IESE Business School
CIIF
University of Navarra

Working Paper

WP No 549
March, 2004

EQUIVALENCE OF TEN DIFFERENT DISCOUNTED
CASH FLOW VALUATION METHODS

Pablo Fernández *

* Professor of Financial Management, PricewaterhouseCoopers Chair of Finance, IESE

IESE Business School - Universidad de Navarra
Avda. Pearson, 21 - 08034 Barcelona. Tel.: (+34) 93 253 42 00 Fax: (+34) 93 253 43 43
Camino del Cerro del Águila, 3 (Ctra. de Castilla, km. 5,180) - 28023 Madrid. Tel.: (+34) 91 357 08 09 Fax: (+34) 91 357 29 13

The CIIF, International Center for Financial Research, is an interdisciplinary center with an international outlook and a focus on teaching and research in finance. It was created at the beginning of 1992 to channel the financial research interests of a multidisciplinary group of professors at IESE Business School and has established itself as a nucleus of study within the School's activities.

Ten years on, our chief objectives remain the same:

- Find answers to the questions that confront the owners and managers of finance companies and the financial directors of all kinds of companies in the performance of their duties
- Develop new tools for financial management
- Study in depth the changes that occur in the market and their effects on the financial dimension of business activity

All of these activities are programmed and carried out with the support of our sponsoring companies. Apart from providing vital financial assistance, our sponsors also help to define the Center's research projects, ensuring their practical relevance.

The companies in question, to which we reiterate our thanks, are:
Aena, A.T. Kearney, Caja Madrid, Fundación Ramón Areces, Grupo Endesa, Telefónica and Unión Fenosa.

http://www.iese.edu/ciif/

EQUIVALENCE OF TEN DIFFERENT DISCOUNTED CASH FLOW VALUATION METHODS

Abstract

This paper shows that ten methods of company valuation using discounted cash flows (WACC; equity cash flow; capital cash flow; adjusted present value; residual income; EVA; business's risk-adjusted equity cash flow; business's risk-adjusted free cash flow; risk-free-adjusted equity cash flow; and risk-free-adjusted free cash flow) always give the same value when identical assumptions are used. This result is logical, since all the methods analyze the same reality using the same assumptions; they differ only in the cash flows taken as the starting point for the valuation. We present all ten methods, allowing the required return to debt to be different from the cost of debt. Seven methods require an iterative process. Only the APV and business risk-adjusted cash flows methods do not require iteration.

JEL Classification: G12, G31, M21

Keywords: discounted cash flow valuation; valuation; equity cash flow; WACC; free cash flow.

EQUIVALENCE OF TEN DIFFERENT DISCOUNTED CASH FLOW VALUATION METHODS

Introduction

To value a company by discounting the expected cash flows at an appropriate discount rate, we may use different expected cash flows, which will have different risks and, therefore, will require different discount rates. As in each case we are valuing the same company, we should get the same valuation, no matter which expected cash flows we use. In this paper we present ten different approaches to valuing companies by discounting cash flows. Although some authors argue that different methods may produce different valuations, we will show that all methods provide the same value under the same assumptions. We were motivated to write this paper because of the question commonly raised by students, faculty and practitioners: "Why do I get different answers from different discounted cash flow valuations and from residual income valuations?"

The most common method for valuing companies is the Free Cash Flow method. In that method, interest tax shields are excluded from the cash flows and the tax deductibility of the interest is treated as a decrease in the weighted average cost of capital (WACC). As the WACC depends on the capital structure, this method requires an iterative process: to calculate the WACC, we need to know the value of the company, and to calculate the value of the company, we need to know the WACC. Seven out of the ten methods presented require an iterative process, while only three (APV and the two business risk-adjusted cash flows methods) do not. The prime advantage of these three methods is their simplicity. Whenever the debt is forecasted in levels, instead of as a percentage of firm value, the APV is much easier to use because the value of the interest tax shields is quite easy to calculate. The two business risk-adjusted cash flows methods are also easy to use because the interest tax shields are included in the cash flows. The discount rate of these three methods is the required return to assets and, therefore, it does not change when capital structure changes.

Section I describes the ten most commonly used methods for valuing companies by discounting cash flows. We show that all ten methods always give the same value. This result is logical, since all the methods analyze the same reality under the same hypotheses; they differ only in the cash flows taken as the starting point for the valuation. The ten methods are as follows:

- equity cash flows discounted at the required return to equity;
- free cash flow discounted at the WACC;
- Capital cash flows discounted at the WACC before tax;
- APV (Adjusted Present Value);
- residual income discounted at the required return to equity;
- EVA discounted at the WACC;

- the business's risk-adjusted free cash flows discounted at the required return to assets;
- the business's risk-adjusted equity cash flows discounted at the required return to assets;
- the risk-free-adjusted free cash flows discounted at the risk-free rate; and
- The risk-free-adjusted equity cash flows discounted at the risk-free rate.

Section II is the application of the ten methods to a specific example.

Section III presents five alternative theories about the value of tax shields and discusses their applicability. The five theories are the following:

1) Harris and Pringle (1985) and Ruback (1995, 2002). All of their equations arise from the assumption that the leverage-driven value creation or value of tax shields (VTS) is the present value of the tax shields discounted at the required return to the unlevered equity (Ku).
2) Myers (1974), who assumes that the value of tax shields (VTS) is the present value of the tax shields discounted at the required return to debt (Kd).
3) Miles and Ezzell (1980). They state that the correct rate for discounting the tax shields (D Kd T) is Kd for the first year, and Ku for the following years.
4) Modigliani and Miller (1963) calculate the value of tax shields by discounting the present value of the tax savings due to interest payments of a risk-free debt (T D R_F) at the risk-free rate (R_F).
5) Damodaran (1994), who assumes that the beta of the debt is zero.

The appendix presents the derivation of the WACC and WACC before tax formulas; proves that the residual income and equity cash flows methods provide the same value; and proves that valuations using EVA and free cash flow provide the same value. The appendix also shows that if EVA is (wrongly) calculated using WACC with book values of debt and equity the result obtained is economic profit, instead of EVA.

Table V contains the formulas used in the paper, and Table VI contains a list of the abbreviations used throughout.

I. Ten discounted cash flow valuation methods

There are four basic discounted cash flow valuation methods:

Method 1. Using the expected equity cash flow (ECF) and the required return to equity (Ke).

Equation (1) indicates that the value of the equity (E) is the present value of the expected equity cash flows[1] (ECF) discounted at the required return to equity (Ke).

$$E_0 = PV_0 [Ke_t; ECF_t] \qquad \textbf{(1)}$$

[1] In actual fact, we are referring to expectations: formula (1) indicates that the shares' value is the present value of the expected equity cash flows. We do not introduce the operator "expected value" in the formulas in order to avoid complicating the expressions any further.

The expected equity cash flow is the sum of all expected cash payments to shareholders, mainly dividends and share repurchases.

Equation (2) indicates that the value of the debt (D) is the present value of the expected debt cash flows (CFd) discounted at the required return to debt (Kd).

$$D_0 = PV_0\ [Kd_t;\ CFd_t] \qquad \textbf{(2)}$$

The expected debt cash flow in a given period is given by equation (3)

$$CFd_t = N_{t-1}\ r_t - (N_t - N_{t-1}) \qquad \textbf{(3)}$$

where N is the book value of the financial debt and r is the cost of debt. $N_{t-1}\ r_t$ is the interest paid by the company in period t. $(N_t - N_{t-1})$ is the increase in the book value of debt in period t. When the required return to debt (Kd) is different than the cost of debt (r), then the value of debt (D) is different than its book value (N). Note that if, for all t, $r_t = Kd_t$, then $N_0 = D_0$. But there are situations in which $r_t > Kd_t$ (e.g. if the company has old fixed-rate debt and interest rates have declined, or if the company can only get expensive bank debt), and situations in which $r_t < Kd_t$ (e.g. if the company has old fixed-rate debt and interest rates have increased).

Method 2. Using the free cash flow and the WACC (weighted average cost of capital)

Equation (4) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the free cash flows (FCF) that the company is expected to generate, discounted at the weighted average cost of debt and shareholders' equity after tax (WACC):

$$E_0 + D_0 = PV_0\ [WACC_t\ ;\ FCF_t] \qquad \textbf{(4)}$$

The free cash flow is the hypothetical equity cash flow when the company has no debt. The expression that relates the FCF with the ECF is:

$$ECF_t = FCF_t + (N_t - N_{t-1}) - N_{t-1}\ r_t\ (1 - T_t) \qquad \textbf{(5)}$$

T_t is the effective tax rate applied to earnings in the levered company in year t.

The WACC is the rate at which the FCF must be discounted so as to ensure that equation (4) gives the same result as that given by the sum of (1) and (2). In the appendix we show that the expression for the WACC is:

$$WACC_t = (E_{t-1}\ Ke_t + D_{t-1}\ Kd_t - N_{t-1}\ r_t\ T_t) \,/\, (E_{t-1} + D_{t-1}) \qquad \textbf{(6)}$$

$(E_{t-1} + D_{t-1})$ is the value of the firm obtained when the valuation is performed using formula (4). Consequently, the valuation is an iterative process: the free cash flows are discounted at the WACC to calculate the value of the firm (D+E) but, in order to obtain the WACC, we first need to know the value of the firm (D+E).[2]

---

[2] Obviously, if the required return to debt (Kd) is assumed equal to the cost of debt (r), then the debt value (D) is equal to its book value (N), and equation (6) is transformed into (6a):

$$WACC_t = (E_{t-1}\ Ke_t + D_{t-1}\ Kd_t\ (1 - T_t)) \,/\, (E_{t-1} + D_{t-1}) \qquad \textbf{(6a)}$$

Some authors, one being Luehrman (1997), argue that equation (4) does not give us the same result as is given by the sum of (1) and (2). That usually happens as a result of an error in calculating equation (6), which requires using the values of equity and debt (E_{t-1} and D_{t-1}) obtained in the valuation. The most common error when calculating the WACC is to use book values of equity and debt, as in Luehrman (1997) and in Arditti and Levy (1977). Other common errors when calculating $WACC_t$ are:

– Using E_t and D_t instead of E_{t-1} and D_{t-1}.
– Using market values of equity and debt, instead of the values of equity and debt (E_{t-1} and D_{t-1}) obtained in the valuation.
– Using formula (6a) instead of formula (6) when the value of debt is not equal to its book value.

Another common error is in calculating the perpetuity value, normally calculated by assuming that the cash flows will grow at a certain rate in perpetuity after a given period. If the assumption is that the balance sheet and the income statement will grow in perpetuity at a rate g after period t_p, then the cash flows will grow at a rate g only after period (t_p+1). But the growth rate of the cash flows in period (t_p+1) will not be g (as assumed by Luehrman (1997)). Section II presents an example in which the balance sheet and the income statement are expected to grow in perpetuity at an annual rate of 2% after year 3, but the expected cash flows grow at a rate of 2% only after year 4.

Method 3. Using the capital cash flow (CCF) and the $WACC_{BT}$ (weighted average cost of capital, before tax)

The capital cash flows[3] are the cash flows available for all holders of the company's securities, whether these be debt or shares. They are equivalent to the equity cash flow (ECF) plus the cash flow corresponding to the debt holders (CFd).

Equation (6) indicates that the value of the debt today (D) plus that of the shareholders' equity (E) is equal to the capital cash flow (CCF) discounted at the weighted average cost of debt and shareholders' equity before tax[4] ($WACC_{BT}$).

$$E_0 + D_0 = PV[WACC_{BTt}; CCF_t] \quad \textbf{(7)}$$

$WACC_{BT}$ represents the discount rate that ensures that the values obtained using equations (7) and (4) are the same. Ruback (2002) also proves in a different way that the CCF method is equivalent to the FCF method. In the appendix, we show that the expression for the $WACC_{BT}$ is:

$$WACC_{BT\,t} = [E_{t-1}\, Ke_t + D_{t-1}\, Kd_t] / [E_{t-1} + D_{t-1}] \quad \textbf{(8)}$$

The expression that relates the CCF with the ECF and the FCF is (9):

$$CCF_t = ECF_t + CFd_t = ECF_t - (N_t - N_{t-1}) + N_{t-1}\, r_t = FCF_t + N_{t-1}\, r_t\, T_t \quad \textbf{(9)}$$

[3] Arditti and Levy (1977) and Ruback (1995 and 2002) suggest that the firm's value could be calculated by discounting the Capital Cash Flows instead of the Free Cash Flows.
[4] Ruback (2002) calls it Pre-Tax WACC.

Method 4. Adjusted present value (APV)

The adjusted present value (APV) equation (10) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is equal to the value of the unlevered company's shareholders' equity Vu plus the value of the tax shields (VTS):

$$E_0 + D_0 = Vu_0 + VTS_0 \quad \textbf{(10)}$$

If Ku is the required return to equity in the debt-free company (also called the required return to assets), Vu is given by (11):

$$Vu_0 = PV_0\ [Ku_t;\ FCF_t] \quad \textbf{(11)}$$

Most descriptions of the APV suggest calculating the VTS by discounting the interest tax shields using some discount rate. Taggart (1991) and Luehrman (1997) propose using the cost of debt (based on the theory that tax shields are about as uncertain as principal and interest payments). However, Harris and Pringle (1985), Kaplan and Ruback (1995), Brealey and Myers (2000) and Ruback (2002) propose using the required return to the unlevered equity as the discount rate.[5] Copeland, Koller and Murrin (2000) assert that "the finance literature does not provide a clear answer about which discount rate for the tax benefit of interest is theoretically correct." They further conclude "we leave it to the reader's judgment to decide which approach best fits his or her situation."

Fernández (2004) shows that the value of tax shields is not the present value of tax shields. He calculates it as the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company, and the present value of taxes for the levered company. Following Fernández (2004), the value of tax shields without cost of leverage is:

$$VTS_0 = PV_0\ [Ku_t;\ D_{t-1}\ Ku_t\ T_t + T_t\ (N_{t-1}\ r_t - D_{t-1}\ Kd_t)] \quad \textbf{(12)}$$

If the cost of debt (r) is equal to the required return to debt, then debt value is equal to its book value, and $VTS_0 = PV_0\ [Ku_t;\ D_{t-1}\ Ku_t\ T_t]$. For no growth perpetuities, equation (12) is VTS = DT. The value of tax shields being DT for no growth perpetuities is quite a standard result. It may be found, for example, in Zvi and Merton (2000), Modigliani and Miller (1963), Myers (1974), Damodaran (2002) and Brealey and Myers (2000).

Consequently, (10) may be rewritten as:

$$E_0 + D_0 = PV_0\ [Ku_t;\ FCF_t + D_{t-1}\ Ku_t\ T_t + T_t\ (N_{t-1}\ r_t - D_{t-1}\ Kd_t)] \quad \textbf{(13)}$$

Relation between Ke and Ku. Equations (1), (2) and (13) may be rewritten in their intertemporal form as:

$$E_t = E_{t-1}\ (1 + Ke_t) - ECF_t \quad \textbf{(1i)}$$

$$D_t = D_{t-1}\ (1 + Kd_t) - CFd_t \quad \textbf{(2i)}$$

$$E_t + D_t = (E_{t-1} + D_{t-1})\ (1 + Ku_t) - [FCF_t + D_{t-1}\ Ku_t\ T_t + T_t\ (N_{t-1}\ r_t - D_{t-1}\ Kd_t)] \quad \textbf{(13i)}$$

---

5 We will discuss these theories in Section III.

Subtracting equation (13i) from the sum of (1i) and (2i), we get:

$$0 = E_{t-1}\ Ke_t + D_{t-1}\ Kd_t - (E_{t-1} + D_{t-1})\ Ku_t + (FCF_t + D_{t-1}\ Ku_t\ T_t + T_t\ (N_{t-1}\ r_t - D_{t-1}\ Kd_t) - ECF_t - CFd_t)$$

From (3) and (5) we know that $FCF_t - ECF_t - CFd_t = - N_{t-1}\ r_t\ T_t$. Therefore,

$$0 = E_{t-1}\ Ke_t + D_{t-1}\ Kd_t - (E_{t-1} + D_{t-1})\ Ku_t + D_{t-1}\ Ku_t\ T_t - D_{t-1}\ Kd_t\ T_t$$

Therefore, the relation between Ke and Ku is:

$$Ke_t = Ku_t + D_{t-1}\ (1 - T_t)\ (Ku_t - Kd_t) / E_{t-1} \quad \textbf{(14)}$$

Relation between WACC and Ku. Substituting (14) in (6), we get:

$$WACC_t\ (E_{t-1} + D_{t-1}) = E_{t-1}\ Ku_t + D_{t-1}\ (1 - T_t)\ (Ku_t - Kd_t) + D_{t-1}\ Kd_t - N_{t-1}\ r_t\ T_t.$$

Therefore, the relation between WACC and Ku is:

$$WACC_t = Ku_t - [D_{t-1}\ T_t\ (Ku_t - Kd_t) + N_{t-1}\ r_t\ T_t] / (E_{t-1} + D_{t-1}) \quad \textbf{(15)}$$

Method 5. Using the residual income[6] and Ke (required return to equity)

Equation (16) indicates that the value of the equity (E) is the equity's book value (Ebv) plus the present value of the expected residual income (RI) discounted at the required return to equity (Ke).

$$E_0 = Ebv_0 + PV_0\ [Ke_t;\ RI_t] \quad \textbf{(16)}$$

The term residual income (RI) is used to define the accounting net income or profit after tax (PAT) minus the equity's book value (Ebv_{t-1}) multiplied by the required return to equity.[7]

$$RI_t = PAT_t - Ke_t\ Ebv_{t-1} \quad \textbf{(17)}$$

In the appendix, we prove that equations (16) and (1) provide the same valuation even if the fiancial statement forecasts do not satisfy the clean surplus relation (i.e., net income less dividends does not equal the change in shareholders' equity).

Penman and Sougiannis (1998), Francis, Olson and Oswald (2000), and Penman (2001) argue that residual income and equity cash flow provide different valuations and that accrual earnings techniques dominate free cash flow and dividend discounting approaches. However, we agree with Lundholm and O'Keefe (2001a), who argue that, properly implemented, both models yield identical valuations for all firms in all years. They identify how prior research has applied inconsistent assumptions to the two models and show how these

---

6 The residual income is also called economic profit, residual earnings, abnormal earnings and excess profit.

7 As $PAT_t = ROE_t\ Ebv_{t-1}$, the residual income can also be expressed as $RI_t = (ROE_t - Ke_t)\ Ebv_{t-1}$.

seemingly minor errors cause surprisingly large differences in the value estimates. Lundholm and O'Keefe (2001b) identify subtle errors in the implementation of the models in prior empirical studies by Penman and Sougiannis (1998) and Francis, Olsson and Oswald (2000).

Method 6. Using the EVA (economic value added) and the WACC (weighted average cost of capital)

Equation (18) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the book value of the shareholders' equity and the debt ($Ebv_0 + N_0$) plus the present value of the expected EVA, discounted at the weighted average cost of capital (WACC):

$$E_0 + D_0 = (Ebv_0 + N_0) + PV_0\ [WACC_t\ ;\ EVA_t] \quad \textbf{(18)}$$

The EVA (economic value added) is the NOPAT (Net Operating Profit After Tax) minus the company's book value ($N_{t-1} + Evc_{t-1}$) multiplied by the weighted average cost of capital (WACC). The NOPAT (Net Operating Profit After Taxes) is the profit of the unlevered (debt-free) company[8].

$$EVA_t = NOPAT_t - (N_{t-1} + Ebv_{t-1})WACC_t \quad \textbf{(19)}$$

In the appendix, we prove that equations (18) and (4) provide the same valuation.

Method 7. Using the business risk-adjusted free cash flow and Ku (required return to assets)

Equation (20) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the business risk-adjusted free cash flows (FCF\\Ku) that the company is expected to generate, discounted at the required return to assets (Ku):

$$E_0 + D_0 = PV_0\ [Ku_t\ ;\ FCF_t\backslash\backslash Ku] \quad \textbf{(20)}$$

The definition of the business risk-adjusted free cash flows is obtained by making (20) equal to (4):

$$FCF_t\backslash\backslash Ku = FCF_t - (E_{t-1} + D_{t-1})\ (WACC_t - Ku_t) = FCF_t + D_{t-1}\ T_t\ (Ku_t - Kd_t) - N_{t-1}\ r_t\ T_t \quad \textbf{(21)}$$

Method 8. Using the business risk-adjusted equity cash flow and Ku (required return to assets)

Equation (22) indicates that the value of the equity (E) is the present value of the expected business risk-adjusted equity cash flows (ECF\\Ku) discounted at the required return to assets (Ku):

$$E_0 = PV_0\ [Ku_t;\ ECF_t\backslash\backslash Ku] \quad \textbf{(22)}$$

---

[8] As $NOPAT_t = ROA_t\ (Ebv_{t-1} + N_{t-1})$, the economic value added (EVA) can also be expressed as:
$EVA_t = (ROA_t - WACC_t)(Ebv_{t-1} + N_{t-1})$

The definition of the business risk-adjusted equity cash flows is obtained by making (22) equal to (1):

$$ECF_t\backslash\backslash Ku = ECF_t - E_{t-1} (Ke_t - Ku_t) = ECF_t - D_{t-1} (1- T_t) (Ku_t - Kd_t) \qquad \textbf{(23)}$$

Method 9. Using the risk-free-adjusted free cash flows discounted at the risk-free rate

Equation (24) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the expected risk-free-adjusted free cash flows ($FCF\backslash\backslash R_F$) that will be generated by the company, discounted at the risk-free rate (R_F):

$$E_0 + D_0 = PV_0 [R_{F\,t} ; FCF_t\backslash\backslash R_F] \qquad \textbf{(24)}$$

The definition of the risk-free-adjusted free cash flows is obtained by making (24) equal to (4):

$$FCF_t\backslash\backslash R_F = FCF_t - (E_{t-1} + D_{t-1}) (WACC_t - R_{F\,t}) \qquad \textbf{(25)}$$

Method 10. Using the risk-free-adjusted equity cash flows discounted at the risk-free rate

Equation (26) indicates that the value of the equity (E) is the present value of the expected risk-free-adjusted equity cash flows ($ECF\backslash\backslash R_F$) discounted at the risk-free rate (R_F):

$$E_0 = PV_0 [R_{F\,t}; ECF_t \backslash\backslash R_F] \qquad \textbf{(26)}$$

The definition of the risk-free-adjusted equity cash flows is obtained by making (26) equal to (1):

$$ECF_t\backslash\backslash R_F = ECF_t - E_{t-1} (Ke_t - R_{F\,t}) \qquad \textbf{(27)}$$

We could also talk about an eleventh method: using the business risk-adjusted capital cash flow and Ku (required return to assets). But the business risk-adjusted capital cash flow is identical to the business risk-adjusted free cash flow ($CCF\backslash\backslash Ku = FCF\backslash\backslash Ku$). Therefore, this method would be identical to Method 7.

And we could talk about a twelfth method: using the risk-free-adjusted capital cash flow and R_F (risk-free rate). But the risk-free-adjusted capital cash flow is identical to the risk-free-adjusted free cash flow ($CCF\backslash\backslash R_F = FCF\backslash\backslash R_F$). Therefore, this method would be identical to Method 9.

The formulas relating the betas to the required returns are:

$$Ke = R + \beta_L P_M \qquad Ku = R_F + \beta u P_M \qquad Kd = R_F + \beta d P_M \qquad \textbf{(28)}$$

R_F is the risk-free rate and P_M is the market risk premium.

In order to operationalize a valuation, very often one begins with assumptions of βd and β_L, not with βu. βu has to be inferred from βd and β_L. The formula that allow us to calculate βu, may easily be derived by substituting (28) in (14):

$$\beta u = [E\, \beta_L + \beta d\, D (1 – T)] / [E + D (1 – T)] \qquad \textbf{(29)}$$

Once the valuation starts with Ku (or βu), all valuation methods require an iterative process except the APV (method 4) and the methods that use business risk-adjusted cash flows (methods 7 and 8). Therefore, from a computational point of view these three valuation methods are much easier to implement.

II. An example. Valuation of the company Tenmethods Inc.

The company Tenmethods Inc. has the balance sheet and income statement forecasts for the next few years shown in Table I. After year 3, the balance sheet and the income statement are expected to grow at an annual rate of 2%. Although the statutory tax rate is 40%, the effective tax rate will be zero in year 1 because the company is forecasting losses, and 36.36% in year 2 because the company will offset the previous year's losses (see line 16). The cost of debt (the interest rate that the bank will charge) is 9%. Using the balance sheet and income statement forecasts in Table I, we can readily obtain the cash flows given at the bottom of Table I. Although the balance sheet and the income statement will grow at an annual rate of 2% after year 3, the cash flows grow at a rate of 2% only after year 4. The growth of the FCF in year 4 is 81.9%.

Table II contains the valuation of the company Tenmethods Inc. using the ten methods described in Section I. The unlevered beta (βu) is 1. The risk-free rate is 6%. The cost of debt (r) is 9%, but the company feels that it is too high. The company thinks that the appropriate required return to debt (Kd) is 8%. The market risk premium is 4%. Consequently, using the CAPM, the required return to assets is 10%. As the cost of debt (r) is higher than the required return to debt (Kd), the value of debt (D, line 2) is higher than its nominal value (N, line 6 of table I). The value of debt also fulfills equation (2i). The first method used is the APV because it does not require an iterative process and, therefore, is easier to implement. To calculate the value of the unlevered equity (line 3) and the value of tax shields (line 4), we only need to compute two present values using Ku (10%). Line 5 is the enterprise value and line 6 is the equity value. Lines 5 and 6 also fulfill equations (1i) and (13i). Line 7 is the required return to equity according to equation (14). Line 8 is the calculation of the equity value as the present value of the expected equity cash flows (equation (1)). Please note that lines 7 and 8 are calculated through an iterative process because for equation (14) we need to know the result of equation (1), and for equation (1) we need to know the result of equation (14). The equity value in lines 8 and 6 is exactly the same. Line 9 is the WACC according to equations (6) and (15). Line 10 is the calculation of the equity value as the present value of the expected free cash flows minus the debt value (equation (4)). Lines 9 and 10 are calculated through an iterative process because for equations (6) and (15) we need to know the result of equation (4), and for equation (4) we need to know the result of equation (6) or (15). Please note that in year 1 WACC = Ku = 10% because the effective tax rate is zero. The equity value in lines 10 and 6 is exactly the same. Line 11 is the $WACC_{BT}$ according to equation (8). Line 12 is the calculation of the equity value as the present value of the expected capital cash flows minus the debt value (equation (7)). Lines 11 and 12 are calculated through an iterative process because for calculating the $WACC_{BT}$ we need to know the equity value and viceversa. Note that in year 1 $WACC_{BT}$ = WACC = Ku = 10% because the effective tax rate is zero. The equity values in lines 12 and 6 are exactly the same. Line 13 is the expected residual income according to equation (17). Line 14 is the calculation of the equity value as the present value of the expected residual income plus the book value of equity (equation (16)). Lines 13 and 14 are calculated through an iterative process because for calculating the residual income we need to know the required return to equity (equation (14)), and for this, we need the equity value. Lines 14 and 6 are equal. Line 15 is the expected economic value added according to equation (19). Line 14 is the calculation of the equity value as the present value of the expected economic value added plus the book value of equity minus the debt value plus the debt book value (equation (18)). Lines 15 and 16 are calculated through an iterative process because for calculating the economic value added, we must know the WACC (equation (6)), and for this, we need the equity value. Lines 16 and 6 are equal. Line 17 is the business's expected risk-adjusted equity cash flows according to equation (23). Line 18 is the calculation of the equity value as the present value of the business's expected risk-adjusted equity cash flows (equation (22)). As the present value is calculated using Ku (10%), there is no need for an iterative process. Lines 18 and 6 are equal. Line 19 is the business's expected risk-adjusted free cash flows according to equation (21). Line 20 is the calculation of the equity value as the present value of the business's expected risk-adjusted free cash flows minus the value of debt (equation (20)). As the present value is calculated using Ku (10%), there is no need for an iterative process. Lines 20 and 6 are equal. Line 21 is the expected risk-free-adjusted equity cash flows according to equation (27). Line 22 is the calculation of the equity value as the present value of the risk-free rate-adjusted equity cash flows (equation (26)). Lines 21 and 22 are calculated through an iterative process because for calculating the risk-free-adjusted equity cash flows we need to know the required return to equity (equation (14)), and for that, we need the equity value. Lines 22 and 6 are equal. Line 23 is the expected risk-free-adjusted free cash flows according to equation (25). Line 24 is the calculation of the equity value as the present value of the risk-free rate-adjusted free cash flows minus the debt value (equation (24)). Lines 23 and 24 are calculated through an iterative process because for calculating the risk-free-adjusted free cash flows we need to know the WACC (equation (6)), and for that, we need the equity value. Lines 24 and 6 are equal. Line 31 is the levered beta according to equation (29). Line 32 is the debt ratio using book values and line 33 is the debt ratio using market values. They are different, as tends to be the case.

Table III shows the sensitivity analysis of the valuation of Tenmethods, Inc. as a function of the growth after period 3 (g) and the required return to debt (Kd). As expected, the equity value, the debt value, the value of tax shields, WACC and Ke increase as growth occurs. The equity value also increases when the required return to debt increases because, as the required return to assets (Ku) is fixed, the required return to equity (Ke) decreases whenever we increase Kd. WACC increases while debt value and the value of tax shields both decrease

III. Comparison to alternative valuation theories

There is a considerable body of literature on the discounted cash flow valuation of firms. The main difference between all of these papers and the approach proposed in sections I and II is that most previous papers calculate the value of tax shields as the present value of the tax savings due to the payment of interest.

Modigliani and Miller (1958 and 1963) studied the effect of leverage on the firm's value. Their famous Proposition 1 states that, in the absence of taxes, the firm's value is independent of its debt, i.e., E + D = Vu, if T = 0. In the presence of taxes and for the case of a perpetuity, but with zero risk of bankruptcy, they calculate the value of tax shields by discounting the present value of the tax savings due to interest payments of a risk-free debt at the risk-free rate (R_F), i.e. VTS= PV[R_F; D T R_F] = D T. As indicated above, this result equals equation (12) for the case of perpetuities, but it is not correct nor applicable for growing perpetuities. Modigliani and Miller explicitly ignore the issue of the riskiness of the cash-flows by assuming that the likelihood of bankruptcy was always zero.

Myers (1974) introduces the APV (adjusted present value) method, but proposes calculating the VTS by discounting the tax savings (N T r) at the required return to debt (Kd). The argument is that the risk of the tax saving arising from the use of debt is the same as the risk of the debt. This approach has also been recommended in later papers in the literature, two being Taggart (1991) and Luehrman (1997). One problem with the Myers (1974) approach is that it does not always give a higher cost of equity than cost of assets. Myers obtains Ke lower than Ku for growing perpetuities when the growth rate is higher than the after-tax cost of debt: $g > Kd\ (1 - T)$. In this situation, as the value of tax shields is higher than the value of debt, the equity (E) is worth more than the unlevered equity (Vu). This hardly makes any economic sense.

Harris and Pringle (1985) propose that the present value of the tax saving due to the payment of interest should be calculated by discounting the interest tax savings (N T r) at the required return to unlevered equity (Ku), i.e. VTS = PV [Ku; N T r]. Their argument is that the interest tax shields have the same systematic risk as the firm's underlying cash flows and, therefore, should be discounted at the required return to assets (Ku). Ruback (1995 and 2002), Kaplan and Ruback (1995), Brealey and Myers (2000, page 555), and Tham and Vélez-Pareja (2001), this last paper following an arbitrage argument, also claim that the appropriate discount rate for tax shields is Ku, the required return to unlevered equity. Ruback (1995 and 2002) presents the Capital Cash Flow (CCF) method and claims that $WACC_{BT} = Ku$. Based on this assumption, Ruback gets the same valuation as Harris and Pringle (1985). Ruback (2002, page 91) also shows that the relation between the beta of the levered equity (β_L), the beta of the unlevered equity (βu) and the beta of debt (βd) is equation (29) assuming $T = 0$.

A large part of the literature argues that the value of tax shields should be calculated in a different manner depending on the debt strategy of the firm. Hence, a firm that wishes to keep a constant D/E ratio must be valued in a different manner from a firm that has a preset level of debt. Miles and Ezzell (1980) indicate that for a firm with a fixed debt target (i.e. a constant [D/(D+E)] ratio), the correct rate for discounting the interest tax shields is Kd for the first year and Ku for the tax saving in later years.[9] Inselbag and Kaufold (1997) and Ruback (2002) argue that when the amount of debt is fixed, interest tax shields should be discounted at the required return to debt. However, if the firm targets a constant debt/value ratio, the value of tax shields should be calculated according to Miles and Ezzell (1980). Finally, Taggart (1991) suggests using Miles & Ezzell (1980) if the company adjusts to its target debt ratio once a year and Harris & Pringle (1985) if the company adjusts to its target debt ratio continuously.

Damodaran (1994, page 31) argues that if all the business risk is borne by the equity, then the formula relating the levered beta (β_L) to the asset beta (βu) is $\beta_L = \beta u + (D/E)\ \beta u\ (1 - T)$. This formula is exactly formula (29) assuming that $\beta d = 0$. One interpretation of this assumption is (see page 31 of Damodaran, 1994) that "all of the firm's risk is borne by the stockholders (i.e., the beta of the debt is zero)". In some cases, it may be reasonable to assume that the debt has a zero beta, but then the required return to debt (Kd) should also be the risk-free rate. However, in several examples in his books Damodaran (1984 and 2002) considers the required return to debt to be equal to the cost of debt, both of which are higher than the risk-free rate.

---

[9] Lewellen and Emery (1986) also claim that this is the most logically consistent method. Although Miles and Ezzell do not mention what the value of tax shields should be, this may be inferred from their equation relating the required return to equity to the required return for the unlevered company (equation 22 in their paper). This relation clearly implies that VTS = PV[Ku; T D Kd] (1 + Ku)/(1 + Kd).

Fernández (2004) shows that the value of tax shields is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company. This is the difference between the present values of two separate cash flows, each with its own risk. He proves that the value of tax shields without cost of leverage is equation (12). When the cost of debt (r) is equal to the required return to debt, then the debt value is equal to its book value, and $VTS_0 = PV_0 [Ku_t; D_{t-1} Ku_t T_t]$. This expression does not mean that the appropriate discount for tax shields is the unlevered cost of equity, since the amount being discounted is higher than the tax shields (it is multiplied by the unlevered cost of equity and not the cost of debt). This result arises as the difference of two present values. In the case of no growth perpetuities, equation (12) is VTS = DT. The value of tax shields being DT for no growth perpetuities is quite a standard result. It may be found, for example, in Zvi and Merton (2000), Modigliani and Miller (1963), Myers (1974), Damodaran (2002) and Brealey and Myers (2000).

Table IV contains the most striking results of the valuation performed on the company Tenmethods, Inc. according to Fernández (2004), Damodaran (1994), Ruback (2002) and Myers (1974). It may be seen that:

– Equity value (E) grows with residual growth (g), except according to Damodaran (1994).

– Required return to equity (Ke) decreases with growth (g), except according to Damodaran (1994) and Ruback (2002).

– The value of tax shields (VTS) decreases with the required return to debt (Kd), except according to Ruback (2002).

– The WACC increases with the required return to debt (Kd), except according to Ruback (2002).

Please note that these exceptions are counterintuitive.

IV. Conclusion

The paper shows that the ten most commonly used discounted cash flow valuation methods always give the same value. This result is logical, since all the methods analyze the same reality under the same hypotheses; they differ only in the cash flows taken as the starting point for the valuation. The ten methods analyzed are:

1) equity cash flows discounted at the required return to equity;

2) free cash flow discounted at the WACC;

3) capital cash flows discounted at the WACC before tax;

4) APV (Adjusted Present Value);

5) residual income discounted at the required return to equity;

6) EVA discounted at the WACC;

7) the business's risk-adjusted free cash flows discounted at the required return to assets;

8) the business's risk-adjusted equity cash flows discounted at the required return to assets;

9) the risk-free-adjusted free cash flows discounted at the risk-free rate; and

10) the risk-free-adjusted equity cash flows discounted at the risk-free rate.

We present all ten methods, allowing the required return to debt to be different from the cost of debt. Seven methods require an iterative process. Only APV and the business risk-adjusted cash flows methods do not require iteration; that makes them the easiest methods to use.

The relevant tax rate is not the statutory tax rate, but the effective tax rate applied to earnings in the levered company in each year.

We also show that if EVA is (wrongly) calculated using WACC with book values of debt and equity, the result obtained is economic profit, instead of EVA.

The value of tax shields is not the present value of tax shields. It is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company. The paper also compares the valuation result with three alternative theories on the calculation of the VTS: Myers (1974), Ruback (2002), and Damodaran (1994). □

Appendix: Proofs

Derivation of the expression of WACC

The intertemporal form of equations (1), (2) and (4) is:

$$E_t = E_{t-1} (1+Ke_t) - ECF_t \quad \textbf{(1i)}$$

$$D_t = D_{t-1} (1+Kd_t) - CFd_t \quad \textbf{(2i)}$$

$$E_t + D_t = (E_{t-1} + D_{t-1}) (1 + WACC_t) - FCF_t \quad \textbf{(4i)}$$

Substracting equation (4i) from the sum of (1i) and (2i), we get:

$$0 = E_{t-1} Ke_t + D_{t-1} Kd_t - (E_{t-1} + D_{t-1}) WACC_t + (FCF_t - ECF_t - CFd_t)$$

From (3) and (5), we know that $FCF_t - ECF_t - CFd_t = - N_{t-1} r_t T_t$. Therefore,

$$WACC_t = [E_{t-1} Ke_t + D_{t-1} Kd_t - N_{t-1} r_t T_t] / (E_{t-1} + D_{t-1}) \quad \textbf{(6)}$$

Derivation of the expression of $WACC_{BT}$

The intertemporal form of equation (7) is:

$$E_t + D_t = (E_{t-1} + D_{t-1}) (1 + WACC_{BTt}) - CCF_t \quad \textbf{(7i)}$$

Substracting equation (7i) from (4i), we get:

$$0 = (E_{t-1} + D_{t-1}) (WACC_t - WACC_{BTt}) + (CCF_t - FCF_t)$$

From (9), we know that $CCF_t - FCF_t = N_{t-1} r_t T_t$. Therefore,

$$WACC_{BTt} = WACC_t + N_{t-1} r_t T_t / (E_{t-1} + D_{t-1}) = (E_{t-1} Ke_t + D_{t-1} Kd_t) / (E_{t-1} + D_{t-1}) \quad \textbf{(8)}$$

Valuations using residual income and cash flow to equity provide the same value

The expected equity cash flow is the sum of all cash payments to shareholders, mainly dividends and share repurchases.

Consequently[10]:

$$ECF_t = PAT_t - (Ebv_t - Ebv_{t-1}) \quad \textbf{(30)}$$

---

[10] If the clean surplus relation does not hold (i.e. $ECF_t \neq PAT_t - \Delta Ebv_t$), for example, because the company allocates a quantity P directly to retained earnings, then Profit After Tax should be adjusted as follows:

$PAT_t = PATbv_t - P$, where $PATbv_t$ is the Profit After Tax shown in the income statement.

Appendix (continued)

By substituting (30) in (1i) we get: $E_t = E_{t-1}(1+Ke_t) - PAT_t - (Ebv_t - Ebv_{t-1})$

Rearranging terms, we get:

$$E_t - Ebv_t = (E_{t-1} - Ebv_{t-1})(1+Ke_t) - (PAT_t - Ke_t\, Ebv_{t-1}) = (E_{t-1} - Ebv_{t-1})(1+Ke_t) - RI_t$$

Valuations using EVA and free cash flow provide the same value

From (30) and (9), the relationship between the FCF and net income or profit after tax (PAT) is:

$$FCF_t = PAT_t - (Ebv_t - Ebv_{t-1}) + N_{t-1}\, r_t\,(1-T_t) - (N_t - N_{t-1}) \qquad \textbf{(31)}$$

As $PAT_t = NOPAT_t - N_{t-1}\, r_t\,(1-T_t)$, equation (31) may be expressed as:

$$FCF_t = NOPAT_t - (Ebv_t - Ebv_{t-1} + N_t - N_{t-1}) \qquad \textbf{(32)}$$

Substituting (32) in (4i), we get:

$$E_t + D_t = (E_{t-1} + D_{t-1})(1 + WACC_t) - NOPAT_t + (Ebv_t - Ebv_{t-1} + N_t - N_{t-1})$$

Rearranging terms, we get:

$$E_t + D_t - (Ebv_t + N_t) =$$

$$= [E_{t-1} + D_{t-1} - (Ebv_{t-1} + N_{t-1})](1+WACC_t) - [NOPAT_t - (N_{t-1} + Ebv_{t-1})WACC_t]$$

EVA calculated using WACC with book values of debt and equity is economic profit

The WACC calculated using book values of equity and debt is:

$$WACCbv_t = [Ebv_{t-1}\, Ke_t + N_{t-1}\, r_t\,(1- T_t)] / (Ebv_{t-1} + N_{t-1}) \qquad \textbf{(33)}$$

Consequently:

$$Ebv_{t-1}\, Ke_t + N_{t-1}\, r_t\,(1- T_t) = WACCbv_t\,(Ebv_{t-1} + N_{t-1}) \qquad \textbf{(34)}$$

As $PAT_t = NOPAT_t - N_{t-1}\, r_t\,(1-T_t)$, the residual income can also be expressed as:

$$RI_t = NOPAT_t - N_{t-1}\, r_t\,(1-T_t) - Ke_t\, Ebv_{t-1} \qquad \textbf{(35)}$$

Taking into consideration that $NOPAT_t = ROA_t\,(Ebv_{t-1} + N_{t-1})$ and replacing (33) and (34) in (35), we get the definition of EVA using WACCbv (WACC calculated with book values of debt and equity):

Appendix (continued)

$$RI_t = (N_{t-1} + Ebv_{t-1})\ (ROA_t - WACCbv_t) \qquad \textbf{(36)}$$

Consequently, another way of expressing (16) is[11]:

$$E_0 = Ebv_0 + PV_0\ [Ke;\ (N_{t-1} + Ebv_{t-1})\ (ROA_t - WACCbv_t)]$$

The difference between residual income and EVA is:

$$RI_t - EVA_t = (N_{t-1} + Ebv_{t-1})\ (WACC_t - WACCbv_t) \qquad \textbf{(37)}$$

[11] ROA *(return on assets)* is also called ROI *(return on investments)*, ROCE *(return on capital employed)*, *ROC (return on capital)* and RONA *(return on net assets)*. ROA = ROI = ROCE = ROC = RONA. ROA is equal to ROE in the unlevered company.

Table I. Balance sheet, income statement and cash flows of Tenmethods, Inc.

Growth of income statement and balance sheet after period 3 = 2%. Cost of debt (r) = 9%

line	*Balance sheet*	0	1	2	3	4	5
1	Working capital requirements (WCR)	800	890	1,000	1,100	1,122.00	1,144.44
2	Gross fixed assets	1,200	1,300	1,450	1,660	1,895.10	2,134.90
3	- accumulated depreciation		200	405	615	829.20	1,047.68
4	Net fixed assets	1,200	1,100	1,045	1,045	1,065.90	1,087.22
5	TOTAL ASSETS	2,000	1,990	2,045	2,145	2,187.90	2,231.66
6	Debt (N)	1,500	1,500	1,500	1,550	1,581.00	1,612.62
7	Equity (book value)	500	490	545	595	606.90	619.04
8	TOTAL LIABILITIES	2,000	1,990	2,045	2,145	2,187.90	2,231.66

line	*Income statement*	1	2	3	4	5
9	EBITDA	325.0	450.0	500.0	510.00	520.20
10	Depreciation	200.0	205.0	210.0	214.20	218.48
11	Interest payments	135.0	135.0	135.0	139.50	142.29
12	PBT (profit before tax)	-10.0	110.0	155.0	156.30	159.43
13	Taxes	0.0	40.0	62.0	62.52	63.77
14	PAT (profit after tax = net income)	**-10.0**	**70.0**	**93.0**	**93.78**	**95.66**
15	NOPAT (Net operating profit after taxes)	125.00	155.91	174.00	177.48	181.03
16	**Tax rate** = line 13 / line 12	**0.0%**	**36.36%**	**40.0%**	**40.0%**	**40.0%**

line	*Cash Flows*	1	2	3	4	5
14	**PAT** (profit after tax)	**-10.00**	**70.00**	**93.00**	**93.78**	**95.66**
17	+ depreciation	200.00	205.00	210.00	214.20	218.48
18	+ increase of debt	0.00	0.00	50.00	31.00	31.62
19	- increase of working capital requirements	-90.00	-110.00	-100.00	-22.00	-22.44
20	- investment in fixed assets	-100.00	-150.00	-210.00	-235.10	-239.80
21	**ECF** (equity cash flow)	**0.00**	**15.00**	**43.00**	**81.88**	**83.52**
22	**FCF** (free cash flow)	**135.00**	**100.91**	**74.00**	**134.58**	**137.27**
23	**CFd** (debt cash flow)	**135.00**	**135.00**	**85.00**	**108.50**	**110.67**
24	**CCF** (capital cash flow)	**135.00**	**150.00**	**128.00**	**190.38**	**194.19**
25	ROE (Return on Equity)	-2.00%	14.29%	17.06%	15.76%	15.76%
26	ROA (Return on Assets)	6.25%	7.83%	8.51%	8.27%	8.27%

Table II. Valuation of Tenmethods, Inc.

This table presents the valuation of the firm in Table I using ten different methods of discounted cash flow valuation: Adjusted present value (lines 3-6); equity cash flows discounted at the required return to equity (lines 7 and 8); free cash flow discounted at the WACC (lines 9 and 10); capital cash flows discounted at the WACC before tax (lines 11 and 12); residual income discounted at the required return to equity (lines 13 and 14); EVA discounted at the WACC (lines 15 and 16); the business's risk-adjusted equity cash flows discounted at the required return to assets (lines 17 and 18); the business's risk-adjusted free cash flows discounted at the required return to assets (lines 19 and 20); the risk-free-adjusted equity cash flows discounted at the risk-free rate (lines 21 and 22); and the risk-free-adjusted free cash flows discounted at the risk-free rate (lines 23 and 24). All ten methods provide the same valuation.

Valuation parameters: R_F = 6%; P_M (market risk premium) = 4%; Kd = 8%; βd = 0.5; βu = 1.0;

line	Formula		0	1	2	3	4	5
1		Ku		10.00%	10.00%	10.00%	10.00%	10.00%
2	(2)	D = PV(Kd; CFd)	1,743.73	1,748.23	1,753.09	1,808.33	1,844.50	1,881.39
3	(11)	Vu = PV (Ku;FCF)	1,525.62	1,543.18	1,596.59	1,682.25	1,715.90	1,750.21
4	(12)	VTS = PV[Ku; D T Ku + T (Nr - DKd)]	762.09	838.30	860.33	878.33	895.90	913.82
5	(10)	E + D = VTS + Vu	2,287.71	2,381.48	2,456.92	2,560.58	2,611.80	2,664.03
6		E = VTS + Vu - D	543.98	633.25	703.83	752.25	767.29	782.64
7	(14)	Ke		16.41%	13.51%	12.99%	12.88%	12.88%
8	(1)	E = PV(Ke;ECF)	543.98	633.25	703.83	752.25	767.29	782.64
9	(6) (15)	WACC		10.000%	7.405%	7.231%	7.256%	7.256%
10	(4)	E = PV(WACC;FCF) - D	543.98	633.25	703.83	752.25	767.29	782.64
11	(8)	$WACC_{BT}$		10.000%	9.466%	9.429%	9.435%	9.435%
12	(7)	E = PV($WACC_{BT}$;CCF) - D	543.98	633.25	703.83	752.25	767.29	782.64
13	(17)	RI (Residual income)		-92.05	3.78	22.21	17.12	17.46
14	(16)	E = PV(Ke;RI) + Ebv	543.98	633.25	703.83	752.25	767.29	782.64
15	(19)	EVA		-75.00	8.55	26.12	21.84	22.28
16	(18)	E = Evc - (D-N) + PV(WACC;EVA)	543.98	633.25	703.83	752.25	767.29	782.64
17	(23)	ECF\\Ku		-34.87	-7.25	21.96	60.18	61.38
18	(22)	E = PV(Ku;ECF\\Ku)	543.98	633.25	703.83	752.25	767.29	782.64
19	(21)	FCF\\Ku		135.00	162.71	142.02	204.85	208.94
20	(20)	E = PV(Ku;FCF\\Ku) - D	543.98	633.25	703.83	752.25	767.29	782.64
21	(27)	ECF\\Rf		-56.63	-32.58	-6.19	30.09	30.69
22	(26)	E = PV(Rf;ECF\\Rf)	543.98	633.25	703.83	752.25	767.29	782.64
23	(25)	FCF\\Rf		43.49	67.46	43.75	102.42	104.47
24	(24)	E = PV(Rf;FCF\\Rf) - D	543.98	633.25	703.83	752.25	767.29	782.64
31	(29)	Levered beta (β_L)		2.602747	1.878406	1.747234	1.721170	1.721170
32		D / (E+D)	76.22%	73.41%	71.35%	70.62%	70.62%	70.62%
33		N / (Ebv+N)	75.00%	75.38%	73.35%	72.26%	72.26%	72.26%

Table III. Sensitivity analysis of the Valuation of Tenmethods, Inc.

Changes in the valuation as a function of the growth after period 3 (g) and the required return to debt (Kd)

g	Equity value (E)	Debt value (D)	Enterprise value (E+D)	Value of tax shields (VTS)	**WACC**		**Ke**		**$WACC_{BT}$**	
					t=1	**t=4**	**t=1**	**t=4**	**t=1**	**t=4**
0.0%	502.08	1692.46	2194.54	625.54	10.00%	7.14%	16.74%	13.02%	10.00%	9.43%
1.0%	521.20	1714.43	2235.63	685.91	10.00%	7.19%	16.58%	12.95%	10.00%	9.43%
2.0%	543.98	1743.73	2287.71	762.09	10.00%	7.26%	16.41%	12.88%	10.00%	9.44%
3.0%	571.24	1784.74	2355.98	861.35	10.00%	7.33%	16.25%	12.82%	10.00%	9.44%
4.0%	603.42	1846.27	2449.69	996.38	10.00%	7.43%	16.12%	12.78%	10.00%	9.44%

Kd	Equity value (E)	Debt value (D)	Enterprise value(E+D)	Value of tax shields (VTS)	**WACC**		**Ke**		**$WACC_{BT}$**	
					t=1	**t=4**	**t=1**	**t=4**	**t=1**	**t=4**
7.00%	328.42	2084.83	2413.25	887.63	10.00%	6.97%	29.04%	17.30%	10.00%	9.04%
7.50%	445.98	1898.79	2344.77	819.15	10.00%	7.12%	20.64%	14.53%	10.00%	9.25%
8.00%	543.98	1743.73	2287.71	762.09	10.00%	7.26%	16.41%	12.88%	10.00%	9.44%
8.50%	626.93	1612.50	2239.43	713.81	10.00%	7.37%	13.86%	11.80%	10.00%	9.60%
9.00%	698.05	1500.00	2198.05	672.43	10.00%	7.48%	12.15%	11.03%	10.00%	9.75%
9.50%	759.70	1402.48	2162.18	636.56	10.00%	7.57%	10.92%	10.45%	10.00%	9.88%

Table IV. Sensitivity analysis of the Valuation of Tenmethods, Inc. according to Damodaran (1994), Harris and Pringle (1985), and Myers (1974)

Changes in the valuation as a function of the growth after period 3 (g) and the required return to debt (Kd).

	E				VTS			
g	Fernández (2004)	Damodaran (1994)	Ruback (2002)	Myers (1974)	Fernández (2004)	Damodaran (1994)	Ruback (2002)	Myers (1974)
0.00%	502.08	281.03	376.92	515.20	625.54	404.48	500.38	638.65
1.00%	521.20	279.02	382.25	553.04	685.91	443.73	546.96	717.75
2.00%	**543.98**	**274.29**	**387.07**	**605.11**	**762.09**	**492.40**	**605.18**	**823.22**
3.00%	571.24	264.13	389.93	680.75	861.35	554.25	680.05	970.87
4.00%	603.42	242.28	386.90	799.39	996.38	635.24	779.86	1192.35

	WACC, t=4				Ke, t=4			
g	Fernández (2004)	Damodaran (1994)	Ruback (2002)	Myers (1974)	Fernández (2004)	Damodaran (1994)	Ruback (2002)	Myers (1974)
0.00%	7.14%	7.81%	7.57%	7.14%	13.02%	18.64%	16.29%	13.02%
1.00%	7.19%	7.84%	7.61%	7.14%	12.95%	18.78%	16.29%	12.63%
2.00%	**7.26%**	**7.88%**	**7.66%**	**7.15%**	**12.88%**	**19.02%**	**16.33%**	**12.19%**
3.00%	7.33%	7.93%	7.71%	7.17%	12.82%	19.46%	16.43%	11.66%
4.00%	7.43%	7.99%	7.78%	7.19%	12.78%	20.35%	16.70%	11.04%

	E				VTS			
Kd	Fernández (2004)	Damodaran (1994)	Ruback (2002)	Myers (1974)	Fernández (2004)	Damodaran (1994)	Ruback (2002)	Myers (1974)
7.00%	328.42	166.67	45.97	438.73	887.63	725.88	605.18	997.95
7.50%	445.98	225.37	232.01	529.45	819.15	598.54	605.18	902.62
8.00%	**543.98**	**274.29**	**387.07**	**605.11**	**762.09**	**492.40**	**605.18**	**823.22**
8.50%	626.93	315.68	518.30	669.19	713.81	402.57	605.18	756.07
9.00%	698.05	351.16	630.80	724.18	672.43	325.54	605.18	698.56
9.50%	759.70	381.92	728.32	771.88	636.56	258.77	605.18	648.74

	WACC, t=4				Ke, t=4			
Kd	Fernández (2004)	Damodaran (1994)	Ruback (2002)	Myers (1974)	Fernández (2004)	Damodaran (1994)	Ruback (2002)	Myers (1974)
7.00%	6.97%	7.29%	7.66%	6.81%	17.30%	23.98%	41.04%	15.03%
7.50%	7.12%	7.60%	7.66%	6.99%	14.53%	20.97%	22.12%	13.31%
8.00%	**7.26%**	**7.88%**	**7.66%**	**7.15%**	**12.88%**	**19.02%**	**16.33%**	**12.19%**
8.50%	7.37%	8.14%	7.66%	7.30%	11.80%	17.66%	13.52%	11.40%
9.00%	7.48%	8.38%	7.66%	7.43%	11.03%	16.66%	11.87%	10.81%
9.50%	7.57%	8.61%	7.66%	7.55%	10.45%	15.89%	10.78%	10.36%

Table V

Main formulas used in the paper

$$E_0 = PV_0\ [Ke_t;\ ECF_t] \quad (1)$$
$$D_0 = PV_0\ [Kd_t;\ CFd_t] \quad (2)$$
$$CFd_t = N_{t-1}\ r_t - (N_t - N_{t-1}) \quad (3)$$
$$E_0 + D_0 = PV_0\ [WACC_t\ ;\ FCF_t] \quad (4)$$
$$ECF_t = FCF_t + (N_t - N_{t-1}) - N_{t-1}\ r_t\ (1 - T_t) \quad (5)$$
$$WACC_t = [E_{t-1}\ Ke_t + D_{t-1}\ Kd_t - N_{t-1}\ r_t\ T_t] / [E_{t-1} + D_{t-1}] \quad (6)$$
$$E_0 + D_0 = PV[WACC_{BTt};\ CCF_t] \quad (7)$$
$$WACC_{BT\ t} = [E_{t-1}\ Ke_t + D_{t-1}\ Kd_t] / [E_{t-1} + D_{t-1}] \quad (8)$$
$$CCF_t = ECF_t + CFd_t = ECF_t - (N_t - N_{t-1}) + N_{t-1}\ r_t = FCF_t + N_{t-1}\ r_t\ T_t \quad (9)$$
$$E_0 + D_0 = Vu_0 + VTS_0 \quad (10)$$
$$Vu_0 = PV_0\ [Ku_t;\ FCF_t] \quad (11)$$
$$VTS_0 = PV_0\ [Ku_t;\ D_{t-1}\ Ku_t\ T_t + T_t\ (N_{t-1}\ r_t - D_{t-1}\ Kd_t)] \quad (12)$$
$$E_0 + D_0 = PV_0\ [Ku_t;\ FCF_t + D_{t-1}\ Ku_t\ T_t + T_t\ (N_{t-1}\ r_t - D_{t-1}\ Kd_t)] \quad (13)$$
$$Ke_t = Ku_t + D_{t-1}\ (1 - T_t)\ (Ku_t - Kd_t) / E_{t-1} \quad (14)$$
$$WACC_t = Ku_t - [D_{t-1}\ T_t\ (Ku_t - Kd_t) + N_{t-1}\ r_t\ T_t] / (E_{t-1} + D_{t-1}) \quad (15)$$
$$E_0 = Ebv_0 + PV_0\ [Ke_t;\ RI_t] \quad (16)$$
$$RI_t = PAT_t - Ke_t\ Ebv_{t-1} \quad (17)$$
$$E_0 + D_0 = (Ebv_0 + N_0) + PV_0\ [WACC_t\ ;\ EVA_t] \quad (18)$$
$$EVA_t = NOPAT_t - (N_{t-1} + Ebv_{t-1})WACC_t \quad (19)$$
$$E_0 + D_0 = PV_0\ [Ku_t\ ;\ FCF_t \backslash\backslash Ku] \quad (20)$$
$$FCF_t \backslash\backslash Ku = FCF_t - (E_{t-1} + D_{t-1})\ (WACC_t - Ku_t) = FCF_t + D_{t-1}\ T_t\ (Ku_t - Kd_t) - N_{t-1}\ r_t\ T_t. \quad (21)$$
$$E_0 = PV_0\ [Ku_t;\ ECF_t \backslash\backslash Ku] \quad (22)$$
$$ECF_t \backslash\backslash Ku = ECF_t - E_{t-1}\ (Ke_t - Ku_t) = ECF_t - D_{t-1}\ (1 - T_t)\ (Ku_t - Kd_t) \quad (23)$$
$$E_0 + D_0 = PV_0\ [R_{F\,t}\ ;\ FCF_t \backslash\backslash R_F] \quad (24)$$
$$FCF_t \backslash\backslash R_F = FCF_t - (E_{t-1} + D_{t-1})\ (WACC_t - R_{F\,t}) \quad (25)$$
$$E_0 = PV_0\ [R_{F\,t};\ ECF_t \backslash\backslash R_F] \quad (26)$$
$$ECF_t \backslash\backslash R_F = ECF_t - E_{t-1}\ (Ke_t - R_{F\,t}) \quad (27)$$
$$Ke = R_F + \beta_L\ P_M \qquad Ku = R_F + \beta u\ P_M \qquad Kd = R_F + \beta d\ P_M \quad (28)$$
$$\beta u = [E\ \beta_L + \beta d\ D\ (1 - T)] / [E + D\ (1 - T)] \quad (29)$$
$$ECF_t = PAT_t - (Ebv_t - Ebv_{t-1}) \quad (30)$$
$$FCF_t = PAT_t - (Ebv_t - Ebv_{t-1}) + N_{t-1}\ r_t\ (1 - T_t) - (N_t - N_{t-1}) \quad (31)$$
$$FCF_t = NOPAT_t - (Ebv_t - Ebv_{t-1} + N_t - N_{t-1}) \quad (32)$$
$$WACCbv_t = [Ebv_{t-1}\ Ke_t + N_{t-1}\ r_t\ (1 - T_t)] / (Ebv_{t-1} + N_{t-1}) \quad (33)$$
$$Ebv_{t-1}\ Ke_t + N_{t-1}\ r_t\ (1 - T_t) = WACCbv_t\ (Ebv_{t-1} + N_{t-1}) \quad (34)$$
$$RI_t = NOPAT_t - N_{t-1}\ r_t\ (1 - T_t) - Ke_t\ Ebv_{t-1} \quad (35)$$
$$RI_t = (N_{t-1} + Ebv_{t-1})\ (ROA_t - WACCbv_t) \quad (36)$$
$$RI_t - EVA_t = (N_{t-1} + Ebv_{t-1})\ (WACC_t - WACCbv_t) \quad (37)$$

$$E_t = E_{t-1}\ (1 + Ke_t) - ECF_t \quad (1i)$$
$$D_t = D_{t-1}\ (1 + Kd_t) - CFd_t \quad (2i)$$
$$E_t + D_t = (E_{t-1} + D_{t-1})\ (1 + WACC_t) - FCF_t \quad (4i)$$
$$E_t + D_t = (E_{t-1} + D_{t-1})\ (1 + WACC_{BTt}) - CCF_t \quad (7i)$$
$$E_t + D_t = (E_{t-1} + D_{t-1})\ (1 + Ku_t) - [FCF_t + D_{t-1}\ Ku_t\ T_t + T_t\ (N_{t-1}\ r_t - D_{t-1}\ Kd_t)] \quad (13i)$$
$$E_t - Ebv_t = (E_{t-1} - Ebv_{t-1})\ (1 + Ke_t) - (PAT_t - Ke_t\ Ebv_{t-1}) = (E_{t-1} - Ebv_{t-1})\ (1 + Ke_t) - RI_t \quad (16i)$$

Table VI

Dictionary

βd = Beta of debt
β_L = Beta of levered equity
βu = Beta of unlevered equity = beta of assets
D = Value of debt
E = Value of equity
Ebv = Book value of equity
ECF = Equity cash flow
RI = Residual income
EVA = Economic value added
FCF = Free cash flow
g = Growth rate of the constant growth case
I = Interest paid
Ku = Cost of unlevered equity (required return to unlevered equity)
Ke = Cost of levered equity (required return to levered equity)
Kd = Required return to debt
N = Book value of the debt
NOPAT = Net Operating Profit After Tax = profit after tax of the unlevered company
PAT = Profit after tax
PBT = Profit before tax
P_M = Market premium = E (R_M - R_F)
PV = Present value
r = Cost of debt
R_F = Risk-free rate
R_M = Market return
ROA = Return on Assets = $NOPAT_t$ / (N_{t-1}+Ebv_{t-1})
ROE = Return on Equity = PAT_t / Ebv_{t-1}
T = Corporate tax rate
VTS = Value of the tax shields
Vu = Value of shares in the unlevered company
WACC = Weighted average cost of capital
$WACC_{BT}$ = Weighted average cost of capital before taxes
WCR = Working capital requirements = net current assets

REFERENCES

Arditti, F. and Levy, H. (1977), “The Weighted Average Cost of Capital as a Cutoff Rate: A Critical Examination of the Classical Textbook Weighted Average”, *Financial Management* (Fall): 24-34.

Bodie, Z. and Merton, R. (2000), *Finance*. (Prentice Hall, Upper Saddle River, New Jersey).

Brealey, R. and Myers, S. (2000), *Principles of Corporate Finance*. (McGraw-Hill, New York, sixth edition).

Copeland, T., Koller, T. and Murrin, J. (2000), *Valuation: Measuring and Managing the Value of Companies.* (Wiley, New York, third edition).

Damodaran, A. (1994), *Damodaran on Valuation*. (John Wiley and Sons, New York).

Damodaran, A. (2002), *Investment Valuation*. (John Wiley and Sons, New York, second edition).

Fernández, P. (2002), *Valuation Methods and Shareholder Value Creation.* (Academic Press, San Diego, CA.).

Fernández, P. (2004), “The Value of Tax Shields is not Equal to the Present Value of Tax Shields”, forthcoming in the *Journal of Financial Economics*.

Francis, J., Olsson, P. and Oswald, D. (2000), “Comparing the accuracy and explainability of dividend, free cash flow, and abnormal earnings equity value estimates”, *Journal of Accounting Research* 38 (Spring): 45-70.

Harris, R. and Pringle, J. (1985), “Risk-adjusted discount rates extensions form the average-risk case”, *Journal of Financial Research* (Fall): 237-244.

Inselbag, I. and Kaufold, H. (1997), “Two DCF approaches for valuing companies under alternative financing strategies (and how to choose between them)”, *Journal of Applied Corporate Finance* (Spring): 114-122.

Kaplan, S. and Ruback, R. (1995), “The Valuation of Cash Flow Forecast: An Empirical Analysis”, *Journal of Finance* 50: 1059-1093.

Lewellen, W. and Emery, D. (1986), “Corporate debt management and the value of the firm”, *Journal of Financial Quantitative Analysis* (December): 415-426.

Luehrman, T. (1997), “What’s it worth: A general manager’s guide to valuation”, and “Using APV: A better tool for valuing operations”, *Harvard Business Review* (May-June):132-154.

Lundholm, R. and O’Keefe, T. (2001a), “Reconciling value estimates from the discounted cash flow model and the residual income model”, *Contemporary Accounting Research* 18 (Summer): 311-335.

Lundholm, R. and O’Keefe, T. (2001b), “On comparing residual income and discounted cash flow models of equity valuation: A response to Penman 2001”, *Contemporary Accounting Research* 18 (Winter): 215-220.

Miles, J. and Ezzell, J. (1980), “The weighted average cost of capital, perfect capital markets and project life: a clarification”, *Journal of Financial and Quantitative Analysis* (September): 719-730.

Miles, J. and Ezzell, J. (1985), “Reequationing tax shield valuation: A note”, *Journal of Finance* Vol. XL, 5: 1485-1492.

Modigliani, F. and Miller, M. (1958), “The cost of capital, corporation finance and the theory of investment”, *American Economic Review* 48: 261-297.

Modigliani, F. and Miller, M. (1963), “Corporate income taxes and the cost of capital: A correction”, *American Economic Review* (June): 433-443.

Myers, S. (1974), “Interactions of corporate financing and investment decisions –Implications for capital budgeting”, *Journal of Finance* (March): 1-25.

Penman, S. (2001), “On comparing cash flow and accrual accounting models for use in equity valuation”, *Contemporary Accounting Research* 18 (Winter): 203-214.

Penman, S. and Sougiannis, T. (1998), “A comparison of dividend, cash flow, and earnings approaches to equity valuation”, *Contemporary Accounting Research* 15 (Fall): 343-383.

Ruback, R. (1995), “A note on capital cash flow valuation”, Harvard Business School Note, 9-295-069.

Ruback, R. (2002), “Capital cash flows: a simple approach to valuing risky cash flows”, *Financial Management* 31: 85-103.

Taggart, R. (1991), “Consistent valuation and cost of capital. Expressions with corporate and personal taxes”, *Financial Management* 20: 8-20.

Tham, J. and Vélez-Pareja, I. (2001), “The correct discount rate for the tax shield: the N-period case”, SSRN Working Paper.