformed traders Each uninformed trader only has information $\{P\}$, or equivalently $\{s\}$ and uses it to update their beliefs. $v|s$ is normally distributed with mean μ_U and precision ρ_U given by

$$\mu_U = \frac{\rho_v \bar{v} + \rho_\phi s}{\rho_U}, \rho_U = \rho_v + \rho_\phi. \tag{9}$$

Given her posterior beliefs about the stock (9), each uninformed trader's demand function is

$$D_U(P) = \frac{\mu_U - P}{\gamma \rho_U^{-1}}. \tag{10}$$

In equilibrium, the stock market clears, i.e.,

$$\int_0^\lambda D_I^i(P)\, di + (1-\lambda) D_U(P) = x. \tag{11}$$

We find the REE by solving equation (11) for P and then verifying that P is of the form conjectured in (3). Proposition 1 characterizes the REE. As we elaborate below, one of the financial market equilibria in our model exhibits the same properties as the financial market equilibrium in the model by Grossman and Stiglitz (1980). So, we refer to this equilibrium as the GS-REE (Grossman-Stiglitz REE) and label the corresponding values of the variables by GS. We refer to the other financial market equilibrium as the NGS-REE (non-Grossman-Stiglitz REE) and label the corresponding values of the variables by NGS.

Proposition 1 *If $\gamma^2 > 4\lambda\rho_\varepsilon\rho_\eta > 0$, for any given λ, and $\rho_\varepsilon > 0, \rho_\eta > 0$, there exist two partially revealing rational expectations equilibria in which,*[9]

$$P = a + bv - cx,$$

where

$$a = \frac{\rho_v \bar{v} + \beta \rho_x \bar{x}}{K}, b = \frac{\lambda(\rho_\varepsilon + \rho_\theta) + (1-\lambda)\rho_\phi}{K}, \ c = \frac{\beta \rho_x + \gamma}{K},$$

and $\beta = \frac{b}{c}$ takes one of two values,

$$\beta^{GS} = \frac{\gamma - \sqrt{\gamma^2 - 4\lambda\rho_\varepsilon\rho_\eta}}{2\lambda\rho_\eta}, \ \beta^{NGS} = \frac{\gamma + \sqrt{\gamma^2 - 4\lambda\rho_\varepsilon\rho_\eta}}{2\lambda\rho_\eta}.$$

with $K = \lambda\rho_I + (1-\lambda)\rho_U$, $\rho_I = \rho_v + \rho_\varepsilon + \rho_\theta$, $\rho_\theta = \beta^2(\rho_x + \rho_\eta)$, $\rho_U = \rho_v + \rho_\phi$,

[9] Although all the variables here depend on λ, we do not express the dependence explicitly for simplicity of notation. This convention is followed in the next section also.

$\rho_{\phi} = \beta^2 \rho_x$.

This result suggests that for a linear (partially revealing) equilibrium price function to exist in the financial market, the signals can not be very sharp, that is, we need low ρ_ε or ρ_η.[10]

In the model of Grossman and Stiglitz (1980), β increases as the fraction of informed traders increases, i.e., $\frac{\partial \beta}{\partial \lambda} > 0$. This is true of the equilibrium with GS variables in our model. In contrast, in the equilibrium with NGS variables, β decreases as λ increases.

However, note that for any given value of λ, the NGS-REE price is always more informative than the GS-REE price since $\beta^{NGS} \geq \beta^{GS}$. The GS and NGS labels are used to indicate that the equilibrium price with GS label has similar properties to the equilibrium price in Grossman and Stiglitz (1980), while the equilibrium price with the NGS label does not. The GS-REE is thus a generalization of the equilibrium in Grossman (1976), Grossman and Stiglitz (1980), and Diamond and Verrecchia (1981) in our setup, while the NGS-REE is a new (partially revealing) equilibrium that does not exist in the previous studies.

The GS-REE is the unique partially revealing equilibrium in Diamond and Verrecchia (1981) as a limiting csae when $\rho_\eta \to 0$, while the NGS-REE approaches a price that fully reveals v. Our results show that this result is not robust to small perturbations in the value of ρ_η and that multiple partially revealing REE prices exist for small positive values of ρ_η. In fact, there is a unique partially revealing REE price even when $\rho_\eta > 0$, with $\rho_\eta = (\gamma^2/4\lambda\rho_\varepsilon)$, however, this is also not robust to small perturbations in the value of ρ_η.[11] Figure 1 illustrates the map of partially revealing REE prices in a CARA-normal setup as ρ_η varies, using a renormalization of β on the

[10] Fully revealing REE prices exist irrespective of the sign of $\left(\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta\right)$. However, we focus on the more interesting partially revealling REE prices. Also, we have nothing to say about the existence of a partially revealling REE price when $\gamma^2 < 4\lambda^2\rho_\varepsilon\rho_\eta$ beyond noting that such a linear price function does not exist.

[11] When $\rho_\eta = \left(\gamma^2/4\lambda^2\rho_\varepsilon\right)$, then the quadratic equation determining the coefficient (b/c) of the price function has a unique root. See the proof of proposition 1 in the appendix.

vertical axis so that when the REE price reveals v fully ($\beta = \infty$), the renormalization takes the value 1 and it takes the value 0 when the REE price reveals nothing about v ($\beta = 0$).[12] The map also contains an REE price that reveals v fully. This price always exists as an equilibrium price in the financial market due to the self-fulfilling nature of an REE.

How the price reacts to variations in v and x determines how the informativeness of price measured by $\beta = (b/c)$ changes as the number of informed traders changes. The channel for this is the interaction of the aggregate demand and supply.

The equilibrium price reacts to changes in v because the informed traders are using information y_i when making decisions, and as a result the aggregate demand function shifts when v varies. The more informed traders there are, the more responsive is the price to variations in v. On the other hand, the change in x directly moves the aggregate supply curve. In the previous studies, price responds to the variation in x only through this direct channel, which is obviously independent of the fraction of the informed traders. Therefore, it is not surprising that in the traditional GS-REE, more informed traders would lead to a more informative price function.

However in our model, changes in x also have an effect on the aggregate demand because higher x implies higher v for a given P through (3), and leads to a higher stock demand, given that all traders partly know x based on their supply signals x_i. Thus, variation in x shifts the aggregate demand also. A larger shift of this kind makes price less responsive to the variations in x. For example, an increase in x generates a downward pressure for price from the supply side, but the higher demand due to the supply signals abates this pressure and the price goes down less.

In particular, the usefulness of the supply signals depends on the informativeness of the price system. To see this, note that x_i works through the signal z_i, and the precision of z_i, $\rho_\theta = \beta^2 \left(\rho_x + \rho_\eta\right)$ given by (6), is positively related to the informativeness measure β. Formally, we can show that the traders shift the aggregate demand

[12] The renormalization is $f(\beta) = 1 - e^{-\beta}$.

function by $\left(\lambda\beta\rho_\eta/\gamma\right)x$, in response to the variation in x_i. Using *constant* to denote a term that does not depend on (P, v, x), the aggregate demand function is

$$D\left(P; v, x\right) = constant + \underbrace{\frac{\lambda\rho_\varepsilon}{\gamma}v}_{\text{effect of } y_i} + \underbrace{\frac{\lambda\beta\rho_\eta}{\gamma}x}_{\text{effect of } x_i} - \frac{\lambda\rho_\varepsilon}{\gamma b}P.$$

Therefore, how the price reacts to x in turn depends on the traders' belief about β. If traders conjecure the price function reveals less information as λ increases, they would respond to this conjecture by shifting their demand less, and as a result increase the responsiveness of price to a given change in x, which in turn self-fulfills the initial belief if this effect is sufficiently large. This equilibrium is the NGS-REE.

The change in the informativeness of price about the payoff can also be understood by considering the limiting case when there are no informed traders in the market, i.e., $\lambda = 0$. When $\lambda = 0$, there is no information about v or x among the traders other than their prior beliefs and there is no linear partially revealing REE price in the financial market. There are only two fully revealing REE prices, which exist due to the self-fulfilling nature of an REE, one of which fully reveals the value of v while the other fully reveals the value of x. As λ converges to zero, the GS-REE approaches the REE which fully reveals x and provides no information on v, while the NGS-REE approaches the REE that fully reveals v and provides no information on x.

As λ increases above zero, there is more information about v and x present among the traders. This introduces more information about v into the GS-REE and more information about x into the NGS-REE, making these partially revealing. As a consequence, the GS-REE price is more informative about v as λ increases and the NGS-REE price less informative about v (and more informative about x) as λ increases.

A trader will purchase costly information about v or x only when she expects the price *not* to fully reveal that information so that she can profit from it. This is exactly the paradox considered by Grossman and Stiglitz (1980).

In our model, when $\rho_\eta = 0$, the supply signal provides no information and the informed trader can only use her payoff signal to make profitable trades. In the limit as ρ_η converges to zero, the NGS-REE price approaches an REE price that fully reveals v. The GS-REE on the other hand approaches a partially revealing REE price. Now, traders seeking to profit from information about v, i.e., from direct speculation will purchase costly information when the REE price only partially reveals v and not if it does so fully. Hence, the financial market will be in equilibrium with a positive fraction of informed traders when the traders' decisions lead to the GS-REE. This is just the familiar result that the GS-REE is driven by the direct speculation motives of traders informed about v.

On the other hand, when $\rho_\varepsilon = 0$, there is no information contained in the payoff signal and any profit from private information can only be made using the information about supply x. As ρ_ε converges to zero, the GS-REE approaches an REE price that fully reveals x, while the NGS-REE approaches a partially revealing REE price. Symmetrically to the previous case, the financial market will be in equilibrium with a positive value of λ if the traders' decisions lead to the NGS-REE. We thus have the symmetric result that the NGS-REE exists due to the desire to profit from trades that are based on information about the stock's supply x. The nature of the informativeness of the GS- and NGS-REE in each of the limiting cases is summarized in Table 1.

Table 1 Information characteristics of the financial market equilibria

	β^{GS}	β^{NGS}
$(\lambda, \rho_\varepsilon) >> 0,\ \rho_\eta = 0$	$(\lambda\rho_\varepsilon/\gamma)$ [partially revealing]	∞ [fully reveals v]
$(\lambda, \rho_\eta) >> 0,\ \rho_\varepsilon = 0$	0 [fully reveals x]	$(\gamma/\lambda\rho_\eta)$ [partially revealing]
$(\rho_\varepsilon, \rho_\eta) >> 0,\ \lambda = 0$	0 [fully reveals x]	∞ [fully reveals v]

To ensure that the multiplicity is not an artifact of the continuum of traders, we also considered an economy identical to the one here but with a finite number of traders. Indeed, when $\rho_\eta > 0$ and there is a large, but finite number of traders in the economy there exist three partially revealing REE in the economy. As the number of traders converges to infinity, the three REE of the finite economy converge to the GS-REE, the NGS-REE, and a REE which fully reveals v. So, the multiplicity of equilibria is not an artifact of the continuum economy, it is generated by the presence of supply signals. Indeed, Palomino (2001, proposition 3) also allows traders to have private information about the aggregate supply of the stock but obtains a unique financial market equilibrium in his model. His model differs from ours in that there is no common component across the traders' endowments so that it reduces to that of Diamond and Verrecchia (1981) with price-taking traders.

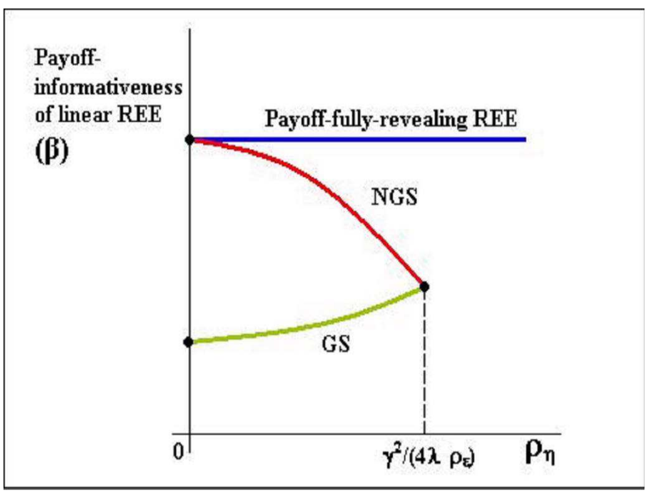

Figure 1 The map of linear REE prices and the precision of the supply signal

Application The (unique) financial market equilibrium in the previous studies of the CARA-normal framework has been extensively used in the finance literature. For example, in Easley and O'Hara (2004) the partially revealing REE is unique and exhibits the same properties as the GS-REE in our model. On the other hand, if the traders coordinate on the NGS-REE in our model, then the policy and theoretical implications are opposite those in Easley and O'Hara (2004). First, on average the

NGS-REE has a higher price and consequently a lower cost of capital than the GS-REE. Second, when the traders coordinate on the NGS-REE, the larger the fraction (λ) of the informed traders, the larger is the cost of capital.[13] In the NGS-REE, when the number of informed traders increases, the information revealed by the price falls, this makes the stock riskier for the uninformed traders, who demand a higher risk premium raising the cost of capital. Finally, in the NGS-REE, when the payoff signal becomes more precise, i.e., ρ_ε increases, then the cost of capital increases, unlike in Easley and O'Hara (2004).

Also, the multiplicity of market-clearing prices in our model means that price crashes can occur and the prices may exhibit excessive volatility relative to fundamentals.

3.1 General equilibrium analysis: removing noise traders

Most previous CARA-normal models are in a partial equilibrium setting, since the aggregate supply of the stock is driven by noise (liquidity) traders who are not modeled. Their trading decisions are simply summarized in a normally distributed random variable. Our extension allows us remove (unmodeled) noise traders from the set up and conduct general equilibrium analysis of the financial market where the aggregate supply of the stock is in fact an aggregate of the endowments of the (rational) traders whose behavior is explicitly analyzed.

Previous attempts at doing away with noise traders by making the random total supply an aggregate of individual endowmenst are restricted to the case of a finite number of traders given the law of large numbers (Diamond and Verrecchia, 1981) or involve assumptions that imply infinite variance of the aggregate supply (Grundy and McNichols, 1989). As pointed out by Hellwig (1980) and Laffont (1985), there is a conceptual problem in using the REE concept with a finite number of traders. In

[13] Here we assume that the traders commit to one financial market equilibrium for all possible values of the parameters.

particular, Helllwig (1980, p478) states that the agents seem slightly schizophrenic as they do not seem to notice the effect they have on the price and act as price takers.

General equilibrium analysis of the financial market with a continuum of traders is possible using a special case of our model, in which the supply signal is available freely to all traders while the payoff signal is available only to a fraction of the traders. The free supply signal for each trader is in fact her (random) endowment of the risky asset, which is only observed by her. The aggregate supply of the risky asset then is just the aggregate of the endowments of the traders. There are no noise or liquidity traders. The traders in the financial market do not know the aggregate supply in the market since the aggregate of traders' endowments is still random. In this case, information about payoff is available as a private noisy signal to only $\lambda > 0$ of the traders who are called *informed.* The private payoff signal is diverse across the traders. Note that the information structure of the model is considered exogenous, since we are interested in general equilibrium analysis of only the financial market.

Trader $i \in [0, 1]$ is endowed with $\bar{n}$ units of the bond and x_i units of the stock, where

$$x_i = x + \eta_i \text{ and } x \sim N\left(\bar{x}, 1/\rho_x\right), \ \eta_i \sim N\left(0, 1/\rho_\eta\right), \ \rho_x > 0, \text{ and } \rho_\eta > 0.$$

As before, $(\eta_i)_{i\in[0,1]}$ are i.i.d across the traders. The realization of x_i is observed only by trader i. The aggregate supply of risky asset is then

$$\int_0^1 x_i di = x.$$

We can interpret x as a common shock due to the fluctuation of the whole economy and η_i as an idiosyncratic shock due to variation in individual ability.[14] For example, if the risky asset is corn, then x can be thought of as the effect of weather on

[14] The existence of multiple equilibria in the financial market does not require a common shock to the endowments of *all* agents. A common shock to a positive fraction of the agents is enough to generate multiple equilibria. A proof of this assertion is available from the authors on request.

the output of corn. In the studies by Grossman and Stiglitz (1980), Diamond and Verrecchia (1981), and Grundy and McNichols (1989) the endowments do not have a common component across traders. This assumption then generates the new result that multiple (partially revealing) REE (prices) exist in the CARA-normal setup.

The analysis can now be done similarly to that for the derivation of proposition 1 and will yield a similar result. Hence, simply closing the model by considering its general equilibrium version yields multiple equilibria in the financial market without resorting to any extra constraints or feedback effects. This suggests that multiple equilibria may in fact be an inherent feature of financial markets based on the CARA-normal framework.

4 Equilibrium in the information market

We now consider the existence and properties of equilibria in the information market. In particular, we show that information acquisition can exhibit strategic complementarity in the CARA-normal framework, which leads to multiple equilibria in this market. The properties of the NGS-REE price with regard to information content indicate that if traders coordinate on this equilibrium then information acquisition can be a strategic complement. We establish this result in proposition 4. However, we also find that information can be a strategic complement when the traders coordinate on the GS-REE. This is an important result since in the previous studies of the CARA-normal framework and its applications, with the exception of Veldkamp (2006a and 2006b), information is a strategic substitute given the (unique) financial market equilibrium.

The next result shows that ex-ante the expected indirect utility of becoming informed is proportional to that of staying uninformed for any trader i, irrespective of the REE coordinated upon, and the respective expected utilities are identical across traders, since the traders are identical ex-ante.

Proposition 2 *Suppose traders coordinate on the GS-REE (or the NGS-REE), then for any given* λ*, the expected indirect utility of the uninformed traders* $(E[V_U])$ *and that of the informed traders* $(E[V_I])$ *is given by*

$$\begin{aligned} E[V_U] &= -\frac{1}{\sqrt{1+A}} e^{-\gamma \bar{N}}, \\ E[V_I] &= e^{\gamma\kappa} \sqrt{\frac{\rho_U}{\rho_I}} E[V_U], \end{aligned}$$

where $A = \rho_U \left(\frac{1}{b} \frac{\rho_\phi}{\rho_U} - 1 \right)^2 \left(\frac{b^2}{\rho_v} + \frac{c^2}{\rho_x} \right)$ *and values of the parameters* $b, c, \rho_\phi, \rho_U, \rho_I$ *are as given in proposition 1 corresponding to the GS-REE (or the NGS-REE, respectively).*

We now make the precise the sense in which information acquisition is a strategic complement (substitute), using the result of proposition 2, which provides that $R(\lambda) = e^{\gamma\kappa} \sqrt{\frac{\rho_U}{\rho_I}}$.

Definition 3 *(Strategic complement / substitute) If* $R'(\lambda) < 0$*, then learning is a strategic complement, and if* $R'(\lambda) > 0$*, then learning is a strategic substitute.*

In other words, strategic complementarity (substitutability) will give the traders more (less) incentive to get informed as the fraction of informed traders is getting larger. This definition corresponds to those in Grossman and Stiglitz (1980) and Barlevy and Veronesi (2000).

We now state our main results, which say that whether information acquisition is a strategic substitute or a strategic complement, depends on the comparison of the relative precision of the payoff and supply signals.

4.1 Complementarity with the GS-REE

As we noted above, the GS-REE is an equilibrium that exists as informed traders seek to profit from their costly information about v, i.e., from direct speculation and also GS-REE price reveals more information about v as λ increases. We now let the traders coordinate on the GS-REE in the financial market, which corresponds to the unique financial market equilibrium of previous studies, and unless otherwise stated, all the variables in period 2 will refer to the GS-REE.

Following definition 4 and proposition 2, if $\sqrt{\rho_I/\rho_U} > e^{\gamma\kappa}$, then a trader would decide to become informed. So, we could measure the benefit from being informed by ρ_I/ρ_U, while $e^{\gamma\kappa}$ is directly related to the cost of information acquisition. In particular, the relative benefit to an informed trader can be decomposed into the benefit from the payoff signal and that from the supply signal, i.e., we have

$$\frac{\rho_I}{\rho_U} = \frac{\rho_\varepsilon}{\rho_U} + \frac{\beta^2 \rho_\eta}{\rho_U} + 1$$

Here, the first term $(\rho_\varepsilon/\rho_U)$ is contributed by the payoff signal, while the second term $\left(\beta^2 \rho_\eta/\rho_U\right)$ is contributed by the supply signal. When the payoff signal is relatively sharper than the supply signal, an informed trader has more precise information about the payoff and so, more to gain from direct speculation in the financial market than from supply-signal-based trading. Now, as more traders seek private information about v to engage in direct speculation, i.e., λ increases, the GS-REE price reveals more information about v, i.e., β rises making it easier for the uninformed traders to free-ride on the information acquired by the informed, i.e., ρ_U becomes larger. Thus, the benefit from the payoff signal decreases making information a substitute.

On the other hand, when the supply signal is more precise than the payoff signal, an informed trader has more to gain from trading based on the information about the stock's supply x. As more traders seek private information about x, λ increases and the GS-REE price reveals less information about x, i.e., $(1/\beta)$ decreases. This makes it harder for the uninformed traders to infer the informed traders private information from the price, and the relative benefit of being uninformed decreases, i.e., when β becomes larger,

$$\frac{\beta^2 \rho_\eta}{\rho_U} = \frac{\rho_\eta}{\left(\rho_v/\beta^2\right) + \rho_x}$$

increases, so that information will be a complement.

Comparing the precision-ratios of the two signals thus indicates which effect is dominant and provides a means of identifying whether information is a substitute or complement as formalized below.

Proposition 3 *Let the traders coordinate on the GS-REE in the financial market. If $\frac{\rho_\varepsilon}{\rho_v} > \frac{\rho_\eta}{\rho_x}$, then information acquisition is a strategic substitute while if $\frac{\rho_\varepsilon}{\rho_v} < \frac{\rho_\eta}{\rho_x}$, then information acquisition is a strategic complement.*

Hence, the presence of diverse payoff signals and diverse supply signals generates complementarities in information acquisition. This is consistent with the idea in Barlevy and Veronesi (2000, p88) that "the crucial component generating learning complementarities is that learning makes identification more complicated for uninformed agents." However, while Barlevy and Veronesi (2007) note that the result for their framework only holds when there is positive correlation between the payoff and the noise term (supply) in the price, our results do not depend on any such assumption.

Figure 2 for an illustration of both cases. In figure 2(a), learning is a strategic substitute, with $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 2$, $\rho_x = 4$, and $\kappa = 1/10$ and there is a unique equilibrium: $\lambda^* = 0.26$. In figure 2(b), learning is a strategic complement, with $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 4$, $\rho_x = 2$, and $\kappa = 1/15$, and there are three equilibria: $\lambda^* \in \{0, 1, 0.96\}$. At $\lambda^* = 0$, the expected indirect utility of a trader from being informed is strictly less than that from being uninformed given that all the other traders are uninformed, i.e., $R(0) > 1$. On the other hand, at $\lambda^* = 1$, the expected indirect utility of a trader from being informed is strictly greater than that from being uninformed given that all the other traders are informed, i.e., $R(1) < 1$.

In Veldkamp (2006a), information is produced by a fixed-cost technology in a competitive production sector and sold at an endogenous price. Once produced, information can be distributed (but not resold) at zero marginal cost and as a result, the equilibrium price of information declines as the number of informed traders (λ) increases. So, an increase in λ makes information cheaper and more desirable to uninformed traders. On the other hand, an increase in λ also makes the financial market price more informative about the payoff, decreasing the value of information. When λ is low, this price-complementarity effect more than offsets the decreased desirability of information causing the net benefit from being informed to increase with λ. A consequence of Veldkamp's (2006a) model is that λ is positive in equilibrium only for high values of v. In contrast, since the complementarity in our model arises from the supply signal, λ can be positive in equilibrium even for low values of v.

In her model, the endogenous price for information generates the complementarity in information acquisition by making the net benefit from being informed non-monotonic in the number of informed traders. In our model, on the other hand, the net benefit curve is always monotonic in the number of informed traders if they commit to one of the two REE and complementarities arise from information revelation in the COM-REE. Veldkamp's (2006a) analysis of data on emerging markets suggests that complementarities in information acquisition are useful in understanding how asset price volatility generates demand for news and how news increases asset price and price dispersion across markets.

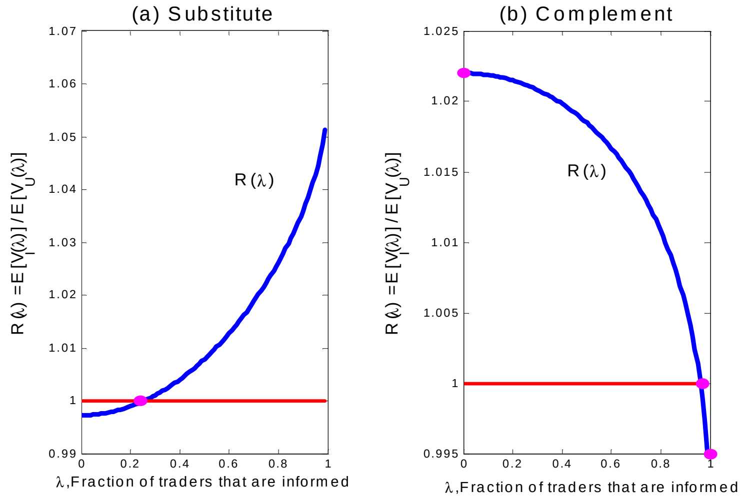

Figure 2 Information market equilibria [GS-REE]

4.2 Complementarity with the NGS-REE

When the traders coordinate on the NGS-REE, arguments analogous to those in the case of coordination on the GS-REE suggest when information is a complement and when it is a substitute. There is a trade-off between trading based on private information about v and x, respectively. However, the NGS-REE price always reveals less information about v and more about x as the fraction of informed traders, λ, increases. This means that it is easier for uninformed traders to free-ride on the information revealed by prices when the informed traders seek to speculate about x, making information a substitute. On the other hand, more traders seeking to speculate using private information about v makes price reveal less informative about v, so that information is a complement. This is formalized as follows.

Proposition 4 *Let the traders coordinate on the NGS-REE. If* $\frac{\rho_\varepsilon}{\rho_v} < \frac{\rho_\eta}{\rho_x}$*, then information acquisition is a strategic substitute while if* $\frac{\rho_\varepsilon}{\rho_v} > \frac{\rho_\eta}{\rho_x}$*, then information acquisition is a strategic complement.*

If the traders coordinate on the NGS-REE, then by proposition 2, $R(\lambda)$ depends on λ only through β^{NGS}. Now, β^{NGS} affects $R(\cdot)$ in exactly the same way as β^{GS}, i.e., $\frac{\partial R(\cdot)}{\partial \beta^{NGS}} = \frac{\partial R(\cdot)}{\partial \beta^{GS}}$ when $\beta^{NGS} = \beta^{GS}$.[15] However, $\frac{\partial \beta^{NGS}}{\partial \lambda}$ has exactly the opposite sign to $\frac{\partial \beta^{GS}}{\partial \lambda}$. So, the complementarity and substitutability result would be exactly the opposite: if the payoff signal is relatively sharper $\left(\frac{\rho_\varepsilon}{\rho_v} > \frac{\rho_\eta}{\rho_x}\right)$, then information acquisition is a complement. This result is illustrated in figure 3(a), where $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 2$, $\rho_x = 4$, and $\kappa = 1/16$, and information is a strategic complement with $\lambda^* \in \{0, 0.85, 1\}$. In figure 3(b), information is a strategic substitute with $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 4$, $\rho_x = 2$, and $\kappa = 1/12$, so that $\lambda^* = 0.88$.

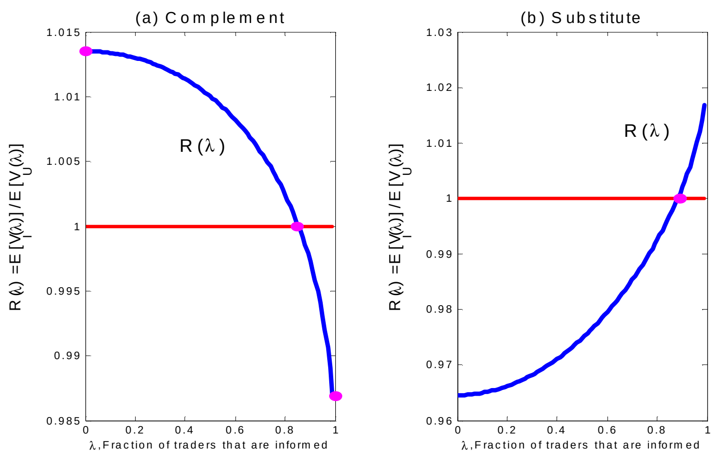

Figure 3 Information market equilibria [NGS-REE]

4.3 Multiplicity of equilibria in the model

Since multiplicity in the financial market occurs independently of multiplicity in the information market and vice versa, there can be a large number of overall equilibria, $\left(\lambda^*, P, (D_I^i)_{i\in[0,\lambda^*]}, (D_U^i)_{i\in(\lambda^*,1]}\right)$ in our model. For example, while there exist two REE for each information market equilibrium and there can exist three information market equilibria for each REE, there are also overall equilibria where each infor-

[15] Obviously $\beta^{NGS} = \beta^{GS}$ is true only for two different values of λ.

mation market equilibrium corresponds to a different REE. Hence, a change in the coordination decision of the traders in financial market could lead to a change in the information market equilibrium. On the other hand, the price of the stock could switch without a change in the fundamentals as the information market equilibrium changes.

Applications Our results provide two sources of excess price volatility - multiplicity in the financial markets and multiplicity in the information market, so that the volatility in stock prices can be excessive relative to that of the fundamentals. A change in the information market equilibrium when the traders coordinate on one of the two REE in the financial market can cause the volatility of the stock price to increase without a change in the volatility of the fundamentals. In the example of figure 2, the price volatility, measured by $Var\left(P^{GS}|\lambda^*\right)$ jumps from 0.125 to 0.18 as λ^* changes from 0 to 0.96 even though the volatility of the payoff $(1/\rho_v)$ and that of the supply $(1/\rho_x)$ do not change from 0.5 and 0.25, respectively. A jump from the GS-REE to the NGS-REE can also cause the volatility of price to increase. For example, the volatility of price increases from $Var\left(P^{GS}|\lambda\right) = 0.15$ to $Var\left(P^{NGS}|\lambda\right) = 0.23$ with a shift from the GS-REE to the NGS-REE for $\lambda = 1/2$, $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 4$, and $\rho_x = 2$.

In multi-asset markets, an implication of the multiplicity of information market equilibria in our model is that the prices of two stocks that have very similar (and independent) fundamentals can differ even if the prices exhibit similar properties. In other words, even if traders coordinate on the same REE for both stocks, for example the GS-REE, the information market could be in different equilibria for the two stocks, for example in one there could be no informed traders and in the other a positive fraction of them, causing the price to be different on average. The information market could be in a different equilibrium for each stock due to historical reasons or extrinsic factors that affect the information market equilibria the traders coordinate on for each stock.

4.4 Application: information and trade frenzies

'Frenzies' in financial markets are occasional surges in asset prices that can be thought of as occurring when "a shift in information suddenly raises an asset's price above what a model without information would predict" (p 577, Veldkamp, 2006a). As noted before, Veldkamp (2006a) generates complementarities in information acquisition in a dynamic model of the CARA-normal framework by endogenizing the price of information. In her model, complementarities can make the change in equilibrium information provision large and abrupt across periods as the fundamentals move beyond a cut-off value. As the unique equilibrium stock price is increasing in the fraction of informed traders, the jump from a no-information equilibrium to a positive-information equilibrium can explain the occurrence of frenzies in financial markets. The equilibrium stock price in her model is similar to that in the equilibrium of Grossman and Stiglitz (1980) and consequently to the GS-REE in our model.

As noted in proposition 5 in the GS-REE the expected price of the stock increases as the number of informed traders increases, while in the NGS-REE the expected price of the stock decreases as the number of informed traders increases. This result suggests that while media frenzies can occur, the reverse phenomenon is also likely.

Proposition 5 *[Information and asset prices] If $\bar{x} > 0$, in the GS-REE, the expected stock price $E[P]$ is a strictly increasing function of λ, and in the NGS-REE, the expected stock price $E[P]$ is a strictly decreasing function of λ.*

Unlike Veldkamp (2006a, 2006b), our model is a static one and multiple equilibria (in the information market) are necessary to explain the occurrence of frenzies. A frenzy in our model occurs when the information market jumps from one equilibrium to another when the traders coordinate on the GS-REE. For example, a jump from $\lambda^* = 0$ to $\lambda^* = 0.96$ in figure 2, causes an abrupt surge in the (expected) price of the stock when the financial market is in the GS-REE. The change in $E[P]$ is 40% when the information market equilibrium changes from $\lambda^* = 0$ to $\lambda^* = 0.96$, specifically $E[P]$ changes from 0.5 to 0.7 as λ^* changes. Another measure of a frenzy is the ratio of the variances of price in the two information market equilibria, $\frac{Var\left(P^{GS}|\lambda^*=0.96\right)}{Var(P^{GS}|\lambda^*=0)} = 1.43$. These values are calculated using $\bar{v} = \bar{x} = 1$.

Hence, it is possible that media frenzies can also occur when information about the stock's supply is available and are caused by an increase in information (measured by λ) like the result of Veldkamp (2006a). However, a fall in the fraction of informed traders when the traders are coordinating on the NGS-REE can also cause a rise in the stock price, the exact opposite of the frenzy phenomenon. For example, a jump from $\lambda^* = 0.85$ to $\lambda^* = 0$ causes a change in $E[P]$ of 17% while the ratio of price variances is $\frac{Var\left(P^{NGS}|\lambda^*=0\right)}{Var(P^{NGS}|\lambda^*=0.85)} = 1.21$ in figure 3.

5 Conclusion

Allowing for supply-based speculation, via a natural extension of a standard CARA-normal asset trading model makes information complementary among the traders and generates multiple equilibria in the financial market. The complementarity exists even for financial market equilibria that share the same informational properties as previous analyses of the framework. Although other studies have explored the consequences of introducing information about supply in the framework, information remains a strategic substitute in those analyses.

Our results mean that the analytically tractable CARA-normal framework can still be used to study financial markets when information complementarity seems a natural phenomenon. Our model also allows for many further extensions. The first extension is already mentioned in footnote 5, which could allow for information market equilibria that have differing fractions of supply- and payoff-informed traders. Our main results still hold in this extension. Muti-asset markets can also be studied using an extension of our model, which could shed light on the comovement of stock prices. Endogenizing the price of information, as in Veldkamp (2006a and 2006b), would also be interesting in our model since that would add an additional source of complementarity in information acquisition.

Another straightforward extension is to consider a model with multi-period trading. Then the prices in consecutive periods are correlated by the (random) asset supply. This may help people infer more through the realization of a price process and can have an effect on the information acquisition decisions. Further, we could allow for borrowing constraints (Yuan, 2005), which may help explain asymmetric and correlated behavior of prices. Finally, we could allow for consumption in the first period in our model, i.e., have the traders care about wealth over two periods and not just the terminal wealth. We believe the intertemporal consumption choice and multiplicity would generate some interesting results as in the study by Muendler (forthcoming).

6 Appendix

Proof of proposition 1. From equations (5) to (11), we solve for P and get the following

$$
\begin{aligned}
& [\lambda \rho_I + (1-\lambda)\rho_U] P \\
= & \left[\rho_v \bar{v} + \lambda \frac{c}{b} \rho_\theta \frac{\rho_x}{\rho_x + \rho_\eta} \bar{x} + (1-\lambda) \frac{c}{b} \rho_\phi \bar{x}\right] \\
& + \left[\lambda(\rho_\varepsilon + \rho_\theta) + (1-\lambda)\rho_\phi\right] v - \left[\lambda \frac{c}{b} \rho_\theta \frac{\rho_x}{\rho_x + \rho_\eta} + (1-\lambda) \frac{c}{b} \rho_\phi + \gamma\right] x
\end{aligned}
$$

Comparing with equation (3), we have the polynomial

$$
\lambda \rho_\eta \left(\frac{b}{c}\right)^2 - \gamma \left(\frac{b}{c}\right) + \lambda \rho_\varepsilon = 0.
$$

Then defining $\beta = b/c$, the result follows directly.

Proof of proposition 2 The ex post indirect utility of informed trader i

$$\begin{aligned} E\left[V_I^i|x_i, y_i, P\right] &= E\left[-e^{-\gamma W_{2I}^i}|x_i, y_i, P\right] \\ &= -\exp\left\{\left[-\gamma\left(\bar{N}_i - \kappa\right) - \frac{\left(E\left[v|x_i, y_i, P\right] - P\right)^2}{2\rho_I^{-1}}\right]\right\} \end{aligned}$$

Of course, conditioning on $\{x_i, y_i, P\}$ is equivalent to conditioning on $\{x_i, y_i, s\}$.

Define $h = Var\left(E\left[v|x_i, y_i, P\right]|s\right)$. By the conditional variance identity formula, we have

$$h = \frac{1}{\rho_U} - \frac{1}{\rho_I}.$$

Define

$$Z_i = \frac{E\left[v|x_i, y_i, P\right] - P}{\sqrt{h}}.$$

So,

$$E\left[V_I^i|s\right] = e^{\gamma\kappa} u\left(W_1^i\right) E\left[\exp\left(-\frac{h}{2\rho_I^{-1}} Z_i^2\right)|P\right].$$

Conditional on P or s, $E\left[v|x_i, y_i, P\right]$ is normally distributed. Hence conditional on P or s, Z_i^2 has a non-central chi-squared distribution. Then, for $t > 0$, the MGF (moment generating function) for Z_i^2 can be written as

$$E\left(e^{-tZ_i^2}|s\right) = \frac{1}{\sqrt{1+2t}} \exp\left[\frac{-\left(E\left[Z_i|s\right]\right)^2 t}{1+2t}\right],$$

where,

$$E\left[Z_i|s\right] = \frac{E\left[v|s\right] - P}{\sqrt{h}}.$$

So, set $t = \frac{h}{2\rho_I^{-1}}$,

$$E\left[\exp\left(-\frac{h}{2\rho_I^{-1}}Z_i^2\right)|P\right] = \sqrt{\frac{\rho_U}{\rho_I}}\exp\left[\frac{-\left(E\left[v|s\right]-P\right)^2}{2\rho_U^{-1}}\right],$$

which implies

$$E\left[V_I^i|s\right] = e^{\gamma\kappa}\sqrt{\frac{\rho_U}{\rho_I}}u\left(W_1^i\right)\exp\left[\frac{-\left(E\left[v|s\right]-P\right)^2}{2\rho_U^{-1}}\right]. \tag{12}$$

The uninformed traders has ex post indirect utility

$$E\left[V_U^i|s\right] = u\left(W_1^i\right)\exp\left[-\frac{\left(E\left[v|s\right]-P\right)^2}{2\rho_U^{-1}}\right]. \tag{13}$$

Then, by (12) and (13),

$$E\left[V_I^i|s\right] - E\left[V_U^i|s\right] = \left(e^{\gamma\kappa}\sqrt{\frac{\rho_U}{\rho_I}} - 1\right)E\left[V_U^i|s\right].$$

Then taking expectation on both sides of the above equation gives us

$$E\left[V_I^i\right] = e^{\gamma\kappa}\sqrt{\frac{\rho_U}{\rho_I}}E\left[V_U^i\right],$$

which proves the second equality in the proposition. Now we turn to the first one.

By (13),

$$E\left[V_U^i|s\right] = -e^{-\gamma\bar{N}_i}\exp\left[-\frac{\left(E\left[v|s\right]-P\right)^2}{2\rho_U^{-1}}\right].$$

By (9),

$$E\left[v|s\right] - P = \left[\frac{1}{b}\frac{\rho_\phi}{\rho_U} - 1\right]P,$$

which has a normal distribution. Then the MGF of Chi-squared distribution gives us the desired result.

Proof of proposition 3 By proposition 1,

$$R(\lambda) = e^{\gamma\kappa}\sqrt{\frac{\rho_U}{\rho_I}} = e^{\gamma\kappa}\sqrt{\frac{\rho_v + \beta^2\rho_x}{\rho_v + \rho_\varepsilon + \beta^2\left(\rho_x + \rho_\eta\right)}}.$$

According to the remark after proposition 1, we know in the GS-REE, $\frac{\partial\beta}{\partial\lambda} > 0$. So, $R'(\lambda)$ has the same sign as $\frac{\partial(\rho_U/\rho_I)}{\partial(\beta^2)}$.

Since

$$\frac{\partial\left(\rho_U/\rho_I\right)}{\partial\left(\beta^2\right)} = \frac{\rho_x\rho_\varepsilon - \rho_\eta\rho_v}{\left[\rho_v + \rho_\varepsilon + \beta^2\left(\rho_x + \rho_\eta\right)\right]^2},$$

then $R'(\lambda) > 0$ if and only if $\rho_x\rho_\varepsilon - \rho_\eta\rho_v > 0$, i.e.,

$$\frac{\rho_\varepsilon}{\rho_v} > \frac{\rho_\eta}{\rho_x}.$$

Proof of proposition 4 The proof is similar to that of proposition 3 and hence omitted.

Proof of proposition 5 By proposition 1,

$$E(P) = a + b\bar{v} - c\bar{x} = \bar{v} - \frac{\gamma}{K}\bar{x}$$

Thus, if $\bar{x} > 0$, then $\frac{\partial E(P)}{\partial\lambda}$ has the same sign as $\frac{\partial K}{\partial\lambda}$.

Note that

$$K = \rho_v + \lambda\rho_\varepsilon + \beta^2\rho_x + \lambda\beta^2\rho_\eta.$$

Thus, obviously $\frac{\partial K^{GS}}{\partial \lambda} > 0$ since $\frac{\partial \beta^{GS}}{\partial \lambda} > 0$. Now,

$$\begin{aligned}\frac{\partial K^{NGS}}{\partial \lambda} &= \left(\rho_\varepsilon + \beta^2 \rho_\eta\right) + 2\left(\rho_x + \lambda \rho_\eta\right)\beta \frac{\partial \beta^{NGS}}{\partial \lambda} = \left(\rho_\varepsilon + \rho_\eta \beta^2\right) - \frac{2\left(\rho_x + \lambda\rho_\eta\right)\gamma}{\lambda\sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}}\beta^2 \\ &= \rho_\varepsilon - \left[\left(\frac{2\gamma}{\sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}} - 1\right)\rho_\eta + \frac{2\rho_x\gamma}{\lambda\sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}}\right]\left[\frac{\gamma + \sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}}{2\lambda\rho_\eta}\right]^2.\end{aligned}$$

Note that

$$\frac{\gamma + \sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}}{2\lambda\rho_\eta} \geq \frac{\gamma}{2\lambda\rho_\eta} \geq \frac{\sqrt{4\lambda^2\rho_\varepsilon\rho_\eta}}{2\lambda\rho_\eta},$$

where the last inequality follows because $\gamma^2 > 4\lambda^2\rho_\varepsilon\rho_\eta$ in order for the REE to exist. So,

$$\begin{aligned}&\left[\left(\frac{2\gamma}{\sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}} - 1\right)\rho_\eta + \frac{2\rho_x\gamma}{\lambda\sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}}\right]\left[\frac{\gamma + \sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}}{2\lambda\rho_\eta}\right]^2 \\ \geq\; &\left(\frac{2\gamma}{\sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}} - 1\right)\rho_\eta\left[\frac{\sqrt{4\lambda^2\rho_\varepsilon\rho_\eta}}{2\lambda\rho_\eta}\right]^2 = \left(\frac{2\gamma}{\sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}} - 1\right)\rho_\varepsilon \geq \rho_\varepsilon.\end{aligned}$$

Thus,

$$\frac{\partial K^{NGS}}{\partial \lambda} = \rho_\varepsilon - \left[\left(\frac{2\gamma}{\sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}} - 1\right)\rho_\eta + \frac{2\rho_x\gamma}{\lambda\sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}}\right]\left[\frac{\gamma + \sqrt{\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta}}{2\lambda\rho_\eta}\right]^2 \leq 0.$$

References

[1] A Admati (1985), A noisy rational expectations equilibrium for multi-asset securities markets, Econometrica, Vol. 53, Issue 3, pp. 629-658

[2] G-M Angeletos and I Werning (forthcoming), Crises and prices: information aggregation, multiplicity, and volatility, American Economic Review, forthcoming

[3] G Barlevy and P Veronesi (2000), Information acquisition in financial markets, Review of Economic Studies, Vol. 67, Issue 1, pp.79-90

[4] T Bewley (1986), Stationary monetary equilibrium with a continuum of independently fluctuating consumers, in Contributions to Mathematical Economics in Honor of Gerard Debreu, edited by W Hildenbrand and A Mas-colell, North-Holland, 1986, pp. 79-102

[5] U Bhattacharya and M Spiegel (1991), Insiders, outsiders, and market breakdowns, Review of Financial Studies, Vol. 4, Issue 2, pp. 255-282

[6] C Chamley (2005), Complementarities in information acquisition with short-term trades, mimeo, Boston University

[7] D Diamond and R Verrecchia (1981), Information aggregation in a noisy rational expectations economy, Journal of Financial Economics, Vol. 9, Issue 3, pp. 221-235

[8] D Easley and M O'Hara (2004), Information and the cost of capital, Journal of Finance, Vol. 59, Issue 4, pp. 1553-1584

[9] G Gennotte and H Leland (1990), Market liquidity, hedging, and crashes, American Economic Review, Vol. 80, Issue 5, pp. 999-1021

[10] S Grossman (1976), On the efficiency of competitive stock markets where trades have diverse information, Journal of Finance, Vol. 31, Issue 2, pp. 573-585

[11] S Grossman and J Stiglitz (1980), On the impossibility of informationally efficient markets, American Economic Review, Vol. 70, Issue 3, pp.393-408

[12] B Grundy and M McNichols (1989), Trade and the revelation of information through prices and direct disclosure, Review of Financial Studies, Vol. 2, Issue 4, pp. 495-526

[13] C Hellwig and L Veldkamp (2006), Knowing what others know: coordination motives in information acquisition, mimeo, UCLA

[14] M Hellwig (1980), On the aggregation of information in competitive markets, Journal of Economic Theory, Vol. 22, Issue 3, pp. 477-498

[15] K Judd (1985), The law of large numbers with a continuum of i.i.d. random variables, Journal of Economic Theory, Vol. 35, Issue 1, pp. 19-25

[16] L Kodres and M Pritsker, (2002), A rational expectations model of financial contagion, Journal of Finance, Vol. 57, Issue 2, pp. 769-799

[17] J-J Laffont (1985), On the welfare analysis of rational expectations equilibria with asymmetric information, Econometrica, Vol. 53, Issue 1, pp. 1-30

[18] A Madhavan (1992), Trading mechanisms in securities markets, Journal of Finance, Vol. 47, Issue 2, pp. 607-641

[19] M-A Muendler (forthcoming), On the possibility of an informationally efficient market, Journal of Economic Theory

[20] F Palomino (2001), Informational efficiency: ranking markets, Economic Theory, Vol. 18, Issue 3, pp. 683-700

[21] Y Sun (2006), The exact law of large numbers via Fubini extension and characterization of insurable risks, Journal of Economic Theory, Vol. 126, Issue 1, pp. 31-69

[22] L Veldkamp (2006a), Media frenzies in markets for financial information, American Economic Review, Vol. 96, Issue 3, pp. 577-601

[23] L Veldkamp (2006b), Information markets and the comovement of asset prices, Review of Economic Studies, Vol. 73, Issue 3, pp. 823-845

[24] K Yuan (2005), Asymmetric price movements and borrowing constraints: a rational expectations equilibrium model of crises, contagion, and confusion, Journal of Finance, Vol. 60, Issue 1, pp. 379-411