When Half the Truth is Better Than the Truth: Theory and Experiments on Aggregate Information Cascades

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Comments welcome.

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Motivation

The canonical model of informational cascades: agents act in sequence and observe the entire history of individual decisions.

In some contexts this may not be true. We may be able to observe only the **aggregate number** of agents who have already made one of two decisions.

Examples:
- Investment in a new technology;
- Raising new capital;
- Restaurant's choice.

Policy relevant example: releasing information on the adoption of a new medical treatment.

The Model

- State space:
  $\Omega=\{0,1\}$ $\omega = 1$ with Pr. r $(0<r<1)$

- Time: $t= 1,2,3,\ldots.n.$

- Agents: finite number (n) of agents.
  Each agent i randomly drawn to act only once, at time i.

- Action Space: Each agent chooses I_i 2$\{0,1\}$
  $I_i = 1 \leftrightarrow$ INVEST

Information

Private Public

Symmetric binary signal with precision q.

$S = \{0, 1\}$

$q = \Pr(s_i=0|\omega=0) = \Pr(s_i=1|\omega=1)$

$r < q < 1;\ q + r > 1$

Information

Private **Public**

Agent i is informed on:

$$T_i = \sum_{j=1}^{i-1} I_j$$

Recall: he does not know his position in the sequence.
Only INVESTMENTS are observable.

Strategy and Payoff

- Agent i chooses I_i 2$\{0,1\}$ to maximize

$$\pi_i = \omega I_i + (1 - \omega)(1 - I_i)$$

- Strategy $J_i : \{0, 1, 2, 3, \ldots, n\text{-}1\} \times \{0,1\} \rightarrow \{0,1\}$
- Mixed strategies: I_i is the probability with which agent i invests.

Informational Cascades

- An Aggregate Up Cascade (AUC) occurs when in equilibrium there is a decision after which all agents decide to invest independently of their signal.

In other words, there exists T^{UP} such that if $T_k = T^{UP}$ then $T_{k+j} = T^{UP} + j - 1$, for j= 1, 2, 3,…

- An Aggregate Down Cascade (ADC) occurs when in equilibrium there is a decision after which all agents decide not to invest independently of their signal.

In other words, there exists T^{DOWN} such that if $T_k = T^{DOWN}$ then $T_{k+j} = T^{DOWN}$, for j = 1,2,3,…

Results

• We restrict the analysis to symmetric PBNE.

Results for $T_i = 0$

• Definition: An initially pure equilibrium (IPE) is an equilibrium that prescribes pure actions for $T_i = 0$ and both possible signals $s_i = 0$ and $s_i = 1$.

• Lemma: In any IPE, an agent follows his own private signal if he observes that nobody has invested: $I_i(0,s_i) = s_i$.

Sketch of Proof for Lemma 1

• By contradiction.

• Case 1: agents never invest after observing $T_i = 0$.

Then $T_i = 0$ is uninformative.

• Case 2: agents always invest after observing $T_i = 0$.

Then $T_i = 0$ means that i is the first decision maker.

• Case 3: agents play against the signal: $I_i(0,s_i) = 1 - s_i$.

Then $T_i = 0$ either because i is the first decision maker or because predecessors had bad signals.

More results on $T_i = 0$

- Lemma 2: An IPE exists if and only if

$$r \geq (1 - q^n) / (2 - (1 - q)^n - q^n)$$

- Lemma 3: In any equilibrium, $I_i(T_i,1) \geq I_i(T_i,0)$ for all T_i

In any equilibrium, if $0 < I_i(T_i,0) < 1$, then $I_i(T_i,1) = 1$, and if $0 < I_i(T_i,1) < 1$, then $I_i(T_i,0) = 0$.

- Lemma 4: In any equilibrium that is not an IPE, $I_i(0,0) = 0$ and $0 < I_i(0,1) < 1$.

Final results on $T_i = 0$

- Proposition 1:

(i) If $r \geq 1/2$, agents who observe $T_i = 0$ follow their signal in all equilibria;

(ii) If $(1 - q^n) / (2 - (1 - q)^n - q^n) < r < 1/2$, there is an equilibrium in which agents who observe $T_i = 0$ follow their signal and there may also be a mixed strategy equilibrium in which agents who observe $T_i = 0$ follow their signal if it is bad and mix if it is good.

(iii) If $r < (1 - q^n) / (2 - (1 - q)^n - q^n)$ there can only be a mixed strategy equilibrium in which agents who observe $T_i = 0$ follow their signal if it is bad and mix if it is good.

Result: Monotonicity

- Lemma 6: In any equilibrium if $T_i < T_i'$, then $l_i(T_i,s_i) \leq l_i(T_i',s_i)$ for any s_i. In particular, if $0 < l_i(T_i,s_i) < 1$, then $l_i(T_i',s_i) = 1$.

Results on Information Cascades

- Proposition 2:

(i) In any equilibrium, $I_i(0,1) > 0$ and $I_i(T_i,1) = 1$ for all $T_i > 0$, i.e., an agent with a good signal always invests with positive probability and an **Aggregate Down Cascade never occurs** in equilibrium.

(ii) In any equilibrium there is at most one T^{MIX} for which $0 < I_i(T^{MIX},0) < 1$. For all $T_i < T^{MIX}$ agents with a bad signal follow their signal and do not invest.

For all $T_i > T^{MIX}$ **an Aggregate Up Cascade occurs**.

Results on Cascades

- Proposition 3: Aggregate Up Cascades are part of any equilibrium. In particular, in any equilibrium, for all signals s_i, $I_i(T_i,s_i) = 1$ for all $T_i > n/2$.

Results

	$\sigma_i = 0$	$\sigma_i = 1$
$T_i = 0$	0	$mixed, 1$
$0 < T_i < T^{MIX}$	0	1
$T_i = T^{MIX}$	$mixed$	1
$T_i > T^{MIX}$	1	1
$T_i \geq T^{UP}$	1	1

Experimental Evidence

- We tested the theory in the lab.
- Two Treatments.
- Treatment A: r = ½; q= 0.7.
- Treatment B: r = ¾; q = 0.8.

	Treatment A		Treatment B	
	$\sigma_i = 0$	$\sigma_i = 1$	$\sigma_i = 0$	$\sigma_i = 1$
$T_i = 0$	0	1	0	1
$T_i = 1$	0	1	1	1
$T_i = 2$	1	1	1	1

The Experiment

- 57 subjects (undergraduate students at UCL).
- 19 sessions (i.e., groups of 3 people).
- In each session the same game was repeated 15 times.
- Strategy method.
- Feedback on the state of the world, T_i, s_i, and payoff.
- Payoff: £3 for the right decision; 0 otherwise.

Results

	Treatment A		Treatment B	
	$\sigma_i = 0$	$\sigma_i = 1$	$\sigma_i = 0$	$\sigma_i = 1$
$T_i = 0$	0.15	0.56	0.21	0.86
	(0.03)	(0.03)	(0.09)	(0.05)
$T_i = 1$	0.24	0.87	0.41	0.94
	(0.04)	(0.03)	(0.06)	(0.03)
$T_i = 2$	0.68	0.85	0.76	0.94
	(0.07)	(0.03)	(0.09)	(0.04)

Probability of an ADC despite 3 good signals in Treatment A = 17%.

Conclusion

- While aggregate up cascades (cascades on the observable action) occur, aggregate down cascades are theoretically impossible. They rarely occur in the lab.

- A good innovation should not remain un-adopted for ever (at least for information reasons).

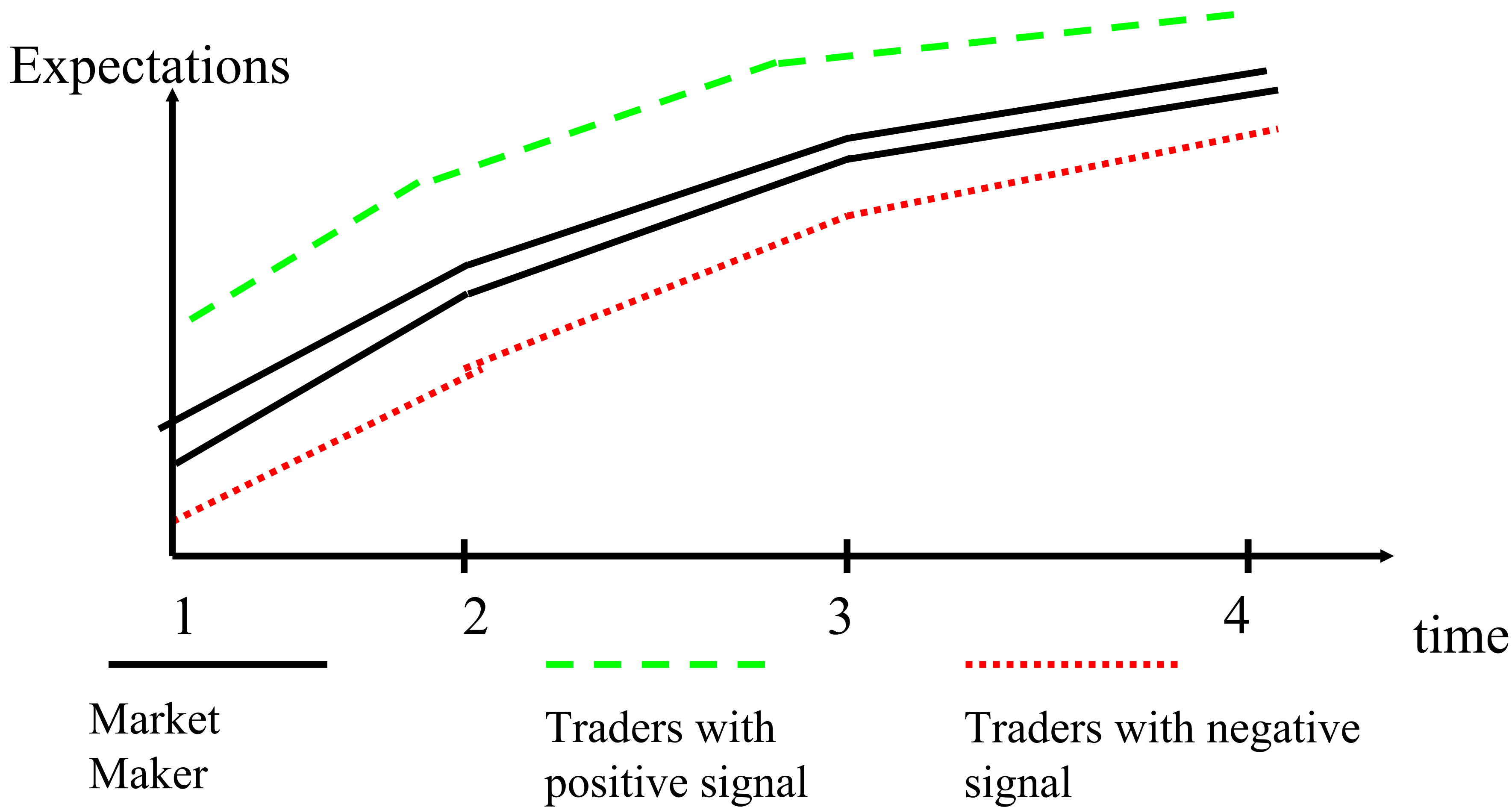
- Telling half the truth can be good to avoid bad cascades.

Herd Behavior

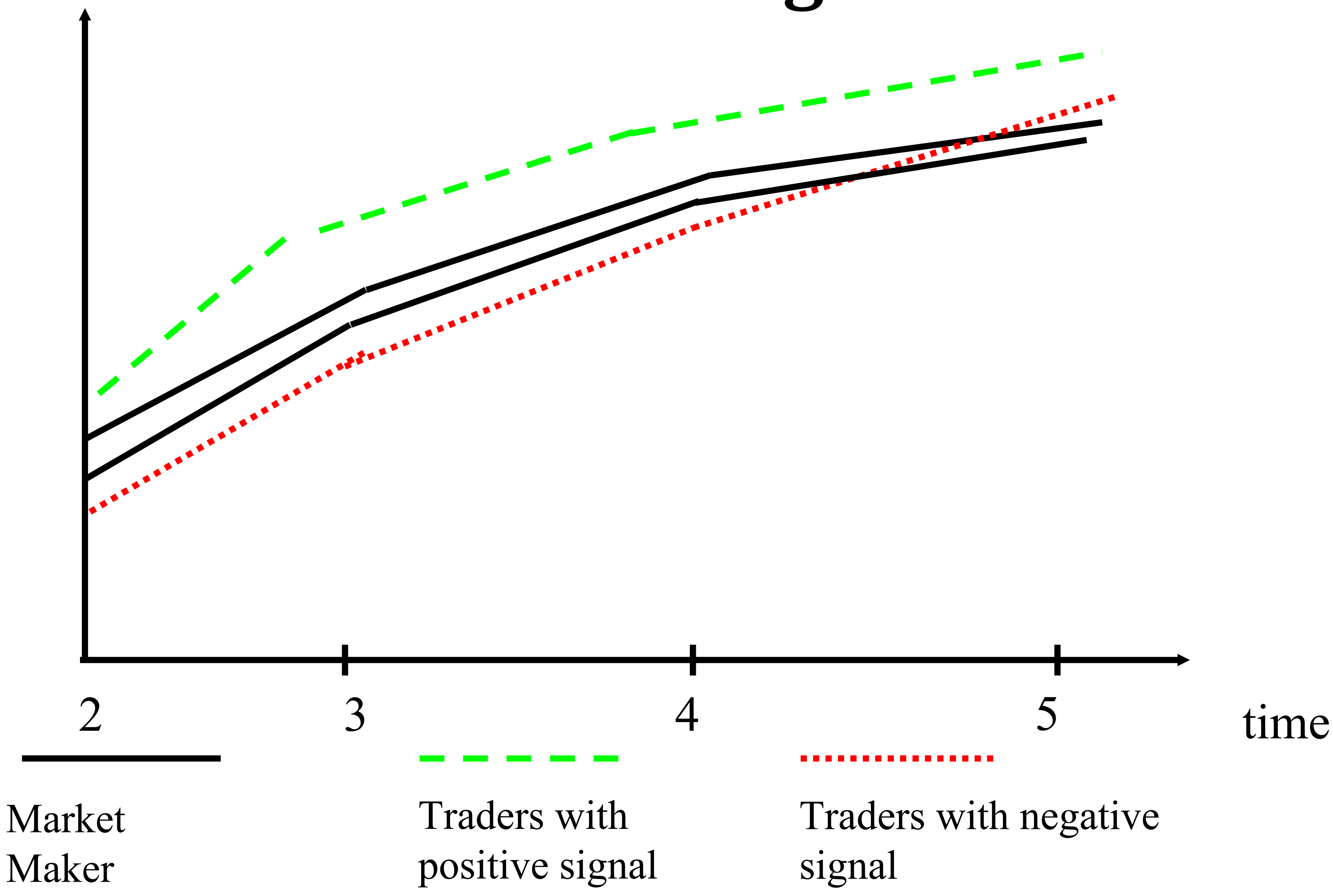
There is herd behavior at time t of day d when an informed trader at that time chooses the same action with probability 1 (<u>independently</u> of his private information).

<u>Result</u>: Herd behavior arises with positive probability. Herd behavior can be misdirected.

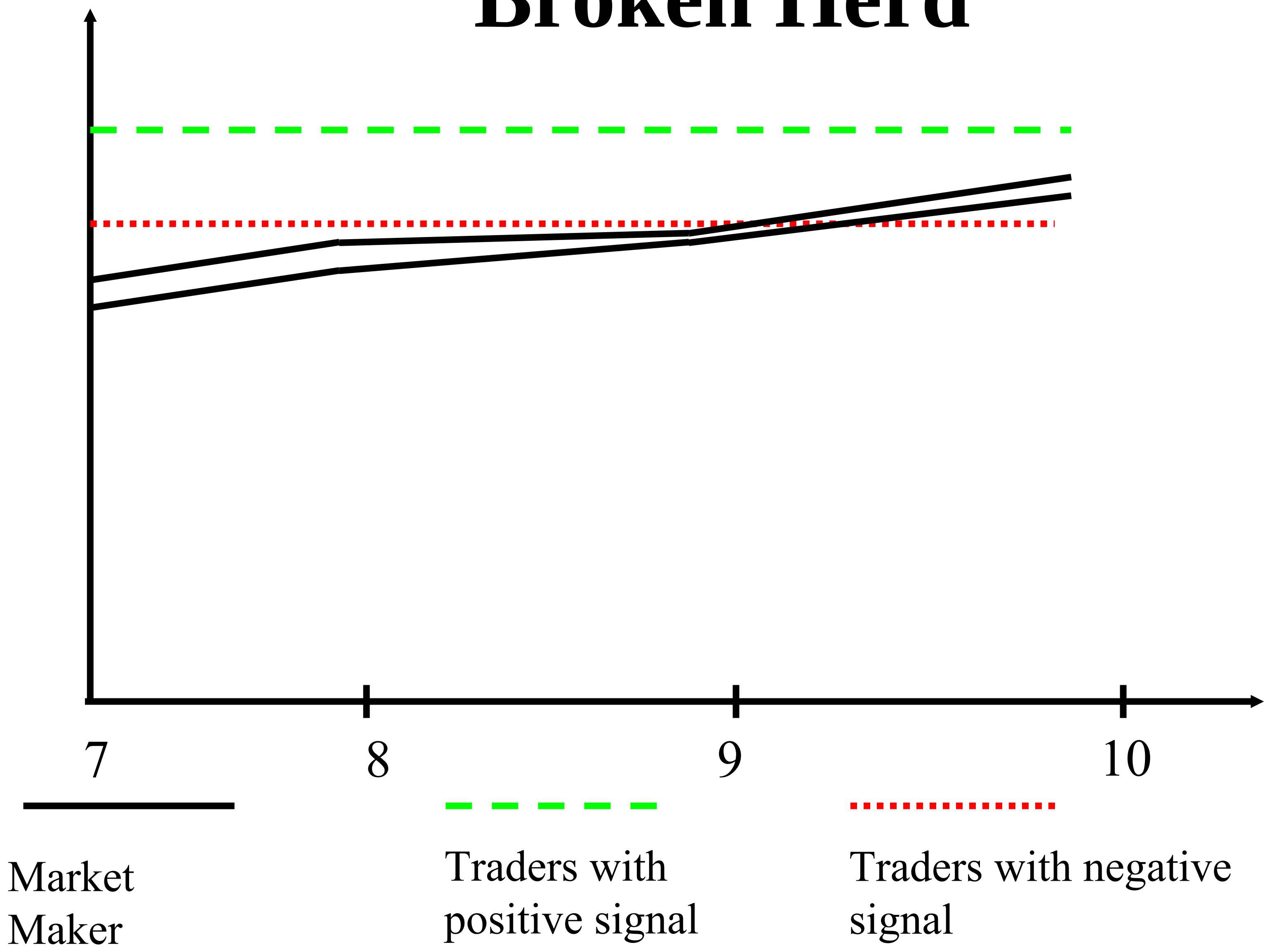
A Heuristic Argument



A Heuristic Argument 2



Broken Herd



7
8
9
10
Market Maker
Traders with positive signal
Traders with negative signal

Price Convergence

Result:

- The price in day d converges almost surely to the true value of the asset v_d.

The Data

- NYSE Transaction data (TAQ dataset)
- Bid and ask quotes and transaction prices.
- Data: Ashland Oil, 1/1/95-31/3/95

The Data

Time	Price
34358	34.2500
34370	34.3750
34915	34.4000
……	…………
……	…………

Time	Bid	Ask
34350	34.0000	34.2500
34360	34.1250	34.3750
34829	34.2500	34.5000
…..	………	……….
…..	………	………

From Prices to Trades

Lee and Ready algorithm (1991, JF).

Price > Mid Quote	$\longrightarrow$	BUY
Price < Mid Quote	$\longrightarrow$	SELL

• One no-trade each 5 minutes of inactivity.

Data

Tuesday 3/1/1995
BUY
NT
SELL
SELL
NT
NT
NT
NT
BUY
BUY
BUY
NT

Number of Days: 63

Average Number of Trades Per Day: 133

Proportion of Buy Trades: 38.7%

Proportion of Sell Trades: 35.7%

Proportion of No Trades : 25.6%

The Likelihood Function

Parameters:

a = probability of an event.

g = probability that the event is good.

m = proportion of informed traders.

q = precision of the signal.

e = probability of a trade by a noise trader.

Goal:

- Compute the probability of each trade.
- Compute likelihood function.

The Likelihood Function 2

➢ Probability of a history of trades over D days:

$$\Pr\left(\left\{h^{d}\right\}_{d=1}^{D} \mid a, g, q, m, e\right) = \prod_{d=1}^{D} \Pr(h^{d} \mid a, g, q, m, e)$$

➢ Probability of a history of trades in any given day? We need to proceed recursively.

$$\Pr(h_{t}^{d} \mid a, g, q, m, e) = \prod_{s=1}^{t} \Pr(x_{s}^{d} \mid h_{s}^{d}, a, g, q, m, e)$$

The Likelihood Function 3

E.g., history: Buy, Buy, Buy…

1) Compute probability of the first trade:

Pr (Buy | a,g,q,m,e).

2) Compute probability of the second trade:

Pr (Buy | Buy, a,g,q,m,e).

3) Compute

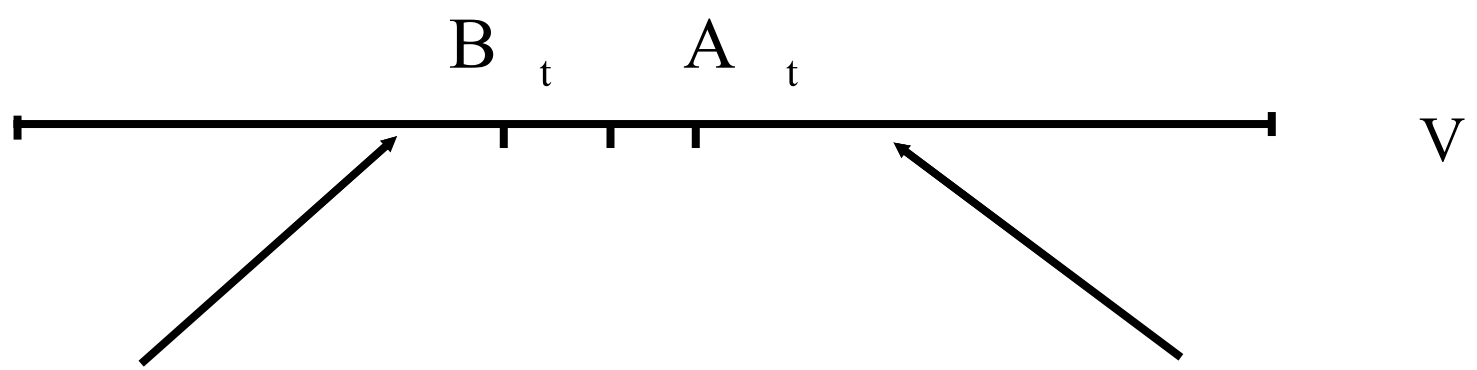
Pr (Buy | Buy, Buy, a,g,q,m,e)

Because of herding these probabilities are not independent.

E.g., if at time 3 traders start to herd,

Pr (Buy|Buy,Buy, a,g,q,m,e) ≠ Pr (Buy| a,g,q,m,e).

No Herding



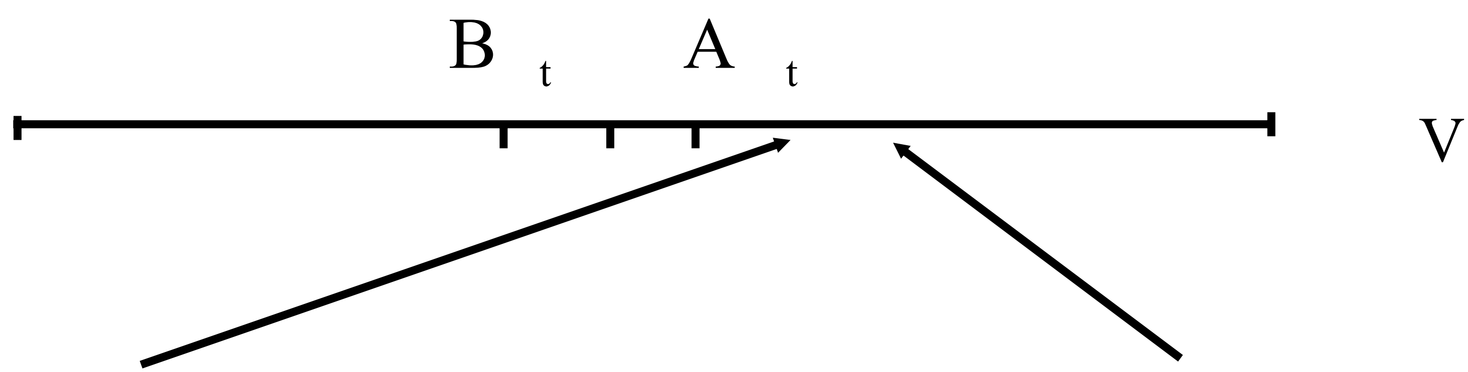
Expected value of a trader with a negative signal.

Expected value of a trader with a positive signal.

<u>Probability of a trade</u>

Computed assuming that a trader with a high signal buys and traders with a low signal sells.

Herd Buy



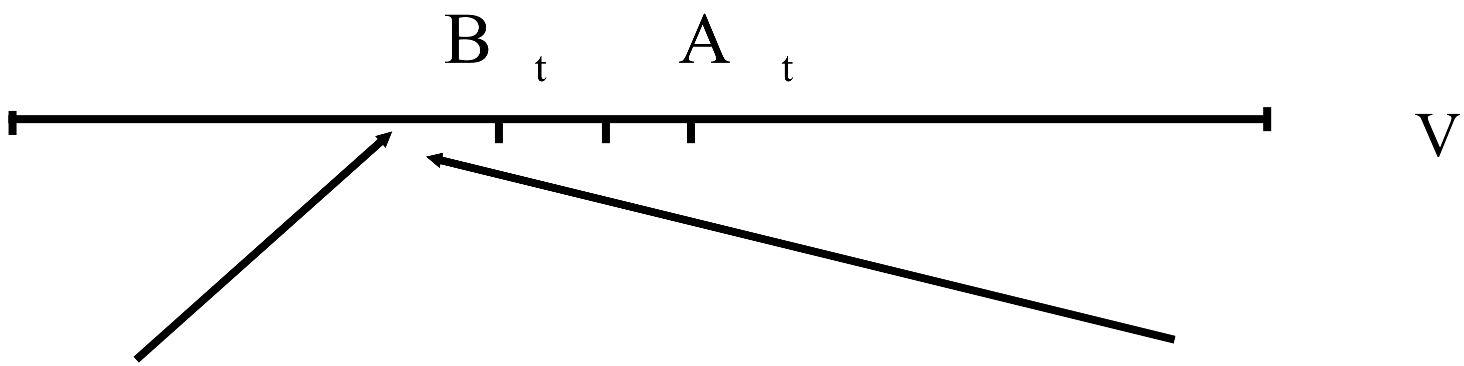
Expected value of a trader with a negative signal.

Expected value of a trader with a positive signal.

Probability of a trade

Computed assuming that all informed traders buy.

Herd Sell



Expected value of a trader with a negative signal.

Expected value of a trader with a positive signal.

<u>Probability of a trade</u>

Computed assuming that all informed traders sell.

Estimation Strategy

Bayesian Estimation

Priors:

•a, g, m, e: random variables with uniform prior on [0,1];

• q: random variable with uniform prior on [1/2,1].

Posterior distributions obtained by MCMC procedure (Hasting-Metropolis algorithm).

Results refer to 1,000,000 draws form the posterior distribution for each parameter (excluding first 100,000 draws: “burn in phase”)

Results

	Probability of	Mean	SD
a	Information Event	.783	.057
g	Asset Value Increase	.685	.097
m	Informed Trader	.236	.018
q	Correct Signal	.659	.027
e	Uninformed Traders Trading	.690	.006

Detecting Herd Behavior

Given (a, g, m, q, e), at each time t we

- compute traders' expectations conditional on receiving a high or a low signal;
- compare them to the market maker's expectations (quotes);

✓ if, for instance, at time t of day d,

$$E\,(V_d |\, H^t_d\,, s^L) > Ask^d_t$$

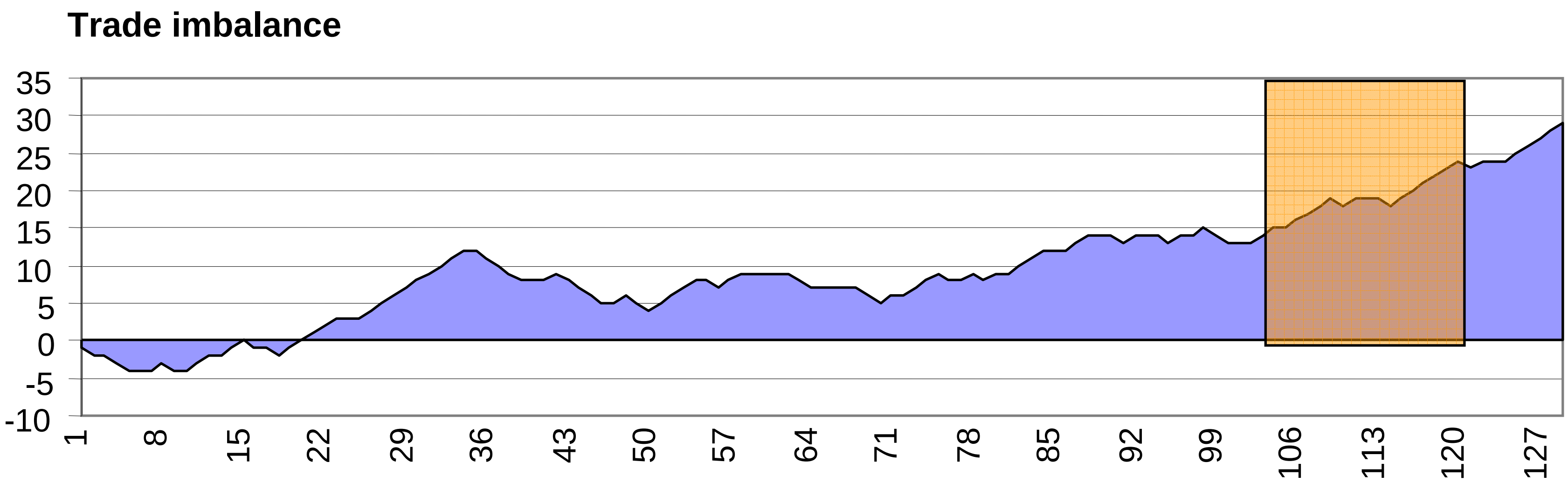
then we are in a herd-buy period.

✓ if, at time t of day d,

$$E\,(V_d |\, H^t_d\,, s^H) < Bid^d_t$$

then we are in a herd-sell period.

A Day of Herding



✓14% percent of trades characterized by herd buying.

Herding

Percentage of Herding Trades

Herd Buy: 7.3%

Herd Sell: 8%

Number of Days When Herding is Relevant

>50%	>30%	>15%
7	15	23

✓63 days

Informational Efficiency

- We simulated our theoretical financial market.
- The parameters are equal to the estimated parameters.
- 100 trades per day, 10000 days.
- We computed the market price.
- We computed the price that would have prevailed if all informed traders had (irrationally) decided never to herd.

Effect on Price

✓Average distance between prices: 0.4%.

✓ Percentage of trading periods where distance is above 10%: 2.8.

Conclusion

• We have shown how to estimate a model of herd behavior in financial markets.

• The sequence of trades does conveys information which is not conveyed by the number of trades.

• In our stock, herding occurs, but not very often.

• There are days where herding is very significant.

Current Research

1. Relevance of herding:
    - for different markets.
    - for different times (crises/tranquil times).

2. Timing of information event. Number of information events per day.

3. Herding due to gains from trade (heterogeneity among traders) (Cipriani and Guarino, 2001).

Related literature

Easley, Kiefer and O'Hara, 1997, RFS.

• They estimate the parameters of a market microstructure model.

But

• the precision of the signal is 1.

✓ No herding.

✓ Sequence of trades does not matter.

✓ Likelihood function is only a function of the number of buys, sells and no trades in any given day.

The Market Maker

•The traders exchange the asset with a market maker.

•The market maker sets the prices at which the asset is exchanged.

• The market maker is assumed to behave competitively: he sets the price to make zero expected profits.

•He sets a bid and an ask price:

$$Bid^d_t = E\,[V_d |\, h^d_t, X^d_t = SELL],$$

$$Ask^d_t = E\,[V_d |\, h^d_t, X^d_t = BUY].$$

“Learning While Herding”

1. The market maker keeps updating his beliefs, while informed traders do not.

2. After some time, herd is broken. Traders will resume trading according to their signal.

Relation to Literature

Social learning theory:

Seminal papers:

Banerjee, QJE 1992.
Bickchandani et al., JPE 1992.
Welch, JoF 1992.

Herding in financial markets:

Avery and Zemsky, AER 1998.
Multidimentional uncertainty
(e.g., uncertainty both on the asset value and on the precision of the signal).

Cipriani and Guarino, mimeo 2001.
Gains from trade - Heterogeneity among traders.

Evidence on Herding

Empirical work:
Lakonishok, Shleifer, and Vishny, JFE, 1992.
Grinblatt, Titman and Wermers, AER, 1995.
Wermers, JoF, 1999.

Different classes of investors (e.g., pension fund or mutual fund managers) cluster their decisions. *Is it real herding?*

Problem for the empiricist: lack of data on private information.

Experimental work:

Cipriani and Guarino, AER 2005

Herding

Informational herding:

✓rational agents disregard private information (and imitate)

✓ imitation is the optimal choice given agents' Bayesian inference from the decisions of their predecessors.

A New Approach: A Structural Estimation

Objectives:

• estimating a theoretical model of herding in financial markets.

• detecting the periods of herding.

• shedding light on private information aggregation by market prices.

• making comparisons across different markets and different economic situations (e.g., tranquil times versus financial crises).

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Gains from trade - Heterogeneity among traders (e.g, different hedging reasons to trade).