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MANAGING TECHNOLOGY DEVELOPMENT FOR SAFETY-CRITICAL SYSTEMS

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Abstract

This paper presents a model that determines the optimal budget allocation strategy for the development of new technologies for safety-critical systems over multiple decision periods. The case of the development of a hypersonic passenger airplane is used as an illustration.

The model takes into account both the probability of technology development success as a function of the allocated budget, and the probability of operational performance of the final system. It assumes that the strategy is to consider (and possibly fund) several approaches to the development of each technology to maximize the probability of development success. The model thus decomposes the system's development process into multiple technology development modules (one for each technology needed), each involving a number of alternative projects. There is a tradeoff between development speed and operational reliability when the budget must be allocated among alternative technology projects with different probabilities of development success and operational reliability (e.g., an easily and quickly developed technology may have little robustness). The probabilities of development and operational failures are balanced by a risk analysis approach which allows the decision maker to optimize the budget allocation among different projects in the development program at the beginning of each budget period.

The model indicates that by considering reliability in the R&D management process, the decision maker can make better decisions, optimizing the balance between development time, cost, and robustness of safety-critical systems.

Key words: technology development, system reliability, risk analysis, failure risk, project management, program management optimization, time to market

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1. Introduction

The development of advanced, safety-critical engineering systems requires meeting stringent goals of technological robustness, schedule and budget. An additional layer of complexity is added when the underlying technologies of the system to be built have not been developed yet. This is a common situation, for example, in the aerospace industry or in the energy sector, where project development often takes months or years. The availability of the technologies under development is crucial to the operational success (and the level of robustness) of the engineering system, but development success is uncertain. As a result, changes can occur late in the development stage and significantly increase the program costs.

A company undertaking this kind of development may face a tradeoff between investing in the most promising technology in terms of system robustness, and the most promising technology in terms of system realization. Modifying a readily available technology on the one hand may increase the probability of development success, but on the other may yield a technology of relatively lower robustness or safety.

In discussions with managers in the aerospace industry, we found that they often handle this tradeoff intuitively for lack of appropriate analytic tools. The goal of this paper is to provide a tool that will guide the budget allocation process by allowing the company undertaking the development effort to balance these two considerations. For example, how should a company that is developing a hypersonic passenger airplane face the tradeoff between funding technology development projects with a low probability of operational success (i.e., this technology is more likely to fail during operation than other technologies) but a high probability of development success (i.e., reduced time-to-market) – and a technology project with opposite characteristics?

The problem of resource allocation and project selection in R&D is similar to other selection and allocation problems. Yet, because it involves engineering performance, it is different enough to warrant a separate treatment here. Teisberg [1995] distinguishes three methods for evaluating strategic investment decisions: dynamic discounted cash flow analysis, decision analysis, and option valuation. The latter has received increased attention since 1994, for example in the works of Dixit and Pindyck [1994], Smith and Nau, [1995], Trigeorgis [1996], Schwartz and Moon [1996], Childs and Triantis [1999] and Shishko and Ebbeler [1999]. We feel, however, that real options are not applicable in the problem under investigation as no twin asset is identifiable. Teisberg goes on to state that in situations where no market information is available or where the available market data are not relevant, decision analysis may be a more appropriate decision aid. Other authors, e.g., Shishko and Ebbeler [1999], have drawn the same conclusion.

The literature in economics and finance that deals with R&D generally does so in the context of competition or value of the firm. This literature frequently derives estimates of technical failure risks from economic or market data, bypassing the knowledge available in the firm itself. A taxonomy of the project selection problem in the economics literature can be found in the work of Ali, Kalwani and Koevenock [1993]. Some of the main considerations are time-to-market, first-mover advantage and industry dynamics, as developed for instance by Aoki [1991] (when do technology followers drop out?), or in a more general context, the value of R&D to society, as in the work of Nelson [1959]. Nelson [1961] also showed that "parallel development of alternative designs seems called for when the technical advances sought are large". Jensen [1987] analyzes the pharmaceutical industry and finds a positive relationship between a firm's research and development intensity and its probability of discovering a new drug. The same applies in the research presented here. The difference is that we focus on the robustness of an engineering system before its operation, using technical information that may not be available to the market at that stage, and risk analysis models that are not always developed by the firm.

The literature in engineering approaches the problem from the inside, i.e., instead of measuring the uncertainty inherent in the development of new technology based on economic data, technical knowledge available in the company is used. A taxonomy of the R&D project selection problem in the realm of applied engineering is provided by Martino [1995]. Balachandra and Friar [1997] provide a review of the literature that deals with critical factors in R&D in a large number of projects. Booker and Bryson [1985] provide an extensive compilation of the literature about decision analysis in project management. Liberatore and Titus [1983] look at the use of management science in R&D project selection but without explicit consideration of expected technical performance. Schmidt and Freeland [1992] review the progress that has been made in the development of quantitative models for R&D project-selection processes. They find that there has been a mismatch between modeling efforts and modeling needs, and state that more interaction is needed between researchers and practitioners. They argue for more research that takes a systems approach to the R&D project selection task, which is one of the goals of the work presented here.

The engineering literature that explicitly addresses management of R&D projects generally deals with the management of development uncertainty. Derman, Lieberman and Ross [1976] address the question of optimal budget allocation for an n-stage development process where several components have to be developed successfully, development success is a continuous non-decreasing function of allocated budget, and a penalty is paid for each undeveloped element. A similar problem is investigated by Chun [1994], but while Derman *et al.* focus on sequential development, Chun also looks at parallel development. The author presents an ordering parameter that indicates which projects to undertake in which period. More recently, Dillon and Paté-Cornell [2001] have developed an Advanced Programmatic Risk Analysis Model (APRAM), balancing development risk and technical failure risk but without explicit consideration of alternatives for each of the technologies necessary.

Using probability distributions for safety parameters and addressing questions of optimal configuration or sequencing, the publications listed above either involve the technological uncertainty present in R&D project management (be it from an economic or an engineering perspective), or the robustness of systems. To the best of our knowledge, no attempt has been made so far to include both the probability of operational success (or technical failure) and the choice of development alternatives in the R&D project selection for funding over time. That is the question that is addressed further in this paper.

Section 2 of this paper presents a formulation of our model. Section 3 provides an illustrative example from the aeronautic industry and analyzes the investment results for different program funding profiles. We conclude in section 4 that the optimal project allocation depends on technical characteristics, development uncertainties, and performance linkages among projects. The appendix contains proofs and formal results.

2. The Model

Our model represents the decision problem of a manufacturing company that aims to develop and build a complex engineering system Sys which will include a number N of yet-to-be-developed technologies. If all necessary technologies are developed successfully, the decision maker can build and operate system Sys. The non-availability of any of these technologies will result in development failure for the system Sys.

2.1. Notation

- i Index of technology module, $i \in [1,2,..., N]$.
- j Index of technology development project, $j \in [1,2,..., m_i]$.
- N Number of technologies under development, therefore of technology development modules.
- m_i Number of projects available for development of technology i.
- $M = \sum_i^N m_i$ Total number of technology development projects under consideration for development of system Sys.
- t Current time period (e.g., year).
- T Number of time periods available for development of system Sys.
- $B_t \leq 1$ Budget available for time period t (as a fraction of the maximum annual budget over time). B_t is discounted to period t=1.
- $x_{i,j,t}$ Budget allocated to project j of module i for period t. For each period t, the sum of all $x_{i,j,t}$ is less than or equal to the available budget B_t of that period, and $x_{i,j,t} \geq 0\ \forall(i,j,t)$.
- $\mathbf{X}_t$ Budget allocation vector for period t. $\mathbf{X}_t = \{x_{1,1,t},...,x_{N,mN,t}\}$.
- $I_{i,j}$ Project state indicator for project (i,j). This indicator is equal to one if project (i,j) has been developed successfully at a considered time t (e.g., at the end of N periods), and zero otherwise. We omit index t to improve readability.
- I_{Sys} System development state indicator. This indicator is equal to one if system Sys can be built at a considered time t (e.g., the end of N periods), and zero otherwise. We omit index t to improve readability.

- $\mathbf{I_{Dev}}(t)$ Development state vector of system Sys at start of period t, indicating which projects have been successfully developed so far. $\mathbf{I_{Dev}}(t) = \{I_{1,1}, ..., I_{N,mN}\}$

- $S(\mathbf{I_{Dev}}(t))$ Set of all possible development states for system Sys *at the end* of period t, given that Sys is in development state $\mathbf{I_{Dev}}(t)$ *at start* of period t. The actual development state vector $U \in S(\mathbf{I_{Dev}}(t))$ represents one of these possible outcomes.

- $d_{i,j}(x_{i,j,t}) = 1 - e^{-a_{i,j}x_{i,j,t}}$ Probability of development success for project j of module i in period t as a function of the allocated budget $x_{i,j,t}$ (budget-probability function).

- $a_{i,j}$ Parameter of the budget-probability function. The larger the value of $a_{i,j}$, the larger the probability of development success for a given budget $x_{i,j,t}$. $a_{i,j} \geq 1$ $\forall(i,j)$.

- $r_{i,j}$ Probability of operating success of subsystem i in system Sys based on the technology developed in project j of module i. $0 < r_{i,j} < 1$.

- R_{OS} Reward (value function) of successful completion of the development program followed by successful operation of system Sys. $R_{OS} > 0$.

- R_{OF} Reward (value function) of successful completion of the development program followed by failure of system Sys in operations. $R_{OS} > R_{OF}$. Note that R_{OF} can be positive or negative (there may be some positive value to lessons learned).

- R_{DF} Reward (value function) of failure of the development program, i.e., system Sys cannot be built. $R_{OS} > R_{OF} > R_{DF}$.

- $\varphi(U|\mathbf{I_{Dev}}(t), \mathbf{X}_t)$ Probability of reaching development state U of the set of all possible development states S *at the end* of period t given the development program's state vector $\mathbf{I_{Dev}}(t)$ *at the beginning* of period t and the budget allocation vector $\mathbf{X}_t$

- $\Phi(\mathbf{I_{Dev}}(t))$ Probability of successful system operation given the development state $\mathbf{I_{Dev}}(t)$ of the program at start of period t, assuming that the system is built based on the best technologies available at that time.

- $K_t(\mathbf{I_{Dev}}(t),\mathbf{X}_t)$ Expected value of the net reward function for stopping development at start of period t, given development state $\mathbf{I_{Dev}}(t)$ and budget allocation vector $\mathbf{X}_t$ of development program.

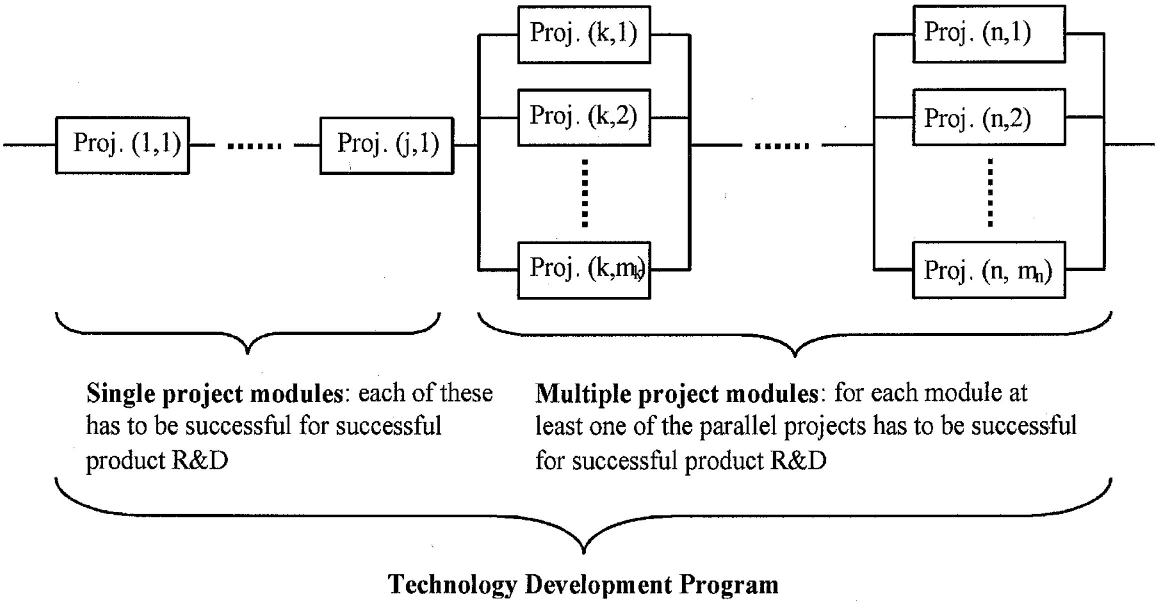
- $L_t(\mathbf{I_{Dev}}(t),\mathbf{X}_t)$ Expected value of the reward function for continuing development at start of period t, given development state $\mathbf{I_{Dev}}(t)$ and budget allocation vector $\mathbf{X}_t$ of development program.

- $V_t(\mathbf{I_{Dev}}(t),\mathbf{X}_t)$ Expected value function for period t, given development state $\mathbf{I_{Dev}}(t)$ and budget allocation vector $\mathbf{X}_t$ of the development program at the beginning of period t.

2.2 Model Formulation

The company has at its disposal a total of $t \leq T$ time periods of equal length (e.g., T years) during which it can develop the necessary technologies and build the system Sys. After successful development the system will be operated during a number of periods. It is assumed here that the first period of operation alone represents proof that the system will work during operation. Each of the N technologies (indexed in i) has a number m_i of different project alternatives available for its development. The total number of projects M available at any time is equal to the sum $M=\sum_{i=1}^{N} m_i$. If at least one project j(i) for each technology i is successfully developed, system Sys can be built. The development projects under consideration for funding can be arranged in a block diagram as shown in Figure 1. In this Figure, each module of projects in parallel represents a number m_i of projects j that aim to develop the same technology i using different approaches, for example, a Diesel engine versus an Otto engine for propulsion, or a vacuum tube versus a semiconductor for electronic switches.

Figure 1: Development Block Diagram



The next question is whether system Sys is robust enough given the available technologies in each module. The probability Φ of operational success of system Sys during a given time period (e.g., first year of operation) can be calculated using Probabilistic Risk Analysis (PRA) as a function of the probabilities of operational success of the system's components. For ease of calculation we assume that system Sys is a single-string system consisting of N subsystems (or components) in series. Each of these subsystems will be based on exactly one of the newly developed technologies.

The next question is how an increase in the amount of development funds for each project increases its probability of development success.

Let B_t be the development budget available for period t. Budgets B_t are expressed in monetary values discounted at the initial period t=0, normalized as a proportion of the largest (discounted) budget B_τ over time; therefore B_τ=1.0 and $0 \le B_t \le B_\tau$ for all other budgets B_t.[3] It is assumed that budgets B_t cannot be saved (ie., no delayed consumption).

Let $x_{i,j,t}$ be the percentage of the development budget B_t that is allocated to a project j aimed at developing technology i during period t. The $x_{i,j,t}$ are allocated so that $x_{i,j,t} \ge 0$ with $\sum_{i,j} x_{i,j,t} \le B_t$. All project's $x_{i,j,t}$ are grouped in the allocation vector $\mathbf{X}(t) = \{x_{1,1,t}, x_{1,2,t},\ldots,x_{N,nN,t}\}$.

Each project's probability of development success in a given time period t is represented by a probability $d_{i,j,t}(x_{i,j,t})$. Assume that this relationship is time invariant (for a given investment during period t, the probability of success does not increase or decrease with past experience) and that it can be represented by an exponential function of the form:

$$d_{i,j,t}(x_{i,j,t}) = 1 - e^{-a_{i,j}x_{i,j,t}} \quad \text{with } a_{i,j} \ge 1 \tag{1}$$

This assumption is reasonable as it implies that the larger $a_{i,j}$, the larger the probability of development success for project (i,j) for a given budget $x_{i,j,t}$, with a decreasing marginal return of the investment $x_{i,j,t}$ in terms of increase in the probability $d_{i,j,t}$ of development success.

The development state of project j in module i can be described by an indicator variable $I_{i,j}$ (0 if unsuccessful, 1 otherwise). These indicator variables can be arranged in a development state vector $\mathbf{I_{Dev}}(t) = \{I_{1,1,t},\ldots, I_{k,l,t}, \ldots, I_{N,mN,t}\}$ which represents the state of system development at the *start* of period t. Given the development state vector $\mathbf{I_{Dev}}$, we define $S(\mathbf{I_{Dev}})$ as the set of development state vectors U that spans all possible outcomes of the development efforts during period t. At the start of period t, the development state vector $\mathbf{I_{Dev}}(t)$ contains $k \ge 0$ indicator variables with $I_{i,j}$=0, i.e., the technology development program contains k undeveloped projects. At the end of the initial period, any of those projects may have been successfully developed. Therefore, the set $S(\mathbf{I_{Dev}})$ contains 2^k different development state vectors U. If, during period t, no project is successfully developed, then $U=\mathbf{I_{Dev}}(t)$, and therefore, $\mathbf{I_{Dev}}(t+1) = U = \mathbf{I_{Dev}}(t)$.

There are three different rewards R that can be realized during any development period, assuming that system Sys is built (or not) based on technologies developed so far: $R_{OS} > 0$ is the reward for successful development and operation of the system. $R_{OF} < R_{OS}$ is the reward for successful development of the system followed by failure during operation.[4] $R_{DF} < R_{OS}$ is the reward for an unsuccessful development program.[5] While R_{OS} is always positive, both R_{OF} and R_{DF} can be negative. We assume here that the reward for successfully developing system Sys but not building it is zero and larger than R_{DF}. All rewards R are discounted to period t=0 and are expressed as multiples (or fractions) of the largest discounted development budget B_τ*. Finally, we assume that the expected reward for developing system Sys, given that system Sys has not been developed so far, is always larger than the reward for development failure, R_{DF}. We also assume that the expected reward for building system Sys, given its successful development represented by development state $\mathbf{I_{Dev}}$, is always larger than the expected reward for not building system Sys, given development state $\mathbf{I_{Dev}}$.

Let $r_{i,j}$ be the probability of operational success (and $1-r_{i,j}$ the probability of operational failure), during the time period under consideration, of any subsystem based on the technology developed in project j of module i. We assume that this probability is determined by the physical characteristics of the technological approach, which in turn can be assessed through engineering models, statistics or expert opinions based on experience of similar components in other environments, even before the development program is initiated. For instance, the Diesel engine may be assessed to be more reliable than the Otto engine, and the transistor may be assessed to be more reliable than the vacuum tube.

We assume that system Sys is a single-string system, i.e., it will have exactly N subsystems i in series, each based on one realization of its corresponding technology i. Assuming independence of technical performance of the different subsystems, the optimal choice is, among all available alternatives for technology i, that which has the lowest probability of technical failure. The probability $\Phi(\mathbf{I_{Dev}}(t))$ of successful operation of system Sys (after a successful development stage) can thus be calculated by multiplying the largest probability of operational success $r_{i,j}$ of all successfully developed projects in each module.

$$\Phi(I_{Dev}(t)) = \prod_{i=1}^{N} \max_{j} \left[r_{i,j} I_{i,j}(I_{Dev}(t)) \right] \tag{2}$$

The sum $\sum_{w=t}^{T} B_w$ represents the R&D budget that is saved by stopping development efforts at the beginning of period t and building (or not building) system Sys at that point. Once the N technologies have been developed successfully, the rational decision maker builds system Sys if the expected value of the rewards of operating the system $EV_{operation}$ exceeds that of not building the system. This reward is:

$$EV_{Operation}(t) = \Phi(I_{Dev})(R_{OS} - R_{OF}) + R_{OF} + \sum_{w=t}^{T} B_w \tag{3}$$

As a result of Equation 3, we can define a lower bound of the probability of operational success Φ for system Sys. Below this value, the rational decision maker will choose not to build the system because the expected costs of failure during operation outweigh the benefits. This lower bound is:

$$\mathbf{\Phi} = \frac{-R_{OF}}{R_{OS} - R_{OF}} \tag{4}$$

At the beginning of each period t, the decision maker wants to maximize his value function V_t. To do so, he has to choose between stopping development and realizing reward K_t, or continuing development and realizing reward L_t. The (simplified) optimal value function at that point is thus obtained as:

$$V_t(I_{Dev}, X_t) = \max_{X_t} \left[K_t(I_{Dev}(t)); L_t(I_{Dev}(t), X_t) \right] \tag{5}$$

subject to the constraints:

$$\sum_{i,j} x_{i,j,t} \le B_t \text{ and } x_{i,j,t} \ge 0 \tag{6}$$

Focusing now on the computation of K_t and L_t, consider the indicator $I_{Sys}(t)$ (Equation 7), which is equal to zero if the system development has been unsuccessful by the start of period t and one otherwise (i.e., at least one option has been successfully developed for each technology).

$$I_{Sys}(t) = \prod_{i} \left[\max_{1 \le j \le m_i} I_{i,j}(I_{Dev}(t)) \right] \tag{7}$$

The expected reward $K_t(\mathbf{I_{Dev}}(t))$ for stopping development at start of period t is a function of the development state indicator $I_{Sys}(t)$. If system development has been unsuccessful so far (i.e., $I_{Sys}(t) = 0$), the decision maker can earn the expected reward R_{DF} and save the remaining development budgets B_t. If system development has been successful ($I_{Sys}=1$), he can earn the expected reward of operating the system and save the remaining development budgets B_t. This result is shown in Equation (8), where the probability $\Phi(\mathbf{I_{Dev}}(t))$ of operational success for system Sys is provided by Equation (2).

$$K_t(I_{Dev}(t)) = I_{Sys}(t)\big[\Phi(I_{Dev}(t))(R_{OS} - R_{OF}) + R_{OF}\big] + (1 - I_{Sys}(t))R_{DF} + \sum_{w=t}^{T} B_w \tag{8}$$

The expected reward for continuing development $L_t(\mathbf{I_{Dev}}(t))$ requires evaluation of the value function $V_{t+1}(U)$ for each possible development outcome U multiplied by its probability $\varphi(U|\mathbf{I_{Dev}}(t), \mathbf{X}_t)$. We compute the probability of each outcome U by explicitly multiplying the probabilities of successful and unsuccessful development outcomes, as shown in Equation (9), for all projects (i,j) given their development state indicators $I_{i,j}(U)$.

$$d_{i,j,t}(x_{i,j,t}, U) = I_{i,j,t}(U) + (-1)^{I_{i,j,t}(U)} e^{-a_{i,j} x_{i,j,t}} \tag{9}$$

Using Equation (9), we can formulate the probability $\varphi(U|\mathbf{I_{Dev}}(t), \mathbf{X}_t)$ of development outcome U, given initial development state $\mathbf{I_{Dev}}(t)$ and budget allocation vector $\mathbf{X}_t$ as shown in Equation (10).[6] In this equation, the only projects (i,j) considered for funding are those that have not yet been successfully developed by the start of period t, i.e., projects j such that j: $I_{i,j}(\mathbf{I_{Dev}}(t))=0$.

$$\varphi(U)|I_{Dev}(t), X_t) = \prod_{i} \left[\prod_{j:1_{i,j}(1_{Dev}(t))=0} d_{i,j,t}(x_{i,j,t}, U) \right] \tag{10}$$

If he chooses to continue development, the decision maker wants to allocate budget B_t so as to maximize the expected reward $L_t(\mathbf{I_{Dev}}(t), \mathbf{X}_t)$. Therefore, he has to find the (optimal) budget allocation vector $\mathbf{X}_t$*. The corresponding optimization problem is thus:

$$I_{Dev}(t), X_t) = \max_{x_t} \left[\sum_{U \in S(1_{Dev})} \varphi(U|I_{Dev}(t), X_t) V_{t+1}(U) \right] \tag{11}$$

Finally, the reward (value) function at the end of the last period t=T (beginning of T+1) is the expected value of operating the system if the development has been successful and the system indicator $I_{Sys}(T+1)=1$. If development has not been successful and $I_{Sys}(T+1)=0$, the value function at the end of the last period is R_{DF} which can be computed from the following equation:

$$V_{T+1}(I_{Dev}(T+1)) = I_{Sys}(T+1)\left[\Phi(I_{Dev}(T+1))(R_{OS}-R_{OF})+R_{OF}\right]+(1-I_{Sys}(T+1))R_{DF} \qquad (12)$$

Equations 5 to 12 represent the decision support model that is at the core of this paper.

2.3 Development

To provide a benchmark, we first consider the problem where the developer wants to develop system Sys within T time periods and is not concerned about the probability of operational success of the system developed. We refer to this as a “development first” strategy, as opposed to a “robustness first” strategy, where the decision maker is mostly concerned about the probability of operational success of the system Sys to be developed. A funding strategy is a sequence of budget allocation vectors (one for each period), where each budget allocation vector depends on the state of the development program at the beginning of the corresponding budget period. In the case of a “development first” strategy, the decision maker would focus only on the probability of development success d_{ij} of each project (i,j) (determined by parameter $a_{i,j}$) and disregard the probability of operational success $r_{i,j}$. Lemma 1 describes the solution structure of this basic problem.[7]

Lemma 1 *For the development-only allocation problem the decision maker will fund at most one project j* in each development module i where j* = maxj {$a_{i,j}$}.*

This result may not be intuitive because it proposes an “all-eggs-in-one-basket” strategy for each technology module. This result comes from the assumption of an exponential relationship between the allocated budget $x_{i,j,t}$ and the probability of development success $d_{i,j}(x_{i,j,t})$.

The multi-stage decision problem in this setting can be formulated as follows: given development state $\mathbf{I_{Dev}}(t)$ at the beginning of period t, the decision maker can either stop development and realize a reward $K_t(\mathbf{I_{Dev}}(t))$, or he can decide to continue the technology development and realize a reward $L_t(\mathbf{I_{Dev}},\mathbf{X}(t))$ by allocating the period’s budget B_t among the projects according to the budget allocation vector $\mathbf{X}(t)$. Since we assume that the developed system’s probability of operational success is always larger than the threshold value defined in Equation (4), the decision maker will always build and operate the system once he completes the development program successfully.

Under the “development first” strategy in period t, let outcome U* be the one where all funded projects meet with success and accordingly $I_{Sys}=1$ with $V^*_{t+1}(U^*)=L_t(U^*,X_t^*)$, as calculated in Equation (11) (with Φ as given in Equation (2)).

Because of the discounting, the value function $V_{t+1}(U)$ for reaching development outcome U in period t+1 is less than the value function $V_t(U)$ for reaching the same development state one (or more) period(s) earlier, as shown in Equation (13).[8]

$$V_t(U(t), X_t) \geq V_{t+1}(U(t+1), X_{t+1}) \tag{13}$$

As a result of Equation (13) and Lemma 1, during each period t and under the "development first" strategy, the decision maker will fund at most one project j* per technology module i, namely the project with the largest parameter $a_{i,j}* = \max_j \{a_{i,j}\}$ in that module. Using this result, we can now formulate the optimization problem for the "development first" strategy decision problem.

The decision problem is reduced to finding a budget allocation vector $\mathbf{X}_t*$ that maximizes the probability $\varphi(U^*|\mathbf{I}_{\mathbf{Dev}}(t), \mathbf{X}_t)$ that all funded projects are successful. The corresponding optimization problem is given in Equation (14):

$$X_t^*(I_{Dev}(t)) = X_t : \max_{X_t} \varphi(U^*|I_{Dev}(t), X_t) \tag{14}$$

Subject to the constraints:

$$\sum_{i,j} x_{i,j,t} \leq B_t \text{ and } x_{i,j,t} \geq 0 \tag{15}$$

With:

$$\varphi(U^*|I_{Dev}(t), X_t) = \prod_{j^*: I_{i,j^*,t}(I_{Dev}(t))=0} \left[1 - e^{-a_{i,j^*} x_{i,j^*,t}}\right] \tag{16}$$

2.3.1 Solution Behavior

Next, we analyze the behavior of the solution for the "development first" strategy. The objective function of the nonlinear programming problem given in Equation (14) is not necessarily pseudo-concave, which means that a local optimum $\mathbf{X}^*_t$ is necessary, but not sufficient for a global optimum.[9] However, the optimization problem of Equation (14) can be transformed into an optimization problem with a strictly concave objective function (over its domain) by applying a log-transform to $\varphi(U^*|\mathbf{I}_{\mathbf{Dev}},\mathbf{X})$.[10] The transformed optimization problem is given in Equation (17).

$$X_t^*(I_{Dev}(t)) = X_t : \max_{X_t} \sum_{j^*: 1_{i,j^*,t}(1_{Dev}(t))=0} \ln\left(1 - e^{-a_{i,j^*} x_{i,j^*,t}}\right) \tag{17}$$

subject to the constraints:

$$\sum_{i,j} x_{i,j,t} \leq B_t \text{ and } x_{i,j,t} \geq 0 \tag{18}$$

Each term of the objective function in Equation (17) is strictly concave and that equation is defined over a convex set.[11] Therefore, the Karush-Kuhn-Tucker conditions are both necessary and sufficient for a global optimum. Accordingly, any converging numeric algorithm (e.g., quasi-Newton) will yield a global optimum for Equation (17). Since we assume that the decision maker is rational in his choice, and therefore that the probability of operational success Φ is larger than the limit given in Equation (4), we can identify a globally optimal strategy. The decision maker will maximize $V_t(\mathbf{I_{Dev}}(t))$ in each period if budget B_t is allocated so that the Karush-Kuhn-Tucker solution to Equation (17), given in Equation (19), holds.

$$\frac{a_{i,j}e^{-a_{i,j}x_{i,j,t}(I_{Dev})}}{1-a_{i,j}e^{-a_{i,j}x_{i,j,t}(I_{Dev})}} = \frac{a_{k,l}e^{-a_{k,l}x_{k,l,t}(I_{Dev})}}{1-a_{k,l}e^{-a_{k,l}x_{k,l,t}(I_{Dev})}} = \text{constant} \tag{19}$$

The available budget B_t for each period t is allocated so that at most one project per technology module is funded. If, in module i, project j* corresponding to $\max_j a_{i,j}$ has already been developed successfully by the start of period t, module i receives no funding. Otherwise, only project j* of module i receives funding. The available budget B_t is allocated among the chosen projects according to Equation (19). Since the structure of the "robustness first" allocation problem is identical to the "development first" allocation problem (assuming that at least one development will be successful for each technology), the solution characteristics presented in the former case also apply to the latter (replacing j*: $\max_j a_{i,j}$ by j*: $\max_j r_{i,j}$). Again, these characteristics are a consequence of the choice of an exponential function that we made to model the relationship between allocated budget and probability of development success of a project.

2.4 Development and Robustness

We now consider the situation where the decision maker allocates the budget for developing system Sys within T time periods, taking into account the probability of development success $d_{i,j}(x_{i,j,t})$ as well as the probability of operational success $r_{i,j}$ of each development project j of technology i. In this case, the decision maker can trade-off the probability of achieving development success within the given time horizon and the probability of operational success. We refer to this as a "mixed" strategy.

In what follows, we assume that there is no dominance among projects of one technology module, i.e., any ordering $r_{i,j} < r_{i,l}$ implies $a_{i,j} > a_{i,l}$. If there were a dominated project k in module i, i.e., a project k such that $a_{i,j} > a_{i,k}$ and $r_{i,j} > r_{i,k}$, then because of the exponential shape of $d_{i,j}$ and $d_{i,k}$, project (i,j) would always be preferred over project (i,k) and the latter would never be funded. It could be dropped from the portfolio of available projects and would not be considered.

In this setting, the decision maker may choose to continue development even though he already has one successful development project for each technology (i.e., $I_{Sys}=1$). This is the case if the value to be gained by seeking development of a system with higher probability of operational success is greater than that of operating the system based on the currently developed projects. The mathematical formulation for this decision problem is given by Equations (2) through (12). Unlike in the development-only problem, we cannot formulate a closed-form solution here because, in general, the objective function cannot be proved to be pseudo-concave. This complicates the task of finding the optimal budget allocation vector $\mathbf{X}^*$ and we have to resort to numerical methods to do so. This approach will only allow finding local optima and one can never be certain that the identified solution is a global optimum. However, by using upper and lower bounds, one can establish whether a solution is a "good" solution or not.

3. Illustrative Example

To illustrate our model, consider the hypothetical example of the development of a High-Speed Civil Transport plane (HSCT) as representative of a complex and safety-critical engineering system. All data used here are hypothetical.

The only supersonic commercial plane in service, the Concorde, does not meet present environmental standards regarding noise and emissions. Furthermore, its limited capacity makes it difficult to operate the plane profitably. Furthermore, the accident that occurred in Paris in July 2000 makes the availability of this vehicle for future service questionable. Any successor to the Concorde must address safety and environmental issues in a satisfactory manner. Significant progress has to be made in many areas of aerospace technology to achieve these goals. The following areas of technology development needs have been identified by Marvis, Bandte and Brewer [1995]:

- Aerodynamics: The lift-to-drag ratio is one of the most important parameters of large commercial aircraft because it has a significant effect on all fuel-related performance (e.g., fuel consumption and maximum range). This ratio decreases significantly at supersonic speeds, resulting in decreased fuel efficiency. Because the HSCT will operate in subsonic as well as supersonic velocity ranges, there is no simple solution to this challenge. To be successful, the HSCT will have to have better cruise efficiency than the Concorde, and must produce higher lift at lower airspeeds to reduce noise reduction (i.e., less reliance on engine thrust for lift).

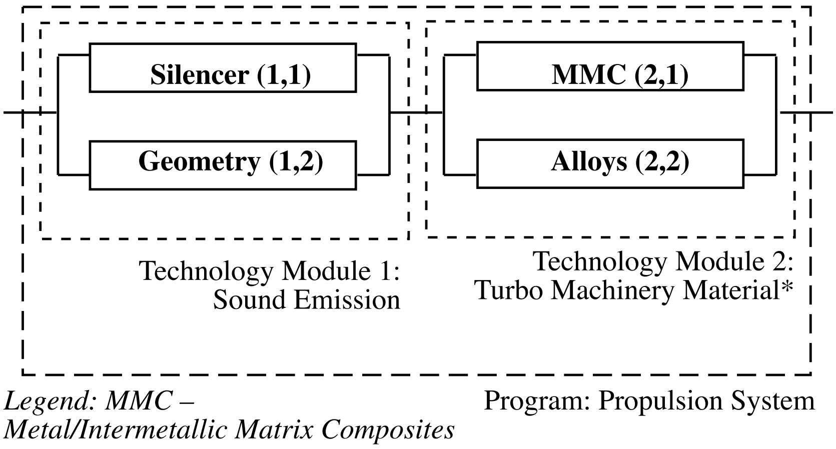
- Airframe: The HSCT has to operate at high altitudes, leading to increased pressure differentials between the cabin and the external environment. Additionally, the main structure of the cabin will be exposed to larger temperature variations (several hundred degrees Fahrenheit) as well as increased radiation from space. Finally, to be economically viable, the plane will have to reach a high level of utilization and safety (i.e., low turn-around time, many passenger miles and system robustness) to be as profitable as subsonic commercial planes, which results in increased loads on the airframe.

- Propulsion: The HSCT must be powered by fuel-efficient and lightweight engines that meet particle- and noise-emission standards while consuming less fuel than the engines of the Concorde. Since the HSCT will also travel at subsonic speeds during descent, engineers will have to balance the conflicting needs of subsonic and supersonic design principles. Combustors will reduce emissions while ensuring long life, new nozzle concepts will reduce noise and weight, material advances will increase the life of turbine blades, and mixed compression inlets will allow stable and efficient operation of the jets over the range of flight speeds.

In our model illustration, we focus on the propulsion sub-system because it is one of the most complex. In particular, we choose, as an example, the development of sound emission reduction technologies and turbo-machinery materials. For each of these development challenges (i.e., technology modules), more than one approach (i.e., development project) are possible.

The decision maker is faced with the problem of developing the propulsion technology within the given time limits and within the allocated budgets. The problem's development block diagram is shown in Figure 2. We assume here that the decision maker is interested in finding the strategy that will maximize his expected revenue for the beginning of the first operating period, subject to budget and time constraints. Again, we assume that the system's performance in the first period of operation after successful development is representative of its robustness in the long run. During each subsequent time period, the decision maker can solve the allocation problem again and, in doing so, incorporate any additional information he might have gathered.

Figure 2. Development block diagram for HSCT propulsion



For each of the four technology development projects shown in Figure 2, experts involved in the project have to be identified to assess, based on their experience, the probability of development and the probability of failure in operations. In a structured interview process, the probabilistic risk analysis (PRA) expert has to elicit the information about the relationship between the portion of the budget allocated to that project's development and its probability of successful development by the end of the period (we assume that clear definitions of what constitutes "successful development" are available during the elicitation process). In a separate step, the experts have to elicit the probabilities of operational success $r_{i,j}$ (i.e., robustness, or conversely of the probability of failure) for each technology I resulting from a development project (I, j).

Once these data have been gathered, one can fit a curve that represents the relationship between allocated development budget and probability of development success. In this case, we used an exponential curve to fit the data points and the following hypothetical data for the model:

- Budgets: All periods *t* have the same budget. Therefore, $B_t = B_\tau = 1.0$ (values discounted to the start of the first period).

- Development Success: The elicited data and the parameters $a_{i,j}$ of the fitted exponential functions linking budget allocation $x_{i,j,t}$ and probability of development success $d_{i,j,t}$ are assumed to be:

  * $a_{1,1} = 3.5$
  * $a_{1,2} = 7.0$
  * $a_{2,1} = 5.0$
  * $a_{2,2} = 8.0$

- Operational Success: The Bernoulli variables for the probability of successful operation of each subsystem i, based on development project j of technology module i, are:

  * $r_{1,1} = 0.85$
  * $r_{1,2} = 0.80$
  * $r_{2,1} = 0.95$
  * $r_{2,2} = 0.80$

- Costs and rewards: The values for the different outcomes are (expressed in present values discounted to the start of period 0)[13]

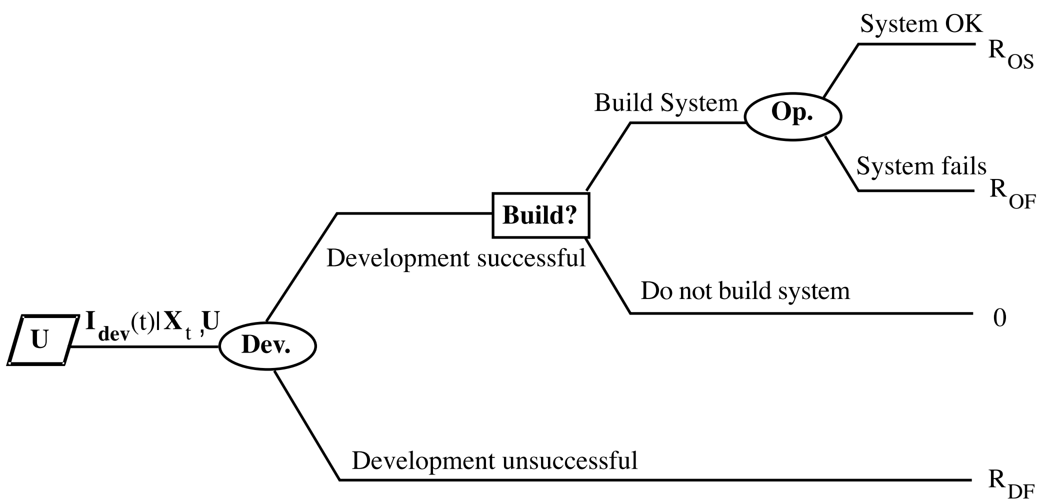
  * R_{OS} = 10.0 for development success and subsequent operation without failure.
  * R_{OF} = –15.0 for development success and subsequent failure during operation.
  * R_{DF} = –1.0 for unsuccessful development.

3.1 Analysis

Without analysis, the decision maker could either follow a "development first" strategy emphasizing development success by time T, or a "robustness first" strategy, focusing on the development of the projects that yield the highest probability of operational success in each technology module. The findings of section 2.3 apply to both of these strategies.

The simplest case would be that of one development period, i.e., T=1. Figure 3 shows a simplified decision tree for this case.

Figure 3: Simplified decision tree for building system Sys within one time period given development state U at the start of that period



The optimal budget allocations for the development strategy alone and the robustness strategy alone require computation of the value function in both cases. The results are: $V_{t=1,\ development}=0.91$ and $V_{t=1,\ robustness}=3.73$. The corresponding budget allocations are $\mathbf{X}_1^*=\{0.000, 0.525, 0.000, 0.475\}$ and $\mathbf{X}_1^*=\{0.552, 0.000, 0.448, 0.000\}$, respectively.[14] The solution vector $\mathbf{X}^*$ of both strategies are given by the closed form solution provided by Equation (19). In one case the projects with the largest parameter $a_{i,j}$ in each module are funded, while in the other case the projects with the largest parameter $r_{i,j}$ in each module are funded.

However, if the decision maker takes into account both development and operational aspects (see section 2.4), he finds a solution that yields an even higher expected value: $V_{t=1}=3.91$ with $\mathbf{X}_1^*=\{0.318, 0.205,0.477, 0.000\}$. The budget allocation vector shows that in this case neither a "development first", nor a "robustness first" strategy can be identified. Instead, in technology module i=1, both projects are funded and a "mixed" strategy is used. The reason for this is that both projects have a relatively low probability of operational success, while their parameters $a_{i,j}$ differ significantly. In module 2, only one project is funded during the first period, i.e., the project with the larger probability of operational success $r_{2,1}$.

Now we assume that the decision maker has two periods to develop the necessary technologies. Again, we calculated the value functions, for the initial period, and for both "development first" and "robustness first" strategies. The results are, respectively, $V_{t=1}=1.95$ with $\mathbf{X}_1^*=\{0.000, 0.525,0.000, 0.475\}$ and $V_{t=1}=5.90$ with $\mathbf{X}_1^*=\{0.561,0.000, 0.439, 0.000\}$ (due to Bellman's principle of optimality the budget allocation vectors $\mathbf{X}_2$ for the second period are identical to the ones found earlier for the one-period problem). Clearly, the "robustness first" strategy produces better results.

If the decision maker takes both development and operational aspects into account, the value function for the initial budgeting period is $V_{t=1}=5.91$ and the budget allocation vector is $\mathbf{X}_1^*=\{0.557, 0.000, 0.443, 0.000\}$. The difference between the "robustness first" and the "mixed" strategy has decreased.

For a time horizon of T=10 the difference between the "robustness first" and the "mixed" strategy has disappeared for all practical purposes: both strategies produce $V_{t=1}=13.94$ with $\mathbf{X}_1^*=\{0.553,0.000, 0.447, 0.000\}$. This is true because during periods one through nine, both the mixed and the robustness-first strategies result in identical budget allocations. Only for the very last period T do they differ. This difference disappears in practice when the values are discounted to period t=1.

These results show that for multi-period decision problems with long time horizons (T >> 1), the decision maker should always choose the "operation first" strategy. Only as he approaches the time horizon T should he switch to a "mixed" strategy that takes both the probability of operational success and the probability of development success into account. Finally, the numerical results also underline the opportunity cost of going for a "quick finish" instead of a high-robustness system.

4. Conclusions

In this paper, we analyzed a budget allocation problem that occurs when tradeoffs between probability of development success and probability of operational success (or safety) of engineering systems under development have to be made. The decision maker has a limited development horizon and needs to achieve development success and a high safety system by that time. We found several interesting results in the results of the model.

First, in the benchmark case of a “development first” approach to R&D (i.e., the decision maker is only concerned with achieving development success as early as possible) an “all eggs in one basket” strategy dominates. The decision maker will fund at most one project per technology module i and the funded project j* will be the one with the largest parameter $a_{i,j}$ in that module. The same result applies to the “robustness first” strategy, where projects j* with the largest parameter $r_{i,j}$ in each module i will be funded. This is counter-intuitive and stands in contrast to previous empirical findings, e.g., Nelson [1961]. It can be explained in part by the decreasing marginal returns characterizing the relationship between the allocated budget and the probability of development success.

Second, when taking into account both the probability of development success and the probability of operational success in the decision process, budget allocation strategies change over time. For the single-period budget allocation problem, the decision maker will allocate funds in a manner that will generally differ from both the “robustness first” and “development first” strategy. As the number of budgeting periods increases, the budget allocation for the initial period t=1 will converge to a “robustness first” strategy, which coincides with general intuition.

Finally, when comparing the results obtained from the two allocation models, we showed that in taking the probability of operational success (or safety) into account from the beginning, the decision maker may save time and money, and obtain a better system sooner.

5. Appendix

5.1 Proof of Dominance

We show here that if in any module i, there is a project such that $a_{i,k} > a_{i,l}$ and $r_{i,k} > r_{i,l}$ for all $l > k$, then all projects $\{l: l=k+1, k+2, ..., n_m\}$ can be removed from the budgeting process.

There exists an ordering for each technology development module i so that $r_{i,k} > r_{i,k+1}$ for all k with $1 \leq k \leq n_i$. Because of the strictly increasing budget probability function $d_{i,j,t}(a_{i,j}, x_{i,j,t})$ any project (j+k) in module i with

$$r_{i,j} > r_{i,j+k} \quad \text{and} \quad a_{i,j} > a_{i,j+k} \tag{20}$$

is dominated by project (i,j) and can be removed from the set of development projects.

Due to the ordering $r_{i,(j)}$ we know that $r_{i,k-1} > r_{i,k}$. The corresponding parameter $a_{i,k}$ can be either smaller, equal to or larger than $a_{i,k-1}$. If it is smaller than or equal to $a_{i,k-1}$, project (i,k) is dominated by project (i,k–1) and can be removed from the set. Therefore, for all projects $1 \leq j \leq (n_i-1)$ in module i, $a_{i,j} < a_{i,j+1}$. The ordering of the $a_{i,k}$ is thus the reverse of the ordering of the $r_{i,(j)}$, where $r_{i,(l)}$ is the ordering of $r_{i,l}$ in decreasing size.

This concludes the proof.

5.2 Proof of Lemma 1

We show here that for the case where the probability of development success $d_{i,j,t}$ has an exponential form, the decision maker will fund at most one project per development module.

Let $d_{i,j} = 1-e^{-a_{i,j}x_{i,j}}$. Let $P_D(i)$ be the probability Φ_D successful development of at least one project in module i, with

$$P_D(i) = 1-\prod_{j=1}^{n_i}(1-d_{i,j,t}) \tag{21}$$

which can be rearranged as:

$$P_D(i) = 1-e^{-\sum_j a_{i,j}x_{i,j,t}} \tag{22}$$

With:

$$x_{i,j,t} \geq 0 \text{ and } a_{i,j} \geq 1 \text{ and } \sum_{j=1}^{n_i} x_{i,j,t} \leq B_t \tag{23}$$

The expression above reaches its maximum for $x_{i,k,t}=B_t$ with $\{k: a_{i,k} = Max_j(a_{i,j})\}$.

This concludes the proof.

5.3 Proof of Karush-Kuhn-Tucker for the One Module Model

We show here that the objective function is concave, defined over a convex set. Therefore, the Karush-Kuhn-Tucker conditions are both necessary and sufficient for a global optimum.

We assume that $r_i \geq r_{i+1}$ for all i and we define $R_k = r_k R_{OS}+(1-r_k)R_{OF}$ as the expected reward for operating system Sys, using the technology developed by project k. Further, we assume a rational development program where the rewards for developing and operating a system that does not exist are larger than those for not doing so. In particular, the reward for successful technology development followed by successful operation is larger than the reward for successful development alone (i.e., the system is developed but not built), which is larger than the reward for development failure: $R_{OS} \geq R_{DS} \geq R_{DF}$. Finally, we assume that the expected reward for developing and operating system Sys is larger than the reward for developing and not operating system Sys: $\Phi(I_{Dev}(t))(R_{OS}-R_{OF}) + R_{OF} \geq R_{DS}$.

Equation (24) formulates the optimization problem for the one module case. The three terms on the right side represent the expected reward for the case that project one is successful, that project two (three, four,..., n-1) is the first successful project and that no development project is successful, respectively.

$$V_t(I_{Dev}(t),X_t) = \max_{x_t} \left[d_t R_1 + \sum_{j=2}^{n} d_j R_j \left[\prod_{k=1}^{j} (1 - d_{k)} \right] + R_{DF} \prod_{k=1}^{n} (1 - d_{k)} \right] \quad (24)$$

subject to the following constraints

$$x_{i,t} \geq 0 \ , \ a_i \geq 1 \ , \ \sum_{j=1}^{n} x_{i,t} \leq B_t \quad (25)$$

$$R_{OS} \geq R_{OF} \quad \text{and} \quad R_{OS} \geq R_{DS} \geq R_{DF} \quad \text{and} \quad r_i(R_{OS} - R_{OF}) + R_{OF} \geq R_{DS} \quad (26)$$

With:

$$d_i = 1 - e^{-a_i x_{i,t}} \quad (27)$$

Using Equations (27), the objective function $V_t(I_{De}v(t), X_t)$ in Equation (24) can be rearranged as follows:

$$V_t(I_{Dev}(t),X_t) = \max_{x_t} \left[R_1 - \sum_{j=1}^{n-1} \left[(R_j - R_{j+1}) e^{-\sum_{k=1}^{j} a_k x_{k,t}} \right] - (R_{n-R_{DF}}) e^{-\sum_{k=1}^{n} a_k x_{k,t}} \right] \quad (28)$$

Each term on the right side of Equation (28) is concave, since $(R_i - R_{i+1}) \geq 0$ for all projects I and all factors $-e^{f(x)}$ are concave. Thus, $V_t(I_{Dev}(t), X_t)$ is a summation of concave functions. Since the concave objective function (28) is defined over the convex set (25), the Karush-Kuhn-Tucker conditions are both necessary and sufficient for global optimality (Bazaraa, Sherali and Shetty [1993]).

This concludes the proof.

5.4 Proof of Karush-Kuhn-Tucker Solution for Multiple Development Modules

We show here that the objective function is concave, defined over a convex set. Therefore, the Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient for global optimality (Bazaraa, Sherali and Shetty [1993]).

Let $d_{i,j} = 1 - e^{-a_{i,j} x_{i,j,t}}$ for all $i \in \{n\}$ and all $j \in \{n_i\}$with $0 \leq x_{i,j,t} \leq B$ and $a_{i,j} \geq 1$.

Lemma 1 *In each technology development module i at most one project j will be funded.*

Using this result, we can now formulate the optimization problem as follows:

$$V^*(X_t) = \max_{x_t} \prod_{i \in N} \left(1 - e^{-a_{i,k} x_{i,k,t}}\right) \quad (29)$$

subject to the constraints:

$$x_{i,j,t} \geq 0 \text{ and } a_{i,j} \geq 1 \text{ and } \sum_{j=1}^{n_i} x_{i,j,t} \leq B_t \tag{30}$$

where k is the index of the project with the largest parameter $a_{i,j}$ in module i . To obtain a concave objective function, we take the natural logarithm ln() of the objective function[16]:

$$\Lambda(V^*) = 1n(V^*(X_t)) = \sum_{i \in N} 1n\left(1 - e^{-a_{i,k} x_{i,k,t}}\right) \tag{31}$$

Each argument $1 - e^{-a_{i,ki} x_{i,ki}}$ in Λ(s) is concave. For $1 - e^{-a_{i,ki} x_{i,k} i} > 0$, each of the summands in Equation (30) is concave (e.g., see Rockafellar [1970]). Therefore, Λ(s) is concave in **X**, and defined over a convex set.[17] The KKT conditions yield the following solution:

$$\frac{a_{i,j} e^{-a_{i,j} x_{i,j,t}(I_{Dev})}}{1 - a_{i,j} e^{-a_{i,j} x_{i,j,t}(I_{Dev})}} = \frac{a_{k,l} e^{-a_{k,l} x_{k,l,t}(I_{Dev})}}{1 - a_{k,l} e^{-a_{k,l} x_{k,l,t}(I_{Dev})}} = \text{const.} \tag{32}$$

This concludes the proof.

Footnotes:

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3 This ensures that we can use the same time invariant function for the relationship between allocated budget and probability of development success across all time periods.
4 It is assumed here that the first period of operation (as defined by the decision maker) alone represents "proof" that the system works.
5 The reward of development failure is generally negative but can be positive if one accounts for lessons learned.
6 If S($\mathbf{I_{Dev}}$(t)), the set of all possible development outcomes for period t, contains 2^k elements U, then we will calculate one probability $\varphi(U|\mathbf{I_{Dev}}(t),\mathbf{X}_t)$ for each of these possible outcomes U. Note that $\sum\varphi = 1$.
7 Proofs can be found in the appendix.
8 This result does not necessarily apply to the "mixed" strategy.
9 See *Nonlinear Programming,* Bazaraa, Sherali and Shetty [1993].
10 Applying a monotone increasing function to the objective function f(**X**) of a maximization problem does not alter the solution vector **X*** of the underlying optimization problem.
11 If f is a (strictly) concave function and ϕ is a strictly increasing function, defined over the same domain as f, then ϕ(f) is a (strictly) concave function.
13 The values for R_{OS}, R_{OF} and R_{DF} can be arbitrary, as long as they meet the constraints of Section 2.
14 All numeric results are rounded to three digits.
15 The final system will employ exactly one project of each technology module, i.e., the one which has the largest reliability $r_{i,k}$ of all successfully developed ones per module i.
16 Applying a monotone increasing function to the objective function of a maximization problem does not alter the location of the optimum, if the transformation is defined over the whole domain of the objective function.
17 Since in each module we fund exactly one project, each of these has to have positive funding for V(s) > 0. Therefore, the ln of V(s) is defined.

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