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EQUIVALENCE OF THE DIFFERENT DISCOUNTED CASH FLOW VALUATION METHODS.

DIFFERENT ALTERNATIVES FOR DETERMINING THE DISCOUNTED VALUE OF TAX SHIELDS AND THEIR IMPLICATIONS FOR THE VALUATION

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Abstract:

This paper addresses the valuation of firms by cash flow discounting.

The first part shows that the four most commonly used discounted cash flow valuation methods (free cash flow discounted at the WACC; cash flow available for equityholders discounted at the required return on the equity flows; capital cash flow discounted at the WACC before taxes; and Adjusted Present Value) always give the same value. This result is logical because all the methods analyse the same reality under the same hypotheses; they only differ in the flows used as the starting point for the valuation.

The disagreements in the various theories on the valuation of the firm arise from the calculation of the discounted value of tax shields (DVTS). The paper shows and analyses 7 different theories on the calculation of the DVTS: Modigliani and Miller (1963), Myers (1974), Miller (1977), Miles and Ezzell (1980), Harris and Pringle (1985), Ruback (1995), Damodaran (1994), and Practitioners method. It is shown that Myers' method (1974) gives inconsistent results.

When analysing the results given by the different theories, it should be remembered that the DVTS is not actually the present value of the tax saving due to the payment of interested discounted at a certain rate but the difference between two present values: the present value of the taxes paid by the firm with no debt minus the present value of the taxes paid by the company with debt. The risk of the taxes paid by the company with no debt is less than the risk of the taxes paid by the company with debt.

The paper also shows the changes that take place in the valuation formulas when the debt's market value does not match its book value.

Section 1 of the paper shows the six most commonly used methods for valuing firms using cash flow discounting: free cash flow discounted at the WACC; cash flow available for equityholders discounted at the required return on the equity flows; capital cash flow discounted at the WACC before taxes; APV; free cash flows adjusted by business risk discounted at the required return on the asset flows; and cash flows available for equity adjusted by business risk discounted at the required return on the asset flows.

Section 2 gives a brief overview of the most significant papers on the valuation of firms using cash flow discounting.

Section 3 gives the main valuation formulas obtained from the most significant papers: Modigliani and Miller (1963), Myers (1974), Miller (1977), Miles and Ezzell (1980), Harris and Pringle (1985), Ruback (1995), Damodaran (1994), and Practitioners method.

Section 4 provides an example to show the valuation differences obtained by the various alternatives discussed in Section 3.

Section 5 analyses in greater detail the cause of the valuation differences obtained by the various authors: the calculation of the discounted value of tax shields (DVTS). It also shows the differences obtained in the valuation using the various theories.

Exhibit 1 lists the abbreviations used in this paper.

Exhibit 2 shows the changes that take place in the valuation formulas when the debt's market value does not match its book value.

Exhibit 3 gives the proof of the equivalence of the valuation formulas.

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1. Discounted cash flow valuation methods

There are four basic discounted cash flow valuation methods (1):

1.A. From the free cash flow and the WACC (Weighted Average Cost of Capital)

Equation [1] shows that the value of the debt (D) plus the value of the equity (E) is the present value of the expected free cash flows (FCF), discounted at the weighted average cost of capital (WACC):

[1] $E_0 + D_0 = PV_0\,[\,WACC_t\,;\,FCF_t] = PV_0\,[\,WACC_t\,;\,FCF_t] = \sum_{t=1}^{\infty} \frac{FCF_t}{\prod_{1}^{t}(1+WACC_t)}$

The definition of WACC (weighted average cost of capital) is [2]:

[2] $WACC_t = [\,E_{t-1}\,Ke_t + D_{t-1}\,Kd_t\,(1-T)]\,/\,[\,E_{t-1} + D_{t-1}\,]$

Ke is the required return on the equity flows, Kd is the required return on the debt flows (cost of debt), and T is the corporate tax rate. $E_{t-1} + D_{t-1}$ are market values (2).

1.B. From the expected cash flow available for equityholders (CFe) and the required return on the firm's equity flows (Ke)

Formula [3] indicates that the value of the equity (E) is the net present value of the expected cash flows available for equityholders (CFe) discounted at the required return on the firm's equity flows (Ke).

[3] $E_0 = PV_0\,[\,Ke_t;\,CFe_t]$

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(1) The formulas given below are valid if the debt's interest rate matches the required return on the debt (Kd), that is, the debt's market value is identical to its book value. See Exhibit 2 and Fernández (1999), pp. 389-391 for the formulas used when this is not so.

(2) In actual fact, the "market values" are the values obtained in the valuation using formula [1]. Consequently, the valuation is an iterative process: the free cash flows are discounted at the WACC to calculate the firm's value (D+E), but the firm's value (D+E) is needed to obtain the WACC.

Formula [4] indicates that the value of the debt (D) is the net present value of the expected cash flows available for the debt (CFd) discounted at the required return on the debt (Kd).

[4] $D_0 = PV_0$ [Kd_t; CFd_t]

The expression that relates the FCF with the CFe is (1):

[5] $CFe_t = FCF_t + \bullet D_t - I_t (1 - T)$

$\bullet D_t$ is the increase in debt. I_t is the interest paid by the firm.

It is obvious that $CFd = I_t - \bullet D_t$

The sum of the values provided by formulas [3] and [4] is identical to the value provided by [1]: (2)

$$E_0 + D_0 = PV_0 \, [\, WACC_t;\, FCF_t] = PV_0 \, [\, Ke_t;\, CFe_t] + PV_0 \, [\, Kd_t;\, CFd_t]$$

1.C. From the capital cash flow (CCF) and the $WACC_{BT}$ (Weighted Average Cost of Capital, before taxes)

The capital cash flows are the cash flows available for all of the firm's stakeholders (debt and equity) and are equivalent to the cash flow available for equityholders (CFe) plus the cash flow available for the debtholders (CFd).

Formula [6] indicates that the value of the debt today (D) plus the value of the equity (E) is equal to the capital cash flow (CCF) discounted at the weighted cost of the debt and equity before taxes ($WACC_{BT}$).

[6] $E_0 + D_0 = PV[WACC_{BT\,t};\, CCF_t]$

The definition of $WACC_{BT}$ is [7]:

[7] $WACC_{BT\,t} = [\, E_{t-1}\, Ke_t + D_{t-1}\, Kd_t] \,/\, [\, E_{t-1} + D_{t-1}\,]$

The expression [7] is obtained by equalling [1] with [6]. $WACC_{BT}$ represents the discount rate that ensures that the value of the firm obtained with both expressions is the same (3): $E_0 + D_0 = PV[WACC_{BT\,t};\, CCF_t] = PV[WACC_t;\, FCF_t\,]$

The expression that relates the CCF with the CFe and with the FCF is [8]:

[8] $CCF_t = CFe_t + CFd_t = CFe_t - \bullet D_t + I_t = FCF_t + I_t\, T$

$\bullet D_t = D_t - D_{t-1}$; $I_t = D_{t-1}\, Kd_t$

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(1) Free cash flow is the cash flow available for equityholders in the hypothetical unlevered firm.
(2) In fact, one way of defining the WACC is: the WACC is the rate at which the FCF must be discounted to obtain the result given by [3] and [4].
(3) Indeed, one way of defining the $WACC_{BT}$ is: the $WACC_{BT}$ is the rate at which the CCF must be discounted to obtain the result given by [3] and [4].

1.D. Adjusted Present Value (APV)

The formula for the Adjusted Present Value (APV) [9] indicates that the value of the debt (D) plus that of the equity (E) of the levered firm is equal to the value of the unlevered firm's equity Vu plus the net present value of the tax saving due to the payment of interest (DVTS):

[9] $E_0 + D_0 = Vu_0 + DVTS_0$

A number of theories exist for calculating the DVTS, which we shall analyse in Section 3 of this paper (1).

Ku is the required return on the firm's unlevered flows (or required return on the asset flows). We calculate Vu using equation [10]:

[10] $Vu_0 = PV_0\ [\ Ku_t;\ FCF_t]$

Combining [9] and [10]:

$$DVTS_0 = E_0 + D_0 - Vu_0 = PV_0\ [\ WACC_t;\ FCF_t] - PV_0\ [\ Ku_t;\ FCF_t]$$

Exhibit 3 shows that the four procedures described always give the same value for the firm, if they are used properly, for any type of forecast (one period, multiperiod, perpetual flows, any flow time structure, constant or variable debt ratios). There is disagreement between various authors regarding calculation of the APV: a number of theories exist about the size of the DVTS, which we will analyse in this paper. The size of the DVTS has implications for the valuation and affects:

– The value of the equity (E) and that of the firm (E+D)

– The relationship between the required return on the asset flows (Ku) and the required return on the equity flows in the levered firm (Ke).

– The relationship between the WACC and the required return on the asset flows (Ku).

We could also talk of a fifth method (from the free cash flow adjusted by business risk), although this is not a new method as such but is derived from the previous ones:

(1) The expressions we will analyse in this paper of the present value of the tax saving due to the payment of interest (DVTS) for a growing perpetuity at the rate g are:

a) Modigliani-Miller (1963):	DVTS = D Ku T / (Ku - g)
b) Myers (1974):	DVTS = D Kd T / (Kd - g)
c) Miller (1977):	DVTS = 0
d) Miles-Ezzell (1980):	DVTS = D Kd T (1 + Ku) / [(1 + Kd) (Ku - g)]
e) Harris-Pringle (1985) and Ruback (1995):	DVTS = D Kd T / (Ku - g)
f) Damodaran (1994):	DVTS = [D Ku T - D (Kd - R_F)(1 - T)] / (Ku - g)
g) Practitioners method:	DVTS = [D Kd T - D (Kd - R_F)] / (Ku - g)

1.E. From the free cash flow adjusted by business risk and the Ku (required return on the asset flows)

Formula [11] indicates that the value of the debt (D) plus that of the equity (E) is the present value of the expected free cash flows adjusted by business risk (FCF\\Ku) that will be generated by the firm, discounted at the required return on the asset flows (Ku):

[11] $E_0 + D_0 = PV_0 [Ku_t ; FCF_t\backslash\backslash Ku]$

The definition of free cash flow adjusted by business risk is:

[12] $FCF_t\backslash\backslash Ku = FCF_t - (E_{t-1} + D_{t-1}) [WACC_t - Ku_t]$

Exhibit 3 shows that equation [12] is obtained from the equivalence of [11] and [1].

Likewise, we could talk of a sixth method (from the cash flow available for equityholders adjusted by business risk), although this is not a new method as such but is derived from the previous ones:

1.F. From the cash flow available for equityholders adjusted by business risk and the Ku (required return on the asset flows)

Formula [13] indicates that the value of the equity (E) is the net present value of the expected cash flows available for equityholders adjusted by business risk (CFe\\Ku) discounted at the required return on the asset flows (Ku):

[13] $E_0 = PV_0 [Ku_t; CFe_t \backslash\backslash Ku]$

The definition of cash flow available for equityholders adjusted by business risk is:

[14] $CFe_t\backslash\backslash Ku = CFe_t - E_{t-1} [Ke_t - Ku_t]$

Exhibit 3 shows that equation [14] is obtained from the equivalence of [13] and [3].

We could also talk of a seventh method; from the capital cash flow adjusted by business risk and the Ku (required return on the asset flows), but the capital cash flow adjusted by business risk is identical to the free cash flow adjusted by business risk (CCF\\Ku = FCF\\Ku). Therefore, this method would be identical to 1.E.

Example. The firm Delta Inc. has forecast its balance sheet and P&L statement for the next few years, which are shown in Table 1. From year 3 onwards, it is forecast that the balance sheet and the P&L statement will grow at an annual rate of 4%.

Table 1. Forecast Balance Sheet and P&L for Delta Inc.

	0	1	2	3	4
WCR (Working Capital Requirements)	400	430	515	550	572.00
Gross Fixed Assets	1,600	1,800	2,300	2,600	2,956.00
- Accumulated depreciation		200	450	720	1,000.80
NFA (Net Fixed Assets)	1,600	1,600	1,850	1,880	1,955.20
TOTAL ASSETS	**2,000**	**2,030**	**2,365**	**2,430**	**2,527.20**
Debt (N)	1,000	1,000	1,100	1,100	1,144.00
Net Worth	1,000	1,030	1,265	1,330	1,383.20
TOTAL	**2,000**	**2,030**	**2,365**	**2,430**	**2,527.20**
P & L					
Margin		300	500	572	603.20
Interests		120	120	132	132.00
PBT (Profit before taxes)		180	380	440	471.20
Taxes		63	133	154	164.92
PAT (Profit After Taxes)		**117**	**247**	**286**	**306.28**

From the balance sheet and P&L forecasts given in Table 1, it is possible to obtain the flows shown in Table 2. Logically, the flows grow at an annual rate of 4% after year 4.

Table 2. Forecast Flows for Delta Inc.

	0	1	2	3	4	5
CFe = Dividends		87.00	12.00	221.00	253.08	263.20
FCF		165.00	-10.00	306.80	294.88	306.68
CFd		120.00	20.00	132.00	88.00	91.52
CCF		207.00	32.00	353.00	341.08	354.72

The asset beta (beta of the shares of the unlevered company) is 1. The risk-free rate is 10%. The cost of debt is 12%. The corporate tax rate is 35%. The market risk premium (P_M) is 8%. Using CAPM, the required return on the asset flows is 18% ($Ku = R_F + ßu\ \ P_M = 10\% + 8\% = 18\%$). With these parameters, the valuation of this firm's equity, using the above formulas, is shown in Table 3. The required return on the equity flows (Ke) is given in the second line of the Table (1). Formula [3] gives the value of the equity by discounting the cash flows available for the equityholders at the required return on the equity flows (Ke) (2). Likewise, formula [4] gives the value of the debt by discounting the cash flows for the debt at the required return on the debt (Kd) (3). Another way to calculate the shares' value is using formula [1]. The present value of the free cash flows discounted at the WACC (formula [2]) gives us the value of the firm, which is the value of the debt plus the value of the shares. By subtracting the value of the debt from this quantity, we obtain the value of the shares. Another way to calculate the shares' value is using formula [6]. The present value of the capital cash flows discounted at the $WACC_{BT}$ (formula [7]) gives us the value of the firm, which is the value of the debt plus the value of the shares. By subtracting the value of the

(1) The required return on the equity flows (Ke) has been calculated in accordance with Modigliani-Miller's theory, which we will see further on.

(2) The relationship beteween the shares' value for two consecutive years is: $E_t = E_{t-1}\ (1+Ke_t) - CFe_t$.

(3) The market value of the debt (D) is equal to its book value (N) in Table 1 because we consider that the required return on the debt (Kd) is equal to the cost of debt (r). Exhibit 2 shows what happens when this is not so.

debt from this quantity, we obtain the value of the shares. The fourth method for calculating the value of the equity is from the Adjusted Present Value, formula [9]. The value of the firm is the sum of the value of the unlevered firm (formula [10]) plus the present value of the tax saving due to the debt (DVTS) (1).

Finally, at the end of Table 3, the cash flow available for equityholders and the free cash flow adjusted by business risk (CFe\\Ku and FCF\\Ku) are calculated using formulas [14] and [12]. Formula [13] gives the value of the shares by discounting the cash flows available for equityholders adjusted by business risk at the required return on the asset flows (Ku). Another method for calculating the value of the shares is using formula [11]. The present value of the free cash flows adjusted by business risk discounted at the required return on the asset flows (Ku) gives us the value of the firm, which is the value of the debt plus the value of the shares. By subtracting the value of the debt from this quantity, we obtain the value of the shares.

The example of Table 3 shows that the result obtained with all six valuations is the same. Thus, the value of the shares today is 1,043. As we have already remarked, these valuations have been carried out in accordance with Modigliani-Miller's theory. The valuations obtained using other theories presented in the following sections of this paper are discussed in Section 4.3.

Table 3. Valuation of Delta Inc.

Equation		**0**	**1**	**2**	**3**	**4**
	Ku	18.00%	18.00%	18.00%	18.00%	18.00%
	Ke	21.74%	21.30%	21.01%	20.86%	20.86%
[1]	E+D = PV(WACC;FCF)	2,043.41	2,183.23	2,523.21	2,601.29	2,705.34
[2]	WACC	14.917%	15.114%	15.253%	15.336%	15.336%
	[1] - D = E	**1,043**	**1,183**	**1,423**	**1,501**	**1,561**
[3]	**E = PV(Ke;CFe)**	**1,043**	**1,183**	**1,423**	**1,501**	**1,561**
[4]	D = PV(CFd;Kd)	1,000	1,000	1,100	1,100	1,144
[6]	D+E=PV($WACC_{BT}$;CCF)	2,043.41	2,183.23	2,523.21	2,601.29	2,705.34
[7]	$WACC_{BT}$	16.972%	17.038%	17.084%	17.112%	17.112%
	[6] - D = E	**1,043**	**1,183**	**1,423**	**1,501**	**1,561**
	DVTS =PV(Ku;D T Ku)	442.09	458.66	478.22	495.00	514.80
[10]	Vu = PV (Ku;FCF)	1,601	1,725	2,045	2,106	2,191
[9]	Vu + DVTS	2,043.41	2,183.23	2,523.21	2,601.29	2,705.34
	[9] - D = E	**1,043**	**1,183**	**1,423**	**1,501**	**1,561**
[11]	D+E = PV(Ku;FCF\\Ku)	2,043.41	2,183.23	2,523.21	2,601.29	2,705.34
[12]	FCF\\Ku		228.00	53.00	376.10	364.18
	[11] - D = E	**1,043**	**1,183**	**1,423**	**1,501**	**1,561**
[14]	CFe\\Ku		48.00	-27.00	178.10	210.18
[13]	**E = PV(Ku;CFe\\Ku)**	**1,043**	**1,183**	**1,423**	**1,501**	**1,561**

(1) As the required return on the equity flows (Ke) has been calculated in accordance with Modigliani-Miller's theory, we must also calculate the DVTS in accordance with Modigliani-Miller's theory, as follows: DVTS = PV(Ku; D T Ku).

2. A brief overview of the most significant papers on the discounted cash flow valuation of firms

There is a considerable body of literature on the discounted cash flow valuation of firms. We will discuss here the most salient papers, concentrating particularly on those which proposed different expressions for the present value of the tax saving due to the payment of interest (DVTS).

Modigliani and Miller (1958) and (1963), studied the effect of leverage on the firm's value. Their proposition 1 (Modigliani-Miller (1958), formula (3)) is that, in the absence of taxes, the firm's value is independent of its debt, i.e.,

$E_0 + D_0 = Vu$, if $T = 0$.

Their second proposition (Modigliani-Miller (1958), formula (8)) is that, in the absence of taxes, the required return on equity flows (Ke) increases at a rate that is directly proportional to the debt (the D/E ratio) at market value:

$Ke = Ku + (D/E) (Ku - Kd)$, if $T = 0$.

In the presence of taxes, their second proposition (Modigliani-Miller (1963), formula (12.c)) is: $Ke = Ku + D (1-T) (Ku - Kd) / E$

In the presence of taxes, their first proposition, in the case of a perpetuity, is transformed into (Modigliani-Miller (1963), formula (3)): $E_0 + D_0 = Vu + D T$

DT is the increase in value due to the leverage (DVTS).

Modigliani-Miller (1963) present several valuation formulas:

Their formula (31.c) is: $WACC = Ku [1 - T D / (E+D)]$.

Their formula (11.c) is: $WACC_{BT} = Ku - D T (Ku - Kd) / (E+D)$.

They also state in their formula (33.c) that, in an investment that can be financed totally by debt, the required return on the debt must be equal to the required return on the asset flows: if $D / (D+E) = 100\%$, $Kd = Ku$.

However, in Modigliani-Miller's last equation (1963), they propose calculating the firm's target financing structure [D / (D+E)] using book values for D and E, instead of market values.

Myers (1974) introduced the APV (adjusted present value). According to Myers, the value of the levered firm is equal to the value of the firm with no debt (Vu) plus the present value of the tax saving due to the payment of interest (DVTS). Myers proposes calculating the DVTS in the following manner:

$DVTS = PV [Kd; D T Kd]$

The argument is that the risk of the tax saving arising from the use of debt is the same as the risk of the debt.

And the firm's value is:

$$APV = E + D = Vu + DVTS = PV [Ku; FCF] + PV [Kd; D T Kd]$$

Arditti and Levy (1977) suggest that the firm's value be calculated by discounting the Capital Cash Flows (cash flow available for equityholders plus cash flow for the debt) instead of the Free Cash Flow. The Capital Cash Flows (CCF) should be discounted at the $WACC_{BT}$ (WACC before taxes). It is easy to show that:

$$E_0 + D_0 = PV_0 [WACC_t ; FCF_t] = PV_0 [WACC_{BTt} ; CCF_t]$$

where the $WACC_{BT}$ is their formula (2):

$$WACC_{BTt} = [E_{t-1} Ke_t + D_{t-1} Kd_t] / [E_{t-1} + D_{t-1}]$$

Arditti and Levy's paper (1977) contains one important error: to calculate the $WACC_{BT}$, they calculate the debt ratio (D / [E+D]) and the equity ratio (E / [E+D]) using book values, instead of market values. This is why they state (p. 28) that the value of the firm obtained by discounting the FCF is different from that obtained by discounting the CCF.

Miller (1977) argues that while there is an optimal debt structure for firms as a whole, this does not exist for firms individually. Miller says that debt does not add any value to the firm due to the clientele effect. Therefore, according to Miller, E+D = Vu.

Miles and Ezzell (1980) argue that the APV and the WACC give different values: "unless the borrowing and, consequently, Ke are exogenous (they do not depend on the firm's value at any given time), the traditional WACC is not appropriate for valuing firms". According to them, a firm that wishes to keep a constant D/E ratio must be valued in a different manner from the firm that has a preset level of debt. Specifically, formula [20] of their paper says that for a firm with a fixed debt target [D/(D+E)], the free cash flow (FCF) must be discounted at the rate:

$$WACC = Ku - [D / (E+D)] [Kd T (1+Ku) / (1+Kd)]$$

Their expression of Ke is their formula [22]:

$$Ke = Ku + D (Ku - Kd) [1 + Kd (1-T)] / [(1+Kd) E]$$

Miles and Ezzell (1985) show in their formula (27) that the relationship between the levered beta and the asset beta (assuming that the debt is risk-free and that the debt beta is zero) is

$$\beta_L = \beta u + D \beta u [1- T R_F / (1+ R_F)] / E$$

Chambers, Harris and Pringle (1982) compare four discounted cash flow valuation methods: updating the cash flow available for equityholders (CFe) at the rate Ke (required return on the equity flows); updating the Free Cash Flow (FCF) at the WACC (weighted cost of debt and equity); updating the Capital Cash Flow (CCF) at the $WACC_{BT}$ (weighted cost of debt and equity before taxes); and Myers' Adjusted Present Value (APV). They say that the first three methods give the same value if the debt level is constant but different values if it is not constant. They also say that the APV only gives the same result as the other three methods in two cases: in firms with only one period and in perpetuities with no growth. The reason for this discrepancy is that they calculate the debt ratio (D/[D+E]) using book values instead of market values.

In their formula (3), **Harris and Pringle (1985)** propose that $WACC_{BT}$ = Ku and, therefore, their expression for the WACC is:

WACC = Ku - D Kd T / (D + E)

They also propose that the present value of the tax saving due to the payment of interest (DVTS) should be calculated by discounting the tax saving due to the debt (Kd T D_{t-1}) at the rate Ku:

DVTS = PV [Ku; D Kd T]

Ruback (1995) assumes in his formula (2.6) that $\beta_L = \beta_U$ (D+E)/E $-\beta_D$ D/E. One can see immediately that with this assumption: $WACC_{BT}$ = Ku. He arrives at formulas that are equivalent to those of Harris-Pringle (1985).

Lewellen and Emery (1986) show that, in the case of a perpetuity with no growth, the value of the levered firm, according to Miles-Ezzell's formulas (1980), is (see their formula (7)):

E + D = Vu + (D Kd T + D Kd T / Ku) / (1 + Kd)

They also show that, in the case of a perpetuity with no growth, the value of the levered firm according to Modigliani-Miller's (1963) and Myers' (1974) formulas matches and is (see their formula (5)): E + D = Vu + T D.

Further on, they show that for a growing perpetuity at a rate g, the value of the levered firm is:

a) Modigliani-Miller (1963): $E_0 + D_0$ = Vu + D T Ku / (Ku - g)
b) Myers' APV (1974): $E_0 + D_0$ = Vu + D T Kd / (Kd - g)
c) Miles-Ezzell (1980): $E_0 + D_0$ = Vu + D T Kd (1+Ku) / [(Ku - g) (1+Kd)]

Taggart (1991) gives a good overview of valuation formulas without personal taxes and with personal taxes. He proposes that Miles-Ezzell's formulas (1980) should be used when the firm adjusts its debt target once a year and Harris-Pringle's formulas (1985) should be used when the firm adjusts its debt target continuously.

Damodaran (1994) argues (1) that if all the business risk is borne by the equity, then the formula relating the levered beta (ßL) with the asset beta (ßU) is: $\beta L = \beta u + (D/E)\ \beta u\ (1 - T)$. Note that this expression arises from the relationship between Modigliani-Miller's levered beta, asset beta and debt beta (2), assuming that the debt beta is zero.

Another way of calculating the levered beta with respect to the asset beta is the following: $\beta L = \beta U + (D/E)$. We will call this method the **Practitioners method**, because it is

(1) p. 31. This expression for the levered beta appears in many books and is often used by consultants and investment banks.
(2) The relationship between Modigliani-Miller's levered beta, asset beta and debt beta is: $\beta L = \beta U + (D/E)\ (\beta U - \beta d)\ (1 - T)$.

often used by consultants and investment banks (1). It is obvious that according to this formula, given the same value for ßu, a higher β_L is obtained than according to Modigliani-Miller and Damodaran (1994).

3. Main valuation formulas

This section gives the main valuation formulas obtained from the papers mentioned in the previous section. Some formulas appear explicitly in the papers. Others are derived from those appearing in the papers.

3.1. Different expressions of the present value of the tax saving due to the payment of interest (DVTS)

The expressions of the value created by debt, i.e., the present value of the tax saving due to the payment of interest (DVTS), are:

[15] Modigliani-Miller (1963): DVTS = PV[Ku ; D Ku T]
[16] Myers (1974): DVTS = PV[Kd ; D Kd T]
[17] Miller (1977): DVTS = 0
[18] Miles-Ezzell (1980): DVTS = PV[Ku ; D Kd T] (1 + Ku) / (1 + Kd)
[19] Harris-Pringle (1985) and Ruback (1995): DVTS = PV[Ku ; D Kd T]
[20] Damodaran (1994): DVTS = PV[Ku ; D Ku T - D (Kd - R_F)(1 - T)]
[21] Practitioners method: DVTS = PV[Ku ; D Ku T - D (Kd - R_F)]

3.2. Relationship of the required return on the equity flows (Ke) with the required return on the asset flows (Ku)

The different equations that link the required return on the equity flows (Ke) with the required return on the asset flows (Ku), according to the above-mentioned theories, are:

a) Modigliani-Miller (1963): Ke = Ku + D (1-T) (Ku - Kd) / E
b) Myers (1974) (2): Ke = Ku + (D -DVTS) (Ku - Kd) / E
c) Miller (1977): Ke = Ku + D [Ku - Kd (1-T)] / E
d) Miles-Ezzell (1980): Ke = Ku + D (Ku - Kd) [1 + Kd (1-T)] / [(1+Kd) E]
e) Harris-Pringle (1985) and Ruback (1995): Ke = Ku + D (Ku - Kd) / E
f) Damodaran (1994): Ke = Ku + D (1-T) (Ku - R_F) / E
g) Practitioners method: Ke = Ku + D (Ku - R_F) / E

3.3. Different expressions of the WACC and the $WACC_{BT}$

The expressions of the WACC (Weighted Average Cost of Capital) corresponding to the values of Ke given in the previous section are:

(1) Two of the many places where it appears are: Ruback (1995), p. 5; and Ruback (1989), p. 2.
(2) According to Myers, for a growing perpetuity at the rate g: DVTS = D T Kd / (Kd-g), and Ke = Ku + D [Kd (1-T) - g] (Ku - Kd) / [E (Kd - g)].

a) Modigliani-Miller (1963): $WACC = Ku [1 - T D / (E+D)]$
b) Myers (1974) (1): $WACC = Ku - [DVTS (Ku-Kd) + D Kd T] / (E+D)$
c) Miller (1977): $WACC = Ku$
d) Miles-Ezzell (1980): $WACC = Ku - [D Kd T (1+Ku) / (1+Kd)] / (E+D)$
e) Harris-Pringle (1985) and Ruback (1995): $WACC = Ku - D Kd T / (E+D)$
f) Damodaran (1994): $WACC = Ku - D [TKu - (1-T) (Kd - R_F)] / (E+D)$
g) Practitioners method: $WACC = Ku - D [R_F - Kd (1-T)] / (E+D)$

The expressions of the $WACC_{BT}$ (Weighted Average Cost of Capital, Before Taxes) corresponding to the values of Ke given in the previous section are:

a) Modigliani-Miller (1963): $WACC_{BT} = Ku - D T (Ku - Kd) / (E+D)$
b) Myers (1974) (2): $WACC_{BT} = Ku - DVTS (Ku - Kd) / (E+D)$
c) Miller (1977): $WACC_{BT} = Ku + D T Kd / (E+D)$
d) Miles-Ezzell (1980): $WACC_{BT} = Ku - D T Kd (Ku - Kd) / [(E+D) (1+Kd)]$
e) Harris-Pringle (1985) and Ruback (1995): $WACC_{BT} = Ku$
f) Damodaran (1994): $WACC_{BT} = Ku + D [(Kd - R_F) - T(Ku - R_F)] / (E+D)$
g) Practitioners method: $WACC_{BT} = Ku + D (Kd - R_F) / (E+D)$

3.4. Different expressions of the beta of the levered firm

The different expressions of the beta of the levered firm (b_L) with respect to the beta of the unlevered firm (bu), according to the various papers, are:

a) Modigliani-Miller (1963): $\beta_L = \beta u + D (1-T) (\beta u - \beta d) / E$
b) Myers (1974) (3): $\beta_L = \beta u + (D - DVTS) (\beta u - \beta d) / E$
c) Miller (1977): $\beta_L = \beta u (D+E) / E - D [\beta d(1- T) - T R_F / P_M] / E$
d) Miles-Ezzell (1980): $\beta_L = \beta u + D (\beta u - \beta d) [1- T Kd / (1+Kd)] / E$
e) Harris-Pringle (1985) and Ruback (1995): $\beta_L = \beta u + D (\beta u - \beta d) / E$
f) Damodaran (1994): $\beta_L = \beta u + D (1-T) \beta u / E$
g) Practitioners method: $\beta_L = \beta u + D \beta u / E$

3.5. Different expressions of the cash flow available for equityholders adjusted by businesss risk

The different expressions of CFe\\Ku (cash flow available for equityholders adjusted by business risk), according to the various papers, are:

a) Modigliani-Miller (1963): $CFe - D (Ku - Kd) (1-T)$
b) Myers (1974) $CFe - (Vu-E) (Ku - Kd)$

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(1) For a growing perpetuity at the rate g, $WACC = Ku - [D Kd T (Ku-g) / (Kd-g)] / (E+D)$.
(2) For a growing perpetuity at the rate g, $WACC_{BT} = Ku - D T Kd (Ku - Kd) / [(E+D) (Kd - g)]$.
(3) For a growing perpetuity at the rate g, $\beta_L = \beta u + D [Kd (1-T) - g] (\beta u - \beta d) / [E (Kd - g)]$.

c)	Miller (1977):	CFe - D [Ku - Kd (1-T)]
d)	Miles-Ezzell (1980):	CFe - D (Ku - Kd) [1+ Kd(1-T)] / (1+Kd)
e)	Harris-Pringle (1985) and Ruback (1995):	CFe - D (Ku - Kd)
f)	Damodaran (1994):	CFe - D (Ku - R_F) (1-T)
g)	Practitioners method:	CFe - D (Ku - R_F)

3.6. Different expressions of the free cash flow adjusted by business risk

The different expressions of FCF\\Ku (free cash flow adjusted by business risk), according to the various papers, are:

a)	Modigliani-Miller (1963):	FCF + D Ku T
b)	Myers (1974)	FCF +T D Kd +DVTS (Ku -Kd)
c)	Miller (1977):	FCF
d)	Miles-Ezzell (1980):	FCF +T D Kd (1+Ku) / (1 + Kd)
e)	Harris-Pringle (1985) and Ruback (1995):	FCF +T D Kd
f)	Damodaran (1994):	FCF + D Ku T - D (Kd - R_F) (1-T))
g)	Practitioners method:	FCF + D [R_F -Kd (1-T)]

Using the expressions of the value of the firm based on the free cash flow adjusted by business risk or the cash flow available for equityholders adjusted by business risk, one can readily find the differences in the valuation of a firm with a preset debt level:

E Modigliani-Miller **- E** Damodaran **=**	PV[Ku; D(Kd- R_F) (1-T)]
E Modigliani-Miller **- E** Practitioners **=**	PV[Ku; D(Kd- R_F) (1-T)+DT(Ku- R_F)]
E Modigliani-Miller **- E** Harris-Pringle **=**	PV[Ku; DT(Ku- Kd)]
E Modigliani-Miller **- E** Myers **=**	PV[Ku; (DT-DVTS Myers)(Ku- Kd)]
E Harris-Pringle **- E** Practitioners **=**	PV[Ku; D(Kd- R_F)]

The table below shows the main formulas for each of the theories.

Table 4. Discounted cash flow valutaion. Main equations
Market value of debt = Book value of debt

	Modigliani-Miller	Damodaran	Practitioners	Harris-Pringle (1985) Ruback (1995)	Myers (1974)	Miles-Ezzell (1980)
Ke	$Ke = Ku + \frac{D(1-T)}{E}(Ku - Kd)$	$Ke = Ku + \frac{D(1-T)}{E}(Ku - R_F)$	$Ke = Ku + \frac{D}{E}(Ku - R_F)$	$Ke = Ku + \frac{D}{E}(Ku - Kd)$	$Ke = Ku + \frac{Vu - E}{E}(Ku - Kd)$	$Ke = Ku + \frac{D}{E}(Ku - Kd)\left[1 - \frac{T\,Kd}{1+Kd}\right]$
β_L	$\beta_L = \beta u + \frac{D(1-T)}{E}(\beta u - \beta d)$	$\beta_L = \beta u + \frac{D(1-T)}{E}\beta u$	$\beta_L = \beta u + \frac{D}{E}\beta u$	$\beta_L = \beta u + \frac{D}{E}(\beta u - \beta d)$	$\beta_L = \beta u + \frac{Vu - E}{E}(\beta u - \beta d)$	$\beta_L = \beta u + \frac{D}{E}(\beta u - \beta d)\left[1 - \frac{T\,Kd}{1+Kd}\right]$
WACC	$Ku\left(1 - \frac{DT}{E+D}\right)$	$Ku\left(1 - \frac{DT}{E+D}\right) + D\frac{(Kd - R_F)(1-T)}{(E+D)}$	$Ku - D\frac{R_F - Kd(1-T)}{(E+D)}$	$Ku - D\frac{Kd\,T}{(E+D)}$	$Ku - \frac{DVTS(Ku - Kd) + D\,Kd\,T}{(E+D)}$	$Ku - D\frac{Kd\,T}{(E+D)}\frac{1+Ku}{1+Kd}$
$WACC_{BT}$	$Ku - \frac{DT(Ku - Kd)}{(E+D)}$	$Ku - D\frac{T(Ku - R_F) - (Kd - R_F)}{(E+D)}$	$Ku + \frac{D(Kd - R_F)}{(E+D)}$	Ku	$Ku - \frac{DVTS(Ku - Kd)}{(E+D)}$	$Ku - \frac{D\,T\,Kd}{(E+D)}\frac{(Ku - Kd)}{(1+Kd)}$
DVTS	VA[Ku; DTKu]	VA[Ku; DTKu - D (Kd- R_F) (1-T)]	VA[Ku; T D Kd - D(Kd- R_F)]	VA[Ku; T D Kd]	VA[Kd; T D Kd]	VA[Ku; T D Kd] (1+Ku)/(1+Kd_0)
CFe_t\\Ku	$CFe_t - D_{t-1}(Ku_t - Kd_t)(1-T)$	$CFe_t - D_{t-1}(Ku - R_F)(1-T)$	$CFe_t - D_{t-1}(Ku_t - R_{Ft})$	$CFe_t - D_{t-1}(Ku_t - Kd_t)$	$CFe_t - (Vu-E)(Ku_t - Kd_t)$	$CFe - D(Ku - Kd)\frac{1 + Kd(1-T)}{(1+Kd)}$
FCF_t\\Ku	$FCF_t + D_{t-1} Ku_t T$	$FCF_t + D_{t-1} Ku\,T - D_{t-1}(Kd - R_F)(1-T)$	$FCF_t + D_{t-1}[R_{Ft} - Kd_t(1-T)]$	$FCF_t + T D_{t-1} Kd_t$	FCF_t +T D Kd +DVTS(Ku -Kd)	FCF+T D Kd(1+Ku) / (1 + Kd)

Common equations to all methods:

WACC y $WACC_{BT}$: $WACC_t = \frac{E_{t-1} Ke_t + D_{t-1} Kd_t (1-T)}{(E_{t-1} + D_{t-1})}$ $\quad WACC_{BTt} = \frac{E_{t-1} Ke_t + D_{t-1} Kd_t}{(E_{t-1} + D_{t-1})}$ $\quad WACC_{BTt} - WACC_t = \frac{D_{t-1} Kd_t T}{(E_{t-1} + D_{t-1})}$

Flows relationships: $CFe_t = FCF_t + (D_t - D_{t-1}) - D_{t-1}Kd_t(1-T)$ $\quad CCF_t = FCF_t + D_{t-1}Kd_t T$ $\quad CCF_t = CFe_t - (D_t - D_{t-1}) + D_{t-1} Kd_t$

Flows\\Ku **CFe\\Ku** = $CFe_t - E_{t-1}(Ke_t - Ku_t)$ $\quad$ **FCF\\Ku** = $FCF_t - (E_{t-1} + D_{t-1})(WACC_t - Ku_t)$ = **CCF\\Ku** = $CCF_t - (E_{t-1} + D_{t-1})(WACC_{BTt} - Ku_t)$

4. Valuation differences according to the most significant papers

4.1. Growing perpetuity with a preset debt level of 30%

Applying the above formulas to a firm with FCF_1 = 100, Ku = 10%, Kd = 7%, [D/(D+E)] = 30%, T = 35%, R_F = 5%, and g = 5%, the values shown in Table 5 are obtained. The value of the unlevered firm (Vu) is 2,000 in all cases. Note that, according to Myers, Ke < Ku = 10%, which does not make sense. According to Myers, DVTS > D when g > Kd (1-T), in the example, when g > 4.55%. As we shall see further on, if g > 4.55%, Ke < Ku, which does not make sense.

Table 5. Example of a valuation of a firm
FCF_1 = 100, Ku = 10%, Kd = 7%, [D/(D+E)] = 30%, T = 35%, R_F = 5%, and g = 5%

	Modigliani-Miller	**Myers**	**Miller**	**Miles-Ezzell**	**Harris-Pringle**	**Damodaran**	**Practitioners**
WACC	8.950%	8.163%	10.000%	9.244%	9.265%	9.340%	9.865%
Ke	10.836%	9.711%	12.336%	11.256%	11.286%	11.393%	12.143%
$WACC_{BT}$	9.685%	8.898%	10.735%	9.979%	10.000%	10.075%	10.600%
E+D	**2,531.65**	**3,162.06**	**2,000.00**	**2,356.05**	**2,344.67**	**2,304.15**	**2,055.50**
Vu	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00
E	**1,772.15**	**2,213.44**	**1,400.00**	**1,649.23**	**1,641.27**	**1,612.90**	**1,438.85**
D	759.49	948.62	600.00	706.81	703.40	691.24	616.65
DVTS	531.65	1.162.06	0.00	356.05	344.67	304.15	55.50
CFe	103.42	104.27	102.70	103.18	103.17	103.11	102.77

In the pages that follow, we will discuss how the valuation's basic parameters change with respect to the growth g.

The firm's value according to Modigliani-Miller and that according to Myers are the same for a perpetuity (when there is no growth). With growth, the firm's value according to Myers is greater than the firm's value according to Modigliani-Miller. All the other theories give values below Modigliani-Miller.

According to all the theories, the firm's WACC is independent of growth, except according to Myers. According to Myers, the WACC decreases when growth increases and is below growth (and, therefore, the firm's value is infinite) when g > Kd [D(1-T)+E]/(E+D); in the example, when g > 6.265%.

According to all the theories, the firm's $WACC_{BT}$ is independent of growth, except according to Myers. According to Myers, the $WACC_{BT}$ decreases when growth increases.

The DVTS according to Modigliani-Miller and that according to Myers are the same for a perpetuity (when there is no growth). With growth, the value of the DVTS according to Myers is greater than the DVTS according to Modigliani-Miller. All the other theories give values below Modigliani-Miller.

According to all the theories, the required return on the equity flows (Ke) is independent of growth, except according to Myers. According to Myers, Ke decreases when growth increases and is less than Ku when g > Kd(1-T), in the example, for g > 4.55%. Obviously, this does not make sense.

Table 6 shows the DVTS with respect to the debt level according to the different theories. The value of the DVTS according to Myers is greater than the DVTS according to Modigliani-Miller. All the other theories give values below Modigliani-Miller. It can be verified that the DVTS according to Myers becomes infinite for a debt level [D/(D+E)] = (Kd-g) / (T Kd), in our example, 81.63%.

Table 6. Present value of the tax saving due to the payment of interest (DVTS) with respect to the debt level (g=5%). [D/(D+E)] = 30%

D/(D+E)	Modigliani-Miller	Myers	Miller	Miles-Ezzell	Harris-Pringle	Damodaran	Practitioners
0%	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10%	150.5	279.2	0.0	106.1	103.0	92.1	18.2
20%	325.6	649.0	0.0	224.1	217.3	193.0	36.7
30%	531.6	1,162.1	0.0	356.0	344.7	304.1	55.5
40%	777.8	1,921.6	0.0	504.7	487.6	427.2	74.7
50%	1,076.9	3,161.3	0.0	673.3	649.0	564.1	94.2
60%	1,448.3	5,547.2	0.0	866.3	832.9	717.4	114.2
70%	1,921.6	12,035.1	0.0	1,089.4	1,044.1	890.2	134.5
80%	2,545.5	98,000.0	0.0	1,350.0	1,289.5	1,086.4	155.2
90%	3,405.4	8	0.0	1,658.7	1,577.8	1,311.3	176.3
100%	4,666.7	8	0.0	2,030.1	1,921.6	1,571.4	197.8

According to all the theories, the firm's WACC decreases with the debt level, except according to Miller, where it is held constant. However, only according to Myers, the WACC becomes less than growth: this happens for debt levels greater than [D/(D+E)] > (Kd-g) / (T Kd), in our example, 81.63%.

According to Modigliani-Miller, Myers and Miles-Ezzell, the firm's $WACC_{BT}$ decreases with the debt level; according to Harris-Pringle, it is constant (equal to Ku); and it increases with the debt level according to Miller, Damodaran and Practitioners method.

4.2. Growing perpetuity with preset debt

Table 7 is identical to Table 5. The only difference is that the initial debt level is set at 759.49 (instead of the debt ratio at 30%). Applying the above formulas, the values shown in Table 14 are obtained. The value of the unlevered firm (Vu) is 2,000 in all cases. Note that, according to Myers, Ke < Ku = 10%, which does not make much sense.

Table 7. Example of the valuation of a firm
FCF_1 = 100, Ku = 10%, Kd = 7%, D = 759,49, T = 35%, R_F = 5%, and g = 5%

	Modigliani-Miller	Myers	Miller	Miles-Ezzell	Harris-Pringle	Damodaran	Practitioners
WACC	8.950%	8.413%	10.000%	9.197%	9.216%	9.284%	9.835%
Ke	10.836%	9.764%	13.337%	11.372%	11.413%	11.568%	12.901%
$WACC_{BT}$	9.685%	9.048%	10.930%	9.978%	10.000%	10.081%	10.734%
E+D	2,531.65	2,930.38	2,000.00	2,382.59	2,372.15	2,334.18	2,068.35
Vu	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00
E	1,772.15	2,170.89	1,240.51	1,623.09	1,612.66	1,574.68	1,308.86
D	759.49	759.49	759.49	759.49	759.49	759.49	759.49
DVTS	531.65	930.38	0.00	382.59	372.15	334.18	68.35
CFe	103.42	103.42	103.42	103.42	103.42	103.42	103.42
D/(D+E)	30.00%	25.92%	37.97%	31.88%	32.02%	32.54%	36.72%

With growth, the value of the firm according to Myers is greater than the value of the firm according to Modigliani-Miller. All the other theories give values below Modigliani-Miller. According to Myers, the firm's value is infinite for growths equal or greater than g = Kd [D(1-T)+E]/(E+D); in the example, when g = 6.265%.

According to Myers, the WACC is equal to growth when g = Kd [D(1-T)+E]/(E+D); in the example, when g = 7%.

According to Modigliani-Miller and Myers, the DVTS is equal for a perpetuity (when there is no growth). With growth, the value of the DVTS according to Myers is greater than the DVTS according to Modigliani-Miller. All the other theories give values below Modigliani-Miller.

According to all the theories, the required return on the equity flows increases with growth. According to Myers, Ke is less than Ku (10%) when g > Kd(1-T), in the example, for g = 4.55%. This obviously does not make sense.

4.3. Differences in the valuation of the firm shown in Table 1

Table 3 gives the valuation of the firm shown in Table 1 according to Modigliani-Miller. This section contains the most significant results of the valuation of the firm Delta Inc. according to Myers (1974), Harris-Pringle (1985), Ruback (1995), Damodaran (1994), and Practitioners method.

Table 8. Valuation of Delta Inc. according to Myers (1974)

	0	***1***	***2***	***3***	***4***
DVTS = PV(Kd;D Kd T)	514.92	534.71	556.88	577.50	600.60
Ke	20.61%	20.22%	20.17%	19.98%	19.98%
E	**1,116.25**	**1,259.28**	**1,501.86**	**1,583.79**	**1,647.14**
WACC	14.555%	14.721%	14.940%	14.987%	14.987%
E + D	2,116.25	2,259.28	2,601.86	2,683.79	2,791.14
$WACC_{BT}$	16.540%	16.580%	16.716%	16.709%	16.709%

Table 9. Valuation of Delta Inc. according to Harris-Pringle (1985), and Ruback (1995)

	0	**1**	**2**	**3**	**4**
DVTS	294.72	305.77	318.81	330.00	343.20
Ke	24.70%	23.82%	23.22%	22.94%	22.94%
E	**896.05**	**1,030.34**	**1,263.80**	**1,336.29**	**1,389.74**
WACC	15.785%	15.931%	16.046%	16.104%	16.104%
E + D	1,896.05	2,030.34	2,363.80	2,436.29	2,533.74
$WACC_{BT}$	18.000%	18.000%	18.000%	18.000%	18.000%

Table 10. Valuation of Delta Inc. according to Damodaran (1994)

	0	1	2	3	4
DVTS	350.86	364.02	379.54	392.86	408.57
Ke	23.46%	22.78%	22.32%	22.09%	22.09%
E	**952.19**	**1,088.58**	**1,324.53**	**1,399.14**	**1,455.11**
WACC	15.439%	15.606%	15.732%	15.799%	15.799%
D + E	1,952.19	2,088.58	2,424.53	2,499.14	2,599.11
$WACC'_{BT}$	17.590%	17.617%	17.637%	17.648%	17.648%

Table 11. Valuation of Delta Inc. according to the Practitioners method

	0	1	2	3	4
DVTS	154.38	160.17	167.00	172.86	179.77
Ke	28.59%	27.04%	25.91%	25.46%	25.46%
E	**755.71**	**884.73**	**1,111.99**	**1,179.14**	**1,226.31**
WACC	16.747%	16.833%	16.906%	16.938%	16.938%
D + E	1,755.71	1,884.73	2,211.99	2,279.14	2,370.31
$WACC_{BT}$	19.139%	19.061%	18.995%	18.965%	18.965%

5. The underlying problem: the discounted value of the tax shield (DVTS)

Gu is the present value of the taxes paid by the unlevered company and GL is the present value of the taxes paid by the levered company. K_{IU} is the required return on the taxes paid by the unlevered company, and K_{IL} is the required return on the taxes paid by the unlevered company. TAXu are the taxes paid by the unlevered company, and TAXL are the taxes paid by the levered company.

FCFo is the free cash flow of the company without taxes, and Kuo is the required return on the asset flows of the company without taxes. Vuo is the value of the unlevered company without taxes. Consequently: Vuo = PV [Kuo; FCFo].

Assuming no bankruptcy costs nor any leverage cost, the total value of the levered company (value of the shares, Vu, plus present value of taxes, Gu) is identical to the total value of the levered company (value of the shares, E, plus value of debt, D, plus present value of taxes, GL):

[22] $Vuo_t = Vu_t + Gu_t = E_t + D_t + GL_t$

Equation [23] shows the equality of the sum of the flows of the unlevered firm and the sum of the flows of the levered firm.

[23] $FCFo = TAXu + FCF = TAX_L + CFe + CFd = TAX_L + CCF$

[22] and [23] imply [24]:

[24] $Vuo\ Kuo = Vu\ Ku + Gu\ K_{IU} = E\ Ke + D\ Kd + G_L\ K_{IL} = (D+E)\ WACC_{BT} + G_L\ K_{IL}$

The so-called "net present value of the tax saving due to the payment of interest" (DVTS) is:

[25] $DVTS_t = Gu_t - G_{Lt}$

DVTS is the difference between two present values of two flows (the present value of the unlevered taxes and the present value of the levered taxes) that have, obviously, different risk. For a growing perpetuity:

[26] $DVTS_t = Gu_t - G_{Lt} = [TAXu_{t+1} / (K_{IU} - g)] - [TAX_{Lt+1} / (K_{IL} - g)]$

The relationship between TAXu and TAX_L is:

[27] $TAXu_{t+1} - TAX_{Lt+1} = D_t\ Kd\ T$

Logically, the taxes of the unlevered firm have less risk than the taxes of the levered firm and therefore:

[28] $\mathbf{K_{IU} < K_{IL}}$

A logical limitation of the value of the DVTS is that

[29] **DVTS < Gu**

In the unlevered firm, the relationship between taxes and profit before taxes (PBT) is:

[30] $TAXu = T\ PBTu$

The relationship between the free cash flow and the taxes of the unlevered firm is

[31] $TAXu = T\ (FCF + H) / (1-T)$

H is a parameter which includes the increase in Working Capital Requirements and the increase in Net Fixed Assets (NFA):

[32] $H = ?WCR + ?NFA$

In the levered firm, the relationship between the cash flow available for equityholders and the taxes of the unlevered firm is

[33] $TAX_L = T\ (CFe + H - ?D) / (1-T)$

Constant perpetuity (no growth)

In a perpetuity with no growth (1): TAXu = T FCF / (1-T); and

TAXL = T CFe / (1-T). TAXu and FCF have the same risk, which implies K_{IU} = Ku.

TAXL and CFe have the same risk, which implies K_{IL} = Ke. Equation [26] is:

DVTS = Gu - GL = (TAXu / Ku) - (TAXL / Ke) = [T / (1-T)] [(FCF/Ku) - (CFe/Ke)]. So, in a perpetuity with no growth: GL = E T / (1-T); Gu = Vu T / (1-T).

DVTS = [T / (1-T)] [Vu- E]. From equation [9]: Vu- E = D - DVTS. Substituting, we obtain:

DVTS = DT

For a constant perpetuity with no growth, equations [15] to [21] are:

[15]	Modigliani-Miller (1963):	DVTS = DT
[16]	Myers (1974):	DVTS = DT
[17]	Miller (1977):	DVTS = 0
[18]	Miles-Ezzell (1980):	DVTS = D Kd T (1 + Ku) / [(1 + Kd) Ku]
[19]	Harris-Pringle (1985) and Ruback (1995):	DVTS = D Kd T / Ku
[20]	Damodaran (1994):	DVTS = D T - D (Kd - R_F)(1 - T) / Ku
[21]	Practitioners method:	DVTS = D T - D (Kd - R_F) / Ku

Only Modigliani-Miller and Myers give the result **DVTS = DT.**

Growing perpetuity

In a growing perpetuity at a rate g:

H = gWCR + gNFA = 0.
Vuo = FCFo / (Kuo -g) = TAXu / (K_{IU} -g) + FCF / (Ku - g)
Vuo = FCFo / (Kuo -g) = TAXL / (K_{IL} -g) + FCF / (WACC - g)

In a growing perpetuity K_{IU} < Ku. As Kuo is the average cost of K_{IU} and Ku, it must also be met that K_{IU} < Kuo. Therefore:

K_{IU} < Kuo < Ku.

In a growing perpetuity, cash flow for debt during the first year is D (Kd - g). For the financing to make sense, it is necessary that:

g < Kd

(1) In a perpetuity with no growth, H = 0. Fixed asset purchases are identical to depreciation, and working capital requirements are constant.

According to Myers,

$DVTS_t = Gu_t - GL_t = [TAXu_{t+1} / (Kd - g)] - [TAXL_{t+1} / (Kd - g)] = D\ Kd\ T / (Kd - g)$, i.e.: $K_{IU} = K_{IL} = Kd$, which does not make sense.

We will now analyse three interesting cases for perpetuities with growth, depending on the size of the parameter H = g (WCR + NFA). We will analyse the cases in which H = 0; H = gD; H=D Kd.

H = 0

When (1) H = 0, TAXu = T FCF / (1-T). Therefore, the risk of FCF and TAXu is the same. Therefore, **Ku =** $\mathbf{K_{IU}}$ and **Gu = T Vu / (1-T).** We immediately see that:

Vuo = Vu / (1-T) = Vu + Gu

From the formula: FCFo = TAXu + FCF, it is apparent that if the risk of FCF and TAXu is identical, the risk of FCFo must also be identical. Therefore:

Kuo = Ku = $\mathbf{K_{IU}}$

Vuo Kuo = Vu Ku+ Gu K_{IU} = Vu Ku / (1-T)

It does not make sense to talk of K_{IL} because if H = 0, the debt must be zero because there is nothing to finance: WCR + NFA = 0.

H = gD

In a growing perpetuity (2), when H = gD, TAXL = T CFe / (1-T). Therefore, the risk of CFe and TAXL is the same. Therefore, **Ke =** $\mathbf{K_{IL}}$ and **GL = T E / (1-T).** We immediately see that:

Vuo = E / (1-T) +D = E + D + GL

The formula Vuo Kuo = E Ke + D Kd + GL K_{IL} is transformed into:

[E / (1-T) +D] Kuo = E Ke / (1-T) + D Kd. Multiplying by (1-T):

[E +D (1-T)] Kuo = E Ke + D Kd(1-T).

According to Modigliani-Miller: E Ke = E Ku + D (1-T) (Ku - Kd). So **Kuo = Ku**

According to Myers: E Ke = E Ku + (Vu-E) (Ku - Kd). Substituting, we obtain:

Kuo = Kd + Vu (Ku-Kd) / [E+D(1-T)]

(1) The condition H = 0, means that WCR + NFA = 0. Therefore D=0 because it is not necessary to finance anything with debt.
(2) The condition H = gD is identical to WCR + NFA = D, which means that all of the firm’s financing is debt.

On the other hand:

Vuo = **E / (1-T) + D** = Vu + Gu

According to *Modigliani-Miller*, the equation: Vuo Kuo = Vu Ku + Gu K_{IU} is transformed into: [E + D (1-T)] Ku / (1-T) = Vu Ku + Gu K_{IU} .

As Gu = Vuo - Vu = E / (1-T) + D - Vu:

[E + D (1-T) - Vu (1-T)] Ku = [E + D (1-T) - Vu (1-T)] K_{IU}

So **K_{IU} = Ku** according to Modigliani-Miller.

According to Myers, K_{IU} = [E Ke /(1-T) + D Kd - Vu Ku] / [E /(1-T) + D - Vu]

Note that H = gD means that WCR + NFA = D, i.e., all of the firm's financing is carried out with debt: the debt ratio at book value is 100%. This situation is different from that studied by Ruback (1986): there, the debt ratio is 100% at market value, all the cash flow generated by the firm corresponds to debt, and the required return on the asset flows (Ku) is equal to the cost of the debt, which must be the risk-free rate R_F.

Table 12 shows the tax risk of a firm with FCF = 100, and with H = gD (assets are fully financed with debt), with respect to growth.

Table 12. Tax risk. H = gD (D = WCR + NFA)
T = 35%. Ku = 10%. Kd = 7%. D = 759.49. FCF of year 1 = 100

g	M-M **Kuo ***	M-M **Ke=K_{IL}**	M-M **Gu**	M-M **G_L**	M-M **DVTS**	Myers **Kuo**	Myers **K_{IU}**	Myers **Ke=K_{IL}**	Myers **Gu**	Myers **G_L**	Myers **DVTS**
0.0%	10.00%	12.93%	538.46	272.64	265.82	10.0%	10.00%	12.93%	538.46	272.64	265.82
1.0%	10.00%	12.29%	643.73	348.37	295.36	9.88%	9.69%	12.04%	666.45	356.32	310.13
2.0%	10.00%	11.80%	775.32	443.04	332.28	9.76%	9.41%	11.35%	836.66	464.51	372.15
3.0%	10.00%	11.41%	944.50	564.75	379.75	9.63%	9.14%	10.78%	1075.95	610.76	465.19
4.0%	10.00%	11.10%	1170.07	727.04	443.04	9.47%	8.87%	10.27%	1442.71	822.46	620.25
5.0%	10.00%	10.84%	1485.88	954.24	531.65	9.25%	8.54%	9.76%	2099.32	1168.94	930.38
6.0%	10.00%	10.62%	1959.59	1295.03	664.56	8.83%	8.06%	9.08%	3799.90	1939.14	1860.76
7.0%	10.00%	10.43%	2749.11	1863.03	886.08	7.00%	7.00%	7.00%	######	######	#####

*** Kuo = K_{IU} in M-M**

The main differences between the two theories are that when growth increases,

	according to M-M	according to Myers
Kuo	constant	decreases
K_{IU}	constant	decreases
Ke = K_{IL}	always > Ku	if g > 4.55%, Ke < Ku

In a growing perpetuity: $TAX_L = T (CFe + H - gD) / (1-T)$. Therefore, the tax risk of the levered company (K_{IL}) will be less than the risk of the cash flow available for equityholders (Ke) if H > gD, (1).

$\mathbf{K_{IL} < Ke}$ if H > gD

H = D Kd

In a growing perpetuity, when H = D Kd, $TAX_L = T\ CCF / (1-T)$. Therefore, the risk of CCF and TAX_L is the same, and $\mathbf{WACC_{BT} = K_{IL}}$. Furthermore,

$\mathbf{G_L = T (D+E) / (1-T)}$. As $FCFo = TAX_L + CCF$, the risk of FCFo must be equal to that of TAX_L and CCF. It is immediately seen that:

FCFo = CCF / (1-T) $= TAX_L + CCF$ and that
$Vuo = (D+E) / (1-T) = G_L + (D+E)$

The equation: $Vuo\ Kuo = G_L K_{IL} + (D+E) WACC_{BT}$ is transformed into:

$(D+E) Kuo / (1-T) = (D+E) WACC_{BT} / (1-T)$

Therefore: $\mathbf{Kuo = WACC_{BT}}$

It must also be met that: $Vuo\ Kuo = G_U K_{IU} + Vu\ Ku$, i.e.:

$(D+E) WACC_{BT} / (1-T) = [(D+E) / (1-T) - Vu] K_{IU} + Vu\ Ku$. Multiplying by (1-T):

$(D+E) WACC_{BT} - Vu\ Ku (1-T) = [(D+E) - Vu (1-T)] K_{IU}$

According to Modigliani-Miller: $(D+E) WACC_{BT} = (D+E) Ku - DT (Ku - Kd)$. Substituting:

$[(D+E) - Vu (1-T)] Ku - DT (Ku - Kd). = [(D+E) - Vu (1-T)] K_{IU}$

So: $\mathbf{K_{IU} = Ku - DT (Ku - Kd) / [(D+E) - Vu (1-T)]}$

According to Myers (1974), $WACC_{BT} = Ku$. So $\mathbf{K_{IU} = Ku > K_{IL}}$; which does not make sense. $(D+E) WACC_{BT} = (D+E) Ku - DVTS (Ku - Kd)$. Substituting:

$[(D+E) - Vu (1-T)] Ku - DVTS (Ku - Kd) = [(D+E) - Vu (1-T)] K_{IU}$

So: $\mathbf{K_{IU} = Ku - DVTS (Ku - Kd) / [(D+E) - Vu (1-T)]}$

Table 13 shows the tax risk of a firm with FCF = 100, and with H = D Kd, with respect to growth.

(1) The condition H > gD, is identical to WCR + NFA > D, which is always met in a growing perpetuity.

Table 13. Tax risk. H = D Kd
T = 35%. Ku = 10%. Kd = 7%. D = 759.49. FCF of year 1 = 100

g	M-M Kuo(*)	M-M K_{IU}	M-M Gu	M-M G_L	M-M DVTS	M-M Ke	Myers Kuo*	Myers K_{IU}	Myers Gu	Myers G_L	Myers DVTS	Myers Ke
0.0%	9.37%	8.71%	947.42	681.60	265.82	12.93%	9.37%	8.71%	947.42	681.60	265.82	12.93%
1.0%	9.43%	8.83%	1052.69	757.33	295.36	12.29%	9.35%	8.67%	1075.41	765.28	310.13	12.04%
2.0%	9.50%	8.96%	1184.27	852.00	332.28	11.80%	9.31%	8.62%	1245.62	873.47	372.15	11.35%
3.0%	9.56%	9.09%	1353.46	973.71	379.75	11.41%	9.26%	8.55%	1484.91	1019.72	465.19	10.78%
4.0%	9.62%	9.22%	1579.03	1135.99	443.04	11.10%	9.19%	8.45%	1851.67	1231.42	620.25	10.27%
5.0%	9.68%	9.35%	1894.84	1363.19	531.65	10.84%	9.05%	8.29%	2508.28	1577.90	930.38	9.76%
6.0%	9.75%	9.48%	2368.55	1703.99	664.56	10.62%	8.72%	7.96%	4208.86	2348.10	1860.76	9.08%
7.0%	9.81%	9.61%	3158.07	2271.99	886.08	10.43%	7.00%	7.00%	#######	#######	#######	7.00%

(*) Kuo = $WACC_{BT}$ = K_{IL}

The main differences between the two theories are that when growth increases,

	according to M-M	according to Myers
Kuo = $WACC_{BT}$ = K_{IL}	increases	decreases
K_{IU}	increases	decreases

In a growing perpetuity: TAXL = T (CCF + H - D Kd) / (1-T). Therefore, the tax risk of the levered firm (K_{IL}) will be less than the risk of the capital cash flow ($WACC_{BT}$) if H > D Kd, and greater in the opposite situation. As: FCFo / (Kuo -g) = TAXL / (K_{IL} -g) + CCF / ($WACC_{BT}$ - g), and furthermore,

FCFo = TAXL + CCF

K_{IL} < Kuo < $WACC_{BT}$ if H > D Kd

References

Arditti, F. D. and H. Levy (1977), " The Weighted Average Cost of Capital as a Cutoff Rate: A Critical Examination of the Classical Textbook Weighted Average", *Financial Management* (Fall), pp. 24-34.

Chambers, D.R., R.S. Harris, and J.J. Pringle (1982), "Treatment of Financing Mix Analyzing Investment Opportunities", *Financial Management* (Summer), pp. 24-41.

Damodaran, A (1994), "Damodaran on Valuation", John Wiley and Sons, New York.

Fernández, Pablo (1999), "Valoración de Empresas", Ediciones Gestión 2.000.

Hamada, R.S. (1972), "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stock", *Journal of Finance* 27 (May), pp. 435-452.

Harris, R.S. and J.J. Pringle (1985), "Risk-Adjusted Discount Rates Extensions form the Average-Risk Case", *Journal of Financial Research* (Fall), pp. 237-244.

Lewellen, W.G. and D.R. Emery (1986), "Corporate Debt Management and the Value of the Firm", *Journal of Financial Quantitative Analysis* (December), pp. 415-426.

Miles, J.A. and J.R. Ezzell (1980), "The Weighted Average Cost of Capital: Perfect Capital Markets and Project Life: A Clarification," *Journal of Financial and Quantitative Analysis* (September), pp. 719-730.

Miles, J.A. and J.R. Ezzell (1985), "Reformulating Tax Shield Valuation: A Note," *Journal of Finance,* Vol. XL, 5 (December), pp. 1485-1492.

Miller, M.H. (1977), "Debt and Taxes", *Journal of Finance* (May), pp. 261-276

Modigliani, F. and M. Miller, (1958), "The Cost of Capital, Corporation Finance and the Theory of Investment", *American Economic Review* 48, pp. 261-297.

Modigliani, F. and M.H. Miller (1963), "Corporate Income Taxes and the Cost of Capital: A Correction", *American Economic Review* (June), pp. 433-443.

Myers, S.C. (1974), "Interactions of Corporate Financing and Investment Decisions - Implications for Capital Budgeting", *Journal of Finance* (March), pp. 1-25.

Ruback, Richard S. (1986), "Calculating the Market Value of Risk-Free Cash Flows", *Journal of Financial Economics* (March), pp. 323-339.

Ruback, Richard S. (1989), "RJR Nabisco. Teaching note", Harvard Business School, 5-289-057.

Ruback, Richard S. (1995), "A Note on Capital Cash Flow Valuation", Harvard Business School, 9-295-069.

Taggart, R.A. Jr. (1991), "Consistent Valuation and Cost of Capital. Expressions With Corporate and Personal Taxes" *Financial Management* (Autumn), pp. 8-20. □

Exhibit 1

EQUIVALENCE OF THE DIFFERENT DISCOUNTED CASH FLOW VALUATION METHODS.
DIFFERENT ALTERNATIVES FOR DETERMINING THE DISCOUNTED
VALUE OF TAX SHIELDS AND THEIR IMPLICATIONS FOR THE VALUATION

Abbreviations used in the document

APV	Adjusted Present Value.
βd	Beta, or systematic risk, of the debt.
βu	Beta, or systematic risk, of the unlevered firm.
β_L	Beta, or systematic risk, of the levered firm.
CAPM	Capital Asset Pricing Model.
CCF	Capital Cash Flow. Cash flow available for all stakeholders: equity and debt.
CFd	Cash flow for the debt.
CFe	Cash flow available for equityholders.
D_t	Market value of debt.
DVTS	Discounted Value of Tax Shields.
E_t	Market value of equity.
FCF	Free Cash Flow. Cash flow available for equityholders in the hypothetical unlevered firm.
FCFo	Free cash flow without taxes. Cash flow available for equityholders in the hypothetical unlevered firm without taxes.
g	Growth
G_L	Present value of taxes of the levered company.
Gu	Present value of taxes of the unlevered company.
TAX_L	Taxes of the levered company.
TAXu	Taxes of the unlevered company.
Kd	Required return on the debt.
Ke	Cost of equity. Required return on the equity flows.
K_{IU}	Required return on the taxes of the unlevered company.
K_{IL}	Required return on the taxes of the levered company.
Ku	Required return on the asset flows. Required return on the firm's unlevered flows. Unlevered, or all-equity, cost of capital.
Kuo	Required return on the firm's unlevered flows without taxes.
NFA	Net Fixed Assets.
NOPAT	Net Operating Profit After Taxes.
PAT	Profit After Taxes.
PBT	Profit Before Taxes.
P_M	Market risk premium = $E(R_M) - R_F$
R_F	Risk-free rate.
T	Corporate tax rate.
PV	Present value.
Vu	Market value of the unlevered firm.
Vuo	Value of the unlevered company without taxes.
WACC	Weighted Average Cost of Capital. Debt and equity ratios at market value.
$WACC_{BT}$	Weighted Average Cost of Capital Before Taxes.
WCR	Working Capital Requirements.

Exhibit 2

EQUIVALENCE OF THE DIFFERENT DISCOUNTED CASH FLOW VALUATION METHODS. DIFFERENT ALTERNATIVES FOR DETERMINING THE DISCOUNTED VALUE OF TAX SHIELDS AND THEIR IMPLICATIONS FOR THE VALUATION

Discounted cash flow valuation methods when the debt's market value (D) is not equal to the debt's book value (N)

This exhibit contains the basic expressions of the four basic discounted cash flow valuation methods, when the debt's market value (D) does not match its book value (N).

If the debt's market value (D) does not match its book value (N), it is because the required return on the debt (Kd) is different from its cost (r).

The interest paid in a period t is: $I_t = N_{t-1} \ r_t$. The increase in debt in a period t is: $\bullet N_t = N_t - N_{t-1}$.

Therefore, the cash flow for the debt in a period t is: $CFd = I_t - \bullet N_t = N_{t-1} \ r_t - (N_t - N_{t-1})$.

Therefore, the value of the debt at t=0 is:

$$D_o = PV\ [Kd_t \ ;\ N_{t-1} \ r_t - (N_t - N_{t-1})]$$

It is easy to show that the relationship between the debt's market value (D) and its book value (N) is:

$$D_t - D_{t-1} = N_t - N_{t-1} + D_{t-1} \ Kd_t - N_{t-1} \, r_t$$

Therefore: $\bullet D_t = \bullet N_t + D_{t-1} \, Kd_t - N_{t-1} \, r_t$

The fact that the debt's market value (D) does not match its book value (N) affects several formulas in section 1 of this paper. Formulas [1], [3], [4], [6], [7], [9] and [10] continue to be valid but the other formulas change.

The expression of the WACC in this case is:

[2*] $\mathbf{WACC = (E\ Ke + D\ Kd - N\ r\ T) / (E + D)}$

The expression relating the CFe with the FCF is:

[5*] $CFe_t = FCF_t + (N_t - N_{t-1}) - N_{t-1} \ r_t \,(1 - T)$

The expression relating the CCF with the CFe and with the FCF is:

[8*] $CCF_t = CFe_t + CFd_t = CFe_t - (N_t - N_{t-1}) + N_{t-1} \, r_t = FCF_t + N_{t-1} \, r_t \, T$

Exhibit 2 (continued)

Different expressions for the present value of the tax saving due to the payment of interest (DVTS)

The expressions of the value created by debt, i.e., the present value of the tax saving due to the payment of interest (DVTS), when the debt's market value (D) does not match its book value (N) are:

a) Modigliani-Miller (1963): $DVTS = PV [Ku_t ;\ D_{t-1} Ku_t T - (N_{t-1} r_t - D_{t-1} Kd_t)T]$
b) Myers (1974): $DVTS = PV [Kd_t ;\ N_{t-1} r_t T]$
c) Miller (1977): $DVTS = 0$
d) Miles-Ezzell (1980): $DVTS = PV [Ku_t ; N_{t-1} r_t T] (1 + Ku) / (1+ Kd)$
e) Harris-Pringle (1985) and Ruback (1995): $DVTS = PV [Ku_t ; N_{t-1} r_t T]$
f) Damodaran (1994): $DVTS = PV [Ku_t ; N_{t-1} r_t T + D_{t-1} T (Ku_t - R_{Ft}) - D_{t-1} (Kd_t - R_{Ft})]$
g) Practitioners method: $DVTS = PV [Ku_t ; N_{t-1} r_t T - D_{t-1} (Kd_t - R_{Ft})]$

The expressions of the WACC (Weighted Average Cost of Capital) when the debt's market value (D) does not match its book value (N) are:

a) Modigliani-Miller (1963): $WACC = Ku - [NrT + DT (Ku - Kd)] / (E+D)$
b) Myers (1974): $WACC = Ku - [NrT + DVTS (Ku - Kd)] / (E+D)$
c) Miller (1977): $WACC = Ku$
d) Miles-Ezzell (1980): $WACC = Ku - [(NrT) (1+Ku)] / [(E+D) (1+Kd)]$
e) Harris-Pringle (1985) and Ruback (1995): $WACC = Ku - (NrT) / (E+D)$
f) Damodaran (1994): $WACC = Ku - [NrT + DT (Ku - R_F) - D (Kd - R_F)] / (E+D)$
g) Practitioners method: $WACC = Ku - [NrT - D (Kd - R_F)] / (E+D)$

Exhibit 3

EQUIVALENCE OF THE DIFFERENT DISCOUNTED CASH FLOW VALUATION METHODS.
DIFFERENT ALTERNATIVES FOR DETERMINING THE DISCOUNTED
VALUE OF TAX SHIELDS AND THEIR IMPLICATIONS FOR THE VALUATION

Proof of the equivalence of the valuation formulas

Equations [3] and [4] can be transformed into [3*] and [4*]:

[3*] $E_t = E_{t-1} (1+Ke_t) - CFe_t$

[4*] $D_t = D_{t-1} (1+Kd_t) - CFd_t$

Equation [1] can be transformed into [1*]:

[1*] $D_t + E_t = (D_{t-1} + E_{t-1}) (1+WACC_t) - FCF_t$

Equation [1*] gives the same value as [3*] + [4*] if:

$(D_{t-1} + E_{t-1}) (1+WACC_t) - FCF_t = E_{t-1} (1+Ke_t) - CFe_t + D_{t-1} (1+Kd_t) - CFd_t$

The relationship between the flows is: $FCF_t = CFd_t + CFe_t - T D_{t-1} Kd_t$

Substituting: $(D_{t-1} + E_{t-1}) (1+WACC_t) = E_{t-1} (1+Ke_t) + D_{t-1} (1+Kd_t) - T D_{t-1} Kd_t$

Thus, we can conclude that equation [1*] gives the same value as [3*] + [4*] (i.e., Equation [1] gives the same value as [3] + [4]) if:

$(D_{t-1} + E_{t-1}) WACC_t = E_{t-1} Ke_t + D_{t-1} Kd_t (1- T)$, i.e., equation [2].

Equation [6] can be transformed into [6*]:

[6*] $D_t + E_t = (D_{t-1} + E_{t-1}) (1+WACC_{BTt}) - CCF_t$

Equation [6*] gives the same value as [3*] + [4*] if:

$(D_{t-1} + E_{t-1}) (1+WACC_{BTt}) - CCF_t = E_{t-1} (1+Ke_t) - CFe_t + D_{t-1} (1+Kd_t) - CFd_t$

The relationship between the flows is: $CCF_t = CFd_t + Cfe_t$

Substituting: $(D_{t-1} + E_{t-1}) (1+WACC_{BTt}) = E_{t-1} (1+Ke_t) + D_{t-1} (1+Kd_t)$

Thus, we can conclude that equation [6*] gives the same value as [3*] + [4*] (i.e., Equation [6] gives the same value as [3] + [4]) if:

$(D_{t-1} + E_{t-1}) WACC_t = E_{t-1} Ke_t + D_{t-1} Kd_t$, i.e., equation [7].

Equation [9] gives the same value as [3] + [4] if:

$E_0 + D_0 = PV_0 [Ke_t; CFe_t] + PV_0 [Kd_t; CFd_t] = PV_0 [Ku_t; FCF_t] + DVTS_0$

Exhibit 3 (continued)

Equation [11] can be transformed into [11*]:

[11*] $D_t + E_t = (D_{t-1} + E_{t-1})\ (1+Ku_t) - FCF_t\backslash\backslash Ku$

Equation [11*] gives the same value as [1*] if:

$(D_{t-1} + E_{t-1})\ (1+Ku_t) - FCF_t\backslash\backslash Ku = (D_{t-1} + E_{t-1})\ (1+WACC_t) - FCF_t$

which means: [12] $FCF_t\backslash\backslash Ku = FCF_t - (E_{t-1} + D_{t-1})\ (WACC_t - Ku_t)$

Equation [13] can be transformed into [13*]:

[13*] $E_t = E_{t-1}\ (1+Ku_t) - CFe_t\backslash\backslash Ku$

Equation [13*] gives the same value as [3*] if:

$E_{t-1}\ (1+Ku_t) - CFe_t\backslash\backslash Ku = E_{t-1}\ (1+Ke_t) - CFe_t$

which means: [14] $CFe_t\backslash\backslash Ku = CFe_t - E_{t-1}\ (Ke_t - Ku_t)$