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CONVERTIBLE BONDS IN SPAIN:
A DIFFERENT SECURITY

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Abstract

Spanish convertible bonds are different from American convertible bonds. First, the conversion price is not fixed in pesetas, but is defined as a percentage discount off the average share price over a number of days before conversion. Second, the conversion option can be exercised only at a few (usually two or three) different dates. Third, the first conversion opportunity is usually only two or three months after the subscription (issue) date.

In the period 1984 to 1990, 248 issues of convertibles accounted for 1.9 trillion pesetas. In this period, companies issued more convertibles than new shares (1.4 trillion pesetas).

Several formulas for valuing Spanish convertible bonds are derived using option theory. Convertibles have been undervalued by an average of 21.6%. The expropriation effect in the period 1984 to 1990 amounts to 125 billion pesetas.

JEL Classification: G10

CONVERTIBLE BONDS IN SPAIN: A DIFFERENT SECURITY*

1. Introduction

Until 1983, almost every Spanish firm that issued new stock used the rights procedure[1]. More recently, a growing number of firms are raising new equity by issuing convertible bonds. However, Spanish convertible bonds are different from American convertible bonds. First, the conversion price of the shares is not fixed in pesetas, but is defined as a percentage discount off the share price on the day before conversion[2]. Second, the conversion option can be exercised at only a few (usually two or three) different dates. Third, the bonds normally do not have call provisions, although a few are callable after the first conversion date. Fourth, the first conversion opportunity is usually only two or three months after the subscription (issue) date.

The usual structure of the convertible bonds issued in Spain is as follows: Prior to the issue date (on which companies issue the convertibles and investors pay the subscription price), shareholders have a period of usually 30 days to decide whether they want to subscribe or not. After this period, non-shareholder investors can subscribe for the bonds that shareholders did not want.The first conversion opportunity is usually 2 to 6 months after the issue date. There is a period of usually 30 days (called average days) in which the average of the share price is computed ($S_{average}$). Then, bondholders have a period of 30 days to decide whether to convert or not. The number of shares they can get in exchange for each bond is $B/(1-d)S_{average}$, where B is the nominal value of the bond and d the discount that is specified in the contract[3].

An example will illustrate the structure of a typical Spanish convertible bond.

Issuer: Asland, S.A. Issue of 30 billion pesetas.

- Bond face value (par): 100,000 pesetas.

- Subscription price: 100% of par value, without fees or commissions for the investor.

- Annual interest of the bonds: 10%. Coupon paid semi-annually.

- Subscription period:

– Preferred for shareholders. 1 bond for each 83 shares. June 14, 1988 to June 25, 1988.

– For non-shareholders: June 26, 1988 to July 15, 1988.

- Issue date: July 15, 1988.

- Maturity of the bonds: July 15, 1991. Bonds not converted will be redeemed at par plus accrued interest.

- Conversion options:

– First: October 10, 1988. Value of the bond: 104,548 pesetas (par plus accrued interest)[4]. Value of the shares: discount of 20% on the average price of the shares during the months of August and September 1988 on the Madrid Stock Exchange. The shares will be valued at least at par value (500 pesetas).

– Second: April 10, 1989. Value of the bond: 104,548 pesetas (par plus accrued interest). Value of the shares: discount of 15% on the average price of the shares during the months of February and March 1989 on the Madrid Stock Exchange.

– Third: July 15, 1991. Value of the bond: 100,000 pesetas (without accrued interest). Value of the shares: discount of 10% on the average price of the shares during the months of February and March 1989 on the Madrid Stock Exchange.

The shares will be valued, in the three conversion options, at least at par value: 500 pesetas[5].

- Liquidity. Trading on the convertible bonds on the secondary market will be requested, but trading will not take place before the first conversion opportunity.

At the first conversion opportunity, the shares were valued at 826.4% (4,132 pesetas) and the bondholders had the period October 10 to October 28, 1988 to decide whether to convert or not. Each bond could be exchanged for 25.302 shares (104,548/4,132). The share price on October 28, 1988 was 998%.

Many authors have derived formulae to find the theoretical value of American convertible bonds[6], that is, convertibles with a fixed conversion price. But Spanish convertibles, that is, convertibles based on a discount for conversion, have not yet been valued.

In this paper, we analyze all the convertibles issued in Spain in the five-year period January 1984-December 1990. 107 companies issued 248 convertibles during this period.

Table 1 offers evidence of the popularity of convertible bonds in Spain in the period 1984-1990. In 1994 there were only 3 issues (Tubacex, Anaya and Mapfre), accounting for 21.5 billion pesetas. In 1993 there were only 4 issues (Tudor, Agromán, Transfesa and Miquel Costas), accounting for 11,5 billion pesetas. In 1992, issues of convertible bonds accounted for 105 billion pesetas.

In 1986, issues of convertible bonds were more than three times the number of issues of new shares. In fact, direct issues of stock went down in 1986 in part because corporations issued a considerable amount of convertible debt. The reduction in convertibles issued in 1987 resulted from a decrease in convertibles from electric utilities, which were 148 billion in 1986 and only 11 in 1987 (see Table 2). The reason for this decline is that, by law, companies cannot issue new stock below par value, and during 1987 the shares of most electric utilities traded below par. Table 2 shows that an increasing number of companies are using convertible bonds. Table 3 shows the most frequent issuers of convertible bonds in the period 1984-1990. 17 companies (mainly banks, electric companies, Telefónica and Asland) account for 74% of the volume issued, but only 42 % of the number of issues.

Most convertibles have more than one conversion opportunity. Convertibles are structured in such a way that the conditions of the first conversion opportunity are the most favorable for bondholders. The only unfavorable circumstance for bondholders occurs when the share price falls substantially during the average period or during the conversion period. Table 4 shows that for most convertibles the first conversion opportunity arises within six months after the issue (subscription) date. Table 5 offers evidence that the first conversion opportunity is ex-ante the most advantageous for bondholders.

Before an issue of convertible bonds can be offered, the company must prepare a prospectus and present it to the Ministry of Economy and Finance. After approval is granted, the shareholders normally have one month in which to subscribe. After this month, there is another month of open subscription, in which non-shareholder investors can subscribe to the rest of the issue. The subscription orders only include the quantity of bonds, because the price is fixed in the prospectus. If there is oversubscription, the issue is allocated among the investors on a pro-rata basis, although orders below one million pesetas are normally fully covered.

An important point to note here is that a bond's issue price remains fixed for a minimum period of two months (assuming approval is given immediately). This is particularly important for issues with a minimum conversion factor, because their values are very sensitive to the share price.

2. An empirical analysis

2.1. Evidence of the Undervaluation of Spanish Convertible Bonds

In section 3, we will derive formulae to value convertible bonds issued in Spain. In this section, we apply these formulae to the 248 issues in our sample. Table 6 shows the results of this procedure. We have calculated the value of the convertibles at the subscription date. The theoretical value is reported as a percentage of the nominal value (par) of the

bonds. This value must be compared with the subscription price of the bonds, which is always 100% of the nominal value[7].

Table 6 shows that all issues of convertible bonds were initially undervalued. Larger issues were less underpriced: the average value was 121.6%, but the average value weighted by volume was 116.4%.

An implication of the undervaluation should be the realization of abnormal returns for bondholders. Table 7 shows the distribution of the discounted gain obtained by bondholders. We define the discounted gain as the gain over an equivalent fixed income instrument[8]. For example, consider a company that simultaneously issued convertible and straight bonds, both with a subscription price of 100. Conversion occurred one year later, and the conversion value was 132. The annual coupon of the fixed income instrument was 10%, so the value of the straight bond at the conversion date was 110. The discounted gain was therefore 20% ([132/110] - 1).

The discounted gain can be directly compared with the value that we calculated: a 20% discounted gain corresponds to a value of 120%. Table 7 shows that bondholders had, on average, substantial abnormal returns. Smaller issues have been more profitable: the average discounted gain was 22.5%, while the average discounted gain weighted by the volume was 16.8%.

The average annual gain of the Madrid Stock Exchange in the period 1984-1988 was 38%. This extraordinary increase in the index accounts for the difference between the average value that we calculated (121.6%) and the average discounted gain[9] (22.5%).

To study the relationship between the valuation (ex-ante) and the discounted gain (ex-post), we have constructed Table 8. It shows that the valuation that we have done is a good predictor of the ex-post performance of the bonds. By buying the bonds that we claim are more valuable, we would have obtained a larger discounted gain.

2.2. Conversion behavior

Most of the issues have a maximum subscription covenant: the maximum volume that can be subscribed by each investor is limited to one million pesetas in many issues. For this reason, we are not considering any sequential optimal exercise. We should expect that all bonds either be converted or not. It is irrational to convert only a portion of the issue. Table 9 shows the proportion of each issue that was converted at the first opportunity, as a function of the discounted gain obtained by bondholders that converted[10].

Table 9 shows that a substantial part of the issues was not converted when it should have been, and that many bondholders converted when they should not have. The losses incurred by bondholders that did not convert when they should have amount to more than 50 billion pesetas, while the losses incurred by bondholders that did convert when they should not have amount to more than 2 billion pesetas.

2.3. Subscription behavior

As shareholders have the privilege of subscribing the undervalued bonds before other investors, we should expect that they would subscribe the entire issue. Table 10 shows

that this is not the case. There are many attractive (undervalued) issues that are only partially subscribed by shareholders. This situation is similar to shareholders having the possibility of buying dollars paying only 80 cents and refusing to do so. From Table 10 we can conclude that shareholders are not fully aware of the undervaluation of the convertible bonds. We can also detect this lack of awareness in the fact that (to the best of our knowledge) shareholders have never asked for a protection from the expropriation of wealth they suffer when outside investors buy the undervalued convertibles.

2.4. Transfer of wealth from shareholders to bondholders

The undervaluation of the bonds, together with the limited subscription of the issues by shareholders, produces a substantial transfer of wealth from shareholders to outside investors that subscribe convertibles. Wealth is taken away from the shareholders that do not subscribe (or subscribe a smaller proportion of the issue than the proportion of shares that they hold) and is given to the outsiders that do subscribe. The shareholders usually have the right to subscribe, but it does not mean that they are fully protected against the undervaluation of the convertibles when issued, because they cannot sell the right to subscribe undervalued convertibles when they do not want (or forget) to subscribe. If the shareholders do not want to subscribe, the bonds are offered to the public, but the shareholders do not receive any compensation. The transfer of wealth for the 248 issues amounts to more than 125 billion pesetas ($1.0 billion).

3. Valuation of zero coupon bonds convertible at a discount

Many authors have derived formulas to find the theoretical value of American convertible bonds[11]. But Spanish convertibles, that is, convertibles with a discount for conversion, have not yet been valued. The following sections of this paper will deal with the valuation of this kind of convertible bond, following the method first developed by Robert Merton (1984)[12].

Useful insights into the valuation of Spanish convertibles can be obtained by first simplifying the instrument and then gradually complicating it. Specific features can then be grafted on to the basic model.

In this section, we consider a simplified convertible zero coupon bond. Consider the following numerical example.

3.1. Numerical example

Company A has 1,000 shares outstanding and 120 convertible bonds. These convertibles are like zero coupon bonds with an option to convert that can be exercised only at the maturity of the bond. At maturity, the owner of a convertible bond has the following options:

– Convert the bond into shares. In the conversion, the bond will be valued at face value ($1,000) and the shares at 75% of the price at the maturity date.

– Not convert and get the face value of the bond ($1,000)

Let S be the share price at the maturity date of the convertible bond and V the total value of Company A[13]. At maturity, a convertible can be exchanged into: 1000 / 0.75 S shares. At maturity, the convertible bond will be converted if its conversion value is higher than its face value:

$$\frac{120 \times \dfrac{1000}{0.75\,S}}{120 \times \dfrac{1000}{0.75\,S} + 1000}\ V > 120{,}000$$

The total value of Company A has to be the sum of the value of the convertibles and the value of the shares. If conversion occurs, then equation (1) has to hold:

$$\frac{120 \times \dfrac{1000}{0.75\,S}}{120 \times \dfrac{1000}{0.75\,S} + 1000}\ V + 1000\,S = V \qquad (1)$$

With a little algebra, equation (1) can be rewritten as:

$$S = (V / 1000) - 160 \qquad (2)$$

Due to the limited liability of the shareholders of Company A, the price of the shares cannot be negative. So, one restriction for the conversion of all the bonds assumed in equation (1) is that V must be at least $160,000. Notice also that for V=$160,000 the shares are worthless. In this extreme situation, every bondholder will convert, getting an infinite number of shares, each with a price of zero.

The value of each convertible at maturity (assuming V > 160,000) will be:

$$\frac{V - 1000\,S}{120} = \frac{160{,}000}{120} = \$1{,}333.33$$

The number of new shares issued will be:

$$120 \times \frac{1000}{0.75\,S} = \frac{160{,}000{,}000}{V - 160{,}000}$$

If the value of Company A is less than $120,000 (the face value of the bonds) at the maturity date, the company will default, the shares will be worthless and the bondholders will be the new owners of the company. But, what happens when the value of Company A is between $120,000 and $160,000? One way to answer this question is to imagine that only some bondholders will convert and others will not. Suppose that c bonds are converted and 120-c are not. Then equation (1) is no longer valid and must be replaced by equation (3):

$$c \, \times \, \cfrac{\cfrac{1000}{0.75\,S}}{c \, \times \, \cfrac{1000}{0.75\,S} \, + \, 1000} \, [\, V - 1000\,(120 - c)\,] \; + \; 1000\,(120 - c) \; + \; 1000\,S = V \qquad (3)$$

Equation (3) indicates that the value of the bonds converted, plus the value of the bonds not converted plus the value of the old shares, must be the total value of the company. Some algebra permits us to rewrite equation (3) as:

$$S \; = \; (\,V\,/\,1000\,) \; - \; 120 \; - \; (\,c/3\,) \qquad (4)$$

The restrictions to equation (4) are: $S \geq 0$; $0 < c < 120$; and $120{,}000 < V < 160{,}000$. After equation (2) we know that the price of the shares must be zero for this range of values of V. For example, if $V = \$130{,}000$, then 30 bonds will be converted, getting an infinite number of shares, each worth nothing. This means that the owners of these 30 convertibles will get the total value of Company A after paying the 90 bonds not converted. So, the bondholders that convert will get $\$40{,}000$, and the bondholders that do not convert will get $\$90{,}000$. This situation creates a problem: some bonds (the non-converted) have a value at maturity of $\$1{,}000$, while others (the converted) have a value of $\$1{,}333.33$. This situation can be solved by forming a bondholders' association that divides the proceeds at maturity. But it can be solved more easily by considering the firm to be in default, unless its value exceeds $\$160{,}000$.

At maturity, the payoff of the 120 convertible bonds can be written as:

$160,000 \qquad\qquad$ if $V > \$160,000$
$V \qquad$ i $\qquad\qquad$ f $V \leq \$160,000$

And the payoff of the 1,000 shares:

$V - \$160,000 \qquad\qquad$ if $V > \$160,000$
$0 \qquad\qquad\qquad\qquad$ if $V \leq \$160,000$

This is exactly the same payoff as an issue of zero coupon bonds with face value of $\$160{,}000$ would have had, had they been issued instead of the convertibles. If the convertibles were issued one year ago, and at that time the discount for straight and risky zero coupon bonds with one year to maturity was 10%, then the price of a convertible one year ago had to be $\$1{,}212.12$ ($\$1{,}333.33/1.1$), if it were properly priced. Note, however, that the capital structure of the company would have been different in the two cases. If it issues convertibles, the company will remain all-equity financed after conversion, whereas if it issued straight bonds, the company would have to decide how to finance the redemption, whether with new equity or new debt. With the convertible bonds, the company already made this choice when the convertibles were issued.

For $V = \$160{,}000$, the share price is zero and the number of shares tends to infinity, but the share price times the number of shares equals $\$160{,}000$.

At any other moment prior to maturity we can consider the value of the stock as a call option on the firm with striking price of $\$160{,}000$.
$$1000\,S_t \; = \; C\,(\,V_t\,,\,\$160{,}000\,)$$

The value of the convertibles will be V_t - C (V_t , \$160,000), where V_t and S_t are the value of the firm and the share price at time t.

Another way of solving the valuation problem of the convertible bonds is to consider each convertible bond as a straight bond with a put option embedded in it[14]. The put option allows the bondholder to sell the bond back to the company at maturity for \$1,333.33. If every bondholder exercises his put, the company will need to pay –at the maturity of the bonds– \$160,000 (\$1,333.33 x 120). If the value of Company A (V) is less than \$160,000, the company will default and the bondholders will become the new owners. Each rational bondholder will exercise his put option because by doing so he gets \$1,333.33 per bond if the value of the company is greater than \$160,000. Otherwise, he gets only \$1,000 per bond.

The valuation of the convertible can also be derived in another way. Each bond can be converted at maturity into 1,000 / (0.75 S^-) shares, where S^- is the share price just before conversion. If S^+ is the share price just after conversion, then the value of a converted bond at maturity will be (1,000 S^+) / (0.75 S^-). But $S^- = S^+ = S$, because otherwise there would be riskless arbitrage opportunities. So, the value of a converted bond at maturity will be 1,000 / 0.75 = \$1,333.33. This approach facilitates the recognition of the fact that the firm will default at maturity unless its value is greater than \$160,000.

3.2. Convertible bonds with accrued interest for conversion

In this section, I will generalize the results that we have already developed in the numerical example of the previous section.

A company has n shares outstanding and q convertible bonds. These convertibles are like zero coupon bonds with an option to convert that can be exercised only at the maturity of the bond. Each convertible was sold for its face value \$b. The total revenue for the company was B = q b. The owner of a convertible bond has -at maturity- the following options:

– Convert the bond into shares. In the conversion, the bond will be valued at face value plus accrued interest (b [1+r]) and the shares at a discount d off the price at the maturity date. So, at maturity, a bond can be exchanged for (b [1+r]) / ([1 - d] S) shares.

– Not to convert but to receive the face value of the bond plus accrued interest (b [1+r]).

Let S be the share price at the maturity date of the convertible bonds. Let V be the total value of the company[15]. The total value of the company has to be the sum of the value of the convertibles and the value of the shares:

$$\frac{\dfrac{B\,(\,1+r\,)}{(\,1-d\,)\,S}}{\dfrac{B\,(\,1+r\,)}{(\,1-d\,)\,S}\;+\;n}\; V \;+\; n\,S \;=\; V \tag{5}$$

With a little algebra we can rewrite equation (5) as:

$$S = \frac{1}{n} \left\{ V - \frac{B(1+r)}{1-d} \right\} \tag{6}$$

Due to the limited liability of shareholders, the price of the shares cannot be negative. So, one restriction for the conversion of all the bonds assumed in equation (5) is that V must be equal to or bigger than $B(1+r)/(1-d)$. Notice also that for $V = [B(1+r)]/[1-d]$ the shares will be worthless. In this extreme situation, every bondholder will convert, receiving an infinite number of shares with price zero.

If converted, the value of the convertible bonds at maturity will be:

$$V - nS = \frac{B(1+r)}{1-d}$$

The number of new shares issued will be:

$$N = \frac{B(1+r)}{(1-d)S} = n\frac{B(1+r)}{V(1-d) - B(1+r)} \tag{7}$$

If the value of the company is smaller than the total payment due to the bondholders at the maturity date, $B(1+r)$, then the company will default, the shares will be worthless and the bondholders will be the new owners of the company. But, what happens when the value of the company at maturity lies between $B[1+r]$ and $B(1+r)/(1-d)$? Again, one possibility is that only some bondholders will convert and others will not. Suppose that c bonds are converted and $[B/b] - c$ are not. Then equation (5) is no longer valid and must be replaced by equation (8):

$$\frac{\dfrac{cb(1+r)}{(1-d)S}}{\dfrac{cb(1+r)}{(1-d)S} + n} [V - (B - bc)(1+r)] + (B - bc)(1+r) + nS = V \tag{8}$$

Some algebra will allow us to rewrite equation (8) as:

$$\frac{cb(1+r)}{(1-d)} + (B - bc)(1+r) + nS = V \tag{9}$$

Equations (8) and (9) indicate that the value of the bonds converted, plus the value of the bonds not converted plus the value of the old shares, must be the total value of the company. After equation (6) we know that the price of the shares must be zero for $V < B(1+r)/(1-d)$. The number of converted bonds will be:

$$c = \frac{V - B(1+r)}{b(1+r)} \frac{(1-d)}{d} \quad \text{for } B(1+r) < V < \frac{B(1+r)}{(1-d)}$$

This situation creates a problem: some bonds have at maturity a value of b (1 + r), while others (the converted) have a value of [b (1 + r)] / [1 - d]. This situation can be solved by forming a bondholders' association that divides the proceeds at maturity. But it can be solved more easily by considering the firm to be in default, unless its value is greater than B(1+ r)/(1-d).

The valuation of the convertible can also be derived in another way. Each bond can be converted at maturity into $b (1 + r) / [(1 - d) S^-]$ shares, where S^- is the share price just before conversion. If S^+ is the share price just after conversion, then the value of a converted bond at maturity will be $[b (1 + r) S^+] / [(1 - d) S^-]$. But $S^- = S^+ = S$, because otherwise there would be riskless arbitrage opportunities. So, the value of a converted bond at maturity will be $b (1 + r) / (1 - d)$. This approach facilitates recognition of the fact that the firm will default unless its value is greater than $B (1 + r) / (1 - d)$.

At maturity, the payoff of the B/b convertible bonds can be written as:

$[B (1 + r)] / [1 - d]$ if $V > [B (1 + r)] / [1 - d]$
V if $V \leq [B (1 + r)] / [1 - d]$

And the payoff of the n shares:

$V - [B (1 + r)] / [1 - d]$ if $V > [B (1 + r)] / [1 - d]$
0 if $V \leq [B (1 + r)] / [1 - d]$

This is exactly the same payoff as an issue of zero coupon bonds paying $[B (1 + r)] / [1 - d]$ at maturity would have had, had they been issued instead of the convertibles. Note, however, that the capital structure of the company would have been different in the two cases. If it issues convertibles, the company will remain all-equity financed after conversion, whereas if it issued straight bonds, the company would have to decide how to finance the redemption: whether with new equity or with new debt. With the convertible bonds, the company already made this choice when the convertibles were issued.

At any other moment prior to maturity we can consider the value of the stock as a call option on the firm with striking price of B (1 + r) / (1 - d).

$$n S_t = C (V_t , [B (1 + r)] / [1 - d]) \tag{10}$$

And the value of the convertibles will be

$$CONV_t = V_t - C (V_t , [B (1 + r)] / [1 - d]), \tag{11}$$

where V_t and S_t are the value of the firm and the share price at time t.

As indicated before, we have assumed that each convertible bond was sold for its face value b and with an interest rate r. If the market interest rate at that time for straight zero coupon bonds with the same maturity and equivalent risk was R, then an investor would be indifferent between buying a convertible and buying a straight bond if their payoffs at maturity were equal, that is, if: $b (1 + R) = b (1 + r) / (1 - d)$. Therefore, if properly priced, the interest rate and the discount of the convertibles must follow relationship (12)

$$r = (1 + R) (1 - d) - 1 \tag{12}$$

3.3. Convertible bonds without accrued interest

These convertibles are exactly like the convertibles in section 2.2, except that for conversion the bonds are not valued at face value plus accrued interest (b [1+ r]) , but only at face value b. Following the same procedure as in the previous section, we derive the following results.

The bonds will be converted only if their conversion value is higher than the face value of the bond plus accrued interest: $B / (1 - d) > B (1 + r)$. This means that the conversion feature will have value only if $(1 - d) (1 + r) < 1$. For any other moment prior to maturity the value of the convertibles will be $C_t = V_t - C (V_t , B / (1 - d))$, where V_t and S_t are the value of the firm and the share price at time t. If the market interest rate at that time for straight zero coupon bonds with the same maturity and equivalent risk were R, then an investor would be indifferent between buying a convertible and buying a straight bond with equal payoffs at maturity, that is, if: $b (1 + R) = b / (1 - d)$. Therefore, if properly priced, the discount of the convertible bonds must follow the relationship $(1 + R) (1 - d) = 1$.

4. Valuation of convertibles with maximum conversion factor

In this section we develop formulas to value convertible zero coupon bonds with maximum conversion factor. These bonds will represent a better approximation to the convertible bonds issued in Spain because, by law, new shares cannot be issued below par value[16].

4.1. Numerical example

Suppose now that Company A issued the same convertibles discussed in 2.1, but with an additional feature:

– For conversion, the shares will be valued at least at $150/share.

This is equivalent to placing a restriction on the number of shares for which a convertible bond can be exchanged. Namely, a bond can be converted into a maximum of 6.67 shares (1,000 / 150). By this means we achieve a closer approximation to real convertibles because, by law, new stock cannot be issued below par, that is, below 100% of the nominal price of the shares.

In this case, equation (1) must be modified to:

$$\frac{120 \times \dfrac{1000}{k}}{120 \times \dfrac{1000}{k} + 1000} V + 1000\, S = V \; ; \; k = MAX [150 , 0.75\, S] \qquad (13)$$

The bond will be converted if its conversion value is greater than $1,000. Then, from equation (13):

$$\frac{V - 1000\,S}{120} = \frac{V}{k + 120} > 1000 \tag{14}$$

If $k = 150$, it means that $150 < S < 200$, and $270,000 < V < $360,000. For this range of values, $V = 1800\,S$. The number of new shares in this interval is constant and equal to 800. As the number of new shares is constant, the value of the convertibles is a constant fraction (44.44%) of the total value of the company ($800 / 1800$). For $V = $270,000, the value of the convertibles is $120,000, the face value. For $V = $360,000, the value of the convertibles is $160,000.

When the value of the company is greater than $360,000, which means that the share price is higher than $200, then $k = 0.75\,S$. In this interval, equation (1) holds.

When the value of Company A is less than $120,000 (the face value of the bonds) at the maturity date, the company will default, the shares will be worthless and the bondholders will be the new owners of the company. When the value of the company lies between $120,000 and $270,000, the bonds will not be converted.

At maturity, the payoff of the 120 convertible bonds can be written as:

$160,000	if $360,000 < V$
0.4444 V	if $270,000 < V \le $360,000
$120,000	if $120,000 < V \le $270,000
V	if $V \le $120,000

And the payoff of the 1000 shares:

V - $160,000	if $360,000 < V$
0.5555 V	if $270,000 < V \le $360,000
V - $120,000	if $120,000 < V \le $270,000
0	if $V \le $120,000

For any other moment prior to maturity, we can consider the value of the convertibles as a combination of call options on the firm with different striking prices. V_t and S_t are the value of the firm and the share price at time t. Note that $800/1800 = 0.4444$.

$$C_t = V_t - C(V_t, \$120{,}000) + \frac{800}{1800}\,C(V_t, \$270{,}000) - \frac{800}{1800}\,C(V_t, \$360{,}000)$$

4.2. Convertible bonds with accrued interest

Now we consider the same convertibles discussed in 3.2, but with an additional feature:

– For conversion, the shares will be valued at least at M/share.

This is equivalent to placing a restriction on the number of shares into which a convertible bond can be converted. Namely, a bond can be converted into a maximum of b (1 + r) / M shares.

In this case, equation (5) must be transformed into:

$$\frac{\dfrac{B (1 + r)}{k}}{\dfrac{B (1 + r)}{k} + n} \, V \; + \; n \, S \; = \; V \; ; \qquad k = MAX \, [\, M \, , \, (1 - d) \, S \,] \qquad\qquad (15)$$

If k = M, it means that S < M/ (1 - d), and conversion will take place for M < S < M/ (1 - d) , and nM + B (1+r) < V < [nM + B(1+r)] / [1-d]. For these values, V = [nM + B (1+r)] S/M. The number of new shares in this interval is constant and equal to B (1 + r) / M. As the number of new shares is constant, the value of the convertibles is a constant fraction of the total value of the company. For V = nM + B (1 + r), the value of the convertibles is B (1 + r). For V = [nM + B (1 + r)] / [1 - d], the value of the convertibles is B (1 + r) / (1 - d).

When the value of the company is greater than [nM + B (1 + r)]/[1 - d], which means that the share price is higher than M/ (1 - d), then k = (1-d) S. In this interval, equation (5) holds.

When the value of Company A is less than B (1 + r) (the promised payment to the bonds) at the maturity date, the company will default, the shares will be worthless and the bondholders will be the new owners of the company. When the value of the company lies between B (1 + r) and nM + B (1 + r), the bonds will not be converted.

At maturity, the payoff of the convertible bonds can be written as:

$$B (1 + r) / (1 - d) \qquad\qquad \text{if} \quad [nM + B (1 + r)] / [1 - d] \; < \; V$$

$$\frac{B (1 + r)}{nM \; + \; B (1 + r)} \, V \qquad\qquad \text{if} \quad nM + B (1 + r) \; < \; V \; \leq \; [nM + B (1 + r)] / [1 - d]$$

$$B (1 + r) \qquad\qquad \text{if} \quad B (1 + r) \; < \; V \; \leq \; nM + B (1 + r)$$

$$V \qquad\qquad \text{if} \quad V \; \leq \; B (1 + r)$$

And the payoff of the n shares:

$$V \; - \; [\, B (1 + r) / (1 - d) \,] \qquad \text{if} \quad [nM + B (1 + r)] / [1 - d] \; < \; V$$

$$\frac{nM}{nM \; + \; B (1 + r)} \, V \qquad\qquad \text{if} \quad nM + B (1 + r) < V \leq [nM + B (1 + r)] / [1 - d]$$

$$V \; - \; B (1 + r) \qquad\qquad \text{if} \quad B (1 + r) \; < \; V \; \leq \; nM + B (1 + r)$$

$$0 \qquad\qquad \text{if} \quad V \; \leq \; B (1 + r)$$

For any other moment prior to maturity, we can consider the value of the convertibles C_t as a combination of call options on the firm with different striking prices. V_t and S_t are the value of the firm and the share price at time t.

$$C_t = V_t - C(V_t, B(1+r)) + \frac{B(1+r)}{nM + B(1+r)} [C(Vt, nM + B(1+r)) - C(Vt, \frac{nM + B(1+r)}{(1-d)})] \quad (16)$$

4.3. Convertible bonds without accrued interest for conversion

These convertibles are exactly like the convertibles in section 4.2, except that for conversion the bonds are not valued at face value plus accrued interest (b [1+ r]) , but only at face value b. Following the same procedure as in the previous section, we derive the following results.

At maturity, the payoff of the convertible bonds can be written as:

$B / (1 - d)$ if $(nM + B) / (1 - d) < V$

$\dfrac{B}{nM + B} V$ if $(nM + B)(1+r) < V \leq (nM + B)/(1 - d)$

$B(1+r)$ if $B(1+r) < V \leq (nM + B)(1+r)$

V if $V \leq B(1+r)$

For any other moment prior to maturity, we can consider the value of the convertibles C_t as a combination of call options on the firm with different striking prices. V_t and S_t are the value of the firm and the share price at time t.

$$C_t = V_t - C(V_t, B(1+r)) + \frac{B}{nM + B} [C(V_t, [nM + B][1+r]) - C(V_t, \frac{nM + B}{(1-d)})] \quad (17)$$

5. Valuation of convertible bonds with maximum and minimum conversion factor

5.1. Numerical example

Suppose now that Company A issued the same convertibles discussed in 3.1, but with two additional features:

– For conversion, the shares will be valued at most at $225/share.

– For conversion, the shares will be valued at least at $150/share.

This is equivalent to placing a restriction on the number of shares that a convertible bond can be exchanged for. Namely, a bond can be converted into a maximum of 6.67 shares (1,000 / 150) and a minimum of 4.44 shares (1,000 / 225).

These two features give a closer approximation to some real convertibles. By law, new stock cannot be issued below par.

In this situation, equation (1) must be transformed into:

$$\frac{120 \times \dfrac{100}{k}}{120 \times \dfrac{1000}{k} + 1000} \, V \; + \; 1000 \, S \; = \; V \; ; \quad k = \text{MIN} \,(\, 225, \, \text{MAX} \,[\, 150 \, , \, 0.75 \, S \,] \qquad (18)$$

Following the same procedure as in the previous sections, k will have different values for different intervals of S and V:

k = $150	S ≤ 200	270,000 < V ≤ 360,000
k = 0.75 S	200 < S ≤ 300	360,000 < V ≤ 460,000
k = $225	300 < S	460,000 < V

At maturity, the payoff of the 120 convertible bonds can be written as:

0.3478 V	if $460,000 < V
$160,000	if $360,000 < V ≤ $460,000
0.4444 V	if $270,000 < V ≤ $360,000
$120,000	if $120,000 < V ≤ $270,000
V	if V ≤ $120,000

For any other moment prior to maturity, we can consider the value of the convertibles as a combination of call options on the firm with different striking prices. V_t and S_t are the value of the firm and the share price at time t.

$$C_t = V_t - C\,(\,V_t\,,\,\$120{,}000\,) \; + \; \frac{800}{1800}\{\, C\,(\,V_t\,,\,\$270{,}000\,) - C\,(\,V_t\,,\,\$360{,}000\,) \,\} \; + $$

$$- \frac{533.33}{1533.33} \; C\,(\,V_t\,,\,\$460{,}000\,)$$

5.2. Convertible bonds with accrued interest for conversion

Now we consider the same convertibles discussed in 3.2, but with two additional features:

– For conversion, the shares will be valued at least at M/share.

– For conversion, the shares will be valued at most at L/share (L > M).

This is equivalent to placing a restriction on the number of shares that a convertible bond can be exchanged for. Namely, a bond can be converted into a minimum of $b(1+r)/L$ shares and into a maximum of $b(1+r)/M$ shares.

In this case, equation (1) must be transformed into:

$$\frac{\dfrac{B(1+r)}{k}}{\dfrac{B(1+r)}{k}+n}\,V + n\,S = V\;;\;\; k = MIN(L, MAX[M,(1-d)S])\qquad(19)$$

Following the same procedure as in the previous sections, k will have different values for different intervals of S and V:

$k = M$	$S \leq M/(1-d)$	$nM + B(1+r) < V \leq [nM + B(1+r)]/[1-d]$
$k = (1-d)S$	$M/(1-d) < S \leq L/(1-d)$	$[nM + B(1+r)]/[1-d] < V \leq [nL + B(1+r)]/[1-d]$
$k = L$	$L/(1-d) < S$	$[nL + B(1+r)]/[1-d] < V$

At maturity, the payoff of the B/b convertible bonds can be written as:

$$\frac{B(1+r)}{nL + B(1+r)}\,V \qquad \text{if } [nL + B(1+r)]/[1-d] < V$$

$$B(1+r)/(1-d) \qquad \text{if } [nM + B(1+r)]/[1-d] < V \leq [nL + B(1+r)]/[1-d]$$

$$\frac{B(1+r)}{nM + B(1+r)}\,V \qquad \text{if } nM + B(1+r) < V \leq [nM + B(1+r)]/[1-d]$$

$$B(1+r) \qquad \text{if } B(1+r) < V \leq nM + B(1+r)$$

$$V \qquad \text{if } V \leq B(1+r)$$

For any other moment prior to maturity, we can consider the value of the convertibles C_t as a combination of call options on the firm with different striking prices. V_t and S_t are the value of the firm and the share price at time t.

$$C_t = V_t - C(V_t, B(1+r)) + \frac{B(1+r)}{B(1+r)+nM}\{C(V_t, B(1+r)+nM) - C(V_t, [nM + B(1+r)]/[1-d])\} +$$

$$(20) \qquad\qquad + \frac{B(1+r)}{B(1+r)+nL}\,C(V_t, [nL + B(1+r)]/[1-d])$$

5.3. Convertible bonds without accrued interest for conversion

Following the same procedure as in the previous section, we derive the following results.

At maturity, the payoff of the B/b convertible bonds can be written as:

$$\frac{B}{nL + B} V \qquad \text{if } [nL + B] / [1 - d] < V$$

$$B / (1 - d) \qquad \text{if } [nM + B] / [1 - d] < V \leq [nL + B] / [1 - d]$$

$$\frac{B}{nM + B} V \qquad \text{if } (nM + B)(1 + r) < V \leq [nM + B] / [1 - d]$$

$$B(1 + r) \qquad \text{if } B(1 + r) < V \leq (nM + B)(1 + r)$$

$$V \qquad \text{if } V \leq B(1 + r)$$

For any other moment prior to maturity, we can consider the value of the convertibles C_t as a combination of call options on the firm with different striking prices.

$$C_t = V_t - C(V_t, B(1+r)) + \frac{B}{B + nM} \{ C(V_t, (B + nM)(1+r)) - C(V_t, [nM + B] / [1 - d] \} -$$

$$(21) \qquad + \frac{B}{B + nL} C(V_t, [nL + B] / [1 - d])$$

6. Extensions of the valuation formulas

In this section we will introduce the different characteristics of Spanish convertible bonds into the valuation procedure. These characteristics were not taken into account in the simplified models considered in sections 3, 4, and 5. In reality, convertible bonds are not zero coupon bonds. Nevertheless, as we shall see in this section, to consider Spanish convertibles as zeros is a very good approximation. Now we shall look at the characteristics that are left out by considering the bonds as zeros:

– The bonds have more than one conversion opportunity

– Conversion occurs before maturity

– Bondholders do not convert immediately, but have a period of 10 to 30 days (conversion period) to decide whether to convert or not.

– The discount is not calculated on the share price of one day, but rather on the average of prices over a number of days.

These four characteristics favor the bondholders. We shall argue that only the average introduces a significant difference to our previous valuation approach.

6.1. Convertibles with more than one conversion opportunity

Suppose now that the convertible bonds in section 4.1. have two conversion dates and that at each one the bond is valued at face value plus accrued interest. The shares are valued at a discount d_1 on the first conversion date and d_2 on the second conversion date. The accrued interest is r_1 at the first conversion date and r_2 at the second one. There are no coupon payments between the two dates.

If the company does not default, the value of the bond converted at the first conversion opportunity is $B (1 + r_1) / (1 - d_1)$ at time 1 (first conversion date). The value of the bond converted at the second conversion date is $b (1 + r_2) / (1 - d_2)$ at time 2 (second conversion date). If R is the appropriate discount rate between time 1 and time 2, every bondholder should convert at time 1 if: $b (1 + r_1) / (1 - d_1) > b (1 + r_2) / [(1 - d_2) (1 + R)]$. In practice, it is normally the case that $(1 + r_2) / (1 + r_1) < (1 + R)$, so a sufficient condition to convert on the first conversion date would be: $d_1 > d_2$

The general condition, however, is:

$$(1 + r_1) / (1 - d_1) > (1 + r_2) / [(1 - d_2) (1 + R)]$$

Allowing for default, suppose that c bonds were converted at time 1. At time 2 the value of the company is V_2 and the value of the remaining $(B / b) - c$ bonds would be:

$$[(B - cb) (1 + r_2)] / [1 - d_2] \quad \text{if } V_2 > [(B - cb) (1 + r_2)] / [1 - d_2]$$
$$V_2 \quad \text{if } V_2 \leq [(B - cb) (1 + r_2)] / [1 - d_2]$$

At any time t between the two conversion dates, we can express the value of the non converted bonds as $V_t - C \{ V_t , (B - cb) (1 + r_2)/ (1 - d_2), \text{time } 2 \}$. At time 1, the value of the company is V_1 and the value of the convertibles is $V_1 - C \{ V_1 , B (1 + r_2) / (1 - d_2) , \text{time } 2 \}$ if all bondholders decide to convert at time 2, and $V_1 - C \{ V_1 , B (1 + r_1) / (1 - d_1), \text{time } 1 \}$ if they decide to convert at time 1. Even for some situations where $d_1 < d_2$ conversion at time 1 can be optimal.

The optimal conversion date can also be contemplated from the point of view of the shareholders. They own a call with two exercise dates. At time 1 they would prefer their call not to be exercised if the strike price at time 1 is bigger than the strike price at time 2. But they also have to consider the time value of the call if exercised at time 2. So, even for some values of the strike price at time 1 ($B [1 + r_1] / [1 - d_1]$) that are smaller than the strike price at time 2 ($B [1 + r_2] / [1 - d_2]$), shareholders would prefer to exercise at time 2. And what is optimal for the shareholders is not optimal for the bondholders, because they share the value of the company.

With real convertibles, as we have already mentioned (see Table 5), it is never the case that $d_1 < d_2$. Then, every bondholder should convert at the first opportunity. Note that in order to decide whether to convert or not at the first conversion opportunity, a rational investor should compare:

(a) the conversion value of the bonds

(b) the value of the bonds considering only the second, third... conversion opportunities.

If (a) > (b), bonds should be converted at the first opportunity.

Given the structure of the convertible bonds issued in Spain, there only two situations in which it can be better not to convert:

– If the share price declines substantially during the average period or during the conversion period.

– If the share price is lower than the minimum price at which the shares are valued for conversion.

6.2. Different maturity and conversion dates

In sections 3, 4, and 5 we valued convertible zero coupon bonds. For these bonds, conversion and maturity occur at the same time. For real convertibles, conversion occurs before maturity. We prove here that when conversion occurs before maturity there is no significant difference from the value of the convertibles derived in previous sections.

Suppose now that the convertible bonds in section 3. 1. can be converted at time 1 and that the maturity is at time 2, three months after time 1. For conversion the bonds are valued at $1,000, and at maturity the promised payment is $1,100.

If the value of the company is greater than $160,000, every bondholder will convert at time 1 and equation (2.1) holds. But if the value of the company at time 1 is lower than $160,000, the bondholders do not receive the value of the company because now the company does not default at time 1. If only c bonds are converted at time 1, equation (22) must hold:

$$\frac{c}{c + 0.75 \, S_1} \, C\left(V_1, 1100\,(120 - c)\right) + V_1 - C(V_1, 1100\,(120 - c)) + 1000 \, S_1 = V_1 \tag{22}$$

Equation (22) states that at time 1, the value of the converted bonds plus the value of the non-converted bonds plus the value of the old shares must equal the value of the company. The call has three months to maturity. Equation (23) is derived from equation (22).

$$S_1 = \frac{C\left(V_1, 1100\,(120 - c)\right)}{1100} - \frac{c}{0.75} \tag{23}$$

The fact that conversion date and maturity (or coupon payment) are not the same introduces a small discrepancy between these and our previous results. Now, the bondholders continue to have the possibility of receiving the total value of the company when $V_1 <$ $160,000, but for this they need to reach an agreement among themselves: some bonds will

be converted and others will not. Now, the company does not default at the conversion date when V_1 < \$160,000 because the bonds mature later. For a payment at maturity of \$1,025/bond, three months from conversion to maturity, volatility of 0.4 and riskless interest rate of 15%, the following values of V_1 and c produce a result of $S_1 = 0$ according to equation (23):

c	1	0.1	0.01	0.001	159
V_1	94,000	79,000	68,000	60,000	159,657

For convertibles with a maximum conversion factor, as is normally the case, the fact that the conversion date is not the maturity date does not produce a large difference either. For the bondholders not to convert requires that $1 + r_2 > (1 + r_1) / (1 - d_1)$, which is never the case. Note also that if the time value of the call is larger than $B(r_2 - r_1)$, then the bondholders will always convert as long as $V_1 > (nM + B)(1 + r_1)$, which is the same result as was found when conversion and maturity were the same date.

6.3 Conversion period

Normally, the bondholders have a period of 20 days to decide whether to convert. This is equivalent to adding a new characteristic to the convertible bonds; namely, the bonds will be converted at a discount on the share price t days before conversion. Conversion date is date zero and the shares are valued at a discount d off the price at time -t .

The bondholders will convert if:

$$\frac{\dfrac{B(1+r)}{(1-d)S_{-t}}}{\dfrac{B(1+r)}{(1-d)S_{-t}} + n} \; V_0 \; > \; B(1+r) ; \tag{24}$$

From equation (24) we know that the bondholders will convert only if $n(1-d)S_{-t} + B(1+r) < V_0$. The payoff of the convertibles at date zero will be:

$$\frac{B(1+r)}{n(1-d)S_{-t} + B(1+r)} \; V_0 \qquad \text{if } n(1-d)S_{-t} + B(1+r) < V_0$$

$$B(1+r) \qquad\qquad \text{if } B(1+r) < V_0 < n(1-d)S_{-t} + B(1+r)$$

$$V_0 \qquad\qquad \text{if } V_0 < B(1+r)$$

At time - t , equation (25) must hold:

$$V_{-t} = nS_{-t} + V_{-t} - C(V_{-t}, B(1+r)) + \frac{B(1+r)}{n(1-d)S_{-t} + B(1+r)} C(V_{-t}, B(1+r) + n(1-d)S_{-t}) \tag{25}$$

Applying equation (25) to company A, we get equation (26):

$$1000 S_{-t} = C(V_{-t}, 120{,}000) - \frac{120}{0.75\,S_{-t} + 120}\, C(V_{-t}, 120{,}000 + 750\,S_{-t}) \qquad (26)$$

Solving equation (26) for r=10%, t= 20 days and volatility = 0.4:

V_{-t}	160,000	180,000	200,000	300,000	400,000
S_{-t}	10^{-8}	19.972	39.984	139.9932	239.9948

Solving equation (26) for r=10%, V_{-t} = 400,000 and volatility = 0.4:

t (days)	0	20	180	365
S_{-t}	240	239.9948	236.5909	232.6687

Solving the implicit equation (26) for r=10%, V_{-t} = 400,000 and volatility=1:

t (days)	0	20	180	365
S_{-t}	240	237.4255	214.3827	207.7503

The numerical analysis shows that for subscription periods of 20 days, so long as the volatility is not very great, the fact that bondholders have a period of time to decide whether to convert or not does not introduce any significant difference into our previous calculations, in which we did not consider this period. This very small difference favors the bondholders.

6.4. Discount off the average

As described in the introduction, the shares are not normally valued at a discount on the share price on one day, but at a discount on an average price. The most common periods used to compute the average have been the previous month (77 issues), the previous three months (16 issues), and the previous fifteen days (13 issues).

The main problem in evaluating the impact of the average is that we have to deal with calls with stochastic striking price[17]. If the price of the shares has lognormally distributed returns, and the share price follows the stochastic equation:

$$d \log (S_t / S) = \mu\, dt + \sigma\, dZ \text{ , where } dZ \text{ is a Wiener Process[18],}$$

then $\log(S_t / S)$ is distributed normally, with mean μt and variance $\sigma^2 t$. We also know[19] that $E(S_t) = S \exp(\mu t + \sigma^2 t /2)$, and that $Var(S_t) = S^2 \exp(2\mu t + \sigma^2 t)[\exp(\sigma^2 t) -1]$.

Assume that the convertible bonds are issued at time -L and can be converted at time 0. For conversion, the shares will not be valued at a discount on the share price at time zero, but at a discount off the average price between day -T and day -1. This means that a convertible bond can be exchanged at time 0 for a number of shares equal to

$$N_{av} = \frac{B(1+r)}{(1-d)S_{av}} ,$$

where $S_{av} = \dfrac{1}{T} \{ S_{-T} + S_{-T+1} + S_{-T+2} + ... + S_{-1} \}.$

The value of a convertible bond at maturity will be $CONV_{av} = N_{av} * S_O$, where S_O is the share price at time zero.

In section 3 we did not consider the average, and we found that a convertible could be exchanged into N shares, where

$$N = \frac{B(1+r)}{(1-d)S_O} , \tag{7}$$

so that its value at conversion was

$$CONV = \frac{B(1+r)}{(1-d)} .$$

It is interesting to explore the difference between the expected value of S_O and the expected value of the average. At time -L, the expected value of S_O is

$$E(S_O) = S_{-L}\, e^{(\mu + s^2/2)L/365} ,$$

and the expected value of the average is

$$E(S_{av}) = S_{-L}\, \frac{1}{T} \left\{ e^{(\mu + \sigma^2/2)(L-T)/365} + e^{(\mu + \sigma^2/2)(L-T+1)/365} + ...+ \right.$$
$$\left. +...+ e^{(\mu + \sigma^2/2)(L-1)/365} \right\}.$$

Some algebra permits us to rewrite this expression as

$$E(S_{av}) = \frac{S_{-L}\,(1 - e^{-(\mu + \sigma^2/2)T/365})\, e^{(\mu + \sigma^2/2)L/365}}{T\,(e^{(\mu + \sigma^2/2)/365} - 1)} \tag{27}$$

We can then compare the expected value of the average with the expected value of the share price at time 0:

$$\frac{E(S_O)}{E(S_{av})} = T\; \frac{e^{(\mu + \sigma^2/2)/365} - 1}{1 - e^{-(\mu + \sigma^2/2)T/365}} \tag{28}$$

It is worth noting that only T, and not L, appears in formula (28). T is measured in days and μ and σ in annual terms. Table 11 provides estimates of equation (28) for different combinations of μ and σ when the average is taken over 30 days. This table shows that valuing the shares at a discount off an average of prices, rather than at a discount off the last day's price, should imply a higher price for convertible bonds. If investors agree that the share price follows an Itô process, then the subscription price of the convertibles should be higher with the average feature than with a discount on a single day's price[20].

7. Conclusion

This paper is a comprehensive study of the convertible bonds used in Spain. Spanish convertible bonds are different from American convertible bonds. First, the conversion price of the shares is not fixed in pesetas, but is defined as a percentage reduction of the average share price over a period of several days before conversion. Second, the conversion option can be exercised on only two or three different dates. Third, the bonds are not callable. Fourth, the first conversion opportunity comes only a few months after the subscription date.

Convertible bonds grew increasingly popular in Spain over the period 1984 to 1988. Issues of convertible bonds accounted for more than $12 billion (1,441 billion pesetas) in that period. In some of these years, issues of convertible bonds accounted for more than three times the issues of new shares.

Several formulas for valuing the different kinds of convertible bonds in Spain are derived using option theory. Optimal conversion policy is also analyzed.

The empirical analysis covers the 248 convertible bonds issued during the period 1984-1990 and offers evidence of a systematic undervaluation of convertible bonds. The undervaluation of the issues in our sample ranges from 0.25% to 170%, with an average undervaluation of 21%. The ex-post analysis also shows that investors made systematically abnormal profits. The comparison between the theoretical value (ex-ante) and the abnormal return (ex-post) shows that our valuation is a good predictor of the performance of Spanish convertible bonds.

The optimal conversion policy for the investor is to convert at the first conversion opportunity, unless the share price declines significantly during the average period or during the conversion period. If the investor wants to lock in the profit, without taking any risk (derived from the illiquidity of the new shares) of future fluctuations of the price of the shares, then, at the conversion date, she must sell short (or sell old shares from her portfolio) a number of shares equal to the number of shares that she is getting in the conversion. The empirical analysis also shows that, on average, investors do not follow the optimal conversion rule.

As shareholders usually have the privilege of subscribing the undervalued bonds before other investors, we would expect that they would subscribe the whole issue. We have seen that this is not the case. Shareholders have subscribed, on average, 46.7% of the issues. This low subscription rate suggests that many shareholders are not aware of the undervaluation of the convertible bonds

A direct consequence of the undervaluation of the convertibles and of the low percentage of the issues subscribed by shareholders is the expropriation effect on the

shareholders who do not subscribe, or subscribe to a smaller proportion of the issue than the proportion of shares that they hold. Wealth (measured as the undervaluation) is taken away from these shareholders and given to the non-shareholders who do subscribe. The expropriation effect in the period 1984 to 1990 adds up to $1.0 billion (125 billion pesetas). This transfer of wealth could be eliminated if shareholders were able to sell the right to buy the convertibles. The opportunity of selling this right would eliminate the wealth expropriation effect.

An explanation for the existence and popularity of convertible bonds in Spain is offered. The funds that companies can raise by issuing stock are very limited because of the role of rights (under Spanish law and practice) as a tax-free distribution to investors. This limitation forces companies to use convertibles to raise equity. □

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[1] Stock dividends are also issued using rights. For example, if a company offers a free new share for every 10 shares owned, an investor (if he is not a shareholder) must purchase 10 rights to get a new share.

[2] The shares are not valued at a discount on the share price of a single day, but rather at a discount on the average of the share price over a number of days before conversion. Fifteen days and one month are the most common periods.

[3] As we will see later, there are several variations to this structure, but this is the most common.

[4] Some of the convertible bonds issued in Spain are valued for conversion at par plus accrued interest (as is this one from Asland), but others are valued only at par.

[5] By law, new shares cannot be issued below par value. Other convertibles also have a maximum conversion price for the shares.

[6] See, for example, Brennan (1980), Brennan and Schwartz (1977), Cox and Rubinstein (1985) and Ingersoll (1977).

[7] The subscription price paid by shareholders was 100% in 240 of the 248 issues in our sample. The subscription price in the remaining 8 issues was 98% and 99%. The subscription price for non-shareholders was 100% in all issues.

[8] We consider an equivalent fixed income instrument to be one that has similar risk and maturity. We have made the comparison using fixed income instruments issued by the same company, or by similar companies in the same industry.

[9] Note that the increase in the share price affects the conversion value of the bonds through calculation of the average and the conversion period.

[10] In order to decide whether to convert at the first conversion opportunity, a rational investor should compare: (a) the conversion value of the bonds, and (b) the value of the bonds considering only the second, third... conversion opportunities. If (a) > (b), bonds should be converted at the first opportunity.

[11] See, for example, Brennan (1980), Brennan and Schwartz (1977), Cox and Rubinstein (1985) and Ingersol (1977).

[12] This method of evaluating contingent claims can also be found, for example, in Black and Cox (1976) and Cox and Rubinstein (1985).

[13] This assumption about the capital structure of the firm is adopted for the sake of convenience; it is not necessary for the validity of the results. If there are more senior securities in the capital structure, we may interpret V as the sum of the market values of the common stock and the convertibles, rather than the value of the entire firm.

[14] I thank Professor Scott Mason for this suggestion.

[15] This assumption about the capital structure of the firm is adopted for the sake of convenience, but is not necessary for the validity of the results. If there are more senior securities in the capital structure, we may interpret V as the sum of the market values of the common stock and the convertibles, rather than the value of the entire firm. The solutions developed in this section would be correct if the sum of the common stock and the convertibles were lognormally distributed; however, they would be inappropriate if the market value of the entire firm were lognormally distributed.

[16] In the prospectus, this constraint appears as: for conversion, the shares will be valued at least at 100%. In Spain, many share prices are still quoted in percentage of the nominal (par) value.

[17] I have to thank Professors Mas-Colell and Mason for their comments.

[18] This is equivalent to saying that log (S_t / S) follows an Itô process.

[19] See J. Cox and Rubinstein, M., *Options Markets*, (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1985), page 204.

[20] Chapter 11 of my dissertation (Fernandez,1989) develops six methods for approximating the impact of the average feature.

Table 1

New issues of corporate (non-convertible) bonds, convertible bonds and shares in Spain
(amounts in billion pesetas)

| Year | Corporate bonds | Convertible bonds | | Shares | | | |
| | | | | Nominal* | | Income | |
		Number	Amount	Number	Amount	Number	Amount
1984	584	*12*	**86**	*67*	169	*36*	95
1985	569	*21*	**79**	*74*	241	*56*	154
1986	493	*37*	**393**	*101*	151	*66*	118
1987	230	*43*	**233**	*131*	243	*74*	398
1988	384	*66*	**651**	*135*	249	*70*	313
1989	383	*49*	**368**	*91*	82	*64*	131
1990	580	*20*	**123**	*67*	103	*39*	182
TOTAL	3,223	*248*	**1,933**	*666*	1,238	*405*	1,391

* Includes stock splits and stock dividends. Changes in nominal and conversions excluded. In Spain, stock splits and stock dividends are done by the rights procedure: an investor willing to subscribe a new share with free subscription price must buy the corresponding number of rights. So, at the time of a stock dividend, shareholders can choose between getting new shares and selling their rights.
Nominal indicates the nominal value of the shares issued. Income indicates the amount raised by companies. The difference between the number of issues is the number of issues with free subscription. For example, in 1988 there were 65 (135-70) issues with free subscription (stock dividends).

Table 2

Issues of convertible bonds in Spain by industry
(Amounts in billion pesetas)

Year	1984	1985	1986	1987	1988	1989	1990	Total
Banks. Volume	41	65	161	135	183	181	33	**799**
Number of issues	*5*	*17*	*15*	*12*	*13*	*14*	*3*	***79***
Electric Co. Volume	4	0	148	11	228	60	30	**482**
Number of issues	*1*	*0*	*7*	*2*	*13*	*3*	*2*	***28***
Telephone. Volume	25	0	55	0	45	0	0	**125**
Number of issues	*1*	*0*	*2*	*0*	*1*	*0*	*0*	***4***
Construction.Volume	9	2	14	16	74	34	21	**170**
Number of issues	*3*	*1*	*3*	*3*	*6*	*5*	*4*	***25***
All other. Volume	7	12	16	72	120	93	49	**369**
Number of issues	*2*	*3*	*10*	*26*	*33*	*27*	*11*	***112***
Total. Volume	86	79	393	233	651	368	123	**1,933**
Number of issues	*12*	*21*	*37*	*43*	*66*	*49*	*20*	***248***
Different Companies	*12*	*10*	*26*	*37*	*51*	*43*	*20*	***107***

Table 3

Most frequent issuers of convertibles (1984-1990)

	Convertibles issued (billion pesetas)	Number of issues
Banco Santander and affiliates	267	30
Fecsa	162	6
Banco Bilbao Vizcaya and affiliates	139	15
Telefonica	125	4
Banesto	105	6
Banco Hispano Americano	89	4
Banco Central	84	3
Hidrola	80	4
Iberduero	75	4
Bankinter	60	3
Fenosa	60	3
Dragados	45	4
Asland	39	4
Hidrocantabrico	29	4
Uralita	27	3
Viesgo	24	2
Banco Exterior and affiliates	24	5
Subtotal	1,435	104
% of total	*74%*	*42%*

Table 4

First conversion opportunity
(Months after subscription)

First conversion opportunity (Months after subscription)	Number of issues
1 - 2	56
2.1 - 4	99
4.1 - 6	45
6.1 - 8	6
8.1 - 10	11
10.1 - 12	15
> 12	16

Table 5

Convertible bonds in Spain
The first conversion opportunity is ex-ante the most advantageous
Convertibles are structured in such a way that the conditions of the first conversion opportunity are the most favorable for bondholders.

Characteristics of convertibles that make the first conversion opportunity the most favorable for investors	Number of issues
- Only one conversion opportunity	31
- Discount at the first conversion opportunity higher than discount at the following conversion opportunities	248
- Maximum* in the first conversion opportunity, but not in the following ones	42
- Maximum in the first conversion opportunity smaller than maximum in the following conversion opportunities	25
- Minimum in the first conversion opportunity smaller than minimum in the following conversion opportunities	14

Issues that have only one of the above characteristics   6

* 88 issues had a maximum share price for conversion (minimum conversion factor) at the first conversion opportunity, 47 at the second and 22 at the third. The maximum at the first conversion opportunity was smaller than the share price at the issue date in 55 of the issues. The average {maximum for conversion / Share price} for these 55 issues was 0.76; the average for the 33 other issues was 1.32.

The minimum at the first conversion opportunity was smaller than the share price at the issue date in 222 of the issues.

Table 6

Undervaluation of convertible bonds
Theoretical value as percentage of par
(To be compared with subscription price of 100% of par for every issue)

Range of theoretical values (% of par)	Number of issues
> 150%	15
140% - 150%	9
130% - 140%	14
120% - 130%	67
110% - 120%	97
100.25% - 110%	46

Average theoretical value 121.6%
Weighted average theoretical value 116.4%.

Table 7

Undervaluation of convertible bonds
Discounted gain* of convertible bonds
(Actual return over fixed income instrument of equivalent risk)

Range of discounted gain	Number of issues
> 50%	31
40% - 50%	7
30% - 40%	20
20% - 30%	31
10% - 20%	47
0% - 10%	46
< 0% (loss)	34

Average discounted gain 22.5%
Weighted average discounted gain 16.8%

*   We define discounted gain as the gain over an equivalent fixed income instrument. We consider a fixed income instrument to be equivalent if it has similar risk and maturity. We have made the comparison using fixed income instruments issued by the same company, or by similar companies in the same industry. For example, consider a company that simultaneously issues convertible bonds and straight bonds, both with subscription price of 100. Conversion occurred one year later, and the conversion value was 132. The annual coupon of the fixed income instruments was 10%, so the value of the straight bond at the conversion date was 110. Therefore, the discounted gain was 20% ([132/110] - 1).

Table 8

Convertible bonds in Spain
Discounted gain by selecting convertibles according to formulas
(1984 - 1990)

Buying convertibles with value*...	...The discounted gain** would have been
150%	55.9%
125 - 150%	37.8%
115 - 125%	20.5%
110 - 115%	16.1%
105 - 110%	7.3%
100 - 105%	1.9%
All	22.5%

*   Value is the result of applying the formulas that we develop in part II to the convertible bonds at the issue date. The value is given as a percentage of the nominal value of the bond. This value can be compared with the subscription price of the convertibles (100% of the nominal value).

** We define discounted gain as the gain over an equivalent fixed income instrument. We consider a fixed income instrument to be equivalent if it has similar risk and maturity. We have made the comparison using fixed income instruments issued by the same company, or by similar companies in the same industry. For example, consider a company that simultaneously issues convertible bonds and straight bonds, both with subscription price of 100. Conversion occurred one year later, and the conversion value was 132. The annual coupon of the fixed income instruments was 10%, so the value of the straight bond at the conversion date was 110. Therefore, the discounted gain was 20% ([132/110] - 1).

Table 9

Bonds converted at the first conversion opportunity

Converted bonds (% of the issue)	% of the issues
90-100%	28.0%
80-90%	14.4%
70-80%	13.6%
60-70%	4.5%
50-60%	8.3%
40-50%	5.3%
30-40%	6.8%
20-30%	5.3%
10-20%	6.8%
0-10%	6.8%
	100.0%

	Converted bonds (% of the issue)	% of the issues	Discounted gain
Issues in which conversion at the first opportunity was <u>not</u> optimal (average)	26.6%	13.2%	-13.6%
Issues in which conversion at the first opportunity was optimal (average)	72.1%	86.8%	31.7%

Average conversion at the first opportunity: 63.84% of the issues.

Table 10

Bonds subscribed by shareholders

Range of subscription by shareholders (% of the issues)	Average value of the bonds (% of face value)
100%	126.4%
80 - 99%	123.1%
60 - 79%	146.0%
40 - 59%	121.9%
20 - 39%	137.2%
2 - 19%	116.7%

Average subscription by shareholders: **52%** of the issues.

Table 11

$E(S_O)/E(S_{average})$ for different values of μ and σ, when the average is calculated over a period of 30 days

			$E(S_O)/E(S_{average})$		
μ	$\sigma=0$	$\sigma=0.2$	$\sigma=0.4$	$\sigma=0.6$	$\sigma=1$
0%	1.00	1.00	1.00	1.01	1.02
1%	1.00	1.00	1.00	1.01	1.02
5%	1.00	1.00	1.01	1.01	1.02
10%	1.00	1.01	1.01	1.01	1.03
15%	1.01	1.01	1.01	1.01	1.03
20%	1.01	1.01	1.01	1.02	1.03
25%	1.01	1.01	1.01	1.02	1.03
30%	1.01	1.01	1.02	1.02	1.03
35%	1.01	1.02	1.02	1.02	1.04
40%	1.02	1.02	1.02	1.02	1.04

$$\frac{E(S_O)}{E(S_{av})} = T \; \frac{e^{(\mu + \sigma^2/2)/365} - 1}{1 - e^{-(\mu + \sigma^2/2)T/365}}$$

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