Platform Competition, Compatibility, and Social Efficiency*

Ramon Casadesus-Masanell† Francisco Ruiz-Aliseda‡

September 30, 2008

Abstract

We study competitive interaction between two platform providers (such as two suppliers of videogame consoles or operating systems) that mediate between sellers of platform-based products (developers of games or applications) and buyers of such products (users of games or applications). Users and developers first trade with one of the platforms (users purchase videogame consoles and developers software development kits) and then with each other (users buy games from developers). We show that the unique equilibrium under platform compatibility exhibits more intense price competition for developers, softer price competition for users, and higher profits than the symmetric equilibrium under incompatibility (which may involve user subsidization to join a platform). Compatibility leads to higher profits because it induces more entry by developers, lower application prices, and larger user surplus that platforms capture through higher user access prices. Incompatibility, however, naturally gives rise to asymmetric equilibria with a dominant platform that captures all users and earns more than in the compatible equilibrium. The model also allows a detailed analysis of social efficiency, and we show that entry by developers is socially excessive (insufficient) if competing platforms are compatible (incompatible).

Key words: Two-sided Platforms, Incompatibility, Network Externalities, Market Dominance, Tipping, Pricing Structure.
JEL code: L11, L15, L40.

*We are very grateful to Hanna Hałaburda, Doh-Shin Jeon, Timothy Van Zandt and seminar participants at Universitat Autònoma de Barcelona and Imperial College's 2008 conference on "Platforms, Markets and Innovation" for helpful comments. Part of this research was conducted while the second author was visiting INSEAD, and he would like to thank all the members of this institution for their hospitality during his stay, as well as for financial support for his research. Casadesus-Masanell is grateful to the HBS Division of Research and IESE Business School's Public-Private Sector Research Center. We thank the NET Institute (www.NETinst.org) for financial support.

†Harvard Business School.

‡Universitat Pompeu Fabra.

1 Introduction

The last three decades have witnessed unprecedented growth in network industries such as video games, computers, credit cards, media, or telecommunications. These industries are often organized around physical or virtual platforms that enable distinct groups of agents to interact with one another and are commonly referred to as two-sided markets or markets with two-sided platforms (Evans, 2003, and Evans and Schmalensee, 2007). An operating systems developer such as Microsoft, for example, provides a software platform that makes possible the completion of value-creating transactions between independent software vendors and users.

It is well-known that platforms are characterized by the presence of inter-group network effects and that these constitute a central feature affecting pricing by platform providers. For example, when pricing its operating systems and software development kits, Microsoft must take into account that the larger the number of applications expected to run on Windows, the more willing users are to adopt it. Likewise, developers' incentives to write Windows applications grow with the number of users who are expected to adopt that operating system.

A key attribute of the market that determines the intensity and scope of network effects is whether competing platforms are compatible or not. In a seminal paper, Katz and Shapiro (1985) study systems compatibility in markets with one-sided platforms.[1] The effects of platform (in)compatibility on market outcomes, however, have largely been ignored by the literature on markets with two-sided platforms.

In this paper, we study competitive interaction between platforms in a two-sided setting under compatibility and incompatibility. Our purpose is two-fold. At the positive level, we are interested in examining the properties of the pricing structures that emerge under the different intellectual property regimes that we consider, to better understand the reasons why markets with two-sided platforms appear to naturally lead to (quasi-)monopolistic industry structures. Prominent examples include the personal computer or digital music distribution industries, which are dominated by Microsoft's Windows and Apple's iTunes, respectively. At the normative level, we would like to shed light on whether equilibrium pricing in markets with two-sided platforms is socially efficient or not in the separate compatibility regimes that we study.

The contribution of this paper is to develop an explanation – by endogenizing the downstream interaction between developers and consumers as an oligopolistic market – why markets such as operating systems and videogame systems are often characterized by incompatibility with one dominant player that perhaps sells access to the platform below cost to one of the two sides.

We model a situation in which two platform providers act as intermediaries between developers of products based on the platform and users of such products. Developers and users first trade with the platform providers by adopting one of the two platforms. Platforms are assumed to be horizontally differentiated from the users' viewpoints. After having gained access to one platform, users buy applications from developers under oligopolistic conditions

[1] In their setting, platforms set access prices to users but *not* to software developers. Moreover, there is no transaction between independent software vendors and users.

that, to a large extent, depend on the pricing structures set by the platform providers.

We compare the nature of platform competition under application compatibility and incompatibility. Under compatibility, active developers sell their applications to all the users, regardless of the platform they have adopted.[2] As a result, developers do not benefit from platform-specific network externalities and perceive both platforms as homogeneous in that they give access to the same pool of users and number of competitors. Platform homogeneity leads to intense competition for developers: platforms set developer access prices equal to the marginal cost to serve them. Marginal cost pricing implies copious entry by developers and low application prices. Under compatibility, platforms cannot vertically differentiate based on the number of applications because users foresee having access to a unique pool of applications. However, because platforms are assumed to be horizontally differentiated, platform providers have some pricing power on the user side. Moreover, because application prices are low in this case, there is large potential value that can be extracted from users through platform access prices. Put differently, access prices to users end up being high because (i) platforms are differentiated horizontally and (ii) inter-group network externalities are exploited very intensely due to fierce competition for developers. We also show that compatibility gives raise to a unique equilibrium and that this equilibrium is symmetric.

The nature of platform competition changes dramatically when platforms are incompatible. To begin, there is a unique symmetric equilibrium under incompatibility but there are also asymmetric equilibria. The symmetric equilibrium under incompatibility exhibits softer price competition for developers compared to the case in which platforms are compatible. Less intense competition for developers leads to reduced entry and higher application prices. This implies that less user surplus is generated and, thus, access prices to users end up being lower than in the case of compatible platforms. In fact, when platform horizontal differentiation is not intense enough, competing providers may try to stimulate developer entry by subsidizing users to join their platform. An important feature of the symmetric incompatibility regime is that it results in lower platform profits than compatibility, even if platform-access fees charged to developers are above marginal cost.

Incompatibility is a pervasive feature of many markets with two-sided platforms. This raises the question of why platform providers do not make their platforms compatible if equilibrium profits are higher in that regime, which is also the socially efficient one. To address this question we study the asymmetric equilibria that are present under incompatibility. In these equilibria one of the platform providers corners the market by pricing the competitor out. Such equilibria yield higher profits for the winning platform compared to profits in the unique equilibrium under compatibility. Hence, we find that it is the quest for market dominance that prevents platforms from agreeing to a common standard. We conclude that incompatibility is associated with market dominance, as happens in Katz and Shapiro (1985). Unlike their one-sided setting, though, our model suggests that tipping may be grounded upon user subsidization (we show that this happens if users do not value applications too much).

[2] More precisely, under (full) platform compatibility, a product sold by a developer is functional on any platform, no matter with which platform provider the developer traded in the first period. Under platform incompatibility, a product sold by a developer is valuable only on the platform sold by the intermediary with which such developer traded.

Our model also allows for detailed social welfare analysis. We find that, unlike a social planner, a locally monopolistic platform provider does not internalize all the positive effect of the network externality exerted on users by developers because it cannot appropriate all the gains from trade that accrue to users. In addition, if the marginal cost of serving a user is negligible (as in operating systems), then a social planner finds it optimal to let users have free access to the platform, whereas a profit-maximizing platform provider charges a positive price in order to extract rents. Therefore, fewer users than socially desirable are served by a locally monopolistic platform provider.

Turning to the developer side, we find that when platforms are incompatible, the under-exploitation of the inter-group network externalities originated on the user side results in fewer developers than socially desirable. Because the providers of incompatible platforms cannot capture all the gains that accrue to users if more developers enter, there is no point in promoting too much entry by developers. However, the result of insufficient entry by developers is reversed under platform compatibility. Compatibility leads to positive externalities across platforms that are not internalized by platform providers when choosing access prices for developers, so the fierce price competition results in marginal cost pricing and, hence, in too many developers relative to what is socially desirable.

We contribute to the literature on systems compatibility and oligopolistic competition,[3] initiated by Katz and Shapiro's (1985) path-breaking work and continued by several papers such as Katz and Shapiro (1986), Economides and Flyer (1997), Crémer, Rey and Tirole (2000) or Malueg and Schwartz (2006). Our paper is most closely related to that of Katz and Shapiro (1985) but differs in several respects from theirs. In particular, our setup deals with two-sided platforms that are horizontally differentiated, unlike theirs in which platforms are one-sided and undifferentiated, which is more appropriate for environments with direct network effects. Because we consider two-sided platforms, we are able to draw richer conclusions on the nature of platform competition under the different intellectual property regimes that we consider. Thus, we show when all users acquire one of the platforms that platforms compete fiercely for developers in the compatibility regime but compete fiercely for users in the incompatibility regime. Indeed, when horizontal differences across platforms are not significant enough, users may acquire an incompatible platform at a price below marginal cost. This result cannot be obtained in a one-sided setting and appears to accord well with the common wisdom that some platforms such as videogame systems are typically priced below marginal cost for users. In particular, our model predicts that platform competition exhibits user subsidization only if platforms are incompatible, both for symmetric and asymmetric equilibria. Another conclusion that cannot be drawn when dealing with one-sided platforms refers to the implications of developer entry on social efficiency under the two compatibility regimes that we study. In contrast, our model shows that incompatibility results in insufficient entry, whereas compatibility leads to excessive developer entry.

The approach that we follow in this paper is that of the recent literature on two-sided platforms, mainly devoted to examining the reasons why competing platform providers may find it optimal to sell their products cheaply to one side of the market while charging a high

[3] See Katz and Shapiro (1994) for a literature review.

price to the other side.[4] Within this literature, our paper is closest to those in which agents on each side of the platform first trade with the platform providers and then with agents on the other side. Caillaud and Jullien (2001, 2003) examine matchmaking intermediation services, such as those provided by dating or real estate agencies, in a model with ex ante identical matchmakers that bear no fixed costs. These papers assume that once a match is made, agents realize all gains from trade, and pay special attention to equilibria in which an intermediary prices a competitor out of the market. Such "dominant firm equilibria" are supported by optimistic rational expectations of agents on both sides of the market, according to which an agent expects everyone to interact with the dominant firm. Our paper also highlights the importance of agents' expectations in the emergence of dominant firm equilibria in a somewhat related context. But in our model trade between users and developers is not efficient and several features of the downstream market structure are endogenously determined.

Our paper is also related to Hagiu (2005), which pioneers the analysis of how platform pricing structures are affected by consumers' preferences for product variety in a setting with monopolistically competitive developers. The focus of his paper is on the efficiency of open (or free access) vs. proprietary platforms, a topic which is of independent interest to that of platform (in)compatibility. Indeed, platform compatibility is studied by a few recent papers dealing with two-sided platforms. Orman (2008) studies the effects of compatibility on competition between proprietary and open two-sided platforms and shows that compatibility may increase profits for the proprietary platform. We focus on the case where both competing platforms are proprietary. Miao (2007) studies two-sided platforms composed of two components supplied by different producers (such as TV sets and TV broadcasting equipment) and examines suppliers' incentives to provide compatibility within platforms. In contrast, our analysis deals with integrated platform suppliers and is concerned with application compatibility between platforms.

The paper is organized as follows. Section 2 describes the two-period platform competition model. In sections 3 and 4 we study competitive interaction between incompatible and compatible platforms, respectively. In both cases we present the analysis under full and partial market coverage. Section 5 compares both intellectual property regimes. We conclude in Section 6.

2 Foundations of the theoretical model

Consider a platform provider, labeled by i, that acts as an intermediary between developers of products based on its platform and users of such products. The platform provider could be a supplier of operating systems, in which case developers would be independent software vendors (ISVs) and users would be individuals or firms that make use of such applications.

[4] This literature has largely flourished on the basis of industry-specific models. Rochet and Tirole (2003), for example, is inspired by the credit card market, Armstrong (2006) captures well the economics of shopping malls or newspapers, and Hagiu (2005) maps to competition between providers of videogame systems. See Rochet and Tirole (2006) for the formal definition of two-sided markets and a general framework that renders much of the earlier literature as special cases.

The platform provider sets access prices for both, users and developers. In the case of operating systems providers, the price charged to users is that of the operating system while that for developers is the price of the software development kit or a license fee.[5] We denote the access price paid by users by p_i^B and that paid by developers by p_i^S. After users and developers have transacted with the platform provider, developers compete against one another to sell their applications. Ceteris paribus, low p_i^B increases the market size for applications, which is desirable to developers. Ceteris paribus, low p_i^S attracts a large numbers of developers and leads to low application prices, which is desirable to users. A forward-looking platform provider sets p_i^B and p_i^S to maximize profits taking into account the resulting structure of the market for applications.

To analyze this situation formally when there exist two platform providers, we set up the following two-period game which extends Church and Gandal's (1993) approach to a market with two-sided platforms. The first period consists of two stages. In the first stage, platform provider $i \in \{1, 2\}$ posts access prices (p_i^B, p_i^S), whereas, in the second stage, all potential users and developers simultaneously decide whether or not to trade with one of the platform providers. In the second period, developers who traded with platform provider i sell applications to users who own platform i (in the case of platform incompatibility) or any of the two existing platforms (in the case of compatibility).

As usual in multi-period contexts, we focus on (pure strategy) subgame-perfect equilibria assuming that all agents have rational expectations. We thus solve the model by backwards induction. Figure 1 illustrates the timing of the game (for the incompatible case):

[5] Introducing royalties per unit of application sold by developers makes the analysis intractable. Notwithstanding, we discuss the likely role of royalties in the conclusion.

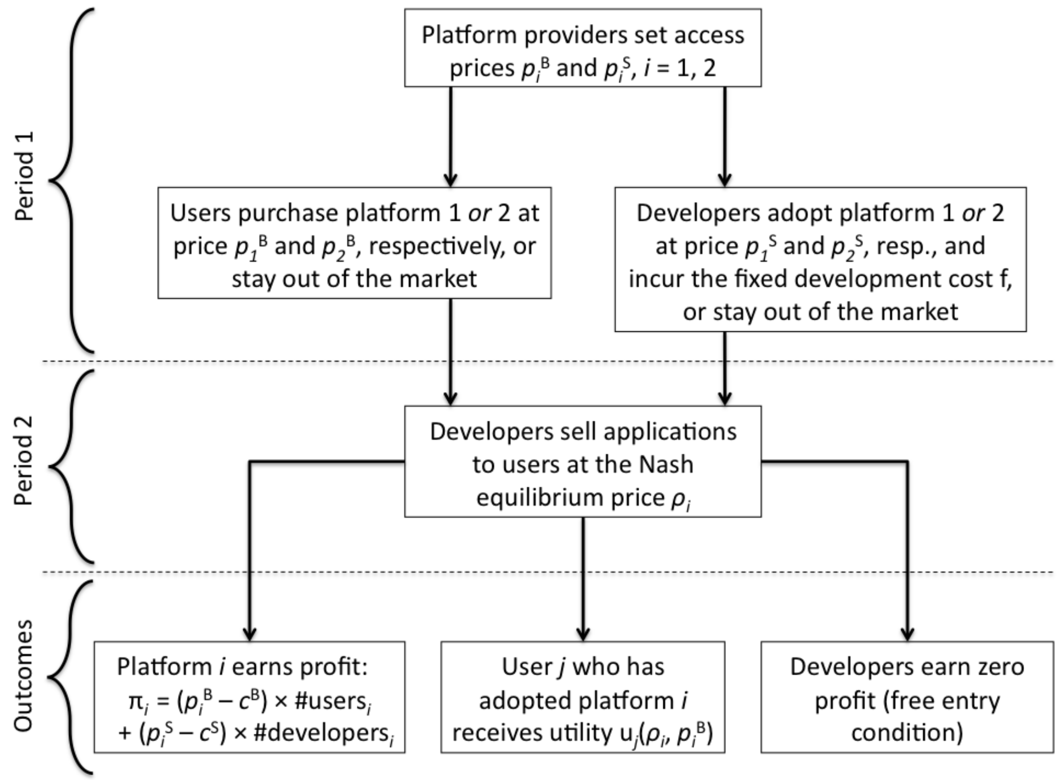


Figure 1

We make the following assumptions about each player group:

Platform providers. There are two competing platforms that have no stand-alone value (e.g., an operating system is valuable only if there are applications that can run on it). The (constant) marginal cost c^B of producing copies of a platform is assumed to be zero.[6] We allow for the possibility that there may be a positive cost of serving developers (these costs are related to the provision of development kits and/or the structuring of licensing contracts). We assume that these costs are identical across platform providers, and we denote them by $c^S \geq 0$.

Users. Users are assumed to have unit demands for platforms and linear demands for applications, which are assumed to be homogeneous goods to simplify the analysis by sidestepping issues related to product variety.[7] Specifically, we assume that a unit mass of (potential) users is uniformly distributed along a Hotelling segment of unit length. We suppose that the platform provider indexed by 1 is located at the left end of the segment, whereas that indexed by 2 is located at the right end. Platform providers differ in their location on the segment but are otherwise identical. It is assumed that a user located at distance $x_i \in [0, 1]$ from platform i has the following demand for applications sold at

[6] This assumption is particularly appropriate for industries involving the production of software such as operating systems. In any case, except for Proposition 1, setting $c^B = 0$ is without loss of generality.

[7] The main arguments are not dependent on the assumption of homogeneity. We discuss the implications of product variety for pricing structures in the concluding section.

price ρ_i if she has acquired platform i: $q_i(\rho_i, x_i) = (1 - tx_i)(a - b\rho_i)$ (where both a and b are positive constants).[8] In combination with the heterogeneous locations of users on $[0, 1]$, parameter $t \geq 0$ captures user differences regarding the perceived performance of the platform by the time it is used with the applications sold by developers: every user has a different demand function for applications (unless $t = 0$), and hence users are heterogeneous even after acquiring a platform.[9] It is worth noting that departing from the widely used unit demand for applications allows us to perform social welfare analysis even if all users purchase one of the platforms and the market is completely covered.

Developers. We suppose that there exist infinitely many potential developers of applications, all ex ante identical, and free entry. Those who have traded with one of the platform providers in the first period must also incur a sunk setup cost $f \geq 0$ to become active in the second period. Developers active in the second period produce applications at non-negative constant marginal cost c, assumed to be smaller than a/b to avoid making the analysis trivial, and compete *à la* Cournot. Cournot competition can be interpreted as a reduced-form for simultaneous capacity choice by developers followed by simultaneous capacity-constrained price competition amongst them, as shown by Kreps and Scheinkman (1983). As customary in the oligopolistic entry literature (Suzumura and Kiyono 1987), we will usually ignore the integer problem and treat the number of developers as a continuous variable.

In what follows, we distinguish two situations, depending on whether platforms are compatible or not. Platforms are compatible if any given application can be used in either platform. In the early eighties, for example, adapters were available that made videogame systems sold by Atari, Mattel, and Coleco fully compatible. And in the operating system space, software such as *Northstar* allows cross-platform functionality of applications (from PC to Mac and the other way around) without undermining performance.

3 Incompatible platforms

When platforms are incompatible, the n_i application developers who have traded with platform provider i develop applications that work exclusively with that platform. As mentioned above, the game is solved by backwards induction. We begin by solving for the second period outcomes, given p_i^B and p_i^S.

Among users who have bought platform $i \in \{1, 2\}$, let x_i denote the distance from i for the individual located farthest away. Monotonicity implies that those individuals whose distance from i is less than x_i must have traded with i as well, so the measure of users

[8] A relevant property of this specification is that the responsiveness of a user's demand to changes in the price of applications does not depend on differences in the perceived performance of the two competing platforms (i.e., it does not depend on x). This property holds regardless of whether the market is fully covered in equilibrium or not, and it is useful in that it simplifies how first-period behavior affects competitive behavior in the second period (e.g., the equilibrium prices of applications do not depend on the number of users attracted by a platform). More complete models must await further research on these issues which seem far from being easily tractable.

[9] Our assumption that horizontal differences are the same even if platforms are compatible can be easily justified by means of examples. For instance, a videoconsole may load faster than another one but be more power-consuming.

served by platform provider i is x_i. If the n_i developers who have traded with platform i charge price ρ_i, then the demand for applications of user $s_i \in [0, x_i]$ is given by $q(\rho_i, s_i) = (1 - ts_i)(a - b\rho_i)$. It follows that aggregate demand for applications functional on i equals $Q(\rho_i, x_i) = \int_0^{x_i} q(\rho_i, s_i)ds_i = x_i(1 - \frac{tx_i}{2})(a - b\rho_i)$, and hence inverse demand is

$$\rho_i(Q, x_i) = \frac{a}{b} - \frac{2Q}{bx_i(2 - tx_i)}.$$

Having obtained aggregate demand, it is standard to show that $\rho_i^{inc} = \frac{a + n_i bc}{b(n_i + 1)}$ is the second-period equilibrium price of platform i's applications under Cournot competition. Each of the n_i developers sells $q_i^{inc} = \frac{x_i(2 - tx_i)(a - bc)}{2(n_i + 1)}$ units of applications, and hence each earns the following post-entry profits:

$$\pi_i^{inc} = \frac{x_i(2 - tx_i)}{2b}\left(\frac{a - bc}{n_i + 1}\right)^2. \tag{1}$$

A user who has acquired platform i and is located at distance $s_i \in [0, x_i]$ from such platform attains the following second-period surplus:

$$u_i^{inc}(s_i) = \int_{\rho_i^{inc}}^{a/b}(1 - ts_i)(a - b\rho_i)d\rho_i = \frac{(1 - ts_i)(a - b\rho_i^{inc})^2}{2b} = \frac{(1 - ts_i)n_i^2}{2b}\left(\frac{a - bc}{n_i + 1}\right)^2. \tag{2}$$

In solving for the first period choices, we first consider the simpler case of partial market coverage. This lays the foundation for the more complex case of full market coverage, which we present in Subsection 3.2.

3.1 Partial market coverage

In this subsection, we suppose that $x_1 < 1 - x_2$, so that the market is not covered. As we show below, a sufficient condition for the market not to be covered under incompatibility and for the platform providers to be active in equilibrium is $\frac{3}{2} < t < \frac{(a - bc)^2}{2b(f + c^S)}$. This case is particularly simple since incompatibility and partial market coverage imply that there are no strategic interactions between platforms: each platform provider is a local monopolist. In contrast, when the market is fully covered, platforms compete for users. Therefore, the results in this section coincide with those in which there is a monopoly platform provider.

To solve the first period of the game, we temporarily ignore the adoption game played by users and developers in the second stage of the first period, and we suppose that the agents on both sides of the platform are able to coordinate on the equilibrium most desired by the platform providers. We will later show that a standard and simple refinement selects that equilibrium.[10]

[10]The issue here is that, given prices (p_i^B, p_i^S) chosen by platform provider i in the first stage, the second-

The location of user x_i indifferent between purchasing provider i's platform and not purchasing at all is such that $u_i^{inc}(x_i) - p_i^B = 0$ or, using (2),

$$\frac{(1-tx_i)n_i^2}{2b}\left(\frac{a-bc}{n_i+1}\right)^2 - p_i^B = 0. \tag{3}$$

In turn, free entry by application developers leads to $\pi_i^{inc} - f - p_i^S = 0$, or using (1):

$$p_i^S = \frac{x_i(2-tx_i)}{2b}\left(\frac{a-bc}{n_i+1}\right)^2 - f. \tag{4}$$

Ceteris paribus, users benefit from competition among developers, whereas a larger market size in the second period makes developers better off. Hence, both sides exhibit positive inter-group network externalities. In addition, the developer side exhibits negative intra-group network externalities, since, for a fixed measure of users, increasing the number of developers destroys second-period profits.

Using equations (3) and (4), the problem faced by a profit-maximizing platform provider that can choose any market allocation (x_i, n_i) that satisfies the two equations can be stated as follows:[11]

$$\begin{aligned}
\max_{x_i,n_i} \Pi_i(x_i, n_i) &= \max_{x_i,n_i}[p_i^B x_i + (p_i^S - c^S)n_i] \\
&= \max_{x_i,n_i}[\frac{(1-tx_i)(a-bc)^2}{2b}\left(\frac{n_i}{n_i+1}\right)^2 x_i + (\frac{x_i(2-tx_i)(a-bc)^2}{2b}\left(\frac{1}{n_i+1}\right)^2 - f - c^S)n_i] \\
&= \max_{x_i,n_i}[\frac{(1-tx_i)(a-bc)^2}{2b}(1 - \frac{1}{(n_i+1)^2} + \frac{tx_i n_i}{(1-tx_i)(n_i+1)^2})x_i - (f + c^S)n_i].
\end{aligned}$$

A profit-maximizing platform provider acknowledges that there are inter- and intra-group externalities, as can be observed in the profit function. Developer surplus is fully extracted via p_i^S (since all developers are symmetric) and user surplus is extracted via p_i^B. Only the marginal user x_i's surplus is fully extracted: all users in $[0, x_i)$ enjoy strictly positive utility in equilibrium.

To understand in greater detail the problem faced by platform provider i, observe that $\frac{(1-tx_i)(a-bc)^2}{2b}$ is the surplus that could be extracted from the marginal user if developers behaved competitively in the second period. Hence,

$$\frac{(1-tx_i)(a-bc)^2}{2b}(1 - \frac{1}{(n_i+1)^2} + \frac{tx_i n_i}{(1-tx_i)(n_i+1)^2})$$

stage subgame will typically have multiple equilibria. The platform will prefer the second-stage equilibirum that results in highest profit. The monotonicity refinement that we use (see Caillaud and Jullien, 2003) is satisfied only by the equilibrium most desired by the platform provider.

[11] Equation (3) shows that, given n_i, there is a one-to-one relationship between x_i and p_i^B and thus we can let the platform provider maximize profit by choice of x_i instead of p_i^B. Likewise, equation (4) shows that, given x_i, there is a one-to-one relationship between n_i and p_i^S, so we can let the platform provider maximize profit by choice of n_i instead of p_i^S.

measures how much (marginal) user surplus platform provider i can capture directly and via developers if it induces entry by n_i developers, taking into account that the *effective* marginal cost of allowing one more developer is $f+c^S$. Although the marginal cost of promoting entry is constant, the marginal value of another entrant may increase or decrease as more developers choose to enter. This happens because second-period price decreases in a convex fashion as the number of second-period competitors increases, which has contradicting effects on users and developers. On the one hand, the negative effect of the intra-group network externalities on the developer side is softer as more developers become active. That is, the reduction in developer surplus due to more intense competition decreases with the number of developers, which is captured by the term $1-\dfrac{1}{(n_i+1)^2}$. On the other hand, the positive marginal impact of the inter-group network externalities on the value of the platform for the indifferent user is lower as more developers enter, which is captured by the term $\dfrac{tx_i n_i}{(1-tx_i)(n_i+1)^2}$.

To solve platform provider i's problem, we compute the first-order conditions so as to get the following set of equations after performing some simple manipulations:

$$x_i = \frac{n_i+2}{2t(n_i+1)} \tag{5}$$

and

$$\frac{x_i[2-tx_i(n_i+1)](a-bc)^2}{2b(n_i+1)^3} = f+c^S. \tag{6}$$

It can be observed from (5) that inducing more entry by developers requires decreasing the number of users targeted by the platform provider, which may seem counterintuitive. This result obtains because attracting more developers lowers the price of applications, which increases the surplus expected by users. Since the increase is greater for those users whose tastes are closer to the location of the platform, the provider chooses to appropriate from the increase in user surplus by targeting those users who benefit most from the inter-group network externality. In other words, the elasticity of demand for i's platform decreases as n_i increases, whence it follows that platform provider i prefers to raise p_i^B, which reduces demand for the platform.

We now solve the system of equations formed by the two first-order conditions, so substitute (5) into (6) to obtain

$$\frac{4-n_i^2}{(n_i+1)^4} = \frac{8bt(f+c^S)}{(a-bc)^2}, \tag{7}$$

whence the optimal value for n_i follows. Letting n_i^* be such a value, it holds that $n_1^* = n_2^* \equiv n^*$. Note that when $\dfrac{(a-bc)^2}{2b(f+c^S)} < t$, there is no $n_i > 0$ that satisfies (7). Intuitively, when t is very large few users enter the market and this implies that the post-entry profits of developers are low. When $\dfrac{(a-bc)^2}{2b(f+c^S)} < t$, these profits are insufficient to cover the fixed costs of promoting developer entry, $f+c^S$, and hence the platform providers are better off staying inactive.

Equation (7) implies that n^* cannot be greater than 2. To understand this result, notice that by lowering p_i^B, the platform provider attracts more users. A larger market for applications implies more entry by developers. And when the number of developers is large, competition for users yields a low applications prices ρ_i and little value to be extracted through p_i^S. Thus there is a trade off. Because the *entire* developers' surplus can be extracted through p_i^S whereas only the surplus of the marginal user can be fully extracted through p_i^B, the platform provider sets (p_i^B, p_i^S) to ensure that competition between developers is mild.[12]

Note also that (5) implies that the optimal number/measure of users to be served by platform provider i is $x^* = \dfrac{n^*+2}{2t(n^*+1)}$. Because the market cannot be covered in equilibrium (by hypothesis), we must have that $x^* < 1/2$, which holds if and only if $\dfrac{n^*+2}{n^*+1} < t$. Although n^* depends on t, a sufficient condition for the inequality to be satisfied is $t > 3/2$. In equilibrium, platform profits are positive, since

$$\Pi_i^* = \frac{(a-bc)^2(n^*)^2(n^*+2)(n^*+4)}{8bt(n^*+1)^4} > 0.$$

We now examine the properties of the equilibrium pricing structure (dropping subscripts). As for the price charged to users, expression (3) yields

$$p^{B*} = u^{inc}(x^*) = \frac{(a-bc)^2(n^*)^3}{4b(n^*+1)^3} > 0 = c^B. \tag{8}$$

Thus, users are never subsidized. In addition, because $\dfrac{8bt(f+c^S)}{(a-bc)^2} = \dfrac{(2-n^*)(2+n^*)}{(n^*+1)^4}$ and $x^* = \dfrac{n^*+2}{2t(n^*+1)}$, equation (4) implies that

$$p^{S*} - c^S = \frac{x^*(2-tx^*)(a-bc)^2}{2b(n^*+1)^2} - (f+c^S) = \frac{4n^*(n^*+2)(a-bc)^2}{8bt(n^*+1)^4} > 0, \tag{9}$$

so $p^{S*} > c^S$. To prevent eroding second-period profits, developers are never subsidized. We summarize the properties of the (optimal) pricing structure in the following proposition.

Proposition 1 *Under platform incompatibility and partial market coverage, platform providers find it optimal to subsidize neither users nor developers: $p^{B*} > 0 = c^B$ and $p^{S*} > c^S$. Moreover, the number of developers targeted by each platform provider is bounded above by 2:*

[12] Note that in our model it holds that $\frac{d\left(\frac{u_i^{inc}(x_i)}{n_i\pi_i^{inc}/x_i}\right)}{dn_i} = \frac{1-tx_i}{2-tx_i} < 1$, so that the utility of the marginal user relative to industry profit generated per user does not grow fast enough as n_i increases. Intuitively, platform provider i cares more about the negative effect of increasing n_i on developer surplus than about the positive impact of an increase in n_i on user surplus. This property explains the result that the optimal number of developers to be served must never exceed 2 (even if $f = c^S = 0$). This result seems robust and indeed it persists in the (untractable) case where second-period demand is given by $q(x, \rho) = a(1-tx) - b\rho$.

$n^* \leq 2$.

Thus far, we have assumed that platform provider i's optimal pricing structure aimed at attaining allocation (x^*, n^*) achieves its objective. Yet, given (p^{B*}, p^{S*}), there might be several solutions to (3) and (4), besides the allocation (x^*, n^*) pursued by the provider when setting such prices.

The following proposition states that the solution that we have focused on is the only one such that the demand functions are not increasing, as required by Caillaud and Jullien's (2003) monotonicity criterion. That is, the solution (x^*, n^*) preferred by a locally monopolistic platform is the only one for which increases in p^B and p^S result in decreases in x and n.

Proposition 2 *Under platform incompatibility and partial market coverage, the profit-maximizing allocation pursued by platform providers (x^*, n^*) when setting prices (p^{B*}, p^{S*}) is the only equilibrium of the second stage subgame such that increases in p^B and p^S lead to decreases in x and n.*

Proof. See Appendix. ■

3.1.1 Social efficiency

We conclude this subsection by studying social welfare under the assumption that the social planner can control the number of users and developers who become active in the first period, but not their subsequent behavior. In this second-best scenario, a welfare-maximizing social planner would face the following problem if it targeted allocation (x, n) for each platform provider:

$$\max_{x,n} W(x,n) = \max_{x,n} 2[\int_0^x (\frac{(1-ts)n^2}{2b}\left(\frac{a-bc}{n+1}\right)^2 - p^B)ds + p^B x + (\frac{x(2-tx)(a-bc)^2}{2b(n+1)^2} - f - c^S)n]$$
$$= \max_{x,n}[\frac{2x(2-tx)(a-bc)^2}{4b}\left(\frac{n}{n+1}\right)^2 + 2(\frac{x(2-tx)(a-bc)^2}{2b(n+1)^2} - f - c^S)n].$$

Denote by (x^e, n^e) the solution to this program. Letting $t > 2$ to make the analysis interesting, it is straightforward to show that $x^e = \frac{1}{t} > x^*$, which implies $p^{Be} = 0$ by (3). As one would expect from the assumption that $c^B = 0$, a social planner finds it optimal to target all users with positive demand for applications by selling the platform for free.

Likewise, n^e is the unique n that solves the following equation:

$$\frac{(a-bc)^2}{2bt(n+1)^3} = f + c^S. \tag{10}$$

Recalling by (7) that $\frac{(2+n^*)(2-n^*)(a-bc)^2}{8bt(n^*+1)^4} = f + c^S$, the fact that $\frac{(2+n)(2-n)}{4(n+1)} < 1$ for $n > 0$ implies that $n^e > n^*$. Unlike a social planner, a monopolistic platform provider does not internalize all the positive effect of the network externality exerted on users by developers because it cannot appropriate all the gains from trade that accrue to users. As a result, the platform provider ends up underexploiting the inter-group network externalities originated on the user side, which in turn explains why it serves fewer developers than socially desirable.

Finally, using equations (4) and (10) leads to the developer entry fee that implements the socially efficient outcome:

$$p^{Se} = \left(\frac{(a-bc)^2(f+c^S)^2}{2bt}\right)^{\frac{1}{3}} - f.$$

To see that $p^{Se} > c^S$, we use (4) and (10) to obtain:

$$p^{Se} - c^S = \frac{(a-bc)^2}{2bt\,(n^e+1)^2} - \left(f + c^S\right) = \frac{(a-bc)^2}{2bt\,(n^e+1)^2} - \frac{(a-bc)^2}{2bt(n^e+1)^3} > 0.$$

We state these results formally.

Proposition 3 *Under platform incompatibility and partial market coverage, platform providers serve too few users and promote insufficient entry by developers from a social efficiency standpoint: $x^* < x^e$ and $n^* < n^e$. A social planner finds it optimal to sell the platforms for free to users and charges an entry fee to developers above marginal cost: $p^{Be} = 0$ and $p^{Se} > c^S$.*

3.2 Full market coverage

We now turn to the situation where the market is fully covered and hence platform providers compete for users. In the case of full market coverage, t is a parameter that captures the intensity of (horizontal) differences in the perceived performance of the platform by the time it is used with the applications sold by developers. Below we show that a sufficient condition for the market to be covered under incompatibility is $t < 2/3$. Because $x_1 = 1 - x_2$, we can drop subscripts so that x denotes the measure of users served by platform provider 1 and $1 - x$ denotes the measure of users served by platform provider 2. The location of the marginal user x is such that $u_1^{inc}(x) - p_1^B = u_2^{inc}(1-x) - p_2^B$, or

$$\frac{(1-tx)n_1^2}{2b}\left(\frac{a-bc}{n_1+1}\right)^2 - p_1^B = \frac{(1-t(1-x))n_2^2}{2b}\left(\frac{a-bc}{n_2+1}\right)^2 - p_2^B. \tag{11}$$

The following free entry condition must hold for developers trading with platform provider $i \in \{1,2\}$:

$$p_i^S = \frac{x_i(2-tx_i)}{2b}\left(\frac{a-bc}{n_i+1}\right)^2 - f, \tag{12}$$

where $x_i = x$ if $i = 1$ and $x_i = 1 - x$ otherwise. Such a condition implies that

$$\left(\frac{(a-bc)n_i}{n_i+1}\right)^2 = \left(a - bc - \sqrt{\frac{2b(p_i^S+f)}{x_i(2-tx_i)}}\right)^2,$$

so expression (11) can be rewritten as:

$$\frac{(1-tx)\left(a-bc-\sqrt{\frac{2b(p_1^S+f)}{x(2-tx)}}\right)^2}{2b}+\frac{(t-1-tx)\left(a-bc-\sqrt{\frac{2b(p_2^S+f)}{(1-x)(2-t(1-x))}}\right)^2}{2b}+p_2^B-p_1^B=0. \tag{13}$$

This equation implicitly defines an expression for x as a function of p_1^B and p_1^S (that is, we have that $x(p_1^B,p_1^S)$, although we will sometimes suppress the dependence to save space).

Using (12) and (13), platform provider 1's problem can be written as follows:

$$\begin{aligned}\max_{p_1^B,p_1^S}\Pi_1(p_1^B,p_1^S) &= \max_{p_1^B,p_1^S}[p_1^B x(p_1^B,p_1^S)+(p_1^S-c^S)n_1(p_1^B,p_1^S)] \\ &= \max_{p_1^B,p_1^S}[p_1^B x(p_1^B,p_1^S)+(p_1^S-c^S)(\frac{(a-bc)\sqrt{x(p_1^B,p_1^S)(2-tx(p_1^B,p_1^S))}}{\sqrt{2b(p_1^S+f)}}-1)].\end{aligned}$$

The first-order conditions are:

$$x(p_1^B,p_1^S)+(p_1^B+\frac{(p_1^S-c^S)(a-bc)(1-tx(p_1^B,p_1^S))}{\sqrt{2b(p_1^S+f)x(p_1^B,p_1^S)(2-tx(p_1^B,p_1^S))}})\frac{\partial x(p_1^B,p_1^S)}{\partial p_1^B}=0 \tag{14}$$

and

$$\begin{aligned}&p_1^B\frac{\partial x(p_1^B,p_1^S)}{\partial p_1^S}+\frac{(a-bc)\sqrt{x(p_1^B,p_1^S)(2-tx(p_1^B,p_1^S))}}{\sqrt{2b(p_1^S+f)}}-1+(p_1^S-c^S)\times \\ &(\frac{(a-bc)(1-tx(p_1^B,p_1^S))}{\sqrt{2b(p_1^S+f)x(p_1^B,p_1^S)(2-tx(p_1^B,p_1^S))}}\frac{\partial x(p_1^B,p_1^S)}{\partial p_1^S}-\frac{(a-bc)\sqrt{x(p_1^B,p_1^S)(2-tx(p_1^B,p_1^S))}}{2\sqrt{2b(p_1^S+f)^3}})=0\end{aligned} \tag{15}$$

Note from expression (14) that there is an additional term relative to traditional settings without network externalities owing to the inter-group network externality exerted by users on developers. The second term within the parenthesis reflects the fact that changing the user price affects the number of users who choose to acquire the platform, which in turn affects the developers' demand for the platform and hence the gain/loss made on the developer side. A similar observation can be made regarding (15), taking into account that changing the entry fee affects the number of developers not only by varying the (sunk) cost of entry but also by changing the size of the second-period market.

When $t<2/3$, a necessary and sufficient condition for a symmetric equilibrium to exist under incompatibility is that $\frac{(a-bc)^2}{2b}\geq\frac{4(f+c^S)(32+8t^2-24t-t^3)^3}{t^3(8-t^2)(4-t)^5}$. Given this assumption, the following proposition establishes that the unique symmetric equilibrium exhibits the property that developers are never subsidized, whereas users may or may not be subsidized depending on the extent of platform differentiation.

Proposition 4 *Suppose there exist two competing platform providers that sell incompatible platforms to a market that is fully covered. In the unique symmetric equilibrium developers* *are never subsidized, and users are subsidized if and only if* $\frac{(a-bc)^2}{2b} < \frac{4(f+c^S)(t^2-6t+12)^3}{(4-t)^2t^3(4+2t-t^2)}$. *In particular, it holds that* $p^{B*} < 0 = c^B$ *and* $p^{S*} > c^S$ *for* $\frac{(a-bc)^2}{2b} < \frac{4(f+c^S)(t^2-6t+12)^3}{(4-t)^2t^3(4+2t-t^2)}$, *whereas* $p^{B*} \geq 0 = c^B$ *and* $p^{S*} > c^S$ *for* $\frac{(a-bc)^2}{2b} \geq \frac{4(f+c^S)(t^2-6t+12)^3}{(4-t)^2t^3(4+2t-t^2)}$.

Proof. See Appendix. ■

Because our model has infinitely many potential developers, incompatible platforms do not compete to attract them via access prices but rather via the size of the user network. When horizontal differentiation is not too strong, platform providers compete fiercely for users by subsidizing their access to the platform with the aim of boosting developer entry and thus profits made on the developer side (as stems from (14) taking into account that $p^S > c^S$ and $\partial x(p_1^B, p_1^S)/\partial p_1^B < 0$ at the symmetric equilibrium prices).

Incompatibility allows platforms to vertically differentiate through the number of applications that they offer. However, since in this section we are studying symmetric equilibria, the number of developers is the same for both platforms. This implies that, in equilibrium, the products end up being horizontally differentiated only. In Section 5 we consider asymmetric equilibria and show that endogenous vertical differences arise on the grounds of the quantity of applications offered on each platform.

3.2.1 Social efficiency

We now turn to social efficiency when platforms are incompatible and the market is fully covered. A social planner which targeted all users *and* chose to let both platform providers operate would solve:[13]

$$\begin{aligned}\max_n W(n) &= \max_n 2[\int_0^{\frac{1}{2}} \frac{(1-ts)n^2}{2b}\left(\frac{a-bc}{n+1}\right)^2 ds + (\frac{4-t}{8b}\left(\frac{a-bc}{n+1}\right)^2 - f - c^S)n] \\ &= \max_n [\frac{(4-t)(a-bc)^2}{8b}(1-\frac{1}{(n+1)^2}) - 2(f+c^S)n].\end{aligned}$$

Therefore,

$$n^e = \left(\frac{(4-t)(a-bc)^2}{8b(f+c^S)}\right)^{\frac{1}{3}} - 1 \qquad \text{and} \qquad p^{Se} = \left(\frac{(4-t)(a-bc)^2(f+c^S)^2}{8b}\right)^{\frac{1}{3}} - f.$$

How do n^e and n^* compare? The following proposition shows that competition between incompatible platforms leads to insufficient entry from a social efficiency point of view. Just as in the case of partial market coverage, providers of incompatible platforms do not internalize all the positive effect of the network externality exerted on users by developers

[13] We should point out that a social planner may prefer having just one platform provider serving both sides of the market so as to avoid underexploitation of network externalities and duplication of costs.

because they cannot appropriate all the gains from trade that accrue to users. As a result, platform providers promote less than socially desirable entry by developers, which results in social welfare losses because of insufficient consumption of applications.

Proposition 5 *Suppose there exist two competing platform providers that sell incompatible platforms to a market that is fully covered. Then the symmetric outcome of duopolistic competition is such that entry by developers is insufficient from a social planner's viewpoint:* $n^* < n^e$. *Moreover, the socially efficient entry fee does not subsidize developers:* $p^{Se} > c^S$.

Proof. See Appendix. ■

4 Compatible platforms

Under compatibility any one of the n_i developers who have traded with platform provider $i \in \{1, 2\}$ sells applications that can be used by users of the competing platform. User demand for the homogeneous good sold by developers depends on the location of the user and the platform that she uses. Letting x_1 denote the location of the marginal user who purchased platform provider 1's platform, it is immediate that those users to the left of x_1 must have traded with 1 as well by monotonicity. Similarly, letting x_2 denote the distance between platform provider 2 and the marginal user who purchased provider 2's platform, the users in between must have traded with 2 too. In addition, note that developers produce homogeneous goods regardless of the platform provider they traded with in the first period, so they must charge the same market price as dictated by the properties of standard Cournot competition. Hence, aggregate demand given a market price ρ is equal to

$$Q(\rho) = [(x_1 - \frac{tx_1^2}{2}) + (x_2 - \frac{tx_2^2}{2})](a - b\rho),$$

and inverse demand is as follows:

$$\rho(Q) = \frac{a}{b} - \frac{2Q}{b[(2x_1 - tx_1^2) + (2x_2 - tx_2^2)]}.$$

Therefore, second-period Cournot competition with $n_1 + n_2 \equiv N$ developers yields the following equilibrium price for an application:

$$\rho^{com} = \frac{a + bcN}{b(N+1)}.$$

Every developer sells q^{com} applications, where

$$q^{com} = \frac{[(2x_1 - tx_1^2) + (2x_2 - tx_2^2)](a - bc)}{2(N+1)},$$

and makes profits equal to

$$\pi^{com} = \frac{[(2x_1 - tx_1^2) + (2x_2 - tx_2^2)]}{2b} \left(\frac{a - bc}{N+1} \right)^2 .$$

For fixed $\{(p_i^B, p_i^S)\}_{i=1}^2$, the effect of compatibility on developers is two-fold. On the one hand, developers have access to a larger pool of users than if platforms were not compatible. On the other, they face more intense competition.

As shown in (2), those users who traded with platform provider $i \in \{1, 2\}$ and who are at distance $s_i \in [0, x_i]$ from such platform expect the following second-period surplus:

$$u^{com}(s_i) = \frac{(1 - ts_i)}{2b} \left(\frac{(a - bc)N}{N+1} \right)^2 .$$

For fixed access prices, the effect of compatibility on users is to enable them to access a larger pool of developers than if platforms were not compatible.

As in the incompatible case, we distinguish two situations when solving the first period of the game, depending on whether the market is covered or not.

4.1 Partial market coverage

Unlike the incompatible case, platform providers that are perceived as local monopolists by users *do* compete for developers and thus best respond to each other when choosing access prices $\left(p_i^B, p_i^S\right)$. Platform compatibility implies that developers will transact with the provider that offers the lowest access fee. And because the equilibrium number of developers also depends on the size of the market for applications, access prices charged to users will also be interdependent across platforms even when the market is not covered. To guarantee that the market is not covered in equilibrium, we assume that $t > 2$ throughout this subsection.

We proceed to solve the first period of the game, taking into account how play unfolds in the second. Because the market is not fully covered, for a fixed price p_i^B, the user x_i indifferent between trading with platform provider $i \in \{1, 2\}$ and not trading at all is given by:

$$u^{com}(x_i) - p_i^B = 0.$$

Hence, user demand for provider i's platform is

$$x_i = \frac{1}{t}(1 - 2bp_i^B \left(\frac{N+1}{(a - bc)N} \right)^2). \tag{16}$$

As for the developer side, the net profit made by a developer who trades with platform provider $i \in \{1, 2\}$ must be zero, that is, the following must hold:

$$\frac{[(2x_1 - tx_1^2) + (2x_2 - tx_2^2)]}{2b} \left(\frac{a - bc}{N+1} \right)^2 - f - p_i^S = 0. \tag{17}$$

Developers' profits in the second period do not depend on the platform they develop for. Thus, both platforms are perceived as homogeneous by developers and they will choose platform based solely on the entry fees being charged (p_1^S and p_2^S). Bertrand competition for developers implies that the *lowest* entry fee will fully determine the number of active developers.[14] Note that there can be no equilibrium in which both platform providers charge an entry fee above c^S. Otherwise, one of the platform providers could slightly decrease p^S, which would discontinuously increase the number of developers she serves, thereby increasing profits.[15] Similarly, there can be no equilibrium in which both platform providers subsidize developers. Otherwise, one of the providers could unilaterally raise her entry fee, which would not affect profits made on the user side, and would stop losses on the developer side. Therefore, if an equilibrium exists, then it must be such that $p_1^S = p_2^S \equiv p^{S**} = c^S$.

The derivation of the equilibrium access fees for users turns out to be quite complex, so we focus on symmetric equilibria in which $p_1^B = p_2^B \equiv p^{B**}$. The following proposition states the main result.

Proposition 6 *In the symmetric equilibrium under platform compatibility and partial market coverage, platform providers find it optimal not to subsidize users and to charge an entry fee to developers equal to marginal cost:* $p^{B**} > 0 = c^B$ *and* $p^{S**} = c^S$*.*

Proof. See Appendix. ■

The main differences with respect to the case of partial market coverage with incompatible platforms are two. On the developer side, platform competition to attract them dissipates profits that can be made from developers. On the user side, a platform provider that lowers user access fees exerts a positive externality on the competing platform by making her user demand for the platform more inelastic, which induces the competing platform to charge higher user prices. As a result, it is no big surprise that compatibility leads to prices above marginal cost for users.

To conclude with the characterization of the symmetric equilibrium, observe that just as in the case of incompatible platforms, we have assumed that platform provider i's equilibrium pricing structure aimed at attaining allocation (x_i^{**}, N^{**}) achieves its objective. However, given (p^{B**}, p^{S**}), there might be several solutions to (16, $i = 1, 2$) and (17). The following proposition states that the solution most preferred by the platform providers is the only one that satisfies the monotonicity criterion.

Proposition 7 *Under platform compatibility and partial market coverage, the profit-maximizing allocation pursued by platform providers* (x_i^{**}, N^{**}) *when setting prices* (p^{B**}, p^{S**}) *is the only equilibrium of the second stage subgame such that increases in* p_i^B *and* p_i^S *lead to decreases in* x_i *and* N*.*

Proof. See Appendix. ■

[14] Although free entry implies that developers end up earning zero profits regardless of the platform they join, it is assumed that the platform charging the lowest p^S attracts all developers, that is, in case of indifference, developers prefer the platform whose access fee is lowest.

[15] Note as well that this would have an arbitrarily small effect on user demand for such provider's platform, which does not affect the thrust of the argument.

4.1.1 Social efficiency

Consider a social planner that maximizes total welfare by choosing the measure of users served by each platform provider x and the total number of developers N. Then it solves the following problem:

$$\max_{x,N} W(x,N) = \max_{x,N}[2\int_0^x \frac{(1-ts)}{2b}\left(\frac{(a-bc)N}{N+1}\right)^2 ds + (\frac{2x-tx^2}{b}\left(\frac{a-bc}{N+1}\right)^2 - f - c^S)N]$$
$$= \max_{x,N}[\frac{(2x-tx^2)(N+2)N}{2b}\left(\frac{a-bc}{N+1}\right)^2 - (f+c^S)N].$$

It is straightforward to show that the welfare-maximizing N^e and x^e satisfy: $\frac{(2x-tx^2)(a-bc)^2}{b(f+c^S)} = (N^e+1)^3$ and $x^e = \frac{1}{t} > \frac{1}{t}(1-2bp^{B**}\left(\frac{N+1}{(a-bc)N}\right)^2) = x^{**}$. For the same reasons as in the incompatibility regime, competition leads to too few users being served (note, in addition, that the socially optimal x is smaller than $\frac{1}{2}$ if $t > 2$). Because $x^e = \frac{1}{t}$ (under the assumption that $t > 2$), the first-order condition for the efficient number of developers can be rewritten as $\frac{(a-bc)^2}{bt(f+c^S)} = (N^e+1)^3$. The solution to this equation gives the socially efficient number of developers N^e.[16] We show that $N^e < N^{**}$.

Proposition 8 *Under platform compatibility and partial market coverage, platform providers serve too few users but promote excessive entry by developers from a social efficiency standpoint: $x^{**} < x^e$ and $N^{**} > N^e$. A social planner finds it optimal to sell the platforms for free to users and charges an entry fee to developers above marginal cost: $p^{Be} = 0$ and $p^{Se} > c^S$.*

Proof. See Appendix. ■

Intuitively, the social planner takes into account that every additional developer "costs" $f + c^S$ but that the marginal benefit that it brings (in terms of lower prices and higher surplus for users) is decreasing in the number of developers. It thus restricts entry by setting $p^{Se} > c^S$. Under competition, however, lack of differentiation on the developer side leads to $p^{S**} = c^S$ and excessive entry. Also, because the market is not covered and the platforms are horizontally differentiated, platform providers do not compete to attract users. Indeed, they act as monopolists when setting p^B. This results in high p^B, low user entry, and a deadweight loss that the social planner avoids.

4.2 Full market coverage

Under full market coverage, which requires that $t \in [0, 2/3)$, we have that $x_1 = 1 - x_2$. We thus drop subscripts. In this case, x is the user indifferent between trading with platform

[16] It can be shown that the socially efficient total number of developers under compatibility does not exceed the socially efficient total number of developers under incompatibility.

providers 1 and 2. Because the market is fully covered, x also denotes the measure of users served by platform provider 1 and $1-x$ that of users served by platform provider 2. For fixed prices p_1^B and p_2^B, the location of the indifferent user is given by:

$$u^{com}(x) - p_1^B = u^{com}(1-x) - p_2^B.$$

Hence, user demand for provider 1's platform is

$$x(p_1^B, p_2^B) = \frac{1}{2} + \frac{b(p_2^B - p_1^B)}{t(a-bc)^2}\left(\frac{N+1}{N}\right)^2. \tag{18}$$

Although both platforms grant access to a single pool of applications, platforms are horizontally differentiated and thus when $p_1^B = p_2^B$ users are not indifferent between both platforms.[17]

As for the developer side, the net profit made by a developer that trades with platform provider $i \in \{1,2\}$ is

$$\pi^{com} - f - p_i^S = \frac{4-t}{4b}\left(\frac{a-bc}{N+1}\right)^2 - f - p_i^S.$$

By the same argument as in Section 4.1, we have $p_1^S = p_2^S \equiv p^{S**} = c^S$. Letting N^{**} be such that $\frac{(4-t)(a-bc)^2}{4b(N^{**}+1)^2} = f + c^S$ (and assuming that $\frac{(a-bc)^2}{2b} \geq \frac{8(f+c^S)}{4-t}$ so that $N^{**} \geq 1$), it is straightforward to show that both platform providers should charge a user price equal to $p^{B**} = \frac{t(a-bc)^2}{2b}\left(\frac{N^{**}}{N^{**}+1}\right)^2 \geq 0$ if they foresee $N^{**} \geq 1$ developers becoming active.

To conclude that both platform providers setting (p^{B**}, p^{S**}) constitutes an equilibrium, it only remains to rule out deviations involving decreases in the entry fee accompanied by changes in the price charged to users.[18] In order to show that one of the platform provider, 1 say, has no incentive to do so, note that if 1's entry fee is the lowest, then it will attract n_1 developers, where n_1 is given by the following free-entry condition:

$$p_1^S = \frac{(4-t)}{4b}\left(\frac{a-bc}{n_1+1}\right)^2 - f. \tag{19}$$

Because there is a one-to-one relationship between p_1^S and n_1, we can let platform provider 1 choose p_1^B and n_1 when keeping p_2^B and p_2^S fixed. That is, using (18) and (19), we have that such platform provider solves

$$\max_{p_1^B, n_1} \Pi_1(p_1^B, n_1) = \max_{p_1^B, n_1}\left[p_1^B\left(\frac{1}{2} + \frac{b(p^{B**} - p_1^B)}{t(a-bc)^2}\left(\frac{n_1+1}{n_1}\right)^2\right) + \frac{(4-t)(a-bc)^2 n_1}{4b(n_1+1)^2} - (f+c^S)n_1\right].$$

[17] Note that, contrary to the case of platform incompatibility, platform providers cannot differentiate vertically their platforms based on the size of the user/developer network.

[18] It is clear that upward changes in the entry fee do not increase profit to be made on developers and do not affect N^{**}, which means that no platform provider has an incentive to change its user price given that its competitor's user price is kept fixed.

It is simple to show that the derivative with respect to p_1^B evaluated at (p^{B**}, N^{**}) is zero. In turn, the (right) derivative with respect to n_1 evaluated at (p^{B**}, N^{**}) is non-positive (since $N^{**} \geq 1$). The question is then whether the decrease in profit from larger n_1 is compensated by the profit increase from an increase in p_1^B. To see that the answer is negative, we solve $\frac{\partial \Pi_1(p_1^B, n_1)}{\partial p_1^B} = 0$ for p_1^B to obtain

$$p_1^B(n_1) = \frac{1}{4}\left(2p^{B**} + \frac{(a-bc)^2 n_1^2 t}{b(n_1+1)^2}\right).$$

We then substitute $p_1^B(n_1)$ in $\Pi_1(p_1^B, n_1)$ to obtain profit as a function of n_1 alone, $\Pi_1(n_1)$. Finally, a little algebra shows that $\frac{\partial \Pi_1(n_1)}{\partial n_1} < 0$ for all parameter values. Hence, both platform providers setting prices (p^{B**}, p^{S**}) constitutes the unique symmetric equilibrium of the game.

The properties of such an equilibrium outcome are driven by the full evaporation of profits on the developer side because developers perceive platforms to be homogeneous. The fact that platforms are not characterized by idiosyncratic network effects implies that providers compete for users as in a traditional Hotelling framework. Hence, profits just accrue because users perceive platforms to be horizontally differentiated. In particular, equilibrium profits are equal to

$$\Pi^{**} = \frac{t(a-bc)^2}{4b}\left(\frac{N^{**}}{N^{**}+1}\right)^2 \geq 0.$$

Note also that the marginal user achieves a positive utility in equilibrium, that is,

$$\frac{(2-3t)(a-bc)^2}{4b}\left(\frac{N^{**}}{N^{**}+1}\right)^2 > 0$$

holds because $t \in [0, 2/3)$.

We summarize all the properties of the unique equilibrium as follows.

Proposition 9 *Suppose that there exist two platform providers that sell compatible platforms in a market that is fully covered. The unique equilibrium is symmetric and yields profits for each firm equal to* $\Pi^{**} = \frac{t(a-bc)^2}{4b}\left(\frac{N^{**}}{N^{**}+1}\right)^2$. *Moreover, users are not subsidized to purchase the platform, whereas developers are charged an entry fee equal to marginal cost:* $p^{B**} \geq 0 = c^B$ *and* $p^{S**} = c^S$.

Proof. See Appendix. ■

We end this section by showing that, contrary to all the other cases, there is only one allocation (x^{**}, N^{**}) that is compatible with the unique symmetric equilibrium (p^{B**}, p^{S**}) derived above.

Proposition 10 *Suppose there exist two competing platform providers that sell compatible platforms in a market that is fully covered. Then there is only one allocation* (x^{**}, N^{**}) *that is compatible with the unique symmetric equilibrium* (p^{B**}, p^{S**}) *derived in Proposition 9.*

Proof. See Appendix. ■

4.2.1 Social efficiency

We proceed to show that price competition for developers is too harsh and results in excessive entry, for exactly the same reasons as in the case of partial market coverage. A social planner that targeted all users and was able to choose the total number of developers active in the second period would solve:

$$\max_N W(N) = \max_N [2\int_0^{\frac{1}{2}} \frac{(1-ts)(a-bc)^2}{2b}\left(\frac{N}{N+1}\right)^2 ds + (\frac{(4-t)(a-bc)^2}{4b}\left(\frac{1}{N+1}\right)^2 - f - c^S)N]$$
$$= \max_N [\frac{(4-t)(a-bc)^2}{8b}(1-\frac{1}{(N+1)^2}) - (f+c^S)N].$$

It is straightforward to show that the efficient number of developers is given by the solution to the first-order condition:

$$N^e = \left(\frac{(4-t)(a-bc)^2}{4b(f+c^S)}\right)^{\frac{1}{3}} - 1. \tag{20}$$

Hence, the fact that $N^{**} = \left(\frac{(4-t)(a-bc)^2}{4b(f+c^S)}\right)^{\frac{1}{2}} - 1$ and the assumption that $\frac{(a-bc)^2}{2b} \geq \frac{8(f+c^S)}{4-t}$ (made earlier to ensure that $N^{**} \geq 1$) yield that $N^e < N^{**}$. Using equation (20), it can be shown that the entry fee that implements the socially efficient outcome exceeds marginal cost c^S. This result, together with the one that fierce price competition for developers induces too much entry, is stated in the following proposition.

Proposition 11 *Suppose there exist two competing platform providers that sell compatible platforms in a market that is fully covered. Then entry by developers is excessive from a social planner's viewpoint:* $N^{**} > N^e$*. Moreover, the socially efficient entry fee does not subsidize developers:* $p^{Se} > c^S$*.*

5 Platform compatibility versus incompatibility

Many industries with two-sided platforms are characterized by platform incompatibility and strong market dominance by one single firm. The purpose of this section is to investigate why this may be so. A natural starting point is to compare the properties of equilibria depending on whether platforms are compatible or not. More specifically, we examine which industry structure leads to highest equilibrium profits. To get a sense, we consider the case in which the market is covered and the only fixed cost incurred by a developer is the entry fee paid to access a platform (i.e., $f = c^S = 0$). In this scenario, it is easy to prove that $p^{S*} = \dfrac{t^2(4-t)(a-bc)^2}{128b}$. Using this result, it is a matter of simple algebra to show that

$$p^{B**} = \frac{t(a-bc)^2}{2b} > p^{B*} = \frac{t(4+2t-t^2)(a-bc)^2}{32b} > 0$$

and

$$\Pi^{**} = \frac{t(a-bc)^2}{4b} > \Pi^{*} = \frac{t(24-t^2-4t)(a-bc)^2}{128b} > 0.$$

This result suggests that the (equilibrium) prices charged to users and profits are greater under platform compatibility than under incompatibility.[19] This raises the question of why platform providers do not somehow negotiate to make their platforms compatible, which is also the socially efficient outcome because it avoids cost duplication. Recalling that the symmetric equilibrium under compatibility is the unique equilibrium, one plausible answer is that there might exist other equilibria when platforms are incompatible in which one of the platform providers earns more than in the unique equilibrium with compatible platforms. This point is reinforced by the fact that, in the incompatible case with full market coverage, playing a symmetric equilibrium becomes more difficult as horizontal differentiation softens. In particular, the requirement that $\dfrac{(a-bc)^2}{2b} \geq \dfrac{4(f+c^S)(t^2-6t+12)^3}{(4-t)^2 t^3 (4+2t-t^2)}$ becomes unrealistically more severe, since the right hand side of the inequality grows without bound as t decreases (for positive $f + c^S$). This fact also suggests that symmetric equilibria may not be representative at all for the case of small t, and asymmetric equilibria with a dominant platform may be the proper benchmark when making comparisons between incompatibility and compatibility.

The question then is whether an asymmetric equilibrium under incompatibility may be preferred by one of the platform providers over the unique equilibrium under compatibility. Although this question is hard to answer in general, a conclusive answer is possible for the case in which $t = 0$. When $t = 0$, the compatibility equilibrium yields zero profits for both providers, so we now show that there exists an equilibrium under incompatibility in which one of the platforms dominates the market and makes positive profits. This result suggests that market dominance by an incompatible platform will arise when users favor the size of the developer network over platform horizontal differentiation. Thus, industry concentration would be favored by significant demand-side economies of scale relative to the intensity of horizontal differences.

In the remainder of this section, we let $t = 0$ to focus on asymmetric equilibria under incompatibility and where one of the platform providers corners the market. The importance of asymmetric equilibria has already been highlighted by Katz and Shapiro's (1985) classical work on one-sided platforms, and more recently, by Caillaud and Jullien (2003) in a setting in which both sides of the platform bargain efficiently, unlike ours in which second-period

[19] We have also performed millions of numerical simulations with $f + c^S > 0$, and we have found no counterexample to this claim.

trade efficiency depends on the platform providers' pricing structures.

Let $\{p_i^B, p_i^S\}_{i=1}^2$ denote a candidate equilibrium strategy profile such that platform provider 1 corners the market. In this case, not only should users derive a positive utility by purchasing the platform from platform provider 1, but also they should prefer trading with 1 rather than 2. Hence, we must have that

$$\frac{n_1^2}{2b}\left(\frac{a-bc}{n_1+1}\right)^2 - p_1^B \geq -p_2^B,$$

where n_1 is given by the following free entry condition: $p_1^S = \frac{1}{b}\left(\frac{a-bc}{n_1+1}\right)^2 - f$.

We now focus on the most profitable deviation platform provider 2 could undertake. In order to avoid infinite losses that would ensue from trying to attract developers, it can only deviate by setting a price for users such that it attracts all of them.[20] This can be done by charging a price (slightly below) $\widetilde{p}_2^B = p_1^B - \frac{n_1^2}{2b}\left(\frac{a-bc}{n_1+1}\right)^2 < 0$. Simultaneously, platform provider 2 could set an entry fee $\widetilde{p}_2^S$ such that $\frac{1}{b}\left(\frac{a-bc}{\widetilde{n}_2+1}\right)^2 - f - \widetilde{p}_2^S = \max(0, -f - p_1^S)$, that is, we must have $\widetilde{p}_2^S = \frac{1}{b}\left(\frac{a-bc}{\widetilde{n}_2+1}\right)^2 - f$. To solve for the optimal number of developers to be attracted, notice that

$$\Pi_2(\widetilde{n}_2) = \widetilde{p}_2^B + (\frac{1}{b}\left(\frac{a-bc}{\widetilde{n}_2+1}\right)^2 - f - c^S)\widetilde{n}_2$$

attains its maximum at some $\widetilde{n}_2 \in (0,1)$, so accounting for the integer constraint yields that either $\widetilde{n}_2 = 1$ or $\widetilde{n}_2 = 0$. This shows that the optimal deviation for platform provider 2 would yield the following profits:

$$\widetilde{\Pi}_2 = p_1^B - \frac{n_1^2}{2b}\left(\frac{a-bc}{n_1+1}\right)^2 + \max(0, \frac{(a-bc)^2}{4b} - f - c^S).$$

Therefore, in order for (p_1^B, p_1^S) to constitute an equilibrium strategy for platform provider 1, p_1^B and n_1 should be the solutions to the following programme (as long as profits are

[20] Such a strategy when agents' beliefs about the network sizes of one of the platform providers are pessimistic is called "divide and conquer" by Caillaud and Jullien (2003).

positive in equilibrium):

$$\max_{\widetilde{p}_1^B, \widetilde{n}_1} \Pi_1(\widetilde{p}_1^B, \widetilde{n}_1) = \max_{\widetilde{p}_1^B, \widetilde{n}_1} [\widetilde{p}_1^B + (\frac{1}{b}\left(\frac{a-bc}{\widetilde{n}_1+1}\right)^2 - f - c^S)\widetilde{n}_1]$$

$$\text{s.t. } \widetilde{p}_1^B \leq \frac{\widetilde{n}_1^2}{2b}\left(\frac{a-bc}{\widetilde{n}_1+1}\right)^2 + \min(0, f + c^S - \frac{(a-bc)^2}{4b})$$

$$\widetilde{p}_1^B \leq \frac{\widetilde{n}_1^2}{2b}\left(\frac{a-bc}{\widetilde{n}_1+1}\right)^2.$$

The first constraint says that platform provider 1 sets a price for users such that 2 cannot make any profit, whereas the second one means that application users find it optimal to trade with platform provider 1 at such prices, given that $\widetilde{n}_1$ developers are attracted. The first constraint is more stringent than the second one if and only if $\frac{(a-bc)^2}{4b} \geq f + c^S$. If this holds, $\frac{(a-bc)^2}{4b} - (f + c^S)$ represents the benefit that accrues to users owing to the existence of an (inactive) platform provider. This is at the expense of the dominant platform provider, which loses some market power owing to the existence of potential competition.

Of course, the most interesting cases from a strategic standpoint are those in which the dominant platform provider would be constrained by the existence of its competitor, so let us assume that $\frac{(a-bc)^2}{4b} \geq f + c^S$, so that we can ignore the second constraint. Clearly, the first constraint must be satisfied with equality in equilibrium, so platform provider 1 solves:

$$\max_{\widetilde{n}_1} \Pi_1(\widetilde{n}_1) = \max_{\widetilde{n}_1} [\frac{(a-bc)^2}{2b}(1 - \frac{1}{(\widetilde{n}_1+1)^2}) - (f + c^S)\widetilde{n}_1 + f + c^S - \frac{(a-bc)^2}{4b}].$$

Hence, the optimal number of developers pursued by platform provider 1 is $\widetilde{n}_1^* = \left(\frac{(a-bc)^2}{b(f+c^S)}\right)^{\frac{1}{3}} - 1$, and $\Pi_1(\widetilde{n}_1^*) \geq \Pi_1(1) > 0$.

It can also be shown that users may be subsidized by the dominant platform provider. Some algebra yields that

$$\widetilde{p}_1^* = \frac{(a-bc)^2}{4b} - \frac{2(a-bc)\left((a-bc)b(f+c^S)\right)^{\frac{1}{3}}}{2b} + \frac{\left((a-bc)b(f+c^S)\right)^{\frac{2}{3}}}{2b} + (f + c^S),$$

so taking into account that $\frac{(a-bc)^2}{b(f+c^S)} \geq 4$, it can be shown that buyers are subsidized if and only if it holds that $8 \leq \frac{(a-bc)^2}{b(f+c_S)} \leq \frac{4}{3\sqrt{3}-5}$. For low values of $(a-bc)^2/b(f+c^S)$, it is pointless for the dominant platform provider to subsidize users since the low surplus that could be extracted by the competing platform provider via developers would not compensate the losses it would make when attracting users. This changes as the surplus grows, so the dominant platform provider prices the competitor out by subsidizing users. However, for large enough surplus, users are not subsidized. The reason is that a "divide and conquer" strategy requires making profits on the developer side and hence the inactive platform provider would not promote too much entry by developers if it deviated, since it would be detrimental to second-period profits. This restriction allows the dominant platform provider to push the user price up.

Noting that the unique equilibrium when platforms are compatible is the symmetric one we exhibited in Section 4.2, and that profits in such an equilibrium vanish as $t \downarrow 0$, we summarize the results of this section as follows.

Proposition 12 *Suppose there exist two competing platform providers that sell incompatible platforms. If $t = 0$, there exist (two) asymmetric equilibria in which one of the platform providers captures all users and achieves a positive profit that exceeds that achieved when platforms are compatible. Moreover, users are subsidized by the dominant platform provider if and only if* $8 \leq \frac{(a-bc)^2}{b(f+c_S)} \leq \frac{4}{3\sqrt{3}-5}$.

In sum, the striking result that platform compatibility yields greater profits than incompatibility in a symmetric equilibrium is highly dependent on the type of equilibrium played. Indeed, asymmetric equilibria when platforms are incompatible may yield greater profits for the dominant platform provider.

6 Conclusion

We have studied price competition between providers of two-sided platforms in a setting where the two sides first pay a price to gain access to the functionalities of a platform and then interact with each other under oligopolistic conditions. The paper provides a theory for why firms may choose to make their platforms incompatible, despite the softer price competition for users that ensues under compatibility and compatibility being socially optimal. Incompatibility might lead to market dominance and high profits by one of the platform providers, even if they are both ex ante identical and there are not fixed costs of operation. We have also shown that symmetric and asymmetric equilibria under incompatibility may exhibit user subsidization in order to spur developer entry, a result that seems in line with analysts' observations in many markets with two-sided platforms. Finally, we have shown that platform competition results in socially insufficient (excessive) developer entry under incompatibility (compatibility).

One limitation of our approach is that we did not allow platform providers to charge royalty fees per unit of output sold to developers (in addition to the fixed access fee p_i^S). There is a strong incentive in our model to set such royalties below zero. On the one hand, incompatible platforms partly internalize the effects of imperfect competition downstream. Therefore, they will want to distort second-period trade as little as possible. Negative royalty fees would move such trade closer to efficiency. Compatible platforms, on the other hand, are likely to set royalty fees below zero for strategic rather than efficiency reasons.[21] Negative

[21]We are implicitly assuming that both types of agents are assumed to trade with a platform provider

royalties would allow developers which traded with a given platform provider to become tougher competitors in the second period, which would boost profits on the developer side because trade by developers with the competing platform provider would be discouraged.

A second shortcoming of our approach is the assumption that products sold by developers are homogeneous. Allowing for product differentiation would introduce an additional incentive for the platform provider/s to promote developer entry. By doing so, developers could better address users' tastes, and hence the platform provider/s could extract the increase in developer profits and user utility by raising license/entry fees and the price charged to users for the platform. However, the main insights regarding the relative advantages of compatibility vs. incompatibility and social efficiency are likely to be qualitatively unaffected.

Perhaps a more important limitation is our focus on the cases in which compatibility requires all platform providers to agree. This may be representative for technological standards, but in several contexts, a platform can be made compatible with another one by means of an adapter. It would be interesting to examine the situations in which the developers for one of the platforms can sell applications to users of both platforms, but the developers for the competing platform can sell applications only to users of their platform. Although technically challenging, analyzing the private and social incentives for partial compatibility seems an important avenue for future research on compatibility in markets with two-sided platforms.

References

[1] Armstrong, M. (2006), "Competition in Two-Sided Markets," *RAND Journal of Economics* 37(3), 668-691.

[2] Caillaud, B. and Bruno Jullien (2001), "Competing Cybermediaries," *European Economic Review* 45(4-6), 797-808.

[3] Caillaud, B. and Bruno Jullien (2003), "Chicken and Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics* 34(2), 521-552.

[4] Church, J. and N. Gandal (1993), "Complementary Network Externalities and Technological Adoption," *International Journal of Industrial Organization*, No. 11, pp. 239-260.

[5] Crémer, J., Patrick Rey and Jean Tirole (2000), "Connectivity in the Commercial Internet," *Journal of Industrial Economics*, 48, pp. 433-472.

[6] Economides, N. and Fredrick Flyer (1997), "Compatibility and Market Structure for Network Goods," Mimeo, New York University.

at the same time so as to focus on the strategic or efficiency reasons why platform providers may want to keep royalties low. We are cognizant that this sidesteps the remarkable result by Hagiu (2006) that platform providers may wish to use royalty fees to alleviate a hold-up problem when developers decide whether or not to trade with a platform provider before users do.

[7] Evans, D. S. (2003), "The Antitrust Economics of Multi-Sided Platform Markets," *Yale Journal on Regulation* 20(2), 325-382.

[8] Evans, D. S. and Richard Schmalensee (2007), "The Industrial Organization of Markets with Two-Sided Platforms," *Issues on Competition Law and Policy*, W.D. Collins ed.

[9] Hagiu, A. (2005), "Two-Sided Platforms: Pricing and Social Efficiency," mimeo, Harvard Business School.

[10] Hagiu, A. (2006), "Pricing and Commitment by Two-Sided Platforms," *RAND Journal of Economics* 37(3), 720-737.

[11] Katz, M. L. and Carl Shapiro (1985), "Network Externalities, Competition, and Compatibility," *American Economic Review* 75(3), 424-440.

[12] Katz, M. L. and Carl Shapiro (1986), "Product Compatibility Choice in a Market with Technological Progress," *Oxford Economic Papers*, New Series, Vol. 38, Supplement: Strategic Behaviour and Industrial Competition, pp. 146-165.

[13] Katz, M. L. and Carl Shapiro (1994), "Systems Competition and Network Effects," *The Journal of Economic Perspectives*, Vol. 8, No. 2, pp. 93-115.

[14] Kreps, D. M. and Jose A. Scheinkman (1983), "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics* 14(2), 326-337.

[15] Malueg, D. A. and Marius Schwartz (2006), "Compatibility Incentives of a Large Network Facing Multiple Rivals," *Journal of Industrial Economics*, 54, pp. 527-567.

[16] Miao, Chun-Hui (2007), "Compatibility in Two-Sided Markets," Mimeo, University of South Carolina.

[17] Orman, Wafa Hakim, (2008) "The Effects of Compatibility on Competition between Proprietary and Open Two-Sided Platforms." Working Paper (3/28/2008).

[18] Rochet, J-C. and Jean Tirole (2002), "Cooperation among Competitors: Some Economics of Payment Card Associations," *RAND Journal of Economics* 33(4), 1-22.

[19] Rochet, J-C. and Jean Tirole (2003), "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association* 1(4), 990-1029.

[20] Rochet, J-C. and Jean Tirole (2006), "Two-Sided Markets: A Progress Report," *RAND Journal of Economics* 37(3), 645-667.

[21] Suzumura K. and K. Kiyono (1987), "Entry Barriers and Economic Welfare," *Review of Economic Studies* 54(1), 157-167.

Appendix

Proof of Proposition 2. We study the system of equations composed of (3) and (4). We begin by solving (3) for x to obtain

$$x_1(n) = \frac{(a-bc)^2 n^2 - 2b(1+n)^2 p^B}{(a-bc)^2 n^2 t}. \tag{21}$$

It is easy to see that (21) as a function of n (for values of n greater than or equal to zero) has the following properties:

- It is increasing and concave.
- Has a horizontal asymptote at
$$\lim_{n\to\infty} x = \frac{(a-bc)^2 - 2bp^B}{(a-bc)^2 t} \equiv \bar{x} < \frac{1}{t}.$$
- It is s zero at $n_{\min} = \dfrac{2bp^B}{\sqrt{2bp^B (a-bc)^2} - 2bp^B}$.

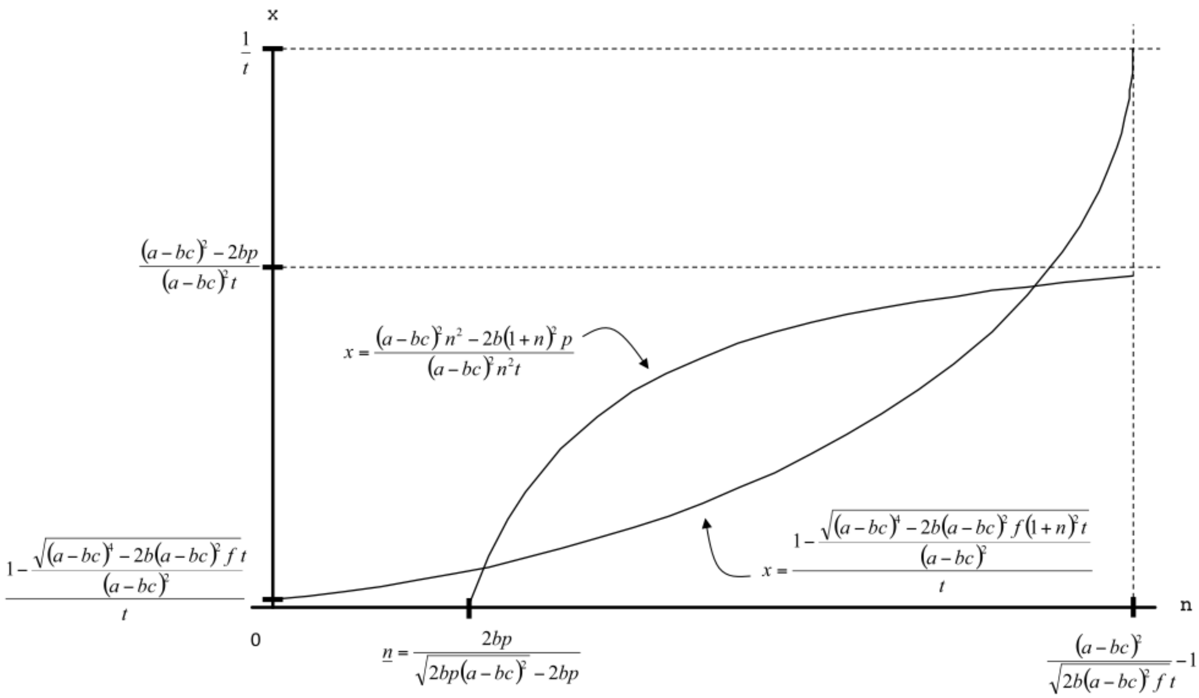
Solving (4) for x we obtain

$$x_2(n) = \frac{1}{t}\left(1 - \frac{\sqrt{(a-bc)^4 - 2b(a-bc)^2(1+n)^2(f+p^S)t}}{(a-bc)^2}\right). \tag{22}$$

$x_2(n)$ has the following properties:

- It is increasing and convex.
- Has a vertical asymptote at
$$\bar{n} = \frac{(a-bc)^2}{\sqrt{2b(a-bc)^2(p^S+f)t}} - 1.$$
- Has no real range for $n > \bar{n}$.
- At $n = \bar{n}$, $x = \dfrac{1}{t}$ and at $n = 0$, $x = \dfrac{1}{t}\left(1 - \sqrt{1 - \dfrac{2b(p^S+f)t}{(a-bc)^2}}\right) > 0$.

We now plot (21) and (22):



Notice that because $\frac{1}{t}$ is always (strictly) larger than the horizontal asymptote, the curves crossing once implies that they cross a second time. Thus, given the equilibrium p^B and p^S (equilibrium implies that the lines cross at (n^*, x^*)), there is always a second solution.

The profit-maximizing allocation pursued by the locally-monopolistic platform provider (x^*, n^*) when setting prices (p^{B*}, p^{S*}), corresponds to the solution furthest away from zero. To see this, notice that the derivative of (22) with respect to n at that point is larger than that of (21) (which is the distinguishing feature of that intersection).

More precisely,

$$\frac{dx_1(n)}{dn} = \frac{4b\,(1+n)\,p^B}{(a-bc)^2\,n^3 t}. \tag{23}$$

and

$$\frac{dx_2(n)}{dn} = \frac{2b\,(1+n)\left(f+p^S\right)}{\sqrt{(a-bc)^4 - 2b\,(a-bc)^2\,(1+n)^2\,(f+p^S)\,t}}. \tag{24}$$

Using (5), (8), and (9), substituting in (23) and (24), and rearranging we obtain,

$$\left.\frac{dx_1(n)}{dn}\right|_{(n^*,x^*)} = \frac{(2tx^*-1)^2}{t}$$

and

$$\left.\frac{dx_2(n)}{dn}\right|_{(n^*,x^*)} = \frac{(2-x^*)\,x^*\,(2tx^*-1)}{\sqrt{1-t\,(2-x^*)\,x^*}}.$$

We now show that

$$\left.\frac{dx_2(n)}{dn}\right|_{(n^*,x^*)} = \frac{(2-x^*)\,x^*\,(2tx^*-1)}{\sqrt{1-t\,(2-x^*)\,x^*}} > \frac{(2tx^*-1)^2}{t} = \left.\frac{dx_1(n)}{dn}\right|_{(n^*,x^*)}.$$

Note that (5) together with $n^* \geq 0$, implies that $2tx^* - 1 \geq 0$. Therefore,

$$\frac{(2-x^*)\,x^*\,(2tx^*-1)}{\sqrt{1-t\,(2-x^*)\,x^*}} > \frac{(2tx^*-1)^2}{t} \Leftrightarrow \frac{(2-x^*)\,x^*t}{\sqrt{1-t\,(2-x^*)\,x^*}} > 2tx^* - 1.$$

Now,

$$\frac{(2-x^*)\,x^*t}{\sqrt{1-t\,(2-x^*)\,x^*}} = \frac{2tx^* - t\,(x^*)^2}{\sqrt{1-t\,(2-x^*)\,x^*}} > \frac{2tx^*-1}{\sqrt{1-t\,(2-x^*)\,x^*}} > 2tx^* - 1.$$

We conclude that

$$\left.\frac{dx_2(n)}{dn}\right|_{(n^*,x^*)} > \left.\frac{dx_1(n)}{dn}\right|_{(n^*,x^*)}.$$

Comparative Statics. By choosing p^B and p^S, the monopolist platform is affecting the location of the curves and thus the intersections. We now see how the curves move as p^B and p^S change. Let's begin with (21). The derivative of x_1 wrt p^B is

$$\frac{dx_1}{dp^B} = -\frac{2b\,(1+n)^2}{(a-bc)^2\,n^2 t} < 0.$$

Thus, the entire curve moves down as p^B grows. Thus, an increase in p^B leads to a reduction in both x and n only for the solution assumed in Section 3.1.

Let's now look at (22). The derivative of x_2 wrt $p^S + f$ is

$$\frac{dx_2}{d(p^S+f)} = \frac{b\,(1+n)^2}{\sqrt{(a-bc)^4 - 2b\,(a-bc)^2\,(p^S+f)\,(1+n)^2\,t}} > 0.$$

The entire curve moves up as p^S raises. Therefore, an increase in p^S leads to a reduction in both x and n only for the solution assumed in Section 3.1. ■

Proof of Proposition 4. Letting

$$H(x) \equiv \frac{2(1-tx)^2(a-bc-\sqrt{\frac{2b(p_1^S+f)}{x(2-tx)}})\sqrt{\frac{2b(p_1^S+f)}{[x(2-tx)]^3}} - t(a-bc-\sqrt{\frac{2b(p_1^S+f)}{x(2-tx)}})^2}{2b} +$$
$$\frac{2(1+tx-t)^2(a-bc-\sqrt{\frac{2b(p_2^S+f)}{(1-x)(2-t(1-x))}})\sqrt{\frac{2b(p_2^S+f)}{[(1-x)(2-t(1-x))]^3}} - t(a-bc-\sqrt{\frac{2b(p_2^S+f)}{(1-x)(2-t(1-x))}})^2}{2b},$$

we have that the implicit function theorem applied on (13) yields

$$\frac{\partial x(p_1^B, p_1^S)}{\partial p_1^B} = \frac{1}{H(x)} \tag{25}$$

and

$$\frac{\partial x(p_1^B, p_1^S)}{\partial p_1^S} = \frac{\frac{(1-tx)(a-bc)}{\sqrt{2bx(2-tx)(p_1^S+f)}} - \frac{(1-tx)}{x(2-tx)}}{H(x)}. \tag{26}$$

Given that we restrict our attention to symmetric equilibria in which $x = \frac{1}{2}$, $p_1^B = p_2^B \equiv p^B$ and $p_1^S = p_2^S \equiv p^S$, the first-order conditions (14) and (15) can be simplified with the aid of expressions (25) and (26) so as to get:

$$\frac{1}{2} = \frac{2b(p^B + \frac{(p^S - c^S)(a-bc)(2-t)}{\sqrt{2b(p^S+f)(4-t)}})}{2t(a-bc-\sqrt{\frac{8b(p^S+f)}{4-t}})^2 - \frac{4(2-t)^2}{4-t}(a-bc-\sqrt{\frac{8b(p^S+f)}{4-t}})\sqrt{\frac{8b(p^S+f)}{4-t}}} \tag{27}$$

and

$$\frac{2b(p^B + \frac{(p^S-c^S)(a-bc)(2-t)}{\sqrt{2b(p^S+f)(4-t)}})(\frac{(2-t)(a-bc)}{\sqrt{2b(4-t)(p^S+f)}} - \frac{2(2-t)}{4-t})}{\frac{4(2-t)^2}{4-t}(a-bc-\sqrt{\frac{8b(p^S+f)}{(4-t)}})\sqrt{\frac{8b(p^S+f)}{4-t}} - 2t(a-bc-\sqrt{\frac{8b(p^S+f)}{4-t}})^2} - \frac{(p^S-c^S)(a-bc)\sqrt{4-t}}{4\sqrt{2b(p^S+f)^3}} = \tag{28}$$
$$1 - \frac{(a-bc)\sqrt{4-t}}{2\sqrt{2b(p^S+f)}}.$$

Plugging equation (27) into (28) yields

$$\frac{(a-bc)\sqrt{4-t}}{8}(\frac{4(f+c^S)+t(p^S-c^S)}{\sqrt{2b(p^S+f)^3}}) = 1, \tag{29}$$

whence it is relatively simple to show that there exists a unique value of p^S that solves such equation.[22] Denoting such a value of p^S by p^{S*}, note that $p^{S*} > c^S$ if and only if $\frac{(a-bc)^2}{2b} > \frac{4(f+c^S)}{4-t}$ holds.

Using (27) and (29), we can find the price p^{B*} charged to users in equilibrium by performing some algebra:

$$p^{B*} = \frac{4t(4+2t-t^2)(p^{S*}+f)(p^{S*}-c^S)^2 - 32(2-t)(3-t)(f+c^S)(p^{S*}+f)(p^{S*}-c^S)}{(4-t)[4(f+c^S)+t(p^{S*}-c^S)]^2}. \tag{30}$$

In order for both platform providers charging a pair of prices (p^{B*}, p^{S*}) to constitute an equilibrium, the values of the parameter space should be such some conditions hold. Thus, the marginal user (i.e., the one located at the middle of the segment) should attain a positive utility, platform providers should make positive profits and a unilateral increase in the user access fee should not increase user demand for the platform that raises such price (as required by the monotonicity refinement). We proceed to derive the parameter restrictions implied by each of these equilibrium conditions.

First, observe that the marginal user attains a positive utility if and only if $u_1^{inc}(1/2) = u_2^{inc}(1/2) > p^{B*}$. Because $u_i^{inc}(1/2) = \frac{(2-t)(a-bc-\sqrt{\frac{8b(p^{S*}+f)}{4-t}})^2}{4b}$, expression (29) implies that this condition is equivalent to the following one:

$$\frac{2(2-t)(4-t)(p^{S*}-c^S)^2(p^{S*}+f)}{[4(f+c^S)+t(p^{S*}-c^S)]^2} > p^{B*}.$$

Making use of (30), we have that this inequality is satisfied if and only if the following holds:

$$(t^3+6t^2-40t+32)(p^{S*}-c^S)^2 + 16(2-t)(3-t)(f+c^S)(p^{S*}-c^S) > 0.$$

The left hand side is strictly convex in p^{S*} (since $t < 2/3$) and takes a negative value for $p^{S*} = -f$ (which clearly is the smallest admissible value of p^{S*}), so it follows that the marginal user makes a positive utility in equilibrium if and only if $p^{S*} > c^S$, that is, if and only if $\frac{(a-bc)^2}{2b} > \frac{4(f+c^S)}{4-t}$ is satisfied. Hence, the parameter constraint that users attain a positive utility implies that developers are not subsidized to enter the market in a symmetric equilibrium with incompatible platforms.

Second, we study the conditions under which platform providers make non-negative profits in a symmetric equilibrium. Profits made by each of them are equal to

$$\Pi_1^* = \Pi_2^* \equiv \Pi^* = \frac{p^{B*}}{2} + (p^{S*}-c^S)(\frac{(a-bc)\sqrt{4-t}}{2\sqrt{2b(p^{S*}+f)}} - 1).$$

[22] Letting $g(p^S) \equiv \frac{(a-bc)\sqrt{4-t}}{8}(\frac{4(f+c^S)+t(p^S-c_S)}{\sqrt{2b(p^S+f)^3}})$ in expression (29), the result follows because $g(-f) = \infty$, $g(\infty) = 0$ and $g' < 0$ (since $t < \frac{2}{3} < 4$).

By (30) and (29), we have

$$\Pi^* = \frac{(p^{S*}-c^S)}{(4-t)[4(f+c^S)+t(p^{S*}-c^S)]^2}[2t(4+2t-t^2)(p^{S*}+f)(p^{S*}-c^S)-$$
$$16(2-t)(3-t)(f+c^S)(p^{S*}+f)+4(4-t)^2(f+c^S)(p^{S*}-c^S)+t(4-t)^2(p^{S*}-c^S)^2],$$

so note that $\Pi^* \geq 0$ if and only if $h(p^{S*}) \geq 0$, where

$$h(p^{S*}) \equiv t(24-4t-t^2)(p^{S*}-c^S)^2+2(4t^2-t^3+32-12t)(p^{S*}-c^S)(f+c^S)-$$
$$16(2-t)(3-t)(p^{S*}+f)(f+c^S).$$

Because $h'' > 0$ and $h(-f) < 0$, it follows that $h(p^{S*}) \geq 0$ for sufficiently large p^{S*}, or equivalently, for large enough $\frac{(a-bc)^2}{2b}$. Let us refer to such a value of $\frac{(a-bc)^2}{2b}$ as W.[23] Because $h(c^S) < 0$, the condition that equilibrium profit is positive is more stringent than the one that makes the utility of the marginal user positive (which simply requires that $p^{S*} > c^S$), so $W > \frac{4(f+c^S)}{4-t}$.

The final condition that must hold in a symmetric equilibrium is the monotonicity condition. It is easy to prove that both $\partial x(p_1^B, p_1^S)/\partial p_1^B$ and $\partial x(p_1^B, p_1^S)/\partial p_1^S$ always have the same sign at the symmetric equilibrium prices. Also, equation (14) implies that $\partial x(p_1^B, p_1^S)/\partial p_1^B < 0$ (evaluated at the symmetric equilibrium prices) if and only if

$$p_1^B + \frac{(p_1^S-c^S)(a-bc)(1-tx(p_1^B,p_1^S))}{\sqrt{2b(p_1^S+f)x(p_1^B,p_1^S)(2-tx(p_1^B,p_1^S))}} > 0. \tag{31}$$

Using expressions (29) and (30) implies that the condition in (31) is equivalent to the following one:

$$\frac{4t(4-t^2+4)(p^{S*}+f)(p^{S*}-c^S)^2-32(2-t)^2(f+c^S)(p^{S*}-c^S)(p^{S*}+f)}{(4-t)[4(f+c^S)+t(p^{S*}-c^S)]^2} > 0.$$

[23] It is straightforward to show that the smallest value of p^{S*} that makes profits non-negative is $c^S + w(f+c^S)$, where

$$w = \frac{2(t^3+4t^2-28t+16)+\sqrt{(t^3+4t^2-28t+16)^2+64t(2-t)(3-t)(24-4t-t^2)}}{2t(24-4t-t^2)}.$$

The lower bound on $\frac{(a-bc)^2}{2b}$ can be obtained by using this value for p^S in equation (29), whence we have that the following condition should hold:

$$\frac{(a-bc)^2}{2b} \geq \frac{64(1+w)^3(f+c^S)}{(4-t)(4+tw)^2} \equiv W.$$

Therefore, the monotonity requirement boils down to assuming that $p^{S*} > c^S + \frac{8(2-t)^2(f+c^S)}{t(8-t^2)}$, which can be shown to directly satisfy the non-negativity constraint on equilibrium profits for $t < 2/3$ (since $h(c^S + \frac{8(2-t)^2(f+c^S)}{t(8-t^2)}) > 0$). Note by (29) that $p^{S*} \geq c^S + \frac{8(2-t)^2(f+c^S)}{t(8-t^2)}$ if and only if $\frac{(a-bc)^2}{2b} \geq Z' \equiv \frac{4(f+c^S)(32+8t^2-24t-t^3)^3}{t^3(8-t^2)(4-t)^5}$.

To conclude the proof, we examine the conditions udner which users of applications may be subsidized when trading with a platform provider. Observe that expression (30) and the fact that $p^{S*} > c^S$ together imply that $p^{B*} \leq 0$ if and only if

$$t(4+2t-t^2)(p^{S*}-c^S) - 8(2-t)(3-t)(f+c^S) \leq 0,$$

that is, users are subsidized in equilibrium if and only if the following inequality holds:

$$p^{S*} \leq c^S + \frac{8(2-t)(3-t)(f+c^S)}{t(4+2t-t^2)}.$$

Letting $p^S = c^S + \frac{8(2-t)(3-t)(f+c^S)}{t(4+2t-t^2)}$ in the left hand side of expression (29) yields

$$\frac{(a-bc)\sqrt{4-t}}{8}\left(\frac{4(f+c^S) + \frac{8(2-t)(3-t)(f+c^S)}{(4+2t-t^2)}}{\sqrt{2b(f+c^S+\frac{8(2-t)(3-t)(f+c^S)}{t(4+2t-t^2)})^3}}\right),$$

which is not larger than 1 for $\frac{(a-bc)^2}{2b} \leq \frac{4(f+c^S)(t^2-6t+12)^3}{(4-t)^2t^3(4+2t-t^2)} \equiv Z$. This shows that $p^{B*} \leq 0$ if and only if $\frac{(a-bc)^2}{2b} \leq Z$. Because $Z > Z'$ for $t < 2/3$, we have that users are subsidized if $\frac{(a-bc)^2}{2b} \in [Z', Z)$, whereas they are not subsidized if $\frac{(a-bc)^2}{2b} \geq Z$. ■

Proof of Proposition 5. The left hand side of the expression in (29) for $p^S = p^{Se}$ is equal to $1 + \frac{t}{4}(\sqrt[3]{\frac{(4-t)(a-bc)^2}{8b(f+c^S)}} - 1)$, which exceeds 1 because it has been assumed that $\frac{(a-bc)^2}{2b} \geq Z'$ and we have shown in the proof of Proposition **??** that $Z' > \frac{4(f+c^S)}{4-t}$, so that inequality $\frac{(a-bc)^2}{2b} > \frac{4(f+c^S)}{4-t}$ is fulfilled. Therefore, it follows that $p^{Se} < p^{S*}$, and hence we have that $n^e > n^*$. In addition, it is easy to see that $p^{Se} > c^S$ because it holds that $\frac{(a-bc)^2}{2b} > \frac{4(f+c^S)}{(4-t)}$. ■

Proof of Proposition 6. We restrict our attention to symmetric equilibria, and we recall that it must hold that $p_1^S = p_2^S = c^S$ in any equilibrium. Let $N = n_1 + n_2$, and in addition, for fixed (p_1^B, p_2^B), let $\{x_1(p_1^B, p_2^B), x_2(p_1^B, p_2^B), N(p_1^B, p_2^B)\}$ denote a solution to the following system of equations:

$$\begin{cases} x_1 - \frac{1}{t}(1 - 2bp_1^B \left(\frac{N+1}{(a-bc)N}\right)^2) = 0 \\ x_2 - \frac{1}{t}(1 - 2bp_2^B \left(\frac{N+1}{(a-bc)N}\right)^2) = 0 \\ \frac{[(2x_1 - tx_1^2) + (2x_2 - tx_2^2)]}{2b} \left(\frac{a-bc}{N+1}\right)^2 - (f + c^S) = 0. \end{cases}.$$

Totally differentiate the system keeping p_2^B fixed at some level p^B so as to get:

$$dx_1 + \frac{2b}{t}\left(\frac{N+1}{(a-bc)N}\right)^2 dp_1^B - \frac{4bp_1^B(N+1)}{t(a-bc)^2 N^3} dN = 0$$

$$dx_2 - \frac{4bp^B(N+1)}{t(a-bc)^2 N^3} dN = 0$$

and

$$\frac{(1-tx_1)}{b}\left(\frac{a-bc}{N+1}\right)^2 dx_1 + \frac{(1-tx_2)}{b}\left(\frac{a-bc}{N+1}\right)^2 dx_2 - \frac{[(2x_1 - tx_1^2) + (2x_2 - tx_2^2)]}{b(N+1)}\left(\frac{a-bc}{N+1}\right)^2 dN = 0.$$

In matrix form:

$$\begin{pmatrix} 1 & 0 & -\frac{4bp_1^B(N+1)}{t(a-bc)^2N^3} \\ 0 & 1 & -\frac{4bp^B(N+1)}{t(a-bc)^2N^3} \\ \frac{(1-tx_1)}{b}\left(\frac{a-bc}{N+1}\right)^2 & \frac{(1-tx_2)}{b}\left(\frac{a-bc}{N+1}\right)^2 & -\frac{[(2x_1-tx_1^2)+(2x_2-tx_2^2)]}{b(N+1)}\left(\frac{a-bc}{N+1}\right)^2 \end{pmatrix} \begin{pmatrix} \frac{dx_1}{dp_1^B} \\ \frac{dx_2}{dp_1^B} \\ \frac{dN}{dp_1^B} \end{pmatrix} = \begin{pmatrix} -\frac{2b}{t}\left(\frac{N+1}{(a-bc)N}\right)^2 \\ 0 \\ 0 \end{pmatrix}.$$

Using Cramer's rule, we have that

$$\frac{dx_1}{dp_1^B} = \frac{\frac{2b}{t}\left(\frac{N+1}{(a-bc)N}\right)^2 \left[\frac{4bp^B(N+1)}{t(a-bc)^2N^3}\frac{(1-tx_2)}{b}\left(\frac{a-bc}{N+1}\right)^2 - \frac{[(2x_1-tx_1^2)+(2x_2-tx_2^2)]}{b(N+1)}\left(\frac{a-bc}{N+1}\right)^2\right]}{\frac{(a-bc)^2[(2x_1-tx_1^2)+(2x_2-tx_2^2)]}{b(N+1)^3} - \frac{4[p_1^B(1-tx_1)+p^B(1-tx_2)]}{tN^3(N+1)}}$$

$$= -\frac{\frac{2[(2x_1-tx_1^2)+(2x_2-tx_2^2)]}{tN^2(N+1)} - \frac{8bp^B(N+1)(1-tx_2)}{N(tN^2)^2(a-bc)^2}}{\frac{(a-bc)^2[(2x_1-tx_1^2)+(2x_2-tx_2^2)]}{b(N+1)^3} - \frac{4[p_1^B(1-tx_1)+p^B(1-tx_2)]}{tN^3(N+1)}}$$

$$= -\frac{b}{tN^2}\left(\frac{N+1}{a-bc}\right)^2 \times$$

$$\left(\frac{2tN^3(a-bc)^2[(2x_1-tx_1^2)+(2x_2-tx_2^2)] - 8bp^B(N+1)^2(1-tx_2)}{(a-bc)^2 tN^3[(2x_1-tx_1^2)+(2x_2-tx_2^2)] - 4b(N+1)^2[p_1^B(1-tx_1)+p^B(1-tx_2)]}\right).$$

Note that

$$\left.\frac{dx_1}{dp_1^B}\right|_{p_1^B=p^B} = -\frac{2b}{tN^2}\left(\frac{N+1}{a-bc}\right)^2(1+\frac{4bp^B(1-tx)(N+1)^2}{2tN^3x(2-tx)\,(a-bc)^2-8bp^B(1-tx)(N+1)^2}),$$

since $x_1 = x_2 \equiv x$ if $p_1^B = p^B$.

When maximizing platform provider 1's profit $p_1^B x_1(p_1^B)$ with respect to p_1^B, the following first-order condition must hold in a symmetric equilibrium:

$$x + p^B \left.\frac{dx_1}{dp_1^B}\right|_{p_1^B=p^B} = 0.$$

That is,

$$x = \frac{2bp^B}{t}\left(\frac{N+1}{(a-bc)N}\right)^2(1+\frac{4bp^B(1-tx)(N+1)^2}{2x(2-tx)\,(a-bc)^2\,tN^3-8bp^B(1-tx)(N+1)^2}). \tag{32}$$

This equation, together with

$$x = \frac{1}{t}(1-2bp^B\left(\frac{N+1}{(a-bc)N}\right)^2) \tag{33}$$

and

$$\frac{2x(2-tx)}{2b}\left(\frac{a-bc}{N+1}\right)^2 = f+c^S, \tag{34}$$

determines the equilibrium values of the triple (p^B, x, N).

To solve for these values, note that substituting (33) into (34) yields that

$$\frac{(1-2bp^B\left(\frac{N+1}{(a-bc)N}\right)^2)(1+2bp^B\left(\frac{N+1}{(a-bc)N}\right)^2)}{bt}\left(\frac{a-bc}{N+1}\right)^2 = f+c^S.$$

Solving for p^B and discarding the negative root (since platforms cannot make negative profits in equilibrium) yields

$$p^B = \frac{1}{2b}\left(\frac{(a-bc)N}{N+1}\right)^2\sqrt{1-bt(f+c^S)\left(\frac{N+1}{a-bc}\right)^2}, \tag{35}$$

which requires that $N \leq \dfrac{(a-bc)}{\sqrt{bt(f+c^S)}} - 1$. It can be shown that x is always positive, but it is smaller than $\frac{1}{2}$ if and only if

$$\frac{2-t}{2} < \sqrt{1 - bt(f + c^S)\left(\frac{N+1}{a-bc}\right)^2},$$

which holds if, for instance, $t > 2$. To find the equilibrium value of N, note that we have that (33) leads to

$$2bp^B\left(\frac{N+1}{(a-bc)N}\right)^2 = 1 - tx,$$

whereas (34) leads to

$$2x(2-tx)\,(a-bc)^2 = 2b(f + c^S)(N+1)^2,$$

so expression (32) can be rewritten as:

$$tx = (1-tx)(1 + \frac{4bp^B(1-tx)(N+1)^2}{2x(2-tx)\,(a-bc)^2\,tN^3 - 8bp^B(1-tx)(N+1)^2}),$$

or, equivalently, as

$$0 = 1 - 2tx + \frac{4bp^B(1-tx)^2}{2bt(f+c^S)N^3 - 8bp^B(1-tx)}.$$

Using (33) on this expression leads to

$$1 = 4bp^B\left(\frac{N+1}{(a-bc)N}\right)^2 + \frac{4bp^B(2bp^B\left(\frac{N+1}{(a-bc)N}\right)^2)^2}{2bt(f+c^S)N^3 - 8bp^B(2bp^B\left(\frac{N+1}{(a-bc)N}\right)^2)},$$

which can be rewritten as

$$2bt(f+c^S)N^3(1-4bp^B\left(\frac{N+1}{(a-bc)N}\right)^2) = 2bp^B\left(\frac{N+1}{(a-bc)N}\right)^2(8bp^B)(1-3bp^B\left(\frac{N+1}{(a-bc)N}\right)^2).$$

Lastly, using (35) leads to the following expression whence N can be derived:

$$N(1-2\sqrt{1-bt(f+c^S)\left(\frac{N+1}{a-bc}\right)^2}) = (\frac{1}{bt(f+c^S)}\left(\frac{a-bc}{N+1}\right)^2 - 1)(2-3\sqrt{1-bt(f+c^S)\left(\frac{N+1}{a-bc}\right)^2}). \tag{36}$$

To simplify, let $A = \frac{bt(f+c^S)}{(a-bc)^2}$ and rewrite (36) as

$$N(1-2\sqrt{1-A\,(N+1)^2}) - (\frac{1}{A\,(N+1)^2} - 1)(2-3\sqrt{1-A\,(N+1)^2}) = 0, \tag{37}$$

with $A \leq 1/(N+1)^2$. Letting $K(N) = A(N+1)^2 \leq 1$, we can further simplify (37) and see that the equilibrium number of developers satisfies:

$$N = \frac{1-K(N)}{K(N)}\left[\frac{2-3\sqrt{1-K(N)}}{1-2\sqrt{1-K(N)}}\right]. \tag{38}$$

Given A, there are two values of N that satisfy (38). To see this, notice that

$$\frac{1-K(N)}{K(N)}\left[\frac{2-3\sqrt{1-K(N)}}{1-2\sqrt{1-K(N)}}\right] \equiv T(N) \tag{39}$$

has a vertical asymptote at $K(N) = \frac{3}{4}$. Furthermore, eq. (39) is a decreasing function of N (and of $K(N)$) with $\lim_{K(N)\to 0^+} T(N) = \infty$, $\lim_{K(N)\to\frac{3}{4}^-} T(N) = -\infty$, $\lim_{K(N)\to\frac{3}{4}^+} T(N) = \infty$ and $\lim_{K(N)\to 1^-} T(N) = 0$. Therefore, there exist two solutions to (38): N_1 and N_2 with $N_1 < N_2$. N_1 is such that $0 < K(N_1) < \frac{3}{4}$ and N_2 such that $\frac{3}{4} < K(N_2) < 1$.

Now, because $p_1^S = p_2^S = c^S$, platforms earn no profits from the developer side regardless of the value of N. However, a larger N implies lower application prices, and more user surplus that can be captured through p^B (since the market is not covered, platform providers do not compete against each other to capture the additional user surplus). Therefore, both platform providers prefer the largest N that solves (38). Let N^{**} be such preferred solution. The solution satisfies $\frac{3}{4} < K(N^{**})$.

Finally, we simplify the expressions for x, p^B, and Π^{**} as follows:

$$x = \frac{1}{t}(1 - \sqrt{1 - A(N^{**}+1)^2}),$$

$$p^{B**} = \frac{t(f+c^S)}{2A}\left(\frac{N^{**}}{N^{**}+1}\right)^2 \sqrt{1-A(N^{**}+1)^2} > 0 = c^B,$$

and

$$\Pi^{**} = \frac{f+c^S}{2A}\left(\frac{N^{**}}{N^{**}+1}\right)^2 \sqrt{1-A(N^{**}+1)^2}(1-\sqrt{1-A(N^{**}+1)^2}).$$

■

Proof of Proposition 7. We look for the solutions (x_1, x_2, N) to the following system of equations:

$$\begin{cases} x_1 - \frac{1}{t}(1 - 2bp^B\left(\frac{N+1}{(a-bc)N}\right)^2) = 0 \\ x_2 - \frac{1}{t}(1 - 2bp^B\left(\frac{N+1}{(a-bc)N}\right)^2) = 0 \\ \frac{[(2x_1 - tx_1^2) + (2x_2 - tx_2^2)]}{2b}\left(\frac{a-bc}{N+1}\right)^2 - (f+c^S) = 0 \end{cases}.$$

Because we study symmetric equilibria, we have imposed $p_1^B = p_2^B \equiv p^B$ and $p_1^S = p_2^S = c^S$. With this, we must have that $x_1 = x_2 \equiv x$. Therefore, the system reduces to one of two equations only:

$$\begin{cases} x - \frac{1}{t}(1 - 2bp^B \left(\frac{N+1}{(a-bc)N}\right)^2) = 0 \\ \frac{(2x - tx^2)}{b}\left(\frac{a-bc}{N+1}\right)^2 - (f + c^S) = 0 \end{cases}.$$

The first equation is the same as (3) and the second is as (4) except that instead of $2b$ in the denominator, we now have b. We thus replicate the proof of Proposition 2. Just as in that case, there are two solutions[24] but only the largest solution has comparative statics that are economically meaningful, as required by Caillaud and Jullien's (2003) monotonicity criterion. It is easy to show that this is the solution that the platform providers prefer. Intuitively, since $p^S = c^S$, platform providers earn no profits from the developers. Thus they prefer N as large as possible so that more value can be extracted from the users through p^B.
■

Proof of Proposition 8. Let $A = \frac{bt(f+c^S)}{(a-bc)^2}$. As shown in the main text, the socially efficient N satisfies $\frac{1}{A} = (N+1)^3$. Let $K(N) = A(N+1)^2$. Then, the socially efficient N^e satisfies

$$N^e = \frac{1 - K(N^e)}{K(N^e)}.$$

In the proof of Proposition 7, we have shown that the equilibrium N under competition (N^{**}) satisfies:

$$N^{**} = \frac{1 - K(N^{**})}{K(N^{**})}\left[\frac{2 - 3\sqrt{1 - K(N^{**})}}{1 - 2\sqrt{1 - K(N^{**})}}\right]$$

and that $K(N^{**}) > \frac{3}{4}$.

Notice finally that $N^e \geq 1$, implies that $K(N^e) \leq \frac{1}{2}$. Therefore, we have $0 < K(N^e) \leq \frac{1}{2} < \frac{3}{4} < K(N^{**}) < 1$. We conclude that $N^{**} > N^e$.

To show that $p^{Se} - c^S > 0$, consider the free-entry condition

$$\frac{2(2x^e - t(x^e)^2)}{2b}\left(\frac{a-bc}{N+1}\right)^2 - f = p^{Se}.$$

We know that $x^e = \frac{1}{t}$. Thus, we may rewrite the condition as:

$$\frac{2/t}{2b}\left(\frac{a-bc}{N+1}\right)^2 - f = p^{Se}.$$

[24] In the proof of Proposition 6 we have also found that there are two values of N that satisfy the optimality condition (36).

Now, subtract c^S from both sides

$$\frac{2/t}{2b}\left(\frac{a-bc}{N+1}\right)^2-\left(f+c^S\right)=p^{Se}-c^S.$$

Notice finally that under competition we have

$$\frac{[(2x_1-tx_1^2)+(2x_2-tx_2^2)]}{2b}\left(\frac{a-bc}{N+1}\right)^2-\left(f+c^S\right)=0$$

and that $(2x_1-tx_1^2)+(2x_2-tx_2^2)<2/t$ for all $0\leq x_i<1/t$. Therefore

$$p^{Se}-c^S=\frac{2/t}{2b}\left(\frac{a-bc}{N+1}\right)^2-\left(f+c^S\right)>\frac{[(2x_1-tx_1^2)+(2x_2-tx_2^2)]}{2b}\left(\frac{a-bc}{N+1}\right)^2-\left(f+c^S\right)=0.$$

Finally, $p^{Be}=0$ follows directly from $x^e=\frac{1}{t}$ (which is derived from the social planner's problem FOCs) and the marginal user condition:

$$x^e-\frac{1}{t}(1-2bp^{Be}\left(\frac{N^e+1}{(a-bc)N^e}\right)^2)=0.$$

■

Proof of Proposition 9. It only remains to show that there are no asymmetric equilibria. Suppose that $p_1^S<p_2^S$, so that platform provider 1 attracts all developers. Using the zero profit condition for developers allows us to write platform provider 2's profit as a function of the price p_2^B it charges to users as follows:

$$\Pi_2(p_2^B)=p_2^B(\frac{1}{2}+\frac{b(p_1^B-p_2^B)}{t(a-bc)^2}(1+\sqrt{\frac{4b(f+p_1^S)}{(4-t)(a-bc)^2}})^2).$$

In turn, platform provider 1's profit as a function of p_1^B and p_1^S is:

$$\Pi_1(p_1^B,p_1^S)=p_1^B(\frac{1}{2}+\frac{b(p_2^B-p_1^B)}{t(a-bc)^2}(1+\sqrt{\frac{4b(f+p_1^S)}{(4-t)(a-bc)^2}})^2)+(p_1^S-c^S)\sqrt{\frac{(4-t)(a-bc)^2}{4b(f+p_1^S)}}.$$

Compute the derivatives of $\Pi_1(p_1^B,p_1^S)$ and $\Pi_2(p_2^B)$ with respect to p_1^B and p_2^B respectively and set them equal to zero:

$$\frac{1}{2}+\frac{b(p_2^B-2p_1^B)}{t(a-bc)^2}(1+\sqrt{\frac{4b(f+p_1^S)}{(4-t)(a-bc)^2}})^2=0$$
$$\frac{1}{2}+\frac{b(p_1^B-2p_2^B)}{t(a-bc)^2}(1+\sqrt{\frac{4b(f+p_1^S)}{(4-t)(a-bc)^2}})^2=0.$$

Clearly, an asymmetric equilibrium must exhibit both firms charging the same price to users and a different one to developers: $p_1^B = p_2^B \equiv p^B$ and $p_1^S < p_2^S$. Now compute the derivative of $\Pi_1(p_1^B, p_1^S)$ wrt p_1^S given that $p_1^B = p_2^B$ so as to get:

$$\left.\frac{\partial \Pi_1(p_1^B, p_1^S)}{\partial p_1^S}\right|_{p_1^B = p_2^B} = \frac{2f + p_1^S + c^S}{4\left(f + p_1^S\right)^2} \sqrt{\frac{(4-t)(a-bc)^2(f+p_1^S)}{b}}.$$

Because this is positive, it holds that p_1^S should be set slightly below p_2^S. However, if $p_2^S \leq c^S$ platform provider 1's profit would jump up if p_1^S were equal to p_2^S rather than just a little less. If $p_2^S > c^S$, then platform provider 2 could profitably deviate by charging p_2^S just below p_1^S. Therefore, no asymmetric equilibrium exists. ■

Proof of Proposition 10. We look for the solutions (x, N) to the following system of equations:

$$\begin{cases} \dfrac{1-tx}{2b}\left(\dfrac{(a-bc)N}{N+1}\right)^2 - p^B = \dfrac{1-t\left(1-x\right)}{2b}\left(\dfrac{(a-bc)N}{N+1}\right)^2 - p^B \\ \dfrac{\left[(2x-tx^2) + \left(2\left(1-x\right) - t\left(1-x\right)^2\right)\right]}{2b}\left(\dfrac{a-bc}{N+1}\right)^2 - \left(f + c^S\right) = 0 \end{cases}.$$

We have imposed $p_1^B = p_2^B = p^B$ and $p_1^S = p_2^S = c^S$ because, as shown in the main text, the case of compatible platforms with full-market coverage has a unique equilibrium which happens to be symmetric.

The first equation implies that $x = \frac{1}{2}$, regardless of the value of N. Given this, N is uniquely determined by the second equation. Therefore, the system has one solution only. ■