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THE VALUE OF TAX SHIELDS IS **NOT** EQUAL TO THE PRESENT VALUE OF TAX SHIELDS

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THE VALUE OF TAX SHIELDS IS NOT EQUAL TO THE PRESENT VALUE OF TAX SHIELDS

Abstract

We show that the value of tax shields is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company. This is the difference between the present values of two separate cash flows, each with its own risk. For constant growth companies, we prove that the value of tax shields in a world with no leverage cost is the present value of the debt, times the tax rate, times the required return to the unlevered equity, discounted at the unlevered cost of equity. Please note, however, that this does not mean that the appropriate discount for tax shields is the unlevered cost of equity, since the amount being discounted is higher than the tax shield (it is multiplied by the unlevered cost of equity and not the cost of debt). Rather, this result arises as the difference of two present values.

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THE VALUE OF TAX SHIELDS IS NOT EQUAL TO THE PRESENT VALUE OF TAX SHIELDS (1)

1. Introduction

There is no consensus in the existing literature regarding the correct way to compute the value of the tax shield. This lack of consensus arises mainly from the fact that most authors think of calculating the value of the tax shield as what is the appropriate present value of the tax savings due to debt. For instance, Myers (1974) proposes to discount the tax savings due to the payment of interest at the cost of debt, while Harris and Pringle (1985) propose discounting these tax savings at the cost of capital for the unlevered firm. Reflecting this perception, Copeland et al. (2000) claim that "the finance literature does not provide a clear answer about which discount rate for the tax benefit of interest is theoretically correct." We show that the consistent way to estimate the value of these tax savings is not by thinking of them as the present value of a cash flow, but as the difference between the present values of two different cash flows: flows to the unlevered firm and flows to the levered firm.

We show that the value of tax shields for perpetuities is equal to the tax rate times the value of debt. For constant growth companies, we prove that the value of tax shields in a world with no leverage cost is the present value of the debt, times the tax rate, times the required return to the unlevered equity, discounted at the unlevered cost of equity (Ku). Please note, however, that this does not mean that the appropriate discount for tax shields is the unlevered cost of equity, since the amount being discounted is higher than the tax shield (it is multiplied by the unlevered cost of equity and not the cost of debt). Rather, this result arises as the difference of two present values.

One could argue that, in practice, working out the present value of tax shields themselves would not necessarily be wrong, provided the appropriate discount rate was used (reflecting the riskiness of the tax shields). The problem here is that it is hard to evaluate the riskiness of tax shields because it is the difference between the present values of two flows (the taxes paid by the unlevered company and those paid by the levered company) with different risk. In this context, evaluating the riskiness of tax shields for a firm is conceptually as hard as evaluating the riskiness of the difference between Microsoft's expected equity cash flow and GE's expected equity cash flow. We may evaluate the riskiness of the expected equity cash flows of each company, but it is difficult (and I think it hardly makes any sense) to try to evaluate the riskiness of the difference between the two expected equity cash flows.

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Perhaps the most important thing to take into account when reading this paper is that the term “discounted value of tax shields” in itself is senseless. The value of tax shields is the difference of two present values of two separate cash flows, each with its own risk.

The rest of the paper is organized as follows. In Section 2, we follow a new method to prove that the value of tax shields for perpetuities in a world without cost of leverage is equal to the tax rate times the value of debt (DT). In Section 3, we derive the relation between the required return to assets and the required return to equity for perpetuities in a world without leverage costs. The corresponding relation between the beta of the levered equity, the beta of the unlevered equity and the beta of debt is also derived. In Section 4, we prove that the value of tax shields in a world with no leverage cost is the present value of the debt times the tax rate times the required return to the unlevered equity, discounted at the unlevered cost of equity. In Section 5, we revise and analyze the existing financial literature on the value of tax shields. We show how most of these existing approaches result in either inconsistent results regarding the cost of equity to the levered and unlevered firm or in estimated tax shields that are too low. Section 6 contains a numerical example for a hypothetical firm that helps to clarify the previous sections. Finally, section 7 concludes.

2. The value of tax shields for perpetuities in a world without leverage cost is DT.

We show in this section that the value of tax shields for a perpetuity is DT, where D indicates the value of the debt today, and T is the tax rate.

The present value of debt (D) plus that of the equity (E) of the levered company is equal to the value of the equity of the unlevered company (Vu) plus the value of tax shields due to interest payments (VTS).

$$E + D = Vu + VTS \qquad [1]$$

VTS (value of tax shields) is the term used in the literature to define the increase in the company’s value as a result of the tax saving obtained by the payment of interest. In this definition the total value of the levered company is equal to the total value of the unlevered company. If leverage costs do not exist, then equation [2] could be stated in terms of:

$$Vu + Gu = E + D + GL \qquad [2]$$

where Vu is the value of the unlevered company, Gu is the present value of the taxes paid by the unlevered company, E is the equity value and D is the debt value. GL is the present value of the taxes paid by the levered company. Equation [2] means that the total value of the unlevered company (left hand side of the equation) is equal to the total value of the levered company (right hand side of the equation). Total value is the enterprise value (often called value of the firm) plus the present value of taxes (2). Please note that equation [2] assumes that expected free cash flows are independent of leverage.

(2) When leverage costs do exist, the total value of the levered company is lower than the total value of the unlevered company. A world with leverage cost is characterized by the following relation:
$Vu + Gu = E + D + GL + \text{Leverage Cost} > E + D + GL$ (3)
Leverage Cost is the reduction in the company’s value due to the use of debt.

From [1] and [2], it is clear that the VTS (value of tax shields) is:

$$VTS = Gu - G_L \qquad [4]$$

We should note that the value of tax shields (VTS) is not the present value (PV) of tax shields. It is the difference between two PVs of two flows with different risk: the PV of the taxes paid in the unlevered company (Gu) and the PV of the taxes paid in the levered company (G_L).

It is quite easy to prove that, in a perpetuity, the profit after tax of the levered company (PAT_L) is equal to the equity cash flow (ECF):

$$PAT_L = ECF \qquad [5]$$

This happens in perpetuities, as the allowed depreciation deduction is exactly equal to cash actually spent to replace capital equipment that wears out.

In a perpetuity, the free cash flow (FCF) is equal to the profit before tax of the unlevered company (PBTu) multiplied by (1 – T):

$$FCF = PBTu\ (1 - T) \qquad [6]$$

We will call FCF_0 the company's free cash flow if there were no taxes. The FCF_0 is equal to the profit before taxes of the unlevered company (PBTu):

$$FCF_0 = PBTu \qquad [7]$$

From [6] and [7] it is clear that the relation between the free cash flow and the company's free cash flow if there were no taxes is:

$$FCF = FCF_0\ (1 - T) \qquad [8]$$

The taxes paid every year by the unlevered company ($Taxes_U$) are:

$$Taxes_U = T\ PBTu = T\ FCF_0 = T\ FCF / (1 - T) \qquad [9]$$

We designate K_{TU} as the required return to tax in the unlevered company, and KTL as the required return to tax in the levered company. The taxes paid by the unlevered company are proportional to FCF_0 and FCF. Consequently, the taxes of the unlevered company have the same risk as FCF_0 (and FCF), and hence must be discounted at the rate Ku (the required return to unlevered equity). In the unlevered company, the required return to tax (K_{TU}) is equal to the required return to equity (Ku). This is only true for perpetuities.

$$KTU = Ku \qquad [10]$$

The present value of the taxes paid by the unlevered company (Gu) is the present value of the taxes paid every year ($Taxes_U$) discounted at the appropriate discount rate (Ku). As Vu = FCF / Ku:

$$Gu = TaxesU / KTU = T\ FCF / [(1 - T)\ Ku] = T\ Vu / (1 - T) \qquad [11]$$

For the levered company, taking into consideration equation [5], the taxes paid each year ($Taxes_L$) are proportional to the equity cash flow (ECF):

$$Taxes_L = T\ PBT_L = T\ PAT_L / (1 - T) = T\ ECF / (1 - T) \qquad [12]$$

PBT_L and PAT_L are the profit before and after tax of the levered company.

The taxes paid by the levered company are proportional to ECF. Consequently, the taxes of the levered company have the same risk as the ECF and thus must be discounted at the rate Ke. So, in the case of perpetuities, the tax risk is identical to the equity cash flow risk and –consequently– the required return to tax in the levered company (K_{TL}) is equal to the required return to equity (Ke). This is only true for perpetuities.

$$K_{TL} = Ke \qquad [13]$$

The relation between profit after tax (PAT) and profit before tax (PBT), is: PAT = PBT (1 – T). Then, the present value of the taxes of the levered company, that is, the value of the taxes paid to the Government, is equal to:

$$G_L = Taxes_L / K_{TL} = T\ ECF / [(1 - T)\ Ke] = T\ E / (1 - T) \qquad [14]$$

The increase in the company's value due to the use of debt is *not* the present value of tax shields due to interest payments, but rather the difference between Gu and G_L, which are the present values of two cash flows with different risks:

$$VTS = Gu - G_L = [T / (1 - T)]\ (Vu - E) \qquad [15]$$

As Vu – E = D – VTS, this gives:

$$VTS = \text{Value of tax shields} = DT \qquad [16]$$

This result is far from being a new idea. Brealey and Myers (2000), Modigliani and Miller (1963), Taggart (1991), Copeland et al. (2000) and many others report it. However, the way of deriving it is new. Most of these papers reach this result by arguing that the appropriate way of computing the value of the tax shield is by considering a certain flow which is DT multiplied by some measure of cost of funds, α, and then discounting that flow at that same rate α. At first glance, α could be anything, related or unrelated to the company that we are valuing. Modigliani and Miller (1963) argue that α is the risk-free rate (R_F). Myers (1974) assumes that α is the cost of debt (Kd) and says that the value of tax shields is the present value of the tax savings (D T Kd) discounted at the cost of debt (Kd). Here, it has been shown that the value of tax shields is the difference between Gu and G_L, which are the present values of two cash flows with different risks: the taxes paid by the unlevered company and the taxes paid by the levered company. In Section 4 we prove that the correct α is the required return to unlevered equity (Ku).

Equation [16] is the difference between the present value of the taxes of the unlevered company (PV[Ku; $Taxes_U$]) and the present value of the taxes of the levered company (PV[Ke; $Taxes_L$]).

$$VTS = PV[Ku;\ Taxes_U] - PV[Ke;\ Taxes_L] \qquad [17]$$

The annual difference between the cash flows to investors in a leveraged firm and the cash flows to investors in an all equity firm are precisely equal to the interest tax shield: Kd T D.

$$Taxes_U - Taxes_L = D\ Kd\ T \quad [18]$$

For perpetuities, and only for perpetuities, it may be argued that the risk of this difference is Kd. Therefore, the present value of this difference should be Kd T D / Kd = T D. But in Section 4, it will be seen that for growing companies the risk of the interest tax shield is not Kd or Ku.

3. Relation between the required return to assets (Ku) and the required return to equity (Ke)

Knowing the value of tax shields in a perpetuity (DT), and considering that Vu = FCF/Ku, we can rewrite equation [1] as [19]:

$$E + D\ (1 - T) = FCF/Ku \quad [19]$$

Taking into consideration that the relation between ECF and FCF for perpetuities is:

$$FCF = ECF + D\ Kd\ (1 - T) \quad [20]$$

and that E = ECF/Ke, we find that the relation between the required return to assets (Ku) and the required return to equity (Ke) in a world without leverage costs is:

$$Ke = Ku + (Ku - Kd)\ D\ (1 - T)\ /E \quad [21]$$

This relation can also be expressed in terms of the systematic risk of each cash flow, i.e. the betas. The formulas relating the betas with the required returns are:

$$Ke = R_F + \beta_L\ P_M \qquad Ku = R_F + \beta u\ P_M \qquad Kd = R_F + \beta d\ P_M$$

RF is the risk-free rate and PM is the market risk premium. The relation between the beta of the levered equity (βL), the beta of the unlevered equity (βu) and the beta of debt (βd) is:

$$\beta_L = \beta u + (\beta u - \beta d)\ D\ (1 - T)\ /E \quad [22]$$

4. The value of tax shields for growing perpetuities

For growing perpetuities, we can calculate the value of taxes for the unlevered and levered firm. In the unlevered company with constant growth, the relation between taxes and free cash flow is different to equation [9], obtained for perpetuities:

$$Taxes_U = T\ PBTu = T\ [FCF + g\ (WCR + NFA)]\ /\ (1 - T) = T\ [FCF + g\ (Ebv + D)]\ /\ (1 - T) \quad [23]$$

where WCR are the net working capital requirements, NFA are the net fixed assets, Ebv is the equity book value and g is the constant growth rate. For any company: WCR + NFA = Ebv + D.

From equation [23] we cannot establish any clear relation between the required return to taxes and the required return to assets (Ku) as we did for perpetuities in equation [9] since taxes are no longer proportional to free cash flow.

Similarly, in the levered company with constant growth, the relation between taxes and equity cash flow is different to equation [12], obtained for perpetuities:

$$\text{Taxes}_L = T\ (\text{ECF} + g\ \text{Ebv})\ /\ (1 - T) \qquad [24]$$

Again, a clear relation cannot be established here between the required return to taxes and the required return to equity as we did for perpetuities in equation [12].

An analogous derivation to the one done in Section 2 for the case of a perpetuity could be done for a growing firm in the extreme situation in which the firm is all-debt financed. In this situation, ECF = E = 0, and all the risk of the assets is borne by the debt (Kd = Ku). In a growing perpetuity, the debt cash flow is D Kd – g D. As ECF = 0, and Kd = Ku, free cash flow in equation [24] is FCF = D Kd (1 – T) – g D. The debt cash flow is D Ku – g D = FCF + D T Ku.

If the firm is all-debt financed, Ebv = 0. In this situation, equation [23] is transformed into:

$$\text{Taxes}_U = T\ [\text{FCF} + g\ (\text{Ebv} + D)]\ /\ (1 - T) = T\ [D\ (Ku - g) - D\ T\ Ku + g\ D)]\ /\ (1 - T) = D\ T\ Ku \qquad [25]$$

And equation [24] is transformed into:

$$\text{Taxes}_L = T\ (\text{ECF} + g\ \text{Ebv})\ /\ (1 - T) = 0 \qquad [26]$$

The appropriate discount rate for the taxes of the unlevered company is Ku because the unlevered taxes are proportional to the interest rate payments (in this case D Ku). Then equation [17] is transformed into:

$$\text{VTS} = Gu - G_L = [\text{Taxes}_U\ /\ (K_{TU} - g)] - [\text{Taxes}_L\ /\ (K_{TL} - g)] = D\ T\ Ku\ /\ (Ku - g) \qquad [27]$$

Once again, here the value of tax shields is not the present value of tax shields due to the payment of interest but rather the difference between Gu and G_L, which are the present values of two cash flows with a different risk. The appropriate way to do an adjusted present value analysis with a growing perpetuity is to calculate the VTS as the present value of D T Ku (*not* the interest tax shield) discounted at the unlevered cost of equity (Ku):

$$\text{VTS} = \text{PV}[Ku;\ D\ T\ Ku] \qquad [28]$$

This result can be derived for the general case by noting that the relation between the equity cash flow of the unlevered firm and the free cash flow, in a company with an expected growth rate of g, is:

$$Vu = \text{FCF}/(Ku - g) \qquad [29]$$

By substituting [29] in [1], we get:

$$E + D = FCF / (Ku - g) + VTS \quad [30]$$

The relation between the equity cash flow and the free cash flow is:

$$FCF = ECF + D\ Kd\ (1 - T) - g\ D \quad [31]$$

By substituting [31] in [30], we get:

$$E + D = [ECF + D\ Kd\ (1 - T) - g\ D] / (Ku - g) + VTS \quad [32]$$

As ECF = E (Ke – g), [32] can be rewritten as:

$$E + D = [E\ (Ke - g) + D\ Kd\ (1 - T) - g\ D] / (Ku - g) + VTS \quad [33]$$

Multiplying both sides of equation [33] by (Ku – g) we get:

$$(E + D)\ (Ku - g) = [E\ (Ke - g) + D\ Kd\ (1 - T) - g\ D] + VTS\ (Ku - g) \quad [34]$$

Eliminating – g (E + D) on both sides of equation [34]:

$$(E + D)\ Ku = [E\ Ke + D\ Kd\ (1 - T)] + VTS\ (Ku - g) \quad [35]$$

Equation [35] can be rewritten as:

$$D\ [Ku - Kd\ (1 - T)] - E\ (Ke - Ku) = VTS\ (Ku - g) \quad [36]$$

Dividing both sides of equation [36] by D (debt value), we get:

$$[Ku - Kd\ (1 - T)] - (E / D)\ (Ke - Ku) = (VTS / D)\ (Ku - g) \quad [37]$$

If (E / D) is constant, the left-hand side of equation [36] does not depend on growth (g) because for any growth rate (E / D), Ku, Kd, and Ke are constant. We know from equation [16] that for g = 0, VTS = DT. Then, equation [37] applied to perpetuities (g = 0) is:

$$[Ku - Kd\ (1 - T)] - (E / D)\ (Ke - Ku) = T\ Ku \quad [38]$$

Substracting [38] from [37] we get

0 = (VTS / D) (Ku – g) – T Ku, solving for the present value of tax-shields we get that:

$$VTS = D\ T\ Ku / (Ku - g) \quad [28]$$

By substituting [28] in [35], we get equation [21]. This implies that equation [21] relating the cost of capital for the levered and unlevered firm also applies to growing perpetuities. In fact, equation [21] also applies to the general case of any type of growth of the firm.

An alternative way of deducing equation [28] is by noting that equation [36] has to hold true also when the company is all-debt financed, that is, when E = 0 and ECF = 0 (the equity cash flow is zero). In this extreme situation, all the risk of the assets is borne by the debt (Kd = Ku), and equation [36] is:

$$D\ [Ku - Ku\ (1 - T)] = VTS\ (Ku - g) \qquad [39]$$

Note that [39] is exactly the same as equation [28].

Equation [28] is valid if the cost of debt (r) is equal to the required return to debt (Kd). In this situation, the value of the debt (D) is equal to the nominal or book value of the debt (N). When the value of the debt (D) is not equal to the nominal or book value of the debt (N), equation [28] changes into:

$$VTS = [D\ T\ Ku + T\ (N\ r - D\ Kd)]\ /\ (Ku - g) \qquad [40]$$

5. Comparison with alternative measures

There is a considerable body of literature on the discounted cash flow valuation of firms. We will discuss in this section the most salient papers, concentrating particularly on those papers that propose alternative expressions for the value of tax shields (VTS). The main difference between all of these papers and the approach proposed above is that most previous papers calculate the value of tax shields as the present value of the tax savings due to the payment of interest. Instead, we indicate that the correct measure of the value of tax shields is the difference between two present values: the present value of taxes paid by the unlevered firm minus the present value of taxes paid by the levered firm. We will show how these proposed methods result in inconsistent valuations of the tax shield or inconsistent relation between the cost of capital to the unlevered and levered firm.

Modigliani and Miller (1958 and 1963) studied the effect of leverage on the firm's value. Their famous proposition 1 states that, in the absence of taxes, the firm's value is independent of its debt, i.e., E + D = Vu, if T = 0. In the presence of taxes and for the case of a perpetuity, but with zero risk of bankruptcy, they calculate the value of tax shields by discounting the present value of the tax savings due to interest payments of a risk-free debt at the risk-free rate (RF), i.e. VTS = PV[RF; D T RF] = D T. As indicated above, this result is the same as our equation [16] for the case of perpetuities, but it is not correct nor applicable for growing perpetuities. Modigliani and Miller explicitly ignored the issue of the riskiness of the cash flows by assuming that the probability of bankruptcy was always zero.

Myers (1974) introduced the APV (adjusted present value) method. According to this method, the value of the levered firm is equal to the value of the firm with no debt plus the present value of the tax saving due to the payment of interest. Myers proposes calculating the VTS by discounting the tax savings (D T Kd) at the cost of debt (Kd). The argument is that the risk of the tax saving arising from the use of debt is the same as the risk of the debt. Then, according to Myers (1974), the value of tax shields is VTS = PV [Kd; D T Kd]. This approach has also been recommended in later papers in the literature, one being Luehrman (1997).

Harris and Pringle (1985) propose that the present value of the tax saving due to the payment of interest should be calculated by discounting the interest tax savings (Kd T D) at the required return to unlevered equity (Ku), i.e. VTS = PV [Ku; D Kd T]. Their argument is that the interest tax shields have the same systematic risk as the firm's underlying cash flows and, therefore, should be discounted at the required return to assets (Ku). Furthermore, Harris and Pringle (1985) believe that "the MM position is considered too extreme by some because it implies that interest tax shields are no more risky than the interest payments themselves" (page 242). Ruback (1995 and 2002), Kaplan and Ruback (1995), Brealey and Myers (2000, page 555), and Tham and Vélez-Pareja (2001), this last paper following an arbitrage argument, also claim that the appropriate discount rate for tax shields is Ku, the required return to unlevered equity.

Ruback (2002) presents the Capital Cash Flow (CCF) method and claims that the appropriate discount rate is Ku. The capital cash flows (3) are the cash flows available for all holders of the company's securities, whether these be debt or shares, and are equivalent to the equity cash flow plus the cash flow corresponding to the debt holders. The capital cash flow is also equal to the free cash flow plus the interest tax shield (D Kd T). According to the Capital Cash Flow (CCF) method, the value of the debt today (D) plus that of the shareholders' equity (E) is equal to the capital cash flow (CCF) discounted at the weighted average cost of capital before tax ($WACC_{BT}$).

$$E + D = PV[WACC_{BT}\, ; CCF] \qquad [41]$$

The definition of $WACC_{BT}$ is [42]:

$$WACC_{BT} = (E\ Ke + D\ Kd) \ / \ (E + D) \qquad [42]$$

because [42] is the discount rate that ensures that the value of the company obtained using the following two expressions is the same:

$$PV[WACC_{BT}\ ; CCF] = PV[WACC; FCF] \qquad [43]$$

But Ruback (1995 and 2002) assumes that $WACC_{BT}$ = Ku. With this assumption, Ruback gets the same valuation as Harris and Pringle (1985) because, according to Ruback,

$$E + D = PV[Ku; CCF] = PV[Ku; FCF] + PV[Ku; D\ Kd\ T] = Vu + PV[Ku; D\ Kd\ T] \quad (44)$$

Note that PV[Ku; D Kd T] is the VTS according to Harris and Pringle (1985).

Ruback (2002, page 91) also shows that the relation between the beta of the levered equity (β_L), the beta of the unlevered equity (βu) and the beta of debt (βd) consistent with [44] is:

$$\beta_L = \beta u + (\beta u - \beta d)\ D / E \qquad [45]$$

Therefore, all Ruback's results (the relation between β_L and βu using no taxes, and Ku being the appropriate discount rate for capital cash flows) come from his method of estimating VTS, which is equal to that of Harris and Pringle (1985).

The enterprise value (E + D) according to our proposed method is:

$$E + D = Vu + PV[Ku; D\ Ku\ T] = PV[Ku; FCF + D\ Ku\ T] \qquad [46]$$

(3) Arditti and Levy (1977) suggested that the firm's value could be calculated by discounting the Capital Cash Flows instead of the Free Cash Flow.

The difference between our proposed method [46] and Ruback's [44] is PV[Ku; D (Ku – Kd) T] (4).

A large part of the literature argues that the value of tax shields should be calculated in a different manner depending on the debt strategy of the firm. A firm that wishes to keep a constant D/E ratio must be valued in a different manner from the firm that has a preset level of debt. Miles and Ezzell (1980) indicate that for a firm with a fixed debt target (i.e. a constant [D/(D+E)] ratio), the correct rate for discounting the tax saving due to debt is Kd for the first year and Ku for the tax saving in later years (5). Although Miles and Ezzell do not mention what the value of tax shields should be, this can be inferred from their equation relating the required return to equity with the required return for the unlevered company (equation 22 in their paper). This relation implies that VTS = PV[Ku; T D Kd] (1 + Ku)/(1 + Kd). Inselbag and Kaufold (1997) and Ruback (2002) argue that if the firm targets the dollar values of debt outstanding, the VTS is given by the Myers (1974) formula. However, if the firm targets a constant debt/value ratio, the value of the tax shield should be calculated according to Miles and Ezzell (1980). Finally, Taggart (1991) proposes to use Miles & Ezzell (1980) if the company adjusts to its target debt ratio once a year and Harris & Pringle (1985) if the company continuously adjusts to its target debt ratio.

Damodaran (1994, page 31) argues that if all the business risk is borne by the equity, then the formula relating the levered beta (βL) with the asset beta (βu) is βL = βu + (D/E) βu (1 – T). This formula is exactly formula [22] assuming that βd = 0. One interpretation of this assumption is (see page 31 of Damodaran, 1994) that "all of the firm's risk is borne by the stockholders (i.e., the beta of the debt is zero)". In some cases, it can be reasonable to assume that the debt has a zero beta. But then, as assumed by Modigliani and Miller (1963), the required return to debt should be the risk-free rate (6). We think that, in general, it is hard to accept that the debt has no risk and that the return on the debt is uncorrelated with the return on assets of the firm. From Damodaran's expression for βL it is easy to deduce the relation between the required return to equity and the required return to assets, i.e. Ke = Ku + (D / E) (1 – T) (Ku – R_F). Although Damodaran does not mention what the value of tax shields should be, his formula relating the levered beta with the asset beta implies that the value of tax shields (7) is VTS = PV[Ku; D T Ku – D (Kd – RF) (1 – T)].

Finally, a common way of calculating the levered beta with respect to the asset beta often used by consultants and investment banks is the following (8):

$$\beta_L = \beta u\ (1+ D/E) \qquad [47]$$

(4) If we start with the levered beta of a company and we unlever that beta, [44] and [46] provide the same answer only if the debt/equity ratio of the company from which we take the levered beta equals the debt/equity ratio of the company we are valuing.
(5) Lewellen and Emery (1986) also claim that this is the most logically consistent method.
(6) This relation for the levered beta appears in many finance books and is widely used by consultants and investment bankers. We think of this formula as an attempt to introduce some leverage cost in the valuation: for a given risk of the assets (βu), by using this formula the resulting βL is higher (and consequently Ke is higher and results in a lower equity value) than with formula [22].
(7) There is a link between the VTS and the relation between the levered beta and the asset beta.
(8) One of the many places where it appears is Ruback (1995, page 5).

It is obvious that according to this formula, given the same value for βu, a higher βL (and a higher Ke and a lower equity value) is obtained than according to [22] and according to Damodaran (1994) (9). Formula [47] relating the levered beta with the asset beta implies that the value of tax shields is VTS = PV[Ku; T D Kd – D(Kd – R_F)]. Given its widespread use in the industry, we will call this method the Practitioners' method.

Given the large number of alternative methods existing in the literature to calculate the value of tax shields, Copeland, Koller and Murrin (2000) assert that "the finance literature does not provide a clear answer about which discount rate for the tax benefit of interest is theoretically correct." They further conclude "we leave it to the reader's judgment to decide which approach best fits his or her situation."

We propose two ways to compare and differentiate among the different approaches. A first way is to calculate what is the value of tax shields for perpetuities for each of the different approaches. A second way is to check what is the implied relation between the unlevered and the levered cost of equity in each of the different approaches proposed in the literature. The levered cost of equity should always be higher than the cost of assets (Ku), since equity cash flows are riskier than the free cash flows.

Table 1 summarizes the implications that each of these approaches has for the value of tax shields in perpetuities. Table 2 summarizes the implications for the relation between the cost of assets and the cost of equity in growing perpetuities. Table 1 shows that only three out of the seven approaches compute the value of the tax shield in perpetuities as DT. The other four theories imply a lower value of the tax shield than DT. Table 2 shows that not all of these approaches also satisfy the relation between the cost of equity and the cost of assets. The Modigliani and Miller (1963) and Myers (1974) approaches do not always give a higher cost of equity than cost of assets. Myers obtains Ke lower than Ku if the value of tax shields is higher than the value of debt. This happens when D T Kd / (Kd – g) > D, that is, when the growth rate is higher than the after-tax cost of debt: g > Kd (1 – T). Please note also that in this situation, as the value of tax shields is higher than the value of debt, the equity (E) is worth more than the unlevered equity (Vu). This hardly makes any economic sense. Modigliani-Miller also provides the inconsistent result of Ke being lower than Ku if the value of tax shields is higher than D [Ku – Kd (1 – T)] / (Ku – g). This happens when either the leverage, the tax rate, the cost of debt or the market risk premium are high. The only approach that provides consistent results for both of these cases is the one proposed above.

Appendix 1 is a compendium of the main valuation formulae according to the different theories.

6. An example of a perpetuity

In this section, we provide a numerical example of how to calculate the value of the tax shield for a simple firm under the different approaches. Let's start by assuming that the cash flows generated by the company are perpetual and constant (there is no growth). The

(9) We interpret formula [46] as an attempt to introduce still higher leverage cost in the valuation. For a given risk of the assets (βu), by using formula [46] we obtain a higher βL (and consequently a higher Ke and a lower equity value) than with the approach of Damodaran (1994) or formula [22]. Damodaran (1994) and Practitioners impose a cost of leverage, but they do so in an ad hoc way.

company must invest to maintain its assets at a level that enables it to ensure constant cash flows: this implies that the book depreciation is equal to the replacement investment. The firm's balance sheet, income statement, cash flows and other valuation parameters of the company may be seen in Table 3. The breakdown of the value of the unlevered and levered companies between all the stakeholders in the firm is summarized in Figure 1. The company has three main stakeholders: shareholders, bondholders and the government. The value of the tax shield is exactly the difference between the present value of the government claim on the levered firm and the present value of the government claim on the unlevered firm.

Table 4 reports the estimated value of the equity for the levered firm, the present value of tax shields, the cost of equity, the WACC for the levered and unlevered firm, and the implied value of the levered beta for each of the seven approaches. Our proposed method, Modigliani and Miller (1963) and Myers (1974) compute the value of the tax shield equal to \$200 (D T = 500 x 0.4 = 200). All the other methods result in a value of the tax shield that is too low.

Now we assume that this hypothetical firm grows at a constant rate of 5%, i.e. g=5%. The firm's Balance Sheet, Income Statement, Cash Flows and other valuation parameters may be seen in Table 5. Note that the Balance Sheet and the Income Statement are identical to those in Table 3 (without growth). However, the cash flows vary.

Table 6 reports the estimated value of the equity for the levered firm, the present value of tax shields, the cost of equity, the WACC, the WACC before taxes, the implied value of the levered beta, and the debt to equity ratio for each of the seven approaches. Our proposed method computes the value of the tax shield equal to \$400. The methods of Modigliani and Miller (1963) and Myers (1974) result in a value of the tax shield that is too high (even higher than the debt value) and correspondingly estimate a higher value of equity than the correct one (10). Please recall that these two methods were the only methods that for the case of a constant perpetuity computed the same value of the tax shield. Furthermore, these two methods reach the implausible conclusion that the beta for the levered firm should be lower than the beta for the unlevered firm (see the fifth column of table 6). All the other methods result in a value of the estimated tax shield in line with their Results for the case of constant perpetuities, i.e. their estimate of the value of the tax shield is too low. As we assume that the value of debt is 500 in both the growth example and the no-growth example, the Debt/Equity ratio is lower with growth because equity is higher. For this reason, the WACC of the growing firm (Table 6) is higher than the WACC of the firm with no growth (Table 4).

(10) One could say, "in practice I do not see why the approach of working out the present value of tax shields themselves would necessarily be wrong, provided the appropriate discount rate was used (reflecting the riskiness of the tax shields)". The problem here is that it is hard to evaluate the riskiness of the tax shield because it is the difference of two present flows (the taxes paid by the unlevered company and those paid by the levered company) with different risk. On the other hand, if we were to follow this approach and calculate the discount rate K* that accomplishes:

$$VTS = D\ T\ Ku / (Ku - g) = D\ T\ Kd / (K^* - g)$$

The solution is $K^* = Kd + g - Kd\ g / Ku$. Obviously this expression is unintuitive.

Table 1. Comparison of value of tax shields (VTS) in perpetuities

Only three out of the seven approaches compute correctly the value of the tax shield in perpetuities as DT. The other four theories imply a lower value of the tax shield than DT.

Theories	**VTS**	**VTS in perpetuities**
Correct method	[28] PV[Ku; D T Ku]	DT
Damodaran (1994)	PV[Ku; DTKu - D (Kd- R_F) (1-T)]	< DT
Practitioners	PV[Ku; T D Kd - D (Kd- R_F)]	< DT
Harris-Pringle (1985), Ruback (1995)	PV[Ku; T D Kd]	< DT
Myers (1974)	PV[Kd; T D Kd]	DT
Miles-Ezzell (1980)	PV[Ku; T D Kd] (1+Ku) / (1+Kd)	< DT
Modigliani-Miller (1963)	PV[R_F; D T R_F]	DT

Ku = unlevered cost of equity
Kd = required return to debt
T = corporate tax rate
D = debt value
R_F = risk-free rate
PV[Ku; D T Ku] = present value of D T Ku discounted at the rate Ku

Table 2. Comparison of the relation between Ke (levered cost of equity) and Ku (unlevered cost of equity) for growing perpetuities

The approaches of Modigliani and Miller (1963) and Myers (1974) do not always result in a higher cost of equity (Ke) than cost of assets (Ku). Myers (1974) obtains Ke lower than Ku if the value of tax shields is higher than the value of debt. This happens when the growth rate (g) is higher than the after-tax cost of debt, i.e. g > Kd (1 – T). Modigliani Miller (1963) also provides the inconsistent result of Ke being lower than Ku if the value of tax shields is higher than D [Ku – Kd (1 – T)] / (Ku – g). This happens when the leverage, the tax rate, the cost of debt or the market risk premium are high.

Theories	**Ke (levered cost of equity)**	**Ke<Ku**
Correct method	[21] Ke = Ku + (D/E) (1 - T) (Ku – Kd)	No
Damodaran (1994)	Ke = Ku + (D/E) (1 - T) (Ku – R_F)	No
Practitioners	Ke = Ku + (D/E) (Ku – R_F)	No
Harris-Pringle (1985), Ruback (1995)	Ke = Ku + (D/E) (Ku – Kd)	No
Myers (1974)	Ke = Ku + (D - VTS) (Ku – Kd) /E	**Yes,** if g > Kd (1 – T)
Miles-Ezzell (1980)	$\mathrm{Ke} = \mathrm{ku} + \frac{D}{E}(\mathrm{Ku} - \mathrm{Kd})\left[1 - \frac{\mathrm{T\,Kd}}{1+\mathrm{Kd}}\right]$	No
Modigliani-Miller (1963)	$\mathrm{Ke} = \mathrm{Ku}\frac{D}{E}\left[\mathrm{Ku} - \mathrm{Kd}(1-\mathrm{T}) - (\mathrm{Ku}-\mathrm{g})\frac{\mathrm{VTS}}{D}\right]$*	**Yes,** if VTS> D [Ku–Kd(1–T)]/(Ku–g)

* Valid only for growing perpetuities
D = debt value
E = equity value
g = growth rate
Kd = required return to debt
R_F = risk-free rate
T = corporate tax rate
VTS = value of tax shields

Table 3. Valuation of a hypothetical firm without growth.
Balance sheet, income statement, cash flows and valuations.

This table presents the valuation of a hypothetical firm without growth using the APV (Adjusted present value). The table also shows that four alternative valuations (equity cash flows discounted at the required return to equity, free cash flows discounted at the WACC, capital cash flows discounted at the WACC before taxes, and value of tax shields as the difference of two net present values) provide the same valuation

Balance Sheet	t = 0	**P & L**	t = 1	**Cash Flows**	t = 1
Working capital requirements (WCR)	0	EBITDA	520	Profit after tax (PAT)	**171**
Net fixed assets	2,000	Depreciation	200	+ Depreciation	200
TOTAL ASSETS	**2,000**	EBIT	320	+ increase of Debt	0
Debt (D)	500	Interest paid (I) = D Kd	35	– increase of WCR	0
Capital (book value)	1,500	Profit before tax (PBT)	285	– investments in fixed assets	–200
Capital (book value)	**1,500**	Taxes (40%)	114	**ECF** (equity cash flow)	171
		Profit after tax (PAT)	171	**FCF** (free cash flow)	192
				CCF (capital cash flow)	206

T = corporate tax rate = 40%
Kd = cost of debt = 7%

Valuation parameters
g = growth rate = 0%
R_F = risk-free rate = 6%
βu = unlevered beta = 1.00
P_M = market risk premium = 4%
Ku = unlevered cost of equity = 6% + 1.00 x 4% = 10%

Valuation. APV (Adjusted present value)
Vu = value of unlevered company = FCF / Ku = 192 / 0.1 = 1,920
Using equation [16]), VTS = D T = 500 x 0.4 = 200
Using equation [1], E + D = Vu + VTS = 1,920 + 200 = 2,120.

As shown in Fernández (2002, chapter 21), the APV (adjusted present value) provides always the same value as the other most commonly used methods for valuing companies by cash flow discounting: equity cash flows discounted at the required return to equity, free cash flows discounted at the WACC, capital cash flows discounted at the WACC before taxes (WACCBT).

Four alternative valuation methods:

Alternative 1. Equity cash flows discounted at the required retum to equity (Ke)

Using equation [21], Ke = Ku + (Ku – Kd) D (1 – T) /E = 0.1 + (0.1 – 0.07) 500 x 0.6 / E = 0.1 + 9/E
E = ECF/Ke = 171 / (0.1 + 9/E). 0.1 E + 9 = 171. E = 1,620. Ke = 10.556%. E + D = 1,620 + 500 = 2,120

Alternative 2. Free cash flows discounted at the WACC

WACC = [E Ke + D Kd (1 – T)] / (E + D) = [1,620 x 0.10556 + 500 x 0.07 x (1 – 0.4)] / (1,620 + 500) = 9.057%
E + D = FCF / WACC = 192 / 0.09057 = 2,120

Alternative 3. Capital cash flows discounted at the $WACC_{BT}$

$WACC_{BT}$ = (E Ke + D Kd) / (E + D) = (1,620 x 0.10556 + 500 x 0.07) / (1,620 + 500) = 9.717%
CCF (Capital cash flow) = ECF + debt cash flow = 171 + 35 = 206
E + D = CCF / $WACC_{BT}$ = 206 / 0.09717 = 2,120

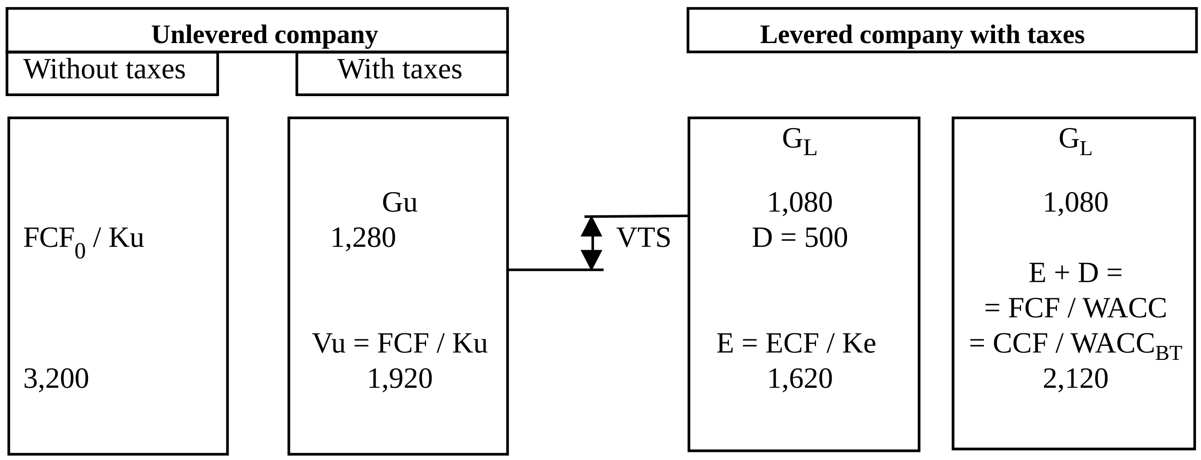
Alternative 4. VTS = Gu - GL= present value of the taxes paid by the unlevered company - present value of the taxes paid by the levered company

Using equation [9], $Taxes_U$ = taxes paid by the unlevered firm = = T FCF / (1 – T) = 0.4 x 192 / 0.6 = 128
Using equation [10], K_{TU} = required return to tax in the unlevered company = Ku = 10%.
Using equation [11],
Gu = present value of the taxes paid by the unlevered company = $Taxes_U$ / K_{TU} = 128 / 0.1 = 1,280
From equation [13], K_{TL} = required return to tax in the levered company = Ke = 10.556%. We know from the income statement that the taxes paid by the levered company ($Taxes_L$) are 114.
Using equation [14],
G_L = present value of the taxes paid by the levered company = $Taxes_L$ / K_{TL} = 114 / 0.10556 = 1,080
Using equation [4], VTS (value of tax shields) = Gu – G_L = 1,280 – 1,080 = 200.
Using equation [1], E + D = Vu + VTS = 1,920 + 200 = 2,120.

Figure 1. Distribution of the company's total value between shareholders, bondholders and the Government for the example of Table 3.

The value of tax shields (VTS) is the difference between the present values (PV) of two cash flows with different risk: the PV of the taxes paid in the unlevered company (Gu) and the PV of the taxes paid in the levered company (G_L).

VTS = Gu - G_L = 1,280 – 1,080 = 200



E + D = Vu + VTS. 1,620 + 500 = 1,920 + 200 = 2,120

CCF = capital cash flow = 206
D = value of the debt = 500
E = value of equity = 1,620
ECF = equity cash flow = 171
FCF = free cash flow = 192
FCF_0 = free cash flow if there were no taxes = 320
Kd = required return to debt = 7%
Ke = levered cost of equity = 10.556%
Ku = unlevered cost of equity = 10%
WACC = weighted average cost of capital = 9.057%
$WACC_{BT}$ = weighted average cost of capital before taxes = 9.717%

Table 4. Value of tax shields (VTS) for a hypothetical firm with no growth.
Application of the seven theories to the example in Table 3. Beta unlevered = 1. Ku = 10%. D = 500.
Our proposed method, Modigliani and Miller (1963) and Myers (1974) compute the value of the tax shield equal to 200 (D T = 500 x 0.4 = 200), the correct value. All the other methods result in a value of the tax shield that is too low.

g = 0% (no growth)	**VTS** value of tax shield	**E** equity value	**Ke** levered cost of equity	**Beta** levered	**D/E** Debt to equity ratio	**WACC** weighted average cost of capital	**$WACC_{BT}$** WACC before taxes
Modigliani-Miller	200.00	1,620.00	10.56%	1.138889	30.86%	9.057%	9.717%
Myers	200.00	1,620.00	10.56%	1.138889	30.86%	9.057%	9.717%
Correct formula	**200.00**	**1,620.00**	**10.56%**	**1.138889**	**30.86%**	**9.057%**	**9.717%**
Damodaran	170.00	1,590.00	10.75%	1.188679	31.45%	9.187%	9.856%
Miles-Ezzell	143.93	1,563.93	10.93%	1.233507	31.97%	9.303%	9.981%
Harris-Pringle & Ruback	140.00	1,560.00	10.96%	1.240385	32.05%	9.320%	10.000%
Practitioners	90.00	1,510.00	11.32%	1.331126	33.11%	9.552%	10.249%

Table 5. Valuation of a hypothetical firm with growth.
Balance sheet, income statement, cash flows and valuation

This table presents the valuation of a hypothetical firm with 5% growth using the APV (Adjusted present value). The table also shows that three alternative valuations (equity cash flows discounted at the required return to equity, free cash flows discounted at the WACC, and capital cash flows discounted at the WACC before taxes) provide the same valuation.

g = 5% (no growth)	**VTS** value of tax shield	**E** equity value	**Ke** levered cost of equity	**Beta** levered	**D/E** Debt to equity ratio	**WACC** weighted average cost of capital	**$WACC_{BT}$** WACC before
Modigliani-Miller	1,200.00	2,540.00	8.78%	0.694882	19.69%	8.026%	8.487%
Myers	700.00	2,040.00	9.71%	0.926471	24.51%	8.622%	9.173%
Correct formula	**400.00**	**1,740.00**	**10.52%**	**1.129310**	**28.74%**	**9.107%**	**9.732%**
Damodaran	340.00	1,680.00	10.71%	1.178571	29.76%	9.220%	9.862%
Miles-Ezzell	287.85	1,627.85	10.90%	1.224337	30.72%	9.324%	9.982%
Harris-Pringle & Ruback	280.00	1,620.00	10.93%	1.231481	30.86%	9.340%	10.000%
Practitioners	180.00	1,520.00	11.32%	1.328947	32.89%	9.554%	10.248%

7. Conclusions

This paper shows that the increase in the company's value due to the use of debt is not the present value of tax shields due to interest payments. It is the difference between the present value of the taxes of the unlevered company and the present value of the taxes of the levered company, which are the present values of two separate cash flows, each with its own risk. The issue of the riskiness of the taxes for both the unlevered and the levered company is addressed. We prove that, in perpetuities, the required return to tax in the unlevered company is equal to the required return to equity in the unlevered company. It is also proven that the required return to tax in the levered company is equal to the required return to equity.

The value of tax shields is the present value of the product of the debt times the tax rate times the required return to the unlevered equity, all of it discounted at the unlevered cost of equity. For a constant perpetuity, this results in a value of the tax shield equal to the value of debt times the tax rate. For the case of growing perpetuities, this number is lower than that proposed by either Myers (1974) or Modigliani and Miller (1963).

The paper also shows that some of the alternative approaches suggested in the literature lead to inconsistent results. Particularly, discounting tax shields at the cost of debt (Myers (1974), Brealey and Myers (2000), and many others) or at the risk-free rate (Modigliani and Miller (1963)) can result, for growing perpetuities, in estimating a lower cost of equity to the levered firm than that of the unlevered firm. This seems clearly unreasonable. On the other hand, discounting tax shields at the required return to the unlevered equity, as suggested by Harris and Pringle (1985), Miles and Ezzell (1980) and Ruback (2002), results in an estimated value of tax shields that is too low. □

Appendix 1

THE VALUE OF TAX SHIELDS IS **NOT** EQUAL
TO THE PRESENT VALUE OF TAX SHIELDS

Main valuation formulas
Market value of the debt = Nominal value

Theories	**VTS**	**Ke**
Correct method	PV[Ku; D T Ku] **[28]**	Ke = Ku + (D/E) (1 - T) (Ku – Kd) **[21]**
Damodaran (1994)	PV[Ku; DTKu - D (Kd- R_F) (1-T)]	Ke = Ku + (D/E) (1 - T) (Ku – R_F)
Practitioners	PV[Ku; T D Kd - D (Kd- R_F)]	Ke = Ku + (D/E) (Ku – R_F)
Harris-Pringle (1985), Ruback (1995)	PV[Ku; T D Kd]	Ke = Ku + (D/E) (Ku – Kd)
Myers (1974)	PV[Kd; T D Kd]	Ke = Ku + (D - VTS) (Ku – Kd) /E
Miles-Ezzell (1980)	PV[Ku; T D Kd] (1+Ku) / (1+Kd)	
Modigliani-Miller (1963)	PV[R_F; D T R_F]	

Theories	β_L	**WACC**
Correct method	$\beta_L = \beta u + \frac{D(1-T)}{E}(\beta u - \beta d)$ **[22]**	$Ku\left(1 - \frac{DT}{E+D}\right)$
Damodaran (1994)	$\beta_L = \beta u + \frac{D(1-T)}{E}\beta u$	$Ku\left(1 - \frac{DT}{E+D}\right) + D\frac{(Kd - R_F)(1-T)}{(E+D)}$
Practitioners	$\beta_L = \beta u + \frac{D}{E}\beta u$ **[47]**	$Ku - D\frac{R_F - Kd(1-T)}{(E+D)}$
Harris-Pringle (1985), Ruback (1995)	$\beta_L = \beta u + \frac{D}{E}(\beta u - \beta d)$ **[45]**	$Ku - D\frac{Kd\,T}{(E+D)}$
Myers (1974)	$\beta_L = \beta u + \frac{D - DVTS}{E}(\beta u - \beta d)$	$Ku - \frac{DVTS(Ku - Kd) + D\,Kd\,T}{(E+D)}$
Miles-Ezzell (1980)	$\beta_L = \beta u + \frac{D}{E}(\beta u - \beta d)\left[1 - \frac{T\,Kd}{1+Kd}\right]$	$Ku - D\frac{Kd\,T}{(E+D)}\frac{1+Ku}{1+Kd}$
Modigliani-Miller (1963)	$\beta_L = \beta u + \frac{D}{E}[\beta u - \beta d + \frac{TKd}{P_M} - \frac{VTS(Ku-g)}{D\,P_M}]$ *	$\frac{D\,Ku - (Ku-g)\,VTS}{(E+D)}$ *

* Valid only for growing perpetuities.

Appendix 1 (continued)

Theories	**WACC$_{BT}$**
Correct method	$Ku - \frac{DT(Ku - Kd)}{(E + D)}$
Damodaran (1994)	$Ku - D\frac{T(Ku - R_F) - (Kd - R_F)}{(E + D)}$
Practitioners	$Ku + \frac{D(Kd - R_F)}{(E + D)}$
Harris-Pringle (1985), Ruback (1995)	Ku
Myers (1974)	$Ku - \frac{DVTS(Ku - Kd)}{(E + D)}$
Miles-Ezzell (1980)	$Ku - \frac{D\ T\ Kd}{(E + D)} \frac{(Ku - Kd)}{(1 + Kd)}$
Modigliani-Miller (1963)	$\frac{D\ Ku\ -\ (Ku - g)\ VTS\ +\ D\ T\ Kd}{(E + D)}$ *

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