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MEAN-SEMIVARIANCE BEHAVIOR (II):
THE D-CAPM

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MEAN-SEMIVARIANCE BEHAVIOR (II): THE D-CAPM

Abstract

For over 30 years academics and practitioners have been debating the merits of the CAPM. One of the characteristics of this model is that it measures risk by beta, which follows from an equilibrium in which investors display mean-variance behavior. In that framework, risk is assessed by the variance of returns, a questionable and restrictive measure of risk. The semivariance of returns is a more plausible measure of risk and can be used to generate an alternative behavioral hypothesis (mean-semivariance behavior), an alternative measure of risk for diversified investors (the downside beta), and an alternative pricing model (the D-CAPM). The empirical evidence discussed in this article for the entire MSCI database of developed and emerging markets clearly supports the downside beta and the D-CAPM over beta and the CAPM.

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MEAN-SEMIVARIANCE BEHAVIOR (II): THE D-CAPM

I. Introduction

For over 30 years academics and practitioners have been debating the merits of the CAPM, focusing on whether beta is an appropriate measure of risk. Most of these discussions are by and large empirical; that is, they focus on comparing the ability of beta to explain the cross-section of returns to that of alternative risk variables. Most of these discussions, however, overlook where beta as a measure of risk comes from, namely, from an equilibrium in which investors display mean-variance behavior (MVB). In other words, from an equilibrium in which investors maximize a utility function that depends on the mean and variance of returns of their portfolio.

The variance of returns, however, is a questionable measure of risk for at least two reasons. First, it is an appropriate measure of risk only when the underlying distribution of returns is symmetric. And second, it can be applied straightforwardly as a risk measure only when the underlying distribution of returns is Normal. However, both the symmetry and the normality of stock returns are seriously questioned by the empirical evidence on the subject.

The semivariance of returns, on the other hand, is a more plausible measure of risk for several reasons: First, investors obviously do not dislike upside volatility; they only dislike downside volatility. Second, the semivariance is more useful than the variance when the underlying distribution of returns is asymmetric and just as useful when the underlying distribution is symmetric; in other words, the semivariance is at least as useful a measure of risk as the variance. And third, the semivariance combines into one measure the information provided by two statistics, variance and skewness, thus making it possible to use a one-factor model to estimate required returns.

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Furthermore, the semivariance of returns can be used to generate an alternative behavioral hypothesis, mean-semivariance behavior (MSB). As shown in Estrada (2002*b*), MSB is almost perfectly correlated with expected utility (and with the utility of expected compound return) and can therefore be defended along the same lines used by Levy and Markowitz (1979) and Markowitz (1991) to defend MVB.

In this article, building on my previous article on MSB (Estrada, 2002*b*), I propose an alternative measure of risk for diversified investors, the downside beta, and an alternative pricing model, the downside CAPM, or D-CAPM for short. I also report evidence from joint and separate samples of developed markets (DMs) and emerging markets (EMs) supporting the downside beta over beta, and, therefore, the D-CAPM over the CAPM.

The rest of the article is organized as follows. Part II discusses the theoretical framework by drawing a parallel between MVB and the CAPM, on the one hand, and MSB and the D-CAPM, on the other. Part III reports and discusses the empirical evidence, which clearly supports the downside beta and, by extension, the D-CAPM and MSB. Finally, part IV contains some concluding remarks. An appendix with exhibits concludes the article.

II- Formal framework: the D-CAPM

I discuss in this part, first, the traditional MVB framework, the pricing model that follows from it (the CAPM), and the relevant magnitudes of this joint framework. Then I discuss the alternative MSB framework, the pricing model that follows from it and that is proposed in this article (the D-CAPM), and the relevant magnitudes of this joint alternative framework. Then I briefly discuss how to estimate the downside beta, the magnitude proposed in this article to replace beta. And, finally, I briefly compare the D-CAPM with previous models based on downside risk proposed in the literature.

1) MVB and the CAPM

In the standard MVB framework, an investor's utility (U) is fully determined by the mean (μ_p) and variance (σ_p^2) of returns of the investor's portfolio; that is, $U = U(\mu_p , \sigma_p^2)$. In such framework, the risk of an asset i taken individually is measured by the asset's standard deviation of returns (σ_i), which is given by

$$\sigma_i = \sqrt{E[(R_i - \mu_i)^2]}\,, \qquad [1]$$

where R and μ represent returns and mean returns, respectively. However, when asset i is just one out of many in a fully diversified portfolio, its risk is measured by its covariance with respect to the market portfolio (σ_{iM}), which is given by

$$\sigma_{iM} = E[(R_i - \mu_i)(R_M - \mu_M)]\,, \qquad [2]$$

where M indexes the market portfolio. Because the covariance is both unbounded and scale-dependent, its interpretation is not straightforward. A more useful measure of risk can be obtained by dividing this covariance by the product of asset i's standard deviation of returns and the market's standard deviation of returns, thus obtaining asset i's correlation with respect to the market (ρ_{iM}), which is given by

$$\rho_{iM} = \frac{\sigma_{iM}}{\sigma_i \cdot \sigma_M} = \frac{E[(R_i - \mu_i)(R_M - \mu_M)]}{\sqrt{E[(R_i - \mu_i)^2] \cdot E[(R_M - \mu_M)^2]}} \ , \qquad [3]$$

Alternatively, the covariance between asset *i* and the market portfolio can be divided by the variance of the market portfolio, thus obtaining asset *i*'s beta (β_i), which is given by

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{E[(R_i - \mu_i)(R_M - \mu_M)]}{E[(R_M - \mu_M)^2]} \ , \qquad [4]$$

This beta can also be expressed as β_i = $(\sigma_i/\sigma_M)\rho_{iM}$ and is the most widely used measure of risk. It is also the only firm-specific magnitude in the model most widely used to estimate required returns on equity, the CAPM, which is given by

$$E(R_i) = R_f + MRP \cdot \beta_i \ , \qquad [5]$$

where $E(R_i)$ and R_f denote the required return on asset *i* and the risk-free rate, respectively, and MRP denotes the market risk premium, defined as $MRP = E(R_M)\text{-}R_f$, where $E(R_M)$ denotes the required return on the market.

2) MSB and the D-CAPM

In the alternative MSB framework, the investor's utility is given by $U = U(\mu_P, \Sigma_P^2)$, where Σ_P^2 denotes the *downside* variance of returns (or semivariance for short) of the investor's portfolio. In this framework, the risk of an asset *i* taken individually is measured by the asset's *downside* standard deviation of returns, or semideviation (Σ_i) for short, which is given by

$$\acute{O}_i = \sqrt{E\{\text{Min}[(R_i - \mu_i)\ ,0]^2\}} \, . \qquad [6]$$

Expression [6] is, in fact, a special case of the semideviation, which can be more generally expressed with respect to any benchmark return *B* (Σ_{Bi}) as

$$\acute{O}_{Bi} = \sqrt{E\{\text{Min}[(R_i - B)\ ,0]^2\}} \, . \qquad [7]$$

Given that throughout this article we will use as the only benchmark for asset *i* the arithmetic mean of its distribution of returns, we will denote the semideviation of asset *i* simply as Σ_i.

In a downside risk framework, the counterpart of asset *i*'s covariance to the market portfolio is given by its downside covariance, or cosemivariance (Σ_{iM}) for short, which is given by

$$\acute{O}_{iM} = E\{\text{Min}[(R_i - \mu_i)\ ,0] \cdot \text{Min}[(R_M - \mu_M)\ ,0]\} \, . \qquad [8]$$

This cosemivariance is also unbounded and scale-dependent, but it can also be standardized by dividing it by the product of asset *i*'s semideviation of returns and the market's semideviation of returns, thus obtaining asset *i*'s downside correlation (Θ_{iM}), which is given by

$$\grave{E}_{iM} = \frac{\acute{O}_{iM}}{\acute{O}_i \cdot \acute{O}_M} = \frac{E\{\text{Min}[(R_i - \mu_i)\,,0] \cdot \text{Min}[(R_M - \mu_M)\,,0]\}}{\sqrt{E\{\text{Min}[(R_i - \mu_i)\,,0]^2\} \cdot E\{\text{Min}[(R_M - \mu_M)\,,0]^2\}}} \ . \qquad [9]$$

Alternatively, the cosemivariance can be divided by the market's semivariance of returns, thus obtaining asset *i*'s downside beta (β_i^D), which is given by

$$\beta_i^D = \frac{\acute{O}_{iM}}{\acute{O}_M^2} = \frac{E\{\text{Min}[(R_i - \mu_i)\,,0] \cdot \text{Min}[(R_M - \mu_M)\,,0]\}}{E\{\text{Min}[(R_M - \mu_M)\,,0]^2\}} \ . \qquad [10]$$

This downside beta, which can also be expressed as $\beta_i^D = (\Sigma_i / \Sigma_M)\Theta_{iM}$, can be articulated into a CAPM-like model based on downside risk. Such model, which is the one proposed in this article, is the *downside CAPM*, or *D-CAPM* for short, and is given by

$$E(R_i) = R_f + MRP{\cdot}\beta_i^D \ . \qquad [11]$$

As can be seen by a straightforward comparison of [5] and [11], the D-CAPM replaces the beta of the CAPM by the downside beta, the appropriate measure of systematic risk in a downside risk framework.

3) A Brief Digression on the Downside Beta

The downside beta of any asset *i* given by [10] can be estimated in at least three ways: First, by dividing the cosemivariance between asset *i* and the market given by [8] by the semivariance of the market given by [6] for $i = M$; that is, $\beta_{iD} = \Sigma_{iM}/\Sigma_{M2}$. Alternatively, it can be estimated by multiplying the ratio of semideviations of asset *i* and the market, the former given by [6] and the latter given by [6] for $i = M$, by the downside correlation between asset *i* and the market, given by [9]; that is, $\beta_{iD} = (\Sigma_i/\Sigma_M)\Theta_{iM}$. Both estimates are mathematically equivalent due to the fact that $\Theta_{iM} = \Sigma_{iM}/(\Sigma_i{\cdot}\Sigma_M)$; hence, $\beta_{iD} = \Sigma_{iM}/\Sigma_{M2} = \Sigma_i{\cdot}\Sigma_M{\cdot}\Theta_{iM}/\Sigma_{M2} = (\Sigma_i/\Sigma_M)\Theta_{iM}$.

Finally, the downside beta of any asset *i* can be estimated using regression analysis, although this estimation is a bit tricky for the following reason. Let $y_t = Min[(R_{it} - \mu_i)\,,\,0]$ and $x_t = Min[(R_{Mt} - \mu_M)\,,\,0]$, and let μ_y and μ_x be the mean of y_t and the mean of x_t, respectively. If a regression is run with y_t as the dependent variable and x_t as the independent variable (that is, $y_t = l0 + \lambda 1{\bullet}x_t + \varepsilon_t$, where ε is an error term and λ_0 and λ_1 are coefficients to be estimated), the estimate of λ_1 would be given by

$$\lambda_1 = \frac{E[(x_t - \mu_x)(y_t - \mu_y)]}{E[(x_t - \mu_x)^2]} \ . \qquad [12]$$

However, note that according to [10], β_{iD} should be given by

$$\beta_i^D = \frac{E[x_t \cdot y_t]}{E[x_t{}^2]} \ . \qquad [13]$$

In other words, the appropriate way to estimate β_i^D using regression analysis is to run a simple linear regression *without a constant* between the dependent variable $y_t = Min[(R_{it} - \mu_i)$, 0] and the independent variable $x_t = Min[(R_{Mt} - \mu_M)\ ,\ 0]$, and obtaining the downside beta as the slope of such regression; that is, run $y_t = \lambda_1 \cdot x_t + \varepsilon_t$, and obtain $\beta_{iD} = \lambda_1$.

4) A Brief Digression on the Downside Risk Framework

Hogan and Warren (1974), Bawa and Lindenberg (1977), and Harlow and Rao (1989) all proposed CAPM-like models based on downside risk measures. Hogan and Warren (1974) called their framework the E-S model and defined a downside beta based on a different definition of cosemivariance; their cosemivariance ($Ó_{iM}^{HW}$) is given by

$$Ó_{iM}^{HW} = E\{(R_i - R_f)\cdot \text{Min}[(R_M - R_f)\ ,0]\}\ . \tag{14}$$

There are three main differences between [14] and [8]. First, under [8], a security adds to the risk of a portfolio only when $R_i<\mu_i$ and $R_M<\mu_M$; under [14], a security adds to the risk of a portfolio when $R_i<\mu_i$ and $R_M<\mu_M$, but reduces the risk of the portfolio when $R_i>\mu_i$ and $R_M<\mu_M$. Second, the benchmark return in [14] is the risk-free rate, whereas the benchmark return in [8] is the mean of each relevant distribution. And third, under [14], the cosemivariance between any two assets *i* and *j* is different from the cosemivariance between assets *j* and *i*, which is an obvious weakness of this definition of cosemivariance (1).

Bawa and Lindenberg (1977) generalize the Hogan-Warren framework and show that, since the CAPM is a special case of their mean-lower partial moment (MLPM) model, their model is guaranteed to explain the data at least as well as the CAPM. In the Bawa-Lindenberg framework, just like in the Hogan-Warren framework, the risk-free rate is the benchmark return in the cosemivariance, and the cosemivariance between any two assets *i* and *j* is different from that between assets *j* and *i*.

Finally, Harlow and Rao (1989) derive an MLPM model for any arbitrary benchmark return, thus rendering the Hogan-Warren and the Bawa-Lindenberg frameworks special cases of their more general model. Their empirical tests reject the CAPM as a pricing model but cannot reject their version of the MLPM model. Interestingly, they argue that the relevant benchmark return that seems to be implied by the data is the mean of the distribution of returns rather than the risk-free rate.

More recently, Estrada (2000, 2001, 2002*a*) proposed to replace the CAPM beta by the ratio between an asset's semideviation of returns and the market's semideviation of returns, and showed that this measure of *total* downside risk explains the cross-section of returns of emerging markets, industries in emerging markets, and Internet stocks. The main difference between the measure of risk proposed in those three articles, the semideviation, and the one proposed here, the downside beta, is that the downside beta is a measure of *systematic* downside risk. For a review of some of the models proposed to estimate required returns on equity in emerging markets, see Estrada (2000) and Pereiro (2001). For a general review of downside risk, see Nawrocki (2000) and Sortino and Satchell (2001).

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(1) The main problem with the Hogan-Warren definition of cosemivariance is that, if the cosemivariance between assets *i* and *j* is different from that between assets *j* and *i*, then it is far from clear how the contribution of each of these two assets to cosemivariance risk should be interpreted. Levkoff (1982) provides a numerical example illustrating the asymmetry between these two cosemivariances.

III- Empirical evidence

The data used in this article are from the Morgan Stanley Capital Indices (MSCI) database of both DMs and EMs available at the end of the year 2001. This database contains monthly data on 23 DMs and 27 EMs for varied sample periods. Because betas and downside betas are computed with respect to the MSCI All Country World Index and this index starts in Jan/1988, that is as far back as the longest time series in this article go; all DMs and some EMs go as far back as Jan/1988, but some other EMs start later. Summary statistics for all markets, together with the earliest month for which data is available for each market, are reported in Exhibit 1 in the appendix.

1) Statistical Significance: The Full Sample

The first step of the analysis consists of computing, over the whole sample period considered for each market, one statistic that summarizes the average (return) performance of each market, and another number that summarizes its risk under each of the four definitions discussed below. Average returns over the whole sample period are summarized by mean monthly arithmetic returns; these estimates are reported in Exhibit 1.

The four risk variables considered are: two for the standard MVB framework (the standard deviation and beta) and two for the alternative MSB framework (the semideviation and downside beta), all four as defined above. An estimate of each of these four variables for each of the 50 markets in the sample is computed over the whole sample period considered for each market; these estimates are also reported in Exhibit 1 (2).

A correlation matrix containing mean returns and the four risk variables under consideration is reported in Table 1 below and provides a preview of some results analyzed in more detail later on. As can be seen in the table, the downside risk measures (the semideviation and the downside beta) outperform the standard risk measures (the standard deviation and beta). In fact, the downside beta, the risk variable proposed in this article to replace beta in the CAPM, thus giving rise to the D-CAPM, outperforms all other risk variables considered (3).

Table 1. Full Sample. Correlation Matrix

	MR	σ	β	Σ	β^D
MR	1.00				
σ	0.58	1.00			
β	0.54	0.53	1.00		
Σ	0.59	0.98	0.59	1.00	
β^D	0.69	0.88	0.83	0.90	1.00

MR: Mean return; σ: Standard deviation; β: Beta (with respect to the world market); Σ: Semideviation; β^D: Downside beta (with respect to the world market).

(2) Semideviations and downside betas for emerging markets are periodically updated in (and can be downloaded from) the 'Emerging Markets' link of my Web page (http://web.iese.edu/jestrada).
(3) It may be interesting to note from Exhibit 1 that, although the standard deviation and the semideviation are almost perfectly correlated (0.98), beta and the downside beta are not as highly correlated (0.83).

More detailed results about the relationship between risk and return across markets can be obtained from regression analysis. I start by running a cross-sectional simple linear regression model relating mean returns to each of the four risk variables considered. More precisely,

$$M_{Ri} = \gamma_0 + \gamma_1 RV_i + u_i \,, \tag{15}$$

where MR_i and RV_i stand for mean return and risk variable, respectively, γ_0 and γ_1 are coefficients to be estimated, u_i is an error term, and i indexes markets. The results of the four regression models (one for each of the four risk variables considered) are reported in panels A and B of Table 2.

Table 2. Full Sample. Simple Regression Analysis

$MR_i = \gamma_0 + \gamma_1 RV_i + u_i$						
RV	γ_0	*t-stat*	γ_1	*t-stat*	R^2	*Adj-*R^2
Panel A: *OLS Estimation*						
σ	0.06	0.28	0.11	4.87	0.33	0.32
β	0.11	0.48	1.04	4.49	0.30	0.28
Σ	–0.07	–0.27	0.18	5.01	0.34	0.33
β^D	–0.45	–1.83	1.24	6.58	0.47	0.46
Panel B: *Heteroskedasticity-Consistent Estimation*						
σ	0.06	0.30	0.11	4.31	0.33	0.32
β	0.11	0.52	1.04	4.50	0.30	0.28
Σ	–0.07	–0.30	0.18	4.70	0.34	0.33
β^D	–0.45	–2.16	1.24	7.17	0.47	0.46

MR: Mean return; *RV*: Risk variable; σ: Standard deviation; β: Beta (with respect to the world market); Σ: Semideviation; β^D: Downside beta (with respect to the world market). Significance on Panel B based on White's heteroskedasticity-consistent covariance matrix. Critical value for a two-sided test at the 5% significance level: 2.01.

Panel A presents the result of OLS regressions, two of which display the presence of heteroskedasticity. Panel B presents the results of regressions in which significance is based on White's heteroskedasticity-consistent covariance matrix. The results in both panels are similar and the qualitative conclusions are the same: All four risk variables are clearly significant (though they differ in their explanatory power). As anticipated in Table 1, the downside risk variables outperform in terms of explanatory power the two standard variables. The downside beta, in fact, explains a substantial 47% of the variability in mean returns across international markets.

Exhibit 2 in the appendix reports the results of three multiple regressions: The standard deviation and the semideviation together; beta and downside beta together; and the four risk variables all together. As can be seen in that exhibit, when beta and downside beta are considered together, it is only the latter that comes out significant. When the four risk variables are considered all together, again the downside beta is the only variable that comes out significant. In other words, the variable proposed in this article to replace beta as the single explanatory variable of the cross-section of stock returns does clearly outperform beta (and the other risk variables) in terms of explanatory power (4).

2) Statistical Significance: Developed Markets and Emerging Markets

I consider in this section two subsamples, one of DMs and the other of EMs, and re-assess the significance and explanatory power of each of the four risk variables considered. Note that a downside risk framework makes more sense the more skewed the distributions of returns are. If all distributions were symmetric, then the semideviation and the standard deviation would contain the same information (5), and MSB would lose most of its appeal as a behavioral model. As Exhibit 1 shows, the distributions of returns are much more skewed in EMs than in DMs; hence, the downside risk measures are expected to perform better in EMs.

Table 3 reports the results of simple linear regression models splitting the sample into DMs (panel A) and EMs (panel B). As the figures clearly show, unsurprisingly, all risk measures perform much better in EMs than in DMs. In fact, no risk measure significantly explains the cross-section of returns in DMs; the best-performing variable is the downside beta, but it is non-significant and explains only 8% of the variability in returns. In EMs, in contrast, all four risk variables are clearly significant and explain no less than one third of the variability in returns. The downside beta is, again, the best performing variable, explaining a whopping 55% of the variability in mean returns.

Table 3. DMs v. EMs. Simple Regression Analysis

$$MR_i = \gamma_0 + \gamma_1 RVi + u_i$$

RV	γ_0	*t-stat*	γ_1	*t-stat*	R^2	*Adj-*R^2
Panel A: *DMs*						
σ	0.63	1.95	0.05	1.12	0.06	0.01
β	0.76	1.99	0.22	0.55	0.01	-0.03
Σ	0.54	1.46	0.09	1.18	0.06	0.02
β^D	0.40	0.89	0.54	1.31	0.08	0.03
Panel B: *EMs*						
σ	–0.79	–1.81	0.16	4.79	0.48	0.46
β	0.11	0.34	1.14	3.75	0.36	0.33
Σ	–1.01	–2.20	0.28	5.00	0.50	0.48
β^D	–0.80	–2.11	1.42	5.57	0.55	0.54

MR: Mean return; *RV*: Risk variable; σ: Standard deviation; β: Beta (with respect to the world market); Σ: Semideviation; β^D: Downside beta (with respect to the world market). Critical values for a two-sided test at the 5% significance level: 2.08 and 2.06 in panels A and B, respectively.

(4) The fact that, when jointly considered, neither the standard deviation nor the semideviation are significant is likely to be explained by the fact that both are highly correlated (0.98).
(5) Note that, for symmetric distributions, it is the case that $2\Sigma^2 = \sigma^2$.

Exhibit 3 in the appendix reports the results of multiple linear regression models, again splitting the sample in DMs and EMs. The results in that exhibit confirm that none of the four risk variables has a significant explanatory power in DMs, though again the downside beta is the best performing variable in terms of significance. In EMs, on the other hand, the downside beta is significant when jointly considered with beta (which comes out non-significant), and is also significant when jointly considered with the other three risk variables (none of which comes out significant).

3) Economic Significance: Spreads in Risk and Return

In order to check the robustness of the results discussed in the previous section, I divided all markets into three equally weighted portfolios ranked by beta, and calculated the spread in mean returns between the riskiest portfolio (the one with the largest betas) and the least risky portfolio (the one with the lowest betas). Then I repeated the process by ranking the portfolios by downside beta and calculating again the spread between the riskiest portfolio (the one with the largest downside betas) and the least risky portfolio (the one with the lowest downside betas). Finally, I repeated the whole process for split samples of DMs and EMs. The relevant results are reported in Table 4.

Table 4. Portfolios

	β	*MR*	β^D	*MR*
Panel A: All Markets				
P1	1.35	1.43	1.71	1.65
P2	0.88	1.01	1.16	0.71
P3	0.53	0.77	0.82	0.87
Spread P1-P3	0.81	0.66	0.89	0.78
Annualized Spread		**8.20**		**9.71**
Relative Spread		**0.81**		**0.87**
Panel B: DMs				
P1	1.17	1.03	1.29	1.12
P2	0.87	0.91	1.01	0.83
P3	0.73	0.98	0.86	0.97
Spread P1-P3	0.44	0.06	0.43	0.15
Annualized Spread		**0.70**		**1.77**
Relative Spread		**0.13**		**0.34**
Panel C: EMs				
P1	1.48	1.65	1.97	2.15
P2	0.88	1.34	1.36	0.84
P3	0.41	0.51	0.83	0.51
Spread P1-P3	1.07	1.14	1.14	1.63
Annualized Spread		**14.60**		**21.48**
Relative Spread		**1.07**		**1.43**

Portfolio 1 (P1) is the riskiest portfolio (largest betas or largest downside betas); Portfolio 3 (P3) is the least risky portfolio (lowest betas or lowest downside betas). *MR*: Mean Return; β: Beta (with respect to the world market); β^D: Downside beta (with respect to the world market). "Relative Spread" defined as the ratio between the "Spread P1-P3" in mean returns and the "Spread P1-P3" in the risk measure. MR in %.

Focusing on the joint sample of DMs and EMs first (Panel A), there does not seem to be a large difference in the spread of the two risk variables between portfolios 1 and 3: The difference between betas is 0.81 and that between downside betas is 0.89. Note, however, that the average beta of portfolio 1 (1.35) is over two and a half times larger than the average beta of portfolio 3 (0.53), whereas the average downside beta of portfolio 1 (1.71) is just over twice as large as the average downside beta of portfolio 3 (0.82).

In terms of mean returns, the spread between portfolios 1 and 3 when ranked by beta is 0.66% a month (8.20% annualized), whereas the spread between these two portfolios when ranked by downside beta is a bit larger, 0.78% a month (9.71% annualized). In other words, return differences spanned by downside beta are larger than return differences spanned by beta by roughly 150 basis points a year. Furthermore, dividing the spread in monthly mean returns by the spread in the risk measure we obtain the relative spread, which is 0.81 in the case of portfolios ranked by betas and 0.87 in the case of portfolios ranked by downside betas; that is, mean returns are more sensitive to differences in downside beta than to equal differences in betas.

Panel B shows that the rather small advantage of downside beta over beta in spanning a difference in returns across international markets is due mostly to the poor performance of both risk measures in DMs. A virtually identical spread in beta and downside beta spans a spread in monthly mean returns of 0.06% (0.70% annualized) and 0.15% (1.77% annualized), respectively. However small, it is still the case that the relative spread spanned by downside beta (0.34) is almost three times as large as that spanned by beta (0.13).

Finally, panel C shows that the downside beta clearly outperforms beta in emerging markets. The average beta of portfolio 1 is over three and a half times larger than the average beta of portfolio 3, and that spread spans a difference in mean monthly returns of 1.14% (14.60% annualized). The average downside beta of portfolio 1, however, is less than two and a half times larger than the average downside beta of portfolio 3, and that spread spans a difference in mean monthly returns of 1.63% (21.48% annualized). In other words, return differences spanned by downside beta are larger than return differences spanned by beta by almost 700 basis points a year. Finally, as evidenced by the relative spreads, mean returns are over 30% more sensitive to differences in downside beta than to equal differences in beta (1.43 versus 1.07).

4) Required Returns on Equity: The CAPM v. the D-CAPM

A brief recap is in order at this point. The results reported and discussed so far indicate that, when considering the joint sample of DMs and EMs, 1) the downside risk variables outperform the standard risk variables when explaining the cross-section of returns; 2) the variable that best explains the cross-section of returns is the downside beta ($R^2 = 0.47$); 3) the only variable that significantly explains the cross-section of returns when all risk variables are jointly considered is the downside beta; and 4) mean returns are more sensitive to variations in downside beta than to equal variations in beta.

When considering split samples of DMs and EMs, on the other hand, the results indicate that 5) none of the four risk variables significantly explains the cross-section of returns in DMs, though the four of them significantly explain the cross-section of returns in EMs; 6) the downside beta is the variable that best explains the cross-section of returns in both DMs ($R^2 = 0.08$) and EMs ($R^2 = 0.55$); 7) when jointly considered, none of the four risk variables significantly explains the cross-section of returns in DMs, and only the downside beta does in EMs; and 8) mean returns are more sensitive to variations in downside beta than to equal variations in beta in both DMs and EMs, the difference in sensitivity being much larger in EMs than in DMs.

I turn now to compare the required returns on equity generated by the CAPM, based on beta and given by [5]), and the D-CAPM, based on the downside beta and given by [11]. In both cases, a risk-free rate of 5.03% and a market risk premium of 5.5% are used (6). The estimates for all markets in the sample are reported in Table 5 below, which shows several interesting results.

First, the average beta is almost the same in DMs and EMs (0.93 versus 0.92), though the average downside beta is about 30% larger in EMs (1.38) than in DMs (1.06). Second, and following from the previous result, the average required return on equity in DMs (10.14%) is slightly higher than that in EMs (10.11%) according to the CAPM, though it is larger in EMs (12.65%) than in DMs (10.86%) according to the D-CAPM. In this regard, the results generated by the D-CAPM, unlike those generated by the CAPM, are in line with the intuition that EMs are riskier than DMs. And third, the difference in the required returns on equity generated by the CAPM and the D-CAPM is rather small in DMs (72 basis points a year) but much larger in EMs (254 basis points a year) (7). This result was expected, for, as Exhibit 1 shows, the distributions of returns in EMs are more skewed than those in DMs. In other words, replacing the CAPM with the D-CAPM becomes more relevant the more skewed the underlying distributions of returns are.

(6) The 5.03% risk-free rate is the yield on 10-year U.S. Treasury Notes on Dec/31/2001. The 5.5% market risk premium is similar to that used by Stulz (1995).

(7) Note that in about one third of EMs the difference in required returns generated by the CAPM and the D-CAPM is larger than 300 basis points a year. For a more detailed analysis of these and other results related to EMs, see Estrada (2002*c*).

Table 5. Required Returns on Equity. CAPM v. D-CAPM

Market	***β***	***β^D***	***CAPM***	***D-CAPM***	***Difference***
Australia	0.77	0.89	9.28	9.93	0.65
Austria	0.63	0.98	8.49	10.43	1.94
Belgium	0.69	0.78	8.81	9.30	0.49
Canada	0.89	0.98	9.93	10.43	0.50
Denmark	0.76	0.89	9.19	9.92	0.73
Finland	1.29	1.43	12.13	12.89	0.76
France	0.94	1.02	10.22	10.62	0.40
Germany	0.95	1.14	10.24	11.28	1.04
Greece	0.72	1.28	9.00	12.06	3.06
Hong Kong	1.19	1.26	11.57	11.95	0.38
Ireland	0.90	0.96	9.97	10.32	0.34
Italy	0.88	1.04	9.86	10.73	0.87
Japan	1.29	1.21	12.14	11.67	–0.47
Netherlands	0.82	0.90	9.55	9.99	0.44
New Zealand	0.84	1.06	9.64	10.88	1.24
Norway	0.95	1.13	10.25	11.24	0.99
Portugal	0.74	0.86	9.08	9.76	0.68
Singapore	1.32	1.42	12.28	12.82	0.55
Spain	1.07	1.19	10.93	11.59	0.66
Sweden	1.27	1.40	11.99	12.72	0.73
Switzerland	0.81	0.90	9.48	9.96	0.48
UK	0.87	0.83	9.82	9.60	–0.22
USA	0.79	0.86	9.40	9.77	0.37
Argentina	0.66	1.82	8.64	15.06	6.42
Brazil	1.44	2.16	12.96	16.92	3.95
Chile	0.57	0.95	8.18	10.25	2.08
China	1.13	1.39	11.24	12.67	1.43
Colombia	0.32	0.81	6.80	9.49	2.69
Czech Rep.	0.66	1.29	8.68	12.15	3.47
Egypt	0.53	0.90	7.93	9.98	2.05
Hungary	1.53	1.91	13.42	15.52	2.10
India	0.54	1.10	8.02	11.06	3.04
Indonesia	0.97	1.60	10.37	13.85	3.48
Israel	0.63	0.87	8.49	9.82	1.33
Jordan	0.11	0.32	5.66	6.77	1.11
Korea	1.25	1.34	11.89	12.42	0.53
Malaysia	1.02	1.33	10.65	12.34	1.69
Mexico	1.12	1.47	11.21	13.12	1.91
Morocco	-0.12	0.39	4.38	7.18	2.80
Pakistan	0.49	1.00	7.74	10.54	2.80
Peru	0.74	1.19	9.12	11.60	2.47
Philippines	1.10	1.40	11.06	12.73	1.67
Poland	1.66	2.02	14.16	16.12	1.97
Russia	2.69	2.85	19.82	20.69	0.87
South Africa	1.10	1.33	11.10	12.34	1.24
Sri Lanka	0.61	1.11	8.37	11.14	2.76
Taiwan	0.87	1.49	9.79	13.25	3.46
Thailand	1.41	1.75	12.79	14.64	1.85
Turkey	1.04	2.13	10.77	16.74	5.97
Venezuela	0.85	1.46	9.69	13.07	3.38
Avg. DMs	**0.93**	**1.06**	**10.14**	**10.86**	**0.72**
Avg. EMs	**0.92**	**1.38**	**10.11**	**12.65**	**2.54**
Avg. All	**0.93**	**1.24**	**10.12**	**11.83**	**1.70**

β: Beta (with respect to the world market); β^D: Downside beta (with respect to the world market). Required returns on equity for the CAPM and the D-CAPM follow equations [5] and [11], respectively, a risk-free rate of 5.03%, and a market risk premium of 5.5%. All numbers other than β and β_D in %. Annual figures.

5) A Final Digression: Why Does the Downside Beta Work?

The superiority of the downside beta over beta in explaining the cross-section of stock returns may be a somewhat surprising finding to some. In this final section, I briefly attempt to justify the plausibility of this empirical result.

First, as mentioned above, it is rather obvious that investors do not dislike volatility per se; they only dislike downside volatility. Investors do not shy away from stocks that exhibit large and frequent jumps above the mean; they shy away from stocks that exhibit large and frequent jumps below the mean. Investors are not afraid of obtaining more than their minimum acceptable return (MAR); they are afraid of obtaining less than their MAR.

Second, aversion to the downside is consistent with both the theory and findings in the literature of behavioral finance. It is clearly consistent, for example, with the S-shaped utility function of prospect theory pioneered by Kahneman and Tversky (1979), in which losses of a given magnitude loom larger than gains of the same magnitude. In this framework, utility is determined by gains and losses with respect to the status quo rather than by wealth.

Finally, the superiority of downside beta may be related to the contagion effect in financial markets (8). Note that in the traditional MVB framework, the appropriate measure of risk is beta when markets are integrated, and the standard deviation when markets are segmented. The superiority of the downside beta may therefore be explained by the fact that markets are more integrated on the downside than on the upside due to the contagion effect, something that most data seem to suggest.

IV- Concluding remarks

Beta and the CAPM (and the behavioral model from which they follow, MVB) have been widely used but also widely debated for over 30 years. Most of the arguments against beta have been by and large empirical, focusing on whether beta explains the cross-section of stock returns. Between my previous article (Estrada, 2002b) and this article, I have questioned beta and the CAPM from both a theoretical point of view (by showing that MSB is at least as plausible as MVB) and an empirical point of view (by showing that the data support the downside beta over beta).

In this article, I have drawn a parallel between the standard framework based on MVB, the CAPM, and beta, and an alternative framework based on downside risk; that is, on MSB, the D-CAPM, and the downside beta. I have also shown the appropriate way to estimate the downside beta, the measure of risk proposed in this article, and how to integrate it into an alternative pricing model, the D-CAPM, proposed in this article to replace the CAPM.

The evidence discussed supports the downside risk measures over the standard risk measures, and particularly the downside beta, which explains over 45% of the variability in the cross-section of returns of a joint sample of DMs and EMs, and almost 55% of the variability in the cross-section of EM returns. The evidence also shows that mean returns in both DMs and EMs are much more sensitive to differences in downside beta than to equal

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(8) I would like to thank Mark Kritzman for suggesting this idea to me.

differences in beta. Furthermore, unlike the CAPM, the D-CAPM plausibly generates a higher (average) required return for EMs than for DMs. Finally, in EMs, the D-CAPM generates average required returns on equity over 250 basis points a year higher than those generated by the CAPM, a substantial difference that can make or break many investment projects and significantly affect the valuation of companies. Differences of this magnitude are simply too large for practitioners to ignore.

Finally, the D-CAPM has an advantage over three-factor models in that it is easier to implement; in fact, it is just as easy to implement as the CAPM. Therefore, this article questions the standard framework based on MVB, the CAPM, and beta, and proposes to replace it with an alternative framework based on MSB, the D-CAPM, and the downside beta. And the empirical evidence reported and discussed supports this proposal.

Appendix

Exhibit 1

MEAN-SEMIVARIANCE BEHAVIOR (II): THE D-CAPM

Summary Statistics (Monthly Stock Returns)

Market	*MR*	σ	ρ	β	Σ	Θ	β^D	*SSkw*	*Start*
Australia	0.86	5.63	0.57	0.77	3.94	0.70	0.89	0.24	Jan/88
Austria	0.54	6.92	0.38	0.63	4.81	0.63	0.98	0.67	Jan/88
Belgium	1.05	4.94	0.58	0.69	3.44	0.70	0.78	1.46	Jan/88
Canada	0.83	5.15	0.72	0.89	3.86	0.79	0.98	–3.28	Jan/88
Denmark	1.17	5.37	0.59	0.76	3.81	0.72	0.89	0.13	Jan/88
Finland	1.65	9.74	0.55	1.29	6.66	0.67	1.43	0.95	Jan/88
France	1.13	5.66	0.69	0.94	4.01	0.79	1.02	0.12	Jan/88
Germany	1.00	6.04	0.65	0.95	4.46	0.79	1.14	–1.57	Jan/88
Greece	1.55	11.34	0.27	0.72	6.67	0.60	1.28	8.63	Jan/88
Hong Kong	1.44	8.45	0.59	1.19	5.80	0.67	1.26	1.37	Jan/88
Ireland	0.99	5.69	0.66	0.90	3.98	0.75	0.96	0.53	Jan/88
Italy	0.72	7.06	0.52	0.88	4.79	0.67	1.04	1.41	Jan/88
Japan	–0.01	7.06	0.76	1.29	4.71	0.80	1.21	2.14	Jan/88
Netherlands	1.18	4.50	0.76	0.82	3.42	0.82	0.90	–3.20	Jan/88
New Zealand	0.35	7.08	0.49	0.84	4.86	0.68	1.06	1.59	Jan/88
Norway	0.88	6.74	0.59	0.95	4.93	0.71	1.13	–2.22	Jan/88
Portugal	0.43	6.66	0.46	0.74	4.42	0.60	0.86	3.20	Jan/88
Singapore	0.94	8.55	0.64	1.32	6.06	0.73	1.42	0.45	Jan/88
Spain	0.96	6.36	0.70	1.07	4.48	0.83	1.19	–0.33	Jan/88
Sweden	1.39	7.37	0.72	1.27	5.33	0.81	1.40	–1.29	Jan/88
Switzerland	1.17	5.14	0.66	0.81	3.63	0.77	0.90	–0.25	Jan/88
UK	0.89	4.69	0.77	0.87	3.21	0.80	0.83	1.43	Jan/88
USA	1.22	4.09	0.81	0.79	3.04	0.88	0.86	–2.23	Jan/88
Argentina	2.96	18.19	0.15	0.66	10.17	0.56	1.82	10.78	Jan/88
Brazil	2.91	17.37	0.35	1.44	11.55	0.58	2.16	2.51	Jan/88
Chile	1.74	7.56	0.32	0.57	5.27	0.56	0.95	–0.42	Jan/88
China	–0.72	12.72	0.37	1.13	7.92	0.54	1.39	4.27	Jan/93
Colombia	0.29	9.68	0.14	0.32	6.55	0.38	0.81	1.41	Jan/93
Czech Rep.	0.24	9.28	0.30	0.66	6.59	0.69	1.29	0.23	Jan/95
Egypt	0.46	8.69	0.25	0.53	5.18	0.61	0.90	4.94	Jan/95
Hungary	1.68	11.84	0.54	1.53	8.17	0.82	1.91	0.94	Jan/95
India	0.42	8.88	0.26	0.54	6.04	0.56	1.10	1.09	Jan/93
Indonesia	1.26	17.08	0.24	0.97	9.88	0.50	1.60	10.38	Jan/88
Israel	0.76	7.13	0.37	0.63	5.42	0.49	0.87	–2.01	Jan/93
Jordan	0.16	4.45	0.11	0.11	3.11	0.32	0.32	–0.80	Jan/88
Korea	0.93	12.56	0.41	1.25	7.68	0.54	1.34	6.83	Jan/88
Malaysia	0.95	10.09	0.42	1.02	6.87	0.60	1.33	3.16	Jan/88
Mexico	2.40	10.41	0.45	1.12	7.67	0.60	1.47	–2.23	Jan/88
Morocco	0.70	4.95	–0.10	–0.12	3.35	0.41	0.39	1.62	Jan/93
Pakistan	–0.02	12.08	0.17	0.49	7.91	0.39	1.00	1.96	Jan/93
Peru	0.97	9.47	0.33	0.74	6.55	0.56	1.19	0.76	Jan/93
Philippines	0.71	10.36	0.44	1.10	6.94	0.63	1.40	2.78	Jan/88
Poland	2.59	17.86	0.39	1.66	10.03	0.62	2.02	11.00	Jan/93
Russia	3.59	22.22	0.50	2.69	15.27	0.65	2.85	0.56	Jan/95
South Africa	0.78	8.20	0.56	1.10	6.02	0.68	1.33	–1.90	Jan/93
Sri Lanka	0.10	10.44	0.24	0.61	6.67	0.51	1.11	4.16	Jan/93
Taiwan	1.27	12.47	0.29	0.87	8.19	0.57	1.49	2.44	Jan/88
Thailand	0.72	12.73	0.46	1.41	8.80	0.62	1.75	1.25	Jan/88
Turkey	2.34	18.90	0.23	1.04	11.86	0.56	2.13	4.47	Jan/88
Venezuela	1.33	14.65	0.24	0.85	10.18	0.44	1.46	–0.23	Jan/93
Avg. DMs	**0.97**	**6.53**	**0.61**	**0.93**	**4.54**	**0.74**	**1.06**	**0.43**	**N/A**
Avg. EMs	**1.17**	**11.86**	**0.31**	**0.92**	**7.77**	**0.55**	**1.38**	**2.59**	**N/A**
Avg. All	**1.08**	**9.41**	**0.45**	**0.93**	**6.28**	**0.64**	**1.24**	**1.60**	**N/A**
World	0.78	4.17	1.00	1.00	3.11	1.00	1.00	–2.14	Jan/88

MR: Mean return; σ: Standard deviation; ρ: Correlation (with respect to the world market); β: Beta (with respect to the world market); Σ: Semideviation; Θ: Downside correlation (with respect to the world market); β^D: Downside beta (with respect to the world market); *SSkw*: Coefficient of standardized skewness. *MR*, σ, and Σ in %. All data through Dec/2001.

Exhibit 2

MEAN-SEMIVARIANCE BEHAVIOR (II): THE D-CAPM

Full Sample. Multiple Regression Analysis

Panel A. $MR_i = \gamma_0 + \gamma_1 RV_{1i} + \gamma_2 RV_{2i} + v_i$							
RV_1 / RV_2	γ_0	*t-stat*	γ_1	*t-stat*	γ_2	*t-stat*	R^2
s / S	–0.07	–0.29	–0.01	–0.05	0.19	0.78	0.34
b / bD	–0.46	–2.15	–0.15	–0.37	1.35	3.83	0.48

Panel B. $MR_i = \gamma_0 + \gamma_1 RV_{1i} + \gamma_2 RV_{2i} + \gamma_3 RV_{3i} + \gamma_4 RV_{4i} + v_i$											
$RV_1/RV_2/RV_3/RV_4$	γ_0	*t-stat*	γ_1	*t-stat*	γ_2	*t-stat*	γ_3	*t-stat*	γ_4	*t-stat*	R_2
$\sigma / \Sigma / \beta / \beta^D$	–0.48	–1.86	0.01	0.10	–0.55	–0.89	–0.16	–0.75	2.40	2.62	0.50

MR: Mean return; *RV*: Risk variable; σ: Standard deviation; β: Beta (with respect to the world market); Σ: Semideviation; β^D: Downside beta (with respect to the world market). Significance based on White's heteroskedasticity-consistent covariance matrix. Critical values for a two-sided test at the 5% significance level: 2.01 in both panels.

Exhibit 3

DMs v. EMs. Multiple Regression Analysis

Panel A. $MR_i = \gamma_0 + \gamma_1 RV_{1i} + \gamma_2 RV_{2i} + v_i$							
RV_1 / RV_2	γ_0	*t-stat*	γ_1	*t-stat*	γ_2	*t-stat*	R^2
Panel A1: DMs							
σ / Σ	0.52	1.49	–0.03	–0.13	0.14	0.40	0.06
β / β^D	0.41	1.06	–0.56	–0.65	1.02	1.53	0.11
Panel A2: EMs							
σ / Σ	–1.00	–2.96	0.02	0.11	0.25	1.07	0.50
β / β^D	–1.01	–2.72	–0.65	–1.30	2.01	4.26	0.58

Panel B. $MR_i = \gamma_0 + \gamma_1 RV_{1i} + \gamma_2 RV_{2i} + \gamma_3 RV_{3i} + \gamma_4 R\quad V_{4i} + v_i$											
$RV1/RV_2/RV_3/RV_4$	γ_0	*t-stat*	γ_1	*t-stat*	γ_2	*t-stat*	γ_3	*t-stat*	γ_4	*t-stat*	R^2
Panel B1: DMs											
$\sigma / \Sigma / \beta / \beta^D$	0.38	1.06	0.01	0.03	–0.94	–0.76	–0.17	–0.24	2.04	1.42	0.12
Panel B2: EMs											
$\sigma / \Sigma / \beta / \beta^D$	–0.99	–2.43	–0.02	–0.12	–0.72	–1.13	–0.00	–0.01	2.20	2.33	0.58

MR: Mean return; *RV*: Risk variable; σ: Standard deviation; β: Beta (with respect to the world market); Σ: Semideviation; β^D: Downside beta (with respect to the world market). Significance based on White's heteroskedasticity-consistent covariance matrix. Critical values for a two-sided test at the 5% significance level: 2.09, 2.06, 2.10, and 2.07 in panels A1, A2, B1, and B2, respectively.

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