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INVESTMENT ALLOCATION UNDER LIMITED LIABILITY RULES

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Abstract

In this article we prove that under very mild conditions (limited liability) a company with a real production process should not invest in a financial asset. The value of the real production process and that of the financial asset are greater when held separately.

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The allocation problem is defined as the search for an optimal investment policy. Optimality is defined in terms of firm value maximization. In a simplified world, we assume that the firm's manager has two investment alternatives. The first investment alternative is to invest in some productive process that costs $1, and that yields an uncertain production value x, where x is a random variable. x is the value of gross production. The range of values of x is $-\infty < x < \infty$, and let $f(x)$ be the density function for variable x. The second investment alternative is the purchase of a financial asset which has a random return y with probability density function g (y) and a range of values for the gross return of $0 \le y < \infty$. Assume that this is a taxless world, the returns of these two assets are uncorrelated with the returns of the economy as a whole, and that cov (x, y) is zero.

Proposition

Given the above mentioned conditions, it is possible to prove that the value of a firm A holding a unit of both x and y is *smaller* than the sum of the value of two firms B and C, B holding a unit of x, and C a unit of y.

Proof.

The present values of firms B and C are,

$$B = \int_0^\infty \frac{x\ f(x)\ dx}{(1+rf)} - 1 \qquad (1)$$

$$C = \int_0^\infty \frac{y\ \gamma(y)\ dy}{(1+rf)} - 1$$

where rf is the risk-free discount rate. (1)

(1) Using the risk-free discount rate is allowed, since we have assumed that the returns of these investments are uncorrelated with the market.

The present value of firm A is going to be very similar to the convolution of two risk units.

$$A = \int_0^\infty \int_0^\infty \frac{(x+y)\ f(x)\ \gamma(x)\, dx\, dy}{(1 + rf)} + \int_0^\infty \int_{-y}^0 \frac{(x+y)\ f(x)\ \gamma(y)\, dx\, dy}{(1 + rf)} - 2 \qquad (2)$$

We have split the value of A into two parts for the sake of convenience. Note that because of limited liability the second lower limit of the second term is –y instead of 0, as for B and C.

The first term of A can be transformed into

$$\int_0^\infty \left| \int_0^\infty \frac{x\ f(x)\ dx}{(1+rf)} + \int_0^\infty \frac{y\ f(x)\ dx}{(1 + rf)} \right| \gamma(y)\ dy \qquad (3)$$

The first term is equal to the value of B, so we can rewrite (3) as

$$B + \int_0^\infty y\ \gamma(y)\ dy \int_0^\infty \frac{f\ (x)\ dx}{(1 + rf)} \qquad (4)$$

Call the second term K1.

The second term of A in (2) can be decomposed into

$$\int_0^\infty \int_{-y}^0 \frac{x\ f(x)\ \gamma(y)\ dx\ dy}{(1 + rf)} + \int_0^\infty \int_{-y}^0 \frac{y\ f(x)\ \gamma(y)\ dx\ dy}{(1 + rf)} \qquad (5)$$

The first term is negative because we are integrating over negative values of x. If we denote this term as K2, K2 < 0. Denote the second term K3.

With the new notation we want to prove that

A = B + K1 + K2 + K3 < B + C

As K2 is negative, it would be enough to prove that K1 + K3 < C.

C can be rewritten as

$$\int_0^{\infty} y \int_0^{\infty} \frac{f(x)\ \gamma(y)\ dx\ dy}{(1 + rf)} + \int_0^{\infty} y \int_{\infty}^{0} \frac{f(x)\ \gamma(y)\ dx\ dy}{(1 + rf)} \qquad (6)$$

Because we are integrating over all possible values of x, the resulting value is unaffected. If we call the first term C1 and the second term C2, C1 = K1 as defined in (4) and C2 is larger than K3 because C2 integrates over a larger range of x, i.e. (–∞, 0) instead of over (–y, 0), and therefore C2 > K3. The result is distribution-free.

Summary and Conclusion

Under the assumptions of this model, firms would prefer to invest only in their own business activities, rather than in financial assets from other companies. This analysis does not consider any tax advantages that might accompany fixed income assets. □

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