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ARE CALCULATED BETAS GOOD FOR ANYTHING?

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ARE CALCULATED BETAS GOOD FOR ANYTHING?

Abstract

We calculate betas of 3,813 companies using 60 monthly returns each day of December 2001 and January 2002. The median (average) of the maximum beta divided by the minimum beta was 3.07 (15.7). The median of the percentage daily change (in absolute value) of the betas was 20%.

Industry betas are also unstable. On average, the maximum beta of an industry was 2.7 times its minimum beta in December 2001 and January 2002. The median (average) of the percentage daily change (in absolute value) of the industry betas was 7% (16%).

This dispersion of the calculated betas has important implications for the instability of beta-ranked portfolios.

JEL Classification: G12, G31, M21

Keywords: beta, historical beta, expected beta, systematic risk, cost of equity

ARE CALCULATED BETAS GOOD FOR ANYTHING?

The beta is one of the most important but elusive parameters in finance. According to the CAPM, it is a measure of the so-called systematic risk. We differentiate the *historical beta* from the *expected beta*, the historical beta being the one we get from the regression of historical data, and the expected beta being the relevant one for estimating the cost of equity (the required return on equity).

Historical betas are used for several purposes:

- To calculate the cost of equity of companies
- To rank assets and portfolios with respect to systematic risk
- To test CAPM and mean-variance efficiency

We argue that historical betas (calculated from historical data) are useless for all three purposes.

The capital asset pricing model (CAPM) defines the required return to equity in the following terms:

$$Ke_i = R_F + E(\beta_i)\,[E(R_M) - R_F]$$

R_F = rate of return for risk-free investments (Treasury bonds)
$E(\beta_i)$ = expected equity's beta of company i.
$E(R_M)$ = expected market return.
$[E(R_M) - R_F]$ = market risk premium

Therefore, given certain values for the equity's beta, the risk-free rate and the market risk premium, it is possible to calculate the required return to equity. The market risk premium is the difference between the *expected* return on the market portfolio and the risk-free rate, which in the context of the CAPM is equal to the incremental return demanded by investors on stocks, above that of risk-free investments.

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When estimating betas the standard procedure is to use five years of monthly data and a value-weighted index. This procedure is widely used in academic research and by commercial beta providers such as Merrill Lynch and Ibbotson and Associates. However, different beta sources provide us with different betas, as is shown in **Table 1**. Bruner et al. (1998) also found sizeable differences among beta providers. For their sample the average beta according to Bloomberg was 1.03, whereas according to Value Line it was 1.24.

Table 1. Betas of different companies according to different sources

	AT&T	Boeing	CocaCola	Date
Yahoo	0.61	0.46	0.29	12-febr-03
Multex	0.87	0.66	0.42	12-febr-03
Quicken	1.14	0.66	0.41	12-febr-03
Reuters	0.87	0.68	0.42	12-febr-03
Bloomberg	1.00	1.07	0.64	12-febr-03
Datastream	1.10	1.10	0.37	12-febr-03
Buy&hold	0.84	0.66	0.41	14-febr-03

We show that, in general, it is an enormous error to use the historical beta as a proxy for the expected beta. First, because it is almost impossible to calculate a meaningful beta because historical betas change dramatically from one day to the next; second, because very often we cannot say with a relevant statistical confidence that the beta of one company is smaller or bigger than the beta of another; third, because historical betas do not make much sense in many cases: high-risk companies very often have smaller historical betas than low-risk companies; fourth, because historical betas depend very much on which index we use to calculate them.

Those results are far from being new. For example, Damodaran (2001, page 72) also calculates different betas for Cisco versus the S&P 500:

Beta estimates for Cisco versus the S&P 500.

	Daily	Weekly	Monthly	Quarterly
2 years	1.72	1.74	1.82	2.7
5 years	1.63	1.70	1.45	1.78

Source: Damodaran (2001, page 72)

Damodaran (1994) also makes this point by calculating the beta of Disney. With daily data, he gets 1.33; 1.38 with weekly data; 1.13 with monthly data; 0.44 with quarterly data; and 0.77 with annual data. With a 3-year period, he gets 1.04; 1.13 with 5 years; and 1.18 with 10 years. Also, the beta depends on the index taken as the benchmark; thus, the beta with respect to the Dow 30 is 0.99; with respect to the S&P 500, it is 1.13, and with respect to the Wilshire 5000, it is 1.05.

We calculate the betas using monthly data every day of the month, not only data of the last day of the month as has usually been done. By doing this, the fact that calculated betas change a lot becomes much clearer. We calculate historical betas for 3,813 companies traded on the New York Stock Exchange (1,462) and the Nasdaq (2,351) *each day* in the 2-month period December 1, 2001 – January 31, 2002 using 5 years of *monthly data*[1]. Each day's betas are calculated betas with respect to the S&P 500, using 60 monthly returns. For example, on December 18, 2001, the beta is calculated by running a regression of the 60 monthly returns of the company calculated on the 18th of every month, on the 60 monthly returns of the S&P 500 calculated on the 18th of every month. We have included only companies that traded in December 1996. Because of this criterion, our sample includes only 450 of the 500 companies that were in the S&P 500 in December 2001.

1. Historical betas change dramatically from one day to the next

The results show that historical betas change dramatically from one day to the next.

Tables 2 and **3** report some statistics about the 62 calculated betas of the 3,813 companies in our sample with respect to the S&P 500 in the two-month period of December 2001 and January 2002. **Table 2** shows that only 2,780 companies (73%) had positive betas on the 62 consecutive days. Only 434 companies (11%) had betas bigger than one on the 62 consecutive days. And 2,927 companies (77%) had, in the sample period, a maximum beta more than two times bigger than their minimum beta. Of the 450 companies in the S&P 500, 52% had a maximum beta more than two times bigger than their minimum beta. Of the 30 companies in the DJIA, 40% had a maximum beta more than two times bigger than their minimum beta. Looking at the 101 industry betas, 25% (31%) of the industries had a maximum weighted (unweighted) beta more than two times bigger than their minimum beta.

Table 2. Historical betas of the 3,813 companies in our sample with respect to the S&P 500

	Full sample				S&P 500				DJIA 30			
	Companies		Market Cap.		Companies		Market Cap.		Companies		Market Cap.	
	Number	%	$ bn	%	Number	%	$ bn	%	Number	%	$ bn	%
All betas > 0	2,780	73%	11,956	93%	404	90%	8,980	93%	28	93%	3,223	94%
Average beta > 1	1,242	33%	5,758	45%	157	35%	4,273	44%	13	43%	1,839	54%
All betas > 1	434	11%	3,116	24%	71	16%	2,574	27%	7	23%	1,372	40%
Average beta < 0	124	3%	132	1%	10	2%	102	1%	0	0%		0%
All betas < 0	2	0%	97	1%	0	0%		0%	0	0%		0%
Abs (Beta max/beta min) > 2	2,927	77%	6,417	50%	235	52%	4,484	47%	12	40%	1,225	36%
Total	3,813	100%	12,886	100%	450	100%	9,638	100%	30	100%	3,425	100%

	Industry weighted betas				Industry unweighted betas			
	Industries		Market Cap.		Industries		Market Cap.	
	Number	%	$ bn	%	Number	%	$ bn	%
All betas > 0	98	97%	12,747	99%	95	94%	12,450	129%
Average beta > 1	20	20%	3,630	28%	26	26%	5,381	56%
All betas > 1	12	12%	2,665	21%	18	18%	3,714	39%
Average beta < 0	1	1%	18	0%	0	0%		0%
All betas < 0	0	0%		0%	0	0%		0%
Abs(Beta max/beta min) > 2	25	25%	1,337	10%	31	31%	3,545	37%
Total	101	100%	12,886	100%	101	100%	9,638	100%

Betas are calculated each day in the period 1/12/01-31/1/02 using 5 years of monthly data, i.e. on December 18, 2001, the beta is calculated by running a regression of the 60 monthly returns of the company on the 60 monthly returns of the S&P 500, the returns of each month being calculated on the 18th of each month. The table shows that 2,780 companies (with a combined market capitalization of $11,956 billion) had positive

1 For example, Brealey and Myers (2000, page 224) also calculate historical betas using 60 monthly returns.

betas on the 62 days in the period 1/12/01-31/1/02. 434 companies had the 62 betas bigger than 1.0 in the period 1/12/01-31/1/02. For 2,927 companies (77% of the sample), the maximum beta divided by the minimum beta was bigger than 2.

The table also contains the statistics of the 450 companies in our sample that belonged to the S&P 500, and of the 30 companies in the DJIA Index in December 2001.

The table contains the same statistics for the betas of 101 industries, both weighted and unweighted.

Table 3. Summary statistics of the historical betas of the 3,813 companies in our sample with respect to the S&P 500

		Company betas			Industry betas	
		Full sample	S&P 500	DJIA 30	Weighted	Unweighted
Beta average	Median	0.72	0.82	0.88	0.84	0.74
	Average	0.83	0.91	0.94	0.88	0.79
	Maximum	4.46	3.13	1.66	2.78	1.98
	Minimum	–1.43	–0.17	0.12	0.06	–0.05
Max - Min	Median	0.88	0.63	0.53	0.41	0.35
	Average	1.05	0.68	0.53	0.45	0.38
	Maximum	3.99	2.17	0.94	1.02	1.23
	Minimum	0.12	0.21	0.27	0.14	0.15
Abs (Max / Min)	Median	3.07	2.11	1.77	1.64	1.52
	Average	15.70	4.76	2.72	2.72	2.57
	Maximum	10116.62	251.72	23.58	35.47	42.03
	Minimum	0.10	0.12	1.25	1.12	0.95
(MAX-Min) / Abs (Beta December 31)	Median	1.31	0.76	0.52	0.49	0.44
	Average	6.72	2.32	0.76	1.26	0.71
	Maximum	2997.32	240.01	3.36	55.62	6.77
	Minimum	0.21	0.21	0.24	0.12	0.19

Betas are calculated each day in the period 1/12/01-31/1/02 using 5 years of monthly data, i.e. on December 18, 2001, the beta is calculated by running a regression of the 60 monthly returns of the company on the 60 monthly returns of the S&P 500. The returns of each month are calculated on the 18th of each month. The table contains the median, the average, the maximum, and the minimum, of:

- Beta average: the average of the 62 betas calculated for each company and industry every day in the period 1/12/01-31/1/02.
- Max - Min: maximum beta minus minimum beta of the 62 betas calculated for each company and industry every day in the period 1/12/01-31/1/02.
- Abs (Max / Min): absolute value of the maximum beta divided by the minimum beta of the 62 betas calculated every day in the period 1/12/01-31/1/02.
- (MAX-Min) / Abs (Beta December 31): maximum beta minus minimum beta of the 62 betas calculated for each company and industry every day in the period 1/12/01-31/1/02 divided by the absolute value of the beta calculated on December 31, 2001.

Table 3 shows that the median of the averages of the 62 betas calculated for each company was 0.72 for the 3,813 companies in our full sample, 0.82 for the 450 companies in the S&P 500, and 0.88 for the 30 companies in the DJIA. The median of the difference between the maximum and the minimum of the 62 betas calculated for each company was 0.88 for the 3,813 companies in our full sample, 0.63 for the 450 companies in the S&P 500, and 0.53 for the 30 companies in the DJIA. Note that the difference between the maximum and the minimum is smaller than 4 because we have eliminated 127 companies for which this difference was bigger than 4. The median of the absolute value of the ratio between the maximum and the minimum of the 62 betas calculated for each company was 3.07 for the 3,813 companies in our full sample, 2.11 for the 450 companies in the S&P 500 and 1.77 for the 30 companies in the DJIA. The median of the difference between the maximum and the minimum of the 62 betas calculated for each company, divided by the beta calculated on December 31, 2001, was 1.31 for the 3,813 companies in our full sample, 0.76 for the 450 companies in the S&P 500, and 0.52 for the 30 companies in the DJIA. This statistic was 0.49 for the 101 industry weighted betas, and 0.44 for the 101 industry unweighted betas. From Tables 2 and 3 it is clear that industry betas have less dispersion than company betas. The betas of the 30 companies in the DJIA have, on average, less dispersion than those of the 450 companies in the S&P 500, and these have, on average, less dispersion than those of the 3,813 companies of the full sample. We understand by less dispersion that:

1. the median and the average of the difference between the maximum and the minimum of the 62 betas calculated for each company is closer to zero,
2. the median and the average of the absolute value of the ratio between the maximum and the minimum of the 62 betas calculated for each company is closer to one, and
3. the median and the average of the difference between the maximum and the minimum of the 62 betas calculated for each company, divided by the beta calculated on December 31, 2001, is closer to zero.

Table 4 contains the range of variation of the maximum beta minus the minimum beta of the 62 betas calculated for each company and industry every day in the period 1/12/01-31/1/02. For only seven companies was the difference between the maximum beta and the minimum beta smaller than 0.2. Table 4 also contains the maximum beta minus minimum beta of the 62 betas calculated for each company and industry every day in the period 1/12/01-31/1/02 divided by the absolute value of the beta calculated on December 31, 2001.

Table 4. Historical betas of the 3,813 companies in our sample with respect to the S&P 500.

Betas are calculated each day in the period 1/12/01-31/1/02 using 5 years of monthly data, i.e. on December 18, 2001, the beta is calculated by running a regression of the 60 monthly returns of the company on the 60 monthly returns of the S&P 500. The returns of each month are calculated on the 18th of each month. The table contains the range of variation of:

- the maximum beta minus the minimum beta of the 62 betas calculated for each company and industry every day in the period 1/12/01-31/1/02, and of
- the maximum beta minus the minimum beta of the 62 betas calculated for each company and industry every day in the period 1/12/01-31/1/02, divided by the absolute value of the beta calculated on December 31, 2001.

		Maximum Beta - Minimum Beta						
# companies		3 - 3.99	2 - 2.99	1 - 1.99	0.5 - 0.99	0.2 - 0.49	< 0.2	average
3,813	Full sample	65	268	1,246	1,574	653	7	1.05
450	S&P 500	0	1	56	250	143	0	0.68
3,363	Not in the S&P 500	65	267	1,190	1,324	510	7	1.10
30	DJIA	0	0	6	16	8	0	0.53
101	Industry weighted	0	0	1	37	59	4	0.45
101	Industry unweighted	0	0	1	15	77	8	0.38

		(Maximum Beta - Minimum Beta)/Abs(Beta December 31)						
# companies		> 3	2 - 2.99	1 - 1.99	0.5 - 0.99	0.2 - 0.49	< 0.2	average
3,813	Full sample	800	425	1,208	1,125	255	0	6.72
450	S&P 500	36	23	95	190	106	0	2.32
3,363	Not in the S&P 500	764	402	1,113	935	149	0	7.31
30	DJIA	1	1	3	11	14	0	0.76
101	Industry weighted	3	2	12	31	51	2	1.26
101	Industry unweighted	2	3	10	24	61	1	0.71

Figure 1 shows the historical betas of AT&T, Boeing and Coca-Cola in the two-month period of December 2001 and January 2002 with respect to the S&P 500. It may be seen that the beta of AT&T varies from 0.32 (January 14, 2002) to 1.02 (December 27, 2001), the beta of Boeing varies from 0.57 (January 30, 2002) to 1.22 (January 20, 2002), and the beta of Coca-Cola varies from 0.55 (December 28, 2001) to 1.11 (January 15, 2002). A closer look at the data shows that the beta of AT&T is higher than the beta of Boeing 32% of the days, and is higher than the beta of Coca-Cola 50% of the days. The beta of Boeing is higher than the beta of Coca-Cola 76% of the days. AT&T has the maximum beta (of the three companies) 29% of the days and the minimum beta 47% of the days. Boeing has the maximum beta (of the three companies) 58% of the days and the minimum beta 15% of the days. Coca-Cola has the maximum beta (of the three companies) 13% of the days and the minimum beta 38% of the day

Figure 1. Historical betas of AT&T, Boeing and Coca-Cola

Betas calculated during the two-month period of December 2001 and January 2002 with respect to the S&P 500. Each day, betas are calculated using 5 years of monthly data, i.e. on December 18, 2001, the beta is calculated by running a regression of the 60 monthly returns of the company on the 60 monthly returns of the S&P 500. The returns of each month are calculated on the 18th of the month:

$$\text{monthly return of December 18, 2001} = \frac{\text{total return December 18, 2001}}{\text{total return November 18, 2001}} - 1$$

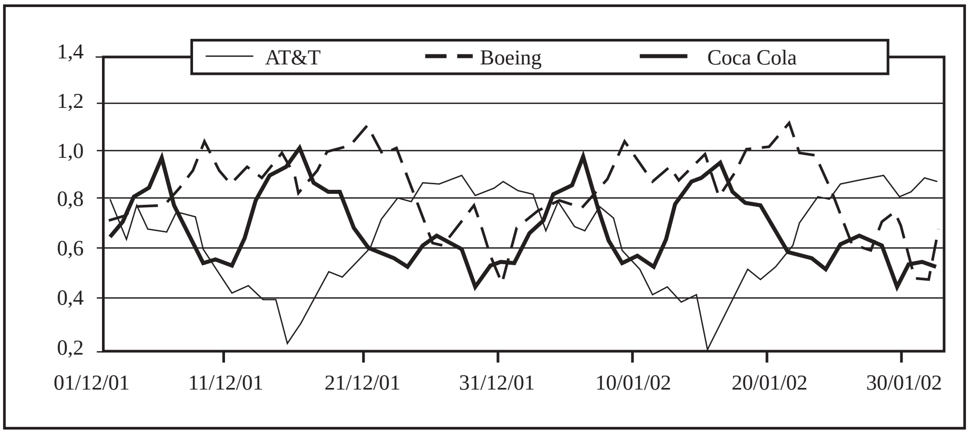


Figure 2 shows the historical betas of Procter & Gamble, Philip Morris and Merck in the two-month period of December 2001 and January 2002 with respect to the S&P 500.

Figure 2. Historical betas of Procter and Gamble, Philip Morris and Merck.
Betas calculated during the two-month period of December 2001 and January 2002 with respect to the S&P 500.

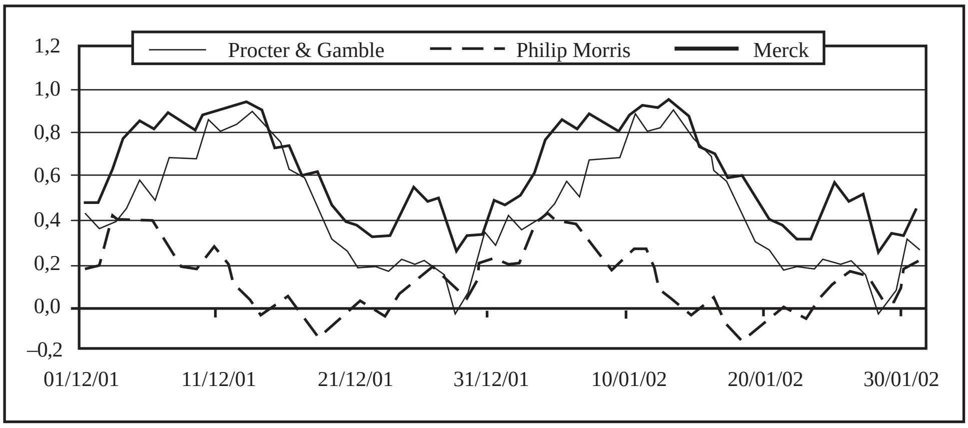
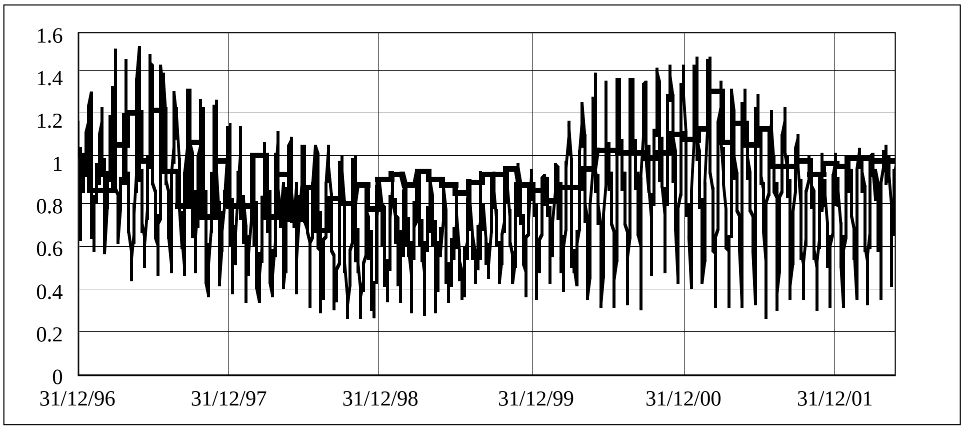


Figure 3 contains the historical betas of AT&T, calculated every day during the period between January 1997 and May 2002. It also contains the historical betas of AT&T, but calculated only the last day of each month.

Figure 3. Historical monthly betas of AT&T

Betas calculated during the 53-month period between January 1997 and May 2002 with respect to the S&P 500. Each day, betas are calculated using 5 years of monthly data, i.e. on December 18, 2001, the beta is calculated by running a regression of the 60 monthly returns of the company on the 60 monthly returns of the S&P 500. The returns of each month are calculated on the 18th of the month.



These three tables and three figures are evidence enough to conclude that calculated betas are very unstable.

Table 5 contains some statistics of the correlation, and of the company volatility divided by the market volatility of the S&P 500 for the 30 companies in the DJIA in the two-month period of December 2001 and January 2002. On average, the maximum divided by its minimum was 2.51 for the correlation, while it was only 1.28 for the ratio of volatilities.

It is clear that the volatility of the betas is mainly a story of volatility of the correlations.

Table 5 shows that, on average, the market price of the shares and the S&P 500 moved in the same direction (both increased or both decreased) only 58% of the months and 48.7% of the days in the 5-year period 1/1/1997-31/12/2001).

Table 5. Some statistics of the correlation and of the company volatility divided by the market volatility of the S&P 500 for the 30 companies in the DJIA in the two-month period of December 2001 and January 2002.

On average, the maximum divided by its minimum was 2.51 for the correlation, while it was only 1.28 for the ratio of volatilities. Correlations and volatilities calculated using 60 monthly data.

$Beta_i$ = correlation ($Return_i$; Return (Market)) x (Company $volatility_i$ / Market volatility)

	Correlation				Company volatility / Market volatility			
	Average	Max	min	Max/min	Average	Max	min	Max/min
3M Co.	0.38	0.50	0.23	2.12	1.51	1.69	1.39	1.22
Alcoa	0.37	0.53	0.21	2.55	2.13	2.47	1.91	1.29
American Express	0.78	0.83	0.70	1.18	1.88	2.17	1.71	1.27
AT&T	0.34	0.44	0.14	3.11	2.22	2.57	1.98	1.29
Boeing	0.45	0.54	0.27	1.97	2.13	2.40	1.92	1.25
Caterpillar	0.40	0.52	0.31	1.65	1.93	2.15	1.74	1.24
Citigroup	0.80	0.85	0.71	1.19	2.07	2.32	1.89	1.23
Coca Cola	0.46	0.61	0.32	1.92	1.73	1.95	1.46	1.33
Du Pont	0.43	0.55	0.31	1.78	1.75	1.97	1.56	1.26
Eastman Kodak	0.28	0.35	0.18	1.97	1.98	2.25	1.67	1.35
Exxon Mobil	0.40	0.58	0.24	2.39	1.06	1.18	0.90	1.31
General Electric	0.79	0.83	0.74	1.11	1.59	1.72	1.46	1.18
Hewlett-Packard	0.53	0.68	0.39	1.76	2.55	2.90	2.23	1.30
Home Depot	0.62	0.71	0.50	1.43	2.01	2.28	1.82	1.25
Honeywell Intl.	0.50	0.59	0.42	1.40	2.44	2.76	2.29	1.20
IBM	0.56	0.66	0.36	1.83	2.11	2.30	1.89	1.22
Intel	0.57	0.64	0.50	1.28	2.72	3.09	2.43	1.27
Intl.Paper	0.38	0.48	0.29	1.67	1.90	2.15	1.65	1.30
Johnson & Johnson	0.37	0.53	0.25	2.14	1.29	1.50	1.10	1.37
J P Morgan Chase	0.72	0.77	0.66	1.18	2.21	2.50	1.89	1.32
McDonalds	0.41	0.49	0.30	1.64	1.60	1.85	1.50	1.23
Merck	0.36	0.55	0.14	4.02	1.67	1.89	1.42	1.33
Microsoft	0.60	0.66	0.52	1.26	2.55	3.10	2.24	1.38
Philip Morris	0.07	0.25	-0.07	3.67	1.96	2.20	1.66	1.32
SBC Comm.	0.30	0.47	0.15	3.16	1.62	1.83	1.36	1.35
United Technologies	0.64	0.72	0.56	1.28	1.89	2.12	1.70	1.25
Wal Mart Stores	0.56	0.67	0.46	1.45	1.78	2.06	1.62	1.27
General Motors	0.52	0.58	0.44	1.31	1.95	2.16	1.78	1.22
Procter & Gamble	0.24	0.45	–0.02	20.37	1.83	2.12	1.58	1.34
Walt Disney	0.55	0.70	0.43	1.61	1.90	2.07	1.69	1.22
Average	0.48	0.59	0.35	2.51	1.93	2.19	1.72	1.28
Max	0.80	0.85	0.74	20.37	2.72	3.10	2.43	1.38
Min	0.07	0.25	–0.07	1.11	1.06	1.18	0.90	1.18

Table 6. Percentage days or months that the share price and the S&P 500 move in the same direction (1/1/1997-31/12/2001)

	All companies		30 companies DJIA	
Percentage range	Monthly data	Daily data	Monthly data	Daily data
0 - 10%	0	10		
10% - 20%	4	32		
20% - 30%	7	126		
30% - 40%	23	598		
40% - 50%	404	1,138		
50% - 60%	2,037	1,406	2	
60% - 70%	1,227	474	16	24
70% - 80%	107	29	11	6
80% - 90%	4	0	1	
90% - 100%	0	0		
Number of companies	3,812	3,812	30	30
Average	58.0%	48.7%	68.3%	65.9%
Median	58.1%	50.0%	66.9%	64.5%

2. Implications for making beta-ranked portfolios

We ordered the 3,813 companies by decreasing betas on December 1, 2001 and constructed 20 portfolios. Portfolio 1 had the companies with the highest betas and portfolio 20 had the companies with the lowest betas. Then we calculated the beta of the portfolios (weighted by market capitalization) each day of the following two months. **Table 7** shows that in the following two months, 300 portfolios were misallocated (i.e. on 26 days, portfolio 5 had lower beta than portfolio 6). On 53 days (out of 62 days) there were portfolios misallocated.

Table 7. Twenty portfolios ranked by decreasing beta on December 1, 2001
Number of misallocated portfolios on the 62 days of the following two months.

Date	misallocated portfolios
December: 10 and 16.	13
December: 12, 13, 14 and 15. January: 12, 13, 15 and 16.	11
January: 14.	10
December: 17 and 19.	9
December: 8 and 21. January: 8, 10, 17 and 19.	8
December: 18. January: 18, 20 and 21.	6
December: 5, 6, 7, 9, 11, 20 and 22. January: 5, 6, 7, 9, 11 and 22.	4
December: 3, 4, 23, 24, 27, 28, 29, 30 and 31. January: 3, 4, 23, 24, 27, 28, 29 and 30.	2
9 days	0

Having ordered the 3,813 companies by decreasing betas on December 1, 2001, we constructed 3,613 portfolios of 200 shares each following a moving window. We also calculated the beta of those 3,613 portfolios on December 15, 2001. **Figure 4** shows the results. Betas of low beta portfolios increased, and betas of high beta portfolios decreased from December 1 to December 15. **Figure 5** shows the difference of the two betas (December 1 and December 15) for each portfolio. **Figure 5** also shows the difference of the betas on December 15 between each portfolio (N) and the portfolio that had the immediate lower beta (N-1) on December 1. On December 15, this difference was negative in 1,520 cases.

Figure 4. 3,613 portfolios of 200 shares ranked by decreasing beta on December 1, 2001. Beta of the portfolios in December 1 (straight line) and beta of the same portfolios on December 15.

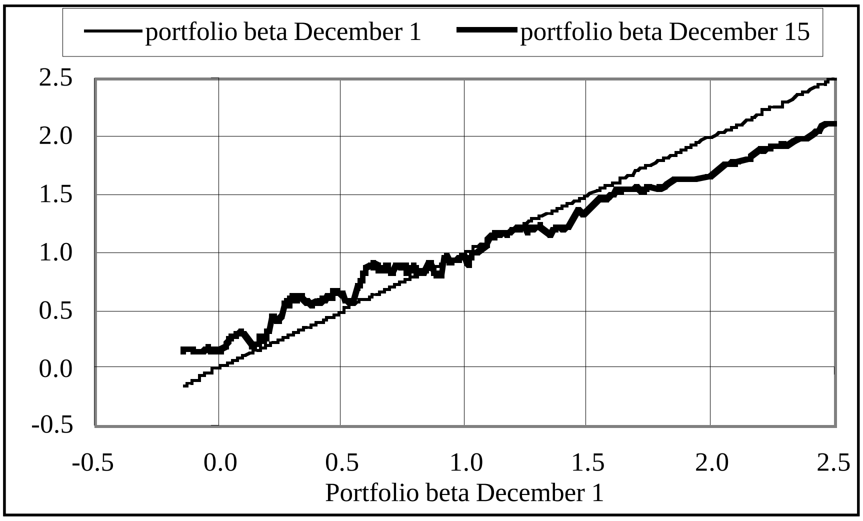
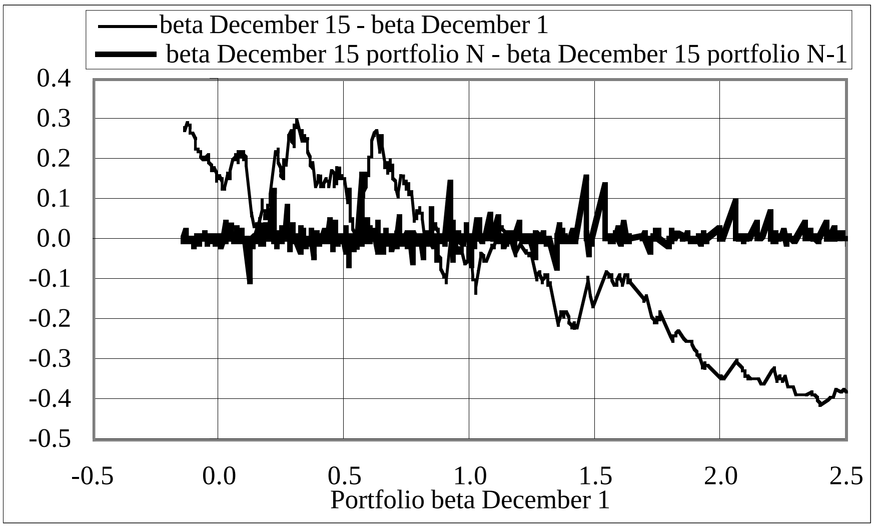


Figure 5. 3,613 portfolios of 200 shares ranked by decreasing beta on December 1, 2001. Difference of the beta of each portfolio in December 15 minus the beta of the same portfolio in December 1.
The chart also shows the difference of the betas on December 15 between each portfolio and the portfolio that had the inmediate lower beta on December 1.



We also formed portfolios in the Fama and French (1992) way on December 1 and on December 15, 2001. **Table 8** shows that on average 71.3% of the companies changed from one portfolio on December 1 to another on December 15.

Table 8. Percentage of the companies in each portfolio formed on December 1 that change portfolio if portfolios formed on December 15, 2001. Change in the betas of each portfolio from December 1 to December 15, 2001. Portfolios formed according to Fama and French (1992)

Portfolios are formed on December 1 and on December 15, 2001. The breakpoints for the size (log of Market Value of Equity, ME, in million $) are determined using all NYSE stocks (1,462) in our sample. All NYSE and Nasdaq stocks are allocated to the 10 size portfolios using the NYSE breakpoints. Then, each size decile is subdivided into 10 ß portfolios using the betas of individual stocks, estimated with 5 years of monthly returns ending in December 1, 2001 in the first case, and in December 15, 2001 in the second.
The betas of the portfolios are estimated with 5 years of monthly returns with respect to the S&P 500.

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel A. Percentage of the companies in each portfolio formed on December 1 that change portfolio if portfolios formed on December 15											
All	71.3%	52.8%	71.9%	78.2%	82.5%	79.3%	80.6%	82.8%	76.1%	71.1%	39.5%
Small-ME	74.7%	61.5%	77.8%	76.9%	83.8%	86.3%	79.5%	82.9%	77.8%	76.9%	45.2%
ME-2	73.3%	62.5%	73.4%	79.7%	87.5%	78.1%	78.1%	78.1%	81.3%	65.6%	48.4%
ME-3	73.1%	52.6%	76.3%	76.3%	81.6%	81.6%	86.8%	89.5%	84.2%	65.8%	40.9%
ME-4	72.9%	62.5%	75.0%	87.5%	71.9%	68.8%	81.3%	93.8%	75.0%	68.8%	45.5%
ME-5	66.6%	51.7%	65.5%	72.4%	82.8%	72.4%	75.9%	86.2%	62.1%	69.0%	31.3%
ME-6	68.4%	47.6%	61.9%	81.0%	85.7%	90.5%	76.2%	66.7%	81.0%	66.7%	30.4%
ME-7	62.6%	19.0%	42.9%	61.9%	81.0%	71.4%	90.5%	85.7%	71.4%	76.2%	27.3%
ME-8	65.4%	31.6%	73.7%	84.2%	78.9%	63.2%	78.9%	78.9%	68.4%	68.4%	30.0%
ME-9	67.5%	26.3%	63.2%	84.2%	78.9%	84.2%	84.2%	84.2%	68.4%	84.2%	26.1%
Large-ME	66.1%	41.2%	76.5%	82.4%	82.4%	70.6%	82.4%	76.5%	70.6%	58.8%	22.2%

Panel B. Portfolio weighted Beta December, 1 - Portfolio weighted Beta December, 15

	All	Low-ß	ß-2	ß-3	ß-4	ß-5	ß-6	ß-7	ß-8	ß-9	High-ß
All		–0.34	–0.22	–0.17	–0.20	–0.09	–0.09	–0.03	0.04	0.04	0.27
Small-ME	–0.09	–0.42	–0.22	–0.17	–0.17	–0.10	–0.13	0.04	0.06	–0.03	0.20
ME-2	–0.11	–0.36	–0.24	–0.21	–0.20	–0.07	–0.14	–0.14	–0.05	0.06	0.26
ME-3	–0.11	–0.29	–0.29	–0.23	–0.26	–0.09	–0.13	–0.18	0.07	0.04	0.19
ME-4	–0.11	–0.48	–0.25	–0.22	–0.17	–0.12	–0.09	–0.03	–0.03	–0.12	0.42
ME-5	–0.06	–0.35	–0.26	–0.17	–0.26	0.01	–0.04	–0.04	0.05	0.15	0.30
ME-6	–0.06	–0.27	–0.16	–0.16	–0.12	–0.17	–0.02	0.00	0.03	0.00	0.25
ME-7	–0.03	–0.20	–0.17	–0.11	–0.24	–0.08	–0.01	–0.01	0.10	0.24	0.19
ME-8	0.01	–0.13	–0.17	–0.06	–0.18	–0.09	–0.03	0.08	0.21	0.09	0.41
ME-9	–0.01	–0.22	–0.14	–0.16	–0.15	–0.16	–0.08	0.12	0.05	0.13	0.42
Large-ME	–0.01	–0.19	–0.18	–0.06	–0.24	–0.07	0.06	–0.03	0.08	0.22	0.34

3. Historical betas depend very much on which index we use to calculate them

Table 9 presents the historical relative betas of the 30 companies in the Dow Jones Industrial Average Index. Relative betas are calculated by dividing the beta with respect to an index on a given day by the beta with respect to another index on the same day. For the 2-month period 1/12/01-31/1/02, the table contains the maximum, the minimum, the average, and maximum divided by the minimum. It may be seen that, on average, the beta with respect to the S&P 500 was smaller than the beta with respect to the DJIA and higher than the beta with respect to the W5000. **Table 9** permits to conclude that relative betas also change dramatically.

Table 9. Historical relative betas of the 30 companies in the Dow Jones Industrial Average Index.

Relative betas are calculated by dividing the beta with respect to one index on a given day by the beta with respect to another index on the same day. For example, the relative beta "Beta S&P 500 / Beta DJ IND" is calculated by dividing the beta with respect to the S&P 500 on a given day by the beta with respect to the Dow Jones Industrial Average (DJIA) index on the same day.

$$\text{relative beta of December 18, 2001} = \frac{\text{calculated beta with respect to the S\&P 500 on December 18, 2001}}{\text{calculated beta with respect to the DJIA on December 18, 2001}}$$

The table contains the maximum, the minimum, the average, and the maximum divided by the minimum of the 62 relative betas calculated in the period 1/12/01-31/1/02

	Beta S&P 500 / Beta DJ IND					Beta S&P 500 / Beta W 5000			
	Max	min	average	max/min		Max	min	average	max/min
3M Co.	0.76	0.51	0.65	1.51		2.02	1.00	1.20	2.02
Alcoa	0.86	0.46	0.66	1.86		1.22	0.94	1.03	1.30
American Express	1.10	0.89	0.99	1.23		1.16	1.04	1.10	1.11
AT&T	2.57	1.19	1.66	2.15		1.09	0.92	1.00	1.19
Boeing	0.84	0.55	0.74	1.51		1.11	0.99	1.05	1.13
Caterpillar	0.78	0.48	0.66	1.62		1.37	1.10	1.21	1.24
Citigroup	1.18	0.96	1.05	1.24		1.16	1.02	1.09	1.13
Coca Cola	1.02	0.80	0.90	1.26		1.30	1.07	1.17	1.21
Du Pont	0.81	0.54	0.70	1.51		1.75	1.09	1.26	1.60
Eastman Kodak	0.90	0.56	0.75	1.60		1.24	0.92	1.10	1.35
Exxon Mobil	0.91	0.69	0.80	1.32		1.38	0.97	1.14	1.43
General Electric	1.23	1.00	1.10	1.23		1.21	1.07	1.12	1.13
Hewlett-Packard	1.33	1.06	1.19	1.26		1.01	0.85	0.93	1.19
Home Depot	1.30	1.00	1.15	1.30		1.18	0.99	1.08	1.19
Honeywell Intl.	0.87	0.68	0.80	1.29		1.31	1.09	1.18	1.20
IBM	1.10	0.80	0.98	1.38		1.10	0.99	1.05	1.12
Intel	1.38	1.06	1.24	1.30		1.04	0.95	1.00	1.09
Intl.Paper	0.80	0.49	0.66	1.62		1.41	1.10	1.22	1.29
Johnson & Johnson	1.12	0.60	0.90	1.86		1.90	1.22	1.40	1.56
J P Morgan Chase	1.24	1.02	1.11	1.21		1.08	0.96	1.04	1.12
McDonalds	1.12	0.73	0.93	1.53		1.81	1.19	1.36	1.51
Merck	1.29	0.70	1.06	1.84		2.67	1.27	1.54	2.11
Microsoft	1.55	1.02	1.34	1.52		1.09	0.99	1.04	1.10
Philip Morris	12.1	–28.1	–0.07	0.43		27.1	–2.86	1.80	9.47
SBC Comm.	1.94	0.98	1.46	1.97		1.74	1.07	1.26	1.62
United Technologies	0.92	0.77	0.86	1.19		1.23	1.07	1.14	1.15
Wal Mart Stores	1.25	0.96	1.10	1.31		1.31	1.11	1.22	1.18
General Motors	1.07	0.85	0.98	1.26		1.07	0.95	1.01	1.13
Procter & Gamble	1.00	–0.16	0.63	6.22		3.09	0.49	1.49	6.34
Walt Disney	1.21	0.90	1.05	1.33		1.14	0.96	1.06	1.18
Average	1.52	0.80	0.97	1.59		2.28	1.01	1.18	1.75

4. We cannot say that the beta of a company is smaller or bigger than the beta of another

Table 10 presents the beta ranking of the 3,813 companies in our sample in the month of December 31, 2001. Each day, companies are ranked from 1 (the company with the lowest beta on that day) to 3,813 (the company with the highest beta on that day). Betas are calculated each day with respect to the S&P 500 using 5 years of monthly data. It may be seen that the average change in ranking for all 3,813 companies in December 2001 is 1,542 ranking positions.

The average beta ranking change was: 233 positions from one day to the next; 479 positions from one day to the next week; and 564 positions over a two-week period.

Table 10. Change in beta ranking order in the month of December, 2001. Statistics of the difference Maximum beta ranking - minimum beta ranking.

Historical betas of 3,813 companies calculated every day during the month of December 2001 with respect to the S&P 500 using 5 years of monthly data. Each day, companies are assigned a beta ranking from 1 (the company with the minimum beta) to 3,813 (the company with the maximum beta). Then, we calculate for each company the difference between the Maximum beta ranking and the minimum beta ranking.

Maximum ranking - minimum ranking	Full sample	S&P 500	DJIA 30
MAX	3,760	2,592	2,041
Min	15	74	467
Average	1,542	1,154	1,001
Median	1,391	1,126	908
Number of companies	3,813	450	30

5. High-risk companies very often have smaller historical betas than low-risk companies

Table 11 reports the calculated betas as of December 31, 2001 of the 30 companies in the Dow Jones Industrial Average Index. Companies are sorted by ascending beta with respect to the S&P 500. According to the S&P 500 betas, Philip Morris is the company with lowest cost of equity, much smaller than GE or Wall Mart. If we assume that the risk-free rate is 5%, that the market risk premium is 4.5%, and that historical betas are a good proxy for expected betas, then the cost of equity of Philip Morris, GE and Wall Mart is 6.0%, 10.2% and 9.1%, respectively. We do not think that this makes much economic sense.

Table 11. Calculated betas as of December 31, 2001 of the 30 companies in the Dow Jones Industrial Average Index

Betas are calculated each day using 5 years of monthly data. Companies are sorted by ascending beta with respect to the S&P 500.

	31/12/2001	Beta S&P 500	Beta DJ IND	Beta W 5000
Philip Morris	MO	0.232	0.378	0.149
Procter & Gamble	PG	0.281	0.460	0.244
Exxon Mobil	XOM	0.358	0.480	0.329
SBC Communications	SBC	0.460	0.367	0.322
Merck	MRK	0.483	0.528	0.299
3M Co.	MMM	0.488	0.778	0.464
Johnson & Johnson	JNJ	0.488	0.578	0.363
Eastman Kodak	EK	0.590	0.791	0.522
Coca Cola	KO	0.654	0.775	0.530
McDonalds	MCD	0.677	0.769	0.500
Caterpillar	CAT	0.731	1.104	0.633
Du Pont	DD	0.771	1.019	0.700
Boeing	BA	0.807	1.178	0.744
Wal Mart Stores	WMT	0.917	0.820	0.749
Walt Disney	DIS	0.928	1.022	0.844
AT&T	T	0.959	0.542	0.942
Intl.Paper	IP	1.011	1.323	0.877
General Motors	GM	1.129	1.167	1.087
Home Depot	HD	1.130	0.891	1.143
General Electric	GE	1.163	0.994	1.063
Honeywell Intl.	HON	1.176	1.564	1.016
Alcoa	AA	1.219	1.487	1.209
IBM	IBM	1.234	1.196	1.171
American Express	AXP	1.245	1.374	1.120
Unite Technologies	UTX	1.323	1.488	1.207
Citigroup	C	1.459	1.525	1.320
Hewlett-Packard	HPQ	1.489	1.270	1.639
J P Morgan Chase & Co.	JPM	1.518	1.431	1.458
Intel	@INTC	1.696	1.434	1.632
Microsoft	@MSFT	1.823	1.379	1.716
average	average	0.948	1.004	0.866

6. Weak correlation between beta and realized return

Table 12 shows the small correlation between the betas and the realized returns of portfolios of 200 companies sorted by realized return.

Table 12. Portfolios of 200 companies sorted by realized return. Correlation of the portfolio return with the beta calculated on December 1, 2001

Correlation Realized Return - Beta.

	1996-2001	1997	1998	1999	2000	2001
Slope	–0.07	0.23	0.64	1.68	–1.00	–0.26
standard error slope	0.076	0.032	0.038	0.013	0.009	0.023
R2	0.0002	0.0145	0.0728	0.8155	0.7667	0.0347
F statistic	0.8	53.2	282.6	15917.3	11835.9	129.4
intercept	0.54	0.02	–0.61	–1.25	1.10	0.43
Returns:						
S&P 500	66.2%	33.4%	28.6%	21.0%	–9.1%	–11.9%
Dow Jones Ind.	68.8%	24.9%	18.1%	26.7%	–4.5%	–5.4%
Wilshire 5000	48.8%	29.2%	21.7%	22.0%	–11.8%	–12.1%

7. About the recommendation of using Industry betas

Some authors recommend using industry betas, instead of company betas. For example, Copeland, Koller and Murrin (2000) recommend "checking several reliable sources because beta estimates vary considerably… If the betas from several sources vary by more than 0.2 or the beta for a company is more than 0.3 from the industry average, consider using the industry average. An industry average beta is typically more stable and reliable than an individual company beta because measurement errors tend to cancel out". But about the CAPM, they conclude (see their page 225), "It takes a better theory to kill an existing theory, and we have not seen the better theory yet. Therefore, we continue to use the CAPM, being wary of all the problems with estimating it." We rather think that a rejection is enough to beat a model.

We have shown that industry betas are quite unstable.

Figure 6 shows the calculated betas of the banking industry in the period December 1, 2001 – January 31, 2002. It shows the evolution of the industry betas (both weighted and unweighted), the maximum beta and the minimum beta of the 409 companies in the industry. The difference between the weighted and unweighted betas is remarkable.

Figure 6. Betas of the banking industry.
409 companies. Market capitalization:$1,150 bn.

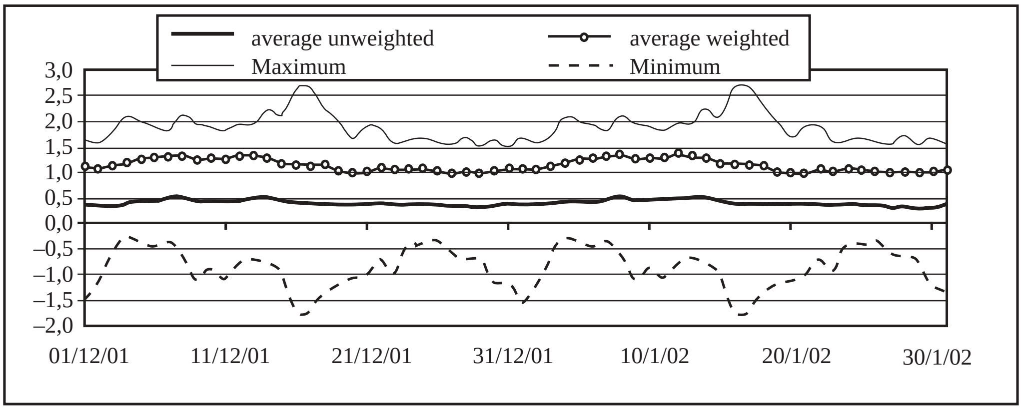


Figure 7 shows the average beta (in the period December 1, 2001 – January 31, 2002), the maximum beta and the minimum beta for each of the 409 companies in the banking industry.

Figure 7. Betas of the banking industry.
Average beta, maximum beta and minimum beta of the 409 banks

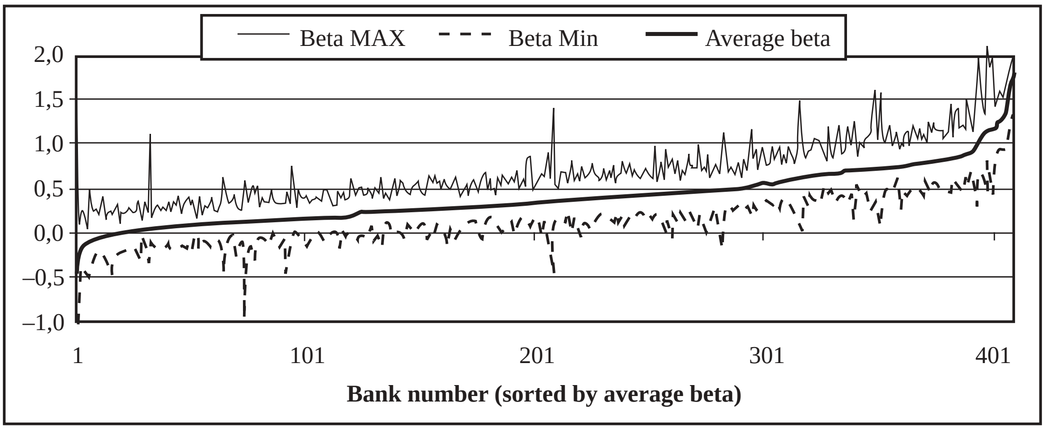


Figure 8 shows the relationship between the average beta (in the period December 1, 2001 – January 31, 2002) and the market capitalization of the 409 companies in the banking industry. On average, bigger companies had higher betas.

Figure 8. Betas of the banking industry.
Average beta and Market capitalization of the 409 banks

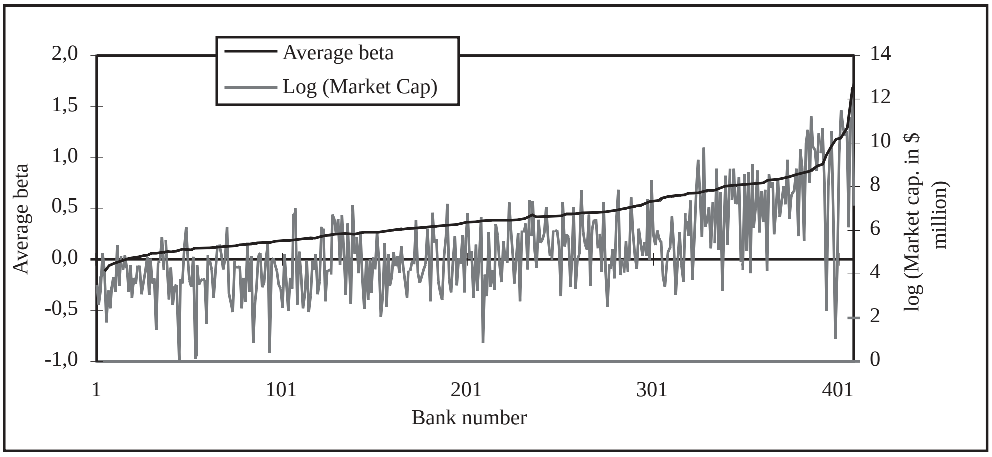
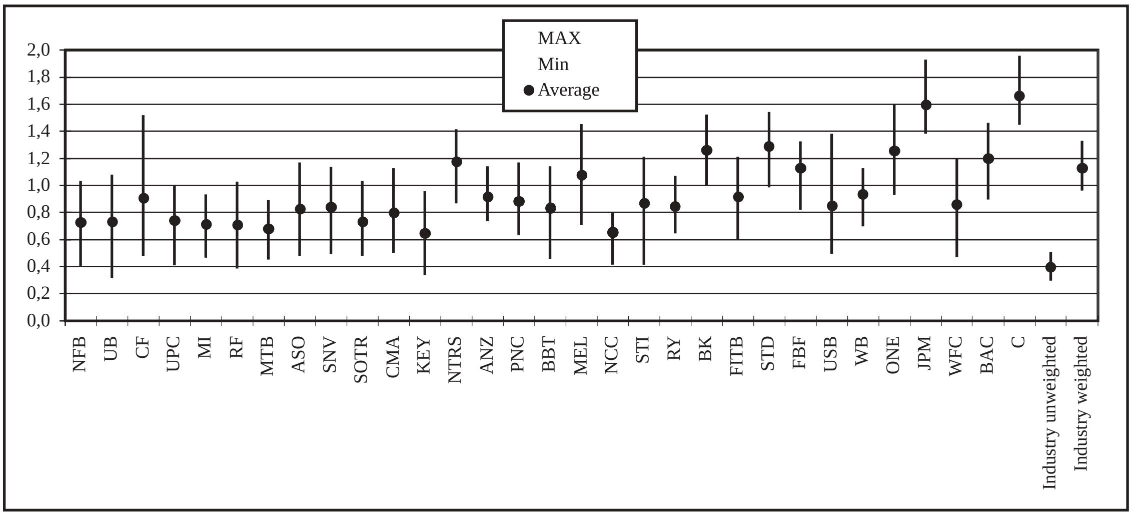


Figure 9 shows the average and the dispersion of the historical betas of the 30 banks with the highest market capitalization and compares them with the average and the dispersion of the industry weighted and unweighted betas.

Figure 9. Betas of the banking industry.
Dispersion of the betas of the 30 banks with the highest market cap.



8. Historical betas and the market-to-book ratio

Figure 10 shows the relationship between the market-to-book ratio and the historical beta of portfolios of 200 companies ranked by MV/BV (market-to-book ratio). The figure plots the 200 companies' rolling average beta on December 1 and on December 15. On average, higher MV/BV (market-to-book ratio) companies had higher beta.

Figure 10. Relationship between market-to-book ratio and historical beta. Portfolios of 200 companies ranked by MV/BV. 200 companies rolling average

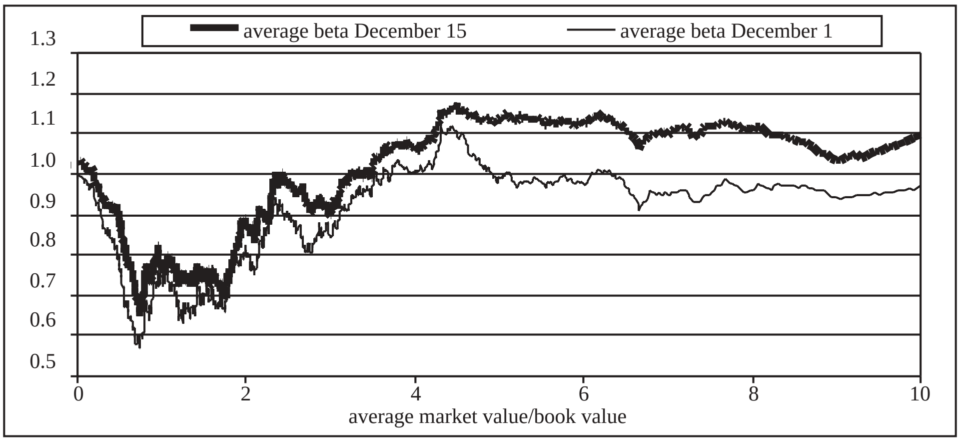


Figure 11 shows the relationship between the market-to-book ratio and the market capitalization of portfolios of 200 companies ranked by market capitalization. The figure plots the 200 companies' rolling average MV/BV (market-to-book ratio) on December 1 and on December 15. On average, bigger companies had higher MV/BV (market-to-book ratio).

Figure 11. Relationship between market-to-book ratio and market capitalization. Portfolios of 200 companies ranked by market capitalization on December 1

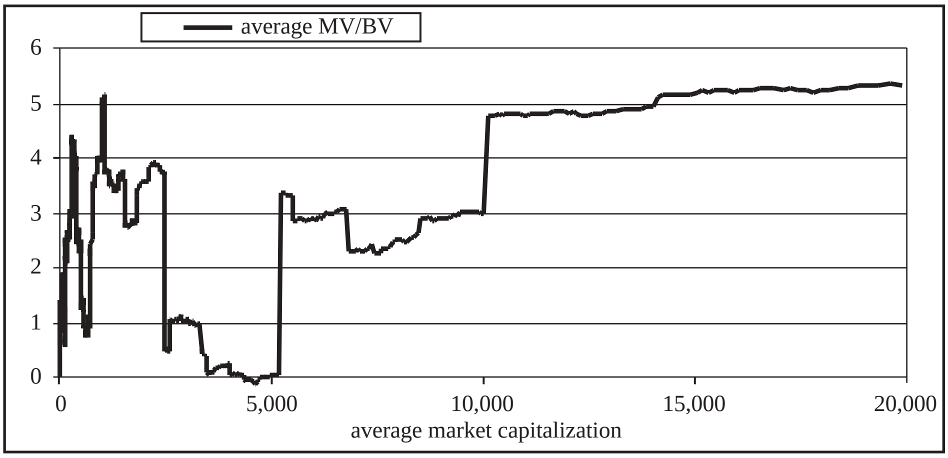
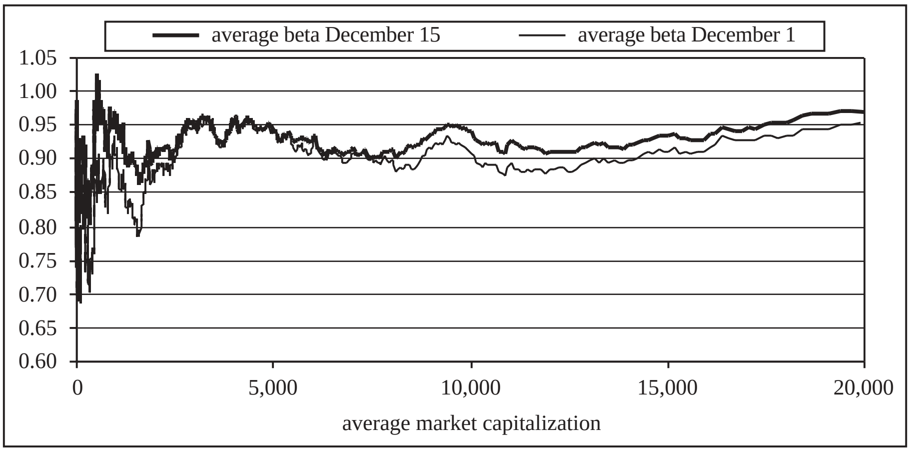


Figure 12 shows the relationship between the market capitalization and the calculated beta of portfolios of 200 companies ranked by market capitalization on December 1. Small caps had low betas.

Figure 12. Relationship between market capitalization and calculated beta. Portfolios of 200 companies ranked by market capitalization on December 1



9. Conclusion

We have shown that, in general, it is an enormous error to use the historical beta as a proxy for the expected beta. First, because it is almost impossible to calculate a meaningful beta because historical betas change dramatically from one day to the next; second, because very often we cannot say with a relevant statistical confidence that the beta of one company is smaller or bigger than the beta of another; third, because historical betas do not make much sense in many cases: high-risk companies very often have smaller historical betas than low-risk companies; fourth, because historical betas depend very much on which index we use to calculate them.

We calculate betas of 3,813 companies using 60 monthly returns each day of December 2001 and January 2002. We report that the maximum beta of a company was, on average, 15.7 times its minimum beta. The median of the maximum beta divided by the minimum beta was 3.07. The median of the percentage daily change (in absolute value) of the betas was 20%, and the median of the percentage (in absolute value) of the betas was 43%.

Industry betas are also unstable. The median (average) of the percentage daily change (in absolute value) of the industry betas was 7% (16%), and the median (average) of the percentage (in absolute value) of the industry betas was 15% (38%). On average, the maximum beta of an industry was 2.7 times its minimum beta in December 2001 and January 2002.

This dispersion of the calculated betas also has important implications for the instability of beta-ranked portfolios. □

Appendix 1

ARE CALCULATED BETAS GOOD FOR ANYTHING?

Literature review about the CAPM

Sharpe (1964) and Lintner (1965) demonstrate that, in equilibrium, a financial asset's return must be positively linearly related to its beta (ß), a measure of systematic risk or co-movement with the market portfolio return:

$$E\ (R_i) = a_1 + a_2\ E\ (\beta_i), \text{ for all assets i,} \qquad [1]$$

where E (R_i)) is the expected return on asset i, E (β_i) is asset i's expected market beta, a_1 is the expected return on a "zero-beta" portfolio, and a_2 is the market risk premium.

The CAPM of Sharpe (1964), Lintner (1965) and Mossin (1966) is predicated on the assumption of a positive systematic risk-return tradeoff and asserts that the expected return for any security is a positive function of three variables: expected beta, expected market return, and the risk-free rate. The basic assumptions of the CAPM are:

1. Investors have homogeneous expectations about asset returns that have a joint normal distribution;

2. Investors are risk-averse individuals who maximize the expected utility of their end-of-period wealth;

3. Markets are frictionless and information is costless and simultaneously available to all investors; there are no imperfections such as taxes, regulations, or restrictions on short selling;

4. There exists a risk-free asset such that investors may borrow and lend unlimited amounts at the risk-free rate.

However, subsequent work by (among many others) Basu (1977), Banz (1981), Reinganum (1981), Litzenberger and Ramaswamy (1979), Keim (1983, 1985)[2] and Fama and French (1992) suggests that either:

1. expected returns are determined not only by the beta and the expected market risk premium but also by non-risk characteristics such as book-to-market ratio, firm size, price-earnings ratio and dividend yield. It implies that the CAPM is misspecified and requires the addition of factors other than beta to explain security returns, or

2. the historical beta has little (or nothing) to do with the expected beta. To put it another way: the problems of measuring beta are systematically related to

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2 Basu (1977) found that low price/earnings portfolios have higher returns than could be explained by the CAPM. Banz (1981) and Reinganum (1981) found that smaller firms tend to have high abnormal rates of return. Litzenberger and Ramaswamy (1979) found that the market requires higher rates of return on equities with high dividend yield. Keim (1983, 1985) reports the January effect, that is, seasonality in stock returns. Tinic and West (1984) reject the validity of the CAPM based on intertemporal inconsistencies due to the January effect.

variables such as firm size and book-to-market ratio. And also the historical market risk premium has little (or nothing) to do with the expected market risk premium, or

3. the heterogeneity of expectations[3] in cross-section returns, volatilities and covariances, and market returns is the reason why it makes no sense to talk about an aggregate market CAPM, although at the individual level expected CAPM does work. It means that while individuals are well characterized by CAPM, and each individual uses an expected beta, an expected market risk premium, and an expected cash flow stream to value each security, all individuals do not agree on these three magnitudes for each security. Consequently, it makes no sense to refer to a "market" expected beta for a security or to a "market" expected market risk premium (or to a "market" expected cash flow stream), for the simple reason that they do not exist.

We may find out an investor's expected IBM beta by asking him. However, it is impossible to determine the expected IBM beta for the market as a whole, because it does not exist. Even if we knew the expected market risk premiums and the expected IBM betas of the different investors who operated on the market, it would be meaningless to talk of an expected IBM beta for the market as a whole. The rationale for this is to be found in the aggregation theorems of microeconomics, which in actual fact are non-aggregation theorems. A model that works well individually for a number of people may not work for all of the people together[4]. For the CAPM, this means that although the CAPM may be a valid model for each investor, it is not valid for the market as a whole, because investors do not have the same return and risk expectations for all shares. The prices are a statement of expected cash flows discounted at a rate that includes the expected market risk premium and the expected beta. Different investors have different cash flow expectations and different future risk expectations (different expected market risk premium and different expected beta). One could only talk of a market risk premium if all investors had the same expectations.

The problem with the expected beta is that investors do not have homogeneous expectations. If they did, it *would* make sense to talk of a market risk premium and of an IBM beta common to all investors because all investors would hold the market portfolio. However, expectations are not homogeneous.

[3] Lintner (1969) argued that the existence of heterogeneous expectations does not critically alter the CAPM in some simplified scenarios. In some cases, expected returns are expressed as complex weighted averages of investors' expectations. But if investors have heterogeneous expectations of expected prices and covariance matrices, the market portfolio is not necessarily efficient and this makes the CAPM non-testable. Lintner (1969) says "in the (undoubtedly more realistic) case with different assessments of covariance matrices, the market's assessment of the expected ending price for any security depends on every investor's assessment of the expected ending price for every security and every element in the investor's assessment of his NxN covariance matrix (N is the number of securities), as well as the risk tolerance of every investor."

[4] As Mas-Colell et al. (1995, page 120) say: "It is not true that whenever aggregate demand can be generated by a representative consumer, this representative consumer's preferences have normative contents. It may even be the case that a positive representative consumer exists but that there is no social welfare function that leads to a normative representative consumer."

CAPM	CAPM only holds at individual level
Homogeneous expectations All investors have equal expectations about asset returns that have a joint normal distribution	Heterogeneous expectations All investors DO NOT have equal expectations about asset returns. Asset returns DO NOT have a joint normal distribution
All investors use the same beta (historical beta) for each share	Each investor uses a different beta (expected beta) for each share
historical beta = expected beta	historical beta NOT EQUAL TO expected beta
All investors hold the market portfolio	Investors hold different portfolios
All investors use the same market risk premium	Investors use different market risk premiums
The market risk premium is the difference between the expected return on the market portfolio and the risk-free rate	The market risk premium is NOT the difference between the expected return on the market portfolio and the risk-free rate

Measurement errors and problems

Original tests of the CAPM focused on whether the intercept in a cross-sectional regression was higher or lower than the risk-free rate, and whether stock individual variance entered into cross-sectional regressions.

Scholes and Williams (1977) found that with nonsyncronous trading of securities, ordinary least squares estimators of beta coefficients using daily data are both biased and inconsistent.

Roll (1977) concludes that the only legitimate test of the CAPM is whether or not the market portfolio (which includes all assets) is mean-variance efficient. The Roll critique does not imply that the CAPM is an invalid theory. However, it does mean that tests of the CAPM must be interpreted with great caution.

Roll (1981) suggests that infrequent trading of shares of small firms may explain much of the measurement error in estimating their betas.

Constantinides (1982) points out that with consumer heterogeneity “in the intertemporal extension of the Sharpe-Lintner CAPM, an asset’s risk premium is determined not only by its covariance with the market return, but also by its covariance with the m-1 state variables” (m is the number of heterogeneous consumers). He also points out that the assumption of complete markets is needed for demand aggregation. But markets are not complete.

Lakonishok and Shapiro (1984, 1986) find an insignificant relationship between beta and returns and a significant relationship between market capitalization and returns

Shanken (1992) presents an integrated econometric view of maximum-likelihood methods and two-pass approaches to estimating historical betas.

Roll and Ross (1994) attribute the observed lack of a systematic relation between risk and return to the possible mean-variance inefficiency of the market portfolio proxies.

Shalit and Yitzhaki (2002) argue that the Ordinary Least-Squares regression estimator is inappropriate for estimating betas. They suggest alternative estimators for beta that are robust with respect to extreme fluctuations in the market return. Using CRSP daily data from 1984 to 1993, they eliminate the highest four and the lowest four market returns and show that the betas of the 75% of the firms change by more than one standard error. For example, the beta of General Electric changes from 1.16 to 1.23 and the beta of Coca-Cola changes from 1.22 to 1.18.

Avramov (2002) uses Bayesian model averaging to analyze the sample evidence on return predictability in the presence of model uncertainty. The analysis reveals in-sample and out-of-sample predictability, and shows that the out-of-sample performance of the Bayesian approach is superior to that of model selection criteria. She finds that market premia are robust predictors. Moreover, small-cap value stocks appear more predictable than large-cap growth stocks. She also investigates the implications of model uncertainty from investment management perspectives. She shows that model uncertainty is more important than estimation risk, and investors who discard model uncertainty face large utility losses.

Zhang, Kogan, and Gomes (2001) reconcile the ability of non-risk characteristics such as firm size and book-to-market to predict returns within a dynamic pricing paradigm. Firm characteristics can appear to predict stock returns because they may be correlated with the true conditional factor loadings, thereby motivating the scaling of betas by firm specific variables. They claim that "size and book-to-market play separate roles in describing the cross-section of returns. These firm characteristics appear to predict stock returns because they are correlated with the true conditional market beta of returns."

Avramov and Chordia (2001) test whether the Zhang, Kogan, and Gomes (2001) scaling procedure improves the performance of the theoretically motivated CAPM and consumption CAPM. The evidence shows that equity characteristics often enter beta significantly. However, "characteristic scaled factor models" do not outperform their unscaled counterparts.

The poor performance of the CAPM has inspired multiple portfolio based factors.

The article that dealt the hardest blow to the CAPM was that published by Fama and French (1992). This article showed that in the period 1963-1990, the correlation between stocks' returns and their betas was very small, while the correlation with the companies' size and their price/book value ratio was greater. They concluded "our tests do not support the most basic prediction of the Sharpe-Lintner-Black Capital Asset Pricing Model that average stock returns are positively related to market betas." The authors divided the shares into portfolios and found that the cross-sectional variation in expected returns may be captured within a three-factor model, the factors being:

- the return on the market portfolio in excess of the risk-free rate
- a zero net investment portfolio that is long in high book-to-market stocks and short in low book-to-market stocks.
- a zero net investment portfolio that is long in small firm stocks and short in large firm stocks.

Table A.1 shows the article's main findings.

However, Griffin (2002) concludes that there are no benefits to extending the Fama and French three-factor model to a global context. Country-specific three-factor models are more useful in explaining stock returns than are world and international versions.

Lakonishok, Shleifer and Vishny (1994) argue that the size and book-to-market effects are due to investor overreaction rather than compensation for risk bearing. According to them, investors systematically overreact to corporate news, unrealistically extrapolating high or low growth into the future. This leads to underpricing of "value" (small capitalization, high book-to-market stocks) and overpricing of "growth" (large capitalization, low book-to-market stocks).

Table A.1. Main findings of Fama and French's article (1992)

Size of the companies	Average beta	Annual average return	Beta of the companies	Average beta	Annual average return	Price / book value	Average beta	Annual average return
1 (biggest)	0.93	10.7%	1 (high)	1.68	15.1%	1 (high)	1.35	5.9%
2	1.02	11.4%	2	1.52	16.0%	2	1.32	10.4%
3	1.08	13.2%	3	1.41	14.8%	3	1.30	11.6%
4	1.16	12.8%	4	1.32	14.8%	4	1.28	12.5%
5	1.22	14.0%	5	1.26	15.6%	5	1.27	14.0%
6	1.24	15.5%	6	1.19	15.6%	6	1.27	15.6%
7	1.33	15.0%	7	1.13	15.7%	7	1.27	17.3%
8	1.34	14.9%	8	1.04	15.1%	8	1.27	18.0%
9	1.39	15.5%	9	0.92	15.8%	9	1.29	19.1%
10 (smallest)	1.44	18.2%	10 (low)	0.80	14.4%	10 (low)	1.34	22.6%

Kothary, Shanken and Sloan (1995) point out that using historical betas estimated from annual rather than monthly returns produces a stronger relation between average return and historical beta. They also claim that the relation between book-to-market equity and average return observed by Fama and French (1992) and others is seriously exaggerated by survivor bias in the COMPUSTAT sample used. They also claim that the Fama and French statistical tests were of such low power that they could not reject a beta-related risk premium of 6% over the post-1940 period. Their most important conclusion, however, is that "our examination of the cross-section of expected returns reveals economically and statistically significant compensation (about 6 to 9% per annum) for beta risk."

Pettengill, Sundaram and Mathur (1995) find a "consistent and highly significant relationship between beta and cross-sectional portfolio returns." They insist that "the positive relationship between returns and beta predicted by CAPM is based on expected rather than realized returns. In periods where excess market returns are negative, an inverse relationship between beta and portfolio returns should exist." They test the following equation:

$$R_{it} = \gamma_{0t} + \gamma_{1t}\,\delta\ \beta_i + \gamma_{2t}\ (1 - \delta)\ \beta_i + \varepsilon_t$$

where $\delta = 1$ if $(R_{Mt} - R_{Ft}) > 0$ (when market excess returns are positive) and
$\delta = 0$ if $(R_{Mt} - R_{Ft}) < 0$ (when market excess returns are negative)
γ_1 is estimated in periods with positive market excess returns and γ_{2t} is estimated in periods with negative market excess returns.

They use 660 monthly CRSP returns from 1936 to 1990. They estimate the betas using a five-year period and the CRSP equally-weighted index as a proxy for the market index. Based on the relative rankings of the estimated betas, securities are divided into 20 portfolios. They find a mean value of 0.0336 (t-statistic = 12.61) for γ_1 and –0.0337 (t-statistic = –13.82) for γ_2. They say, "as expected, high beta portfolios incur lower returns during down markets (280 months) than low beta portfolios… and receive a positive risk premium during up markets (380 months)." They also remark that their results are very similar to those of Lakonishok and Shapiro (1984), who found slope coefficients of $\gamma_1 = –0.0333$ and $\gamma_2 = –0.0354$.

Elsas, El-Shaer and Theissen (2000) follow the Pettengill, Sundaram and Mathur (1995) methodology for the German market and find a positive and statistically significant relation between beta and return in our sample period 1960-1995 as well as in all subperiods we analyze. They claim, "Our empirical results provide a justification for the use of betas estimated from historical return data by portfolio managers."

Fama and French (1996) argue that survivor bias does not explain the relation between book-to-market equity and average return. They conclude that historical beta alone cannot explain expected return.

Kothary and Shanken (1999) insist on the fact that Fama and French (1992) tend to ignore the positive evidence on historical beta and to overemphasize the importance of book-to-market. They claim that, while statistically significant, the incremental benefit of size given beta is surprisingly small economically. They also claim that book-to-market is a weak determinant of the cross-sectional variation in average returns among large firms and it fails to account for return differences related to momentum and trading volume.

Cremers (2001) claims that the data do not give clear evidence against the CAPM because it is difficult to reject the joint hypothesis that the CAPM holds and that the CRSP value-weighted index is efficient or a perfect proxy for the market portfolio. He also claims that the poor performance of the CAPM seems often due to measurement problems of the market portfolio and its beta. He concludes that "according to the data, the CAPM may still be alive."

Bartholdy and Peare (2001) investigate the usefulness of the standard recommendation of using five years of monthly data and a value-weighted index for calculating the historical beta. They find that five years of monthly data and an equal-weighted index provide the most efficient estimate of the historical beta. However, they find that the ability of historical betas to explain differences in returns in subsequent periods ranges from a low of 0.01% to a high of 11.73% across years, and at best 3% on average. Based on these results, they say "it may well be appropriate to declare beta dead."

Chung, Johnson and Schill (2001) use size-sorted portfolio returns at daily, weekly, quarterly and semi-annual intervals and find in every case that the distribution of returns differs significantly from normality. They also show that adding systematic co-moments (not standard) of order 3 through 10 reduces the explanatory power of the Fama-French factors to insignificance in almost every case.

Berglund and Knif (1999) propose an adjustment of the cross-sectional regressions of excess returns against betas to give larger weights to more reliable beta forecasts. They find a significant positive relationship between returns and the beta forecast when the proposed approach is applied to data from the Helsinki Stock Exchange, while the traditional Fama-MacBeth (1973) approach as such finds no relationship at all.

Koutmos and Knif (2002) propose a dynamic vector GARCH model for the estimation of time-varying betas. They find that in 50% of the cases betas are higher during market declines (the opposite is true for the remaining 50%). They claim that the static market model overstates unsystematic risk by more than 10% and that dynamic betas follow stationary, mean reverting processes.

Merrill Lynch and Bloomberg adjust beta estimates in this very simple way:

Expected beta = 0.67 historical beta + 0.33

Appendix 2

ARE CALCULATED BETAS GOOD FOR ANYTHING?

Summary statistics of the historical betas of the 30 companies in the Dow Jones Industrial Average

Betas with respect to the S&P 500, the DJIA and the W 5000. Betas are calculated each day in the period 1/12/01-31/1/02 using 5 years of monthly data, i.e. on December 18, 2001, the beta is calculated by running a regression of the 60 monthly returns of the company on the 60 monthly returns of the corresponding index. The returns of each month are calculated on the 18th of the month. The table contains the maximum, the minimum, the average, maximum minus minimum divided by the average, and maximum divided by the minimum of the 62 betas calculated every day in the period 1/12/01-31/1/02.

	Beta S&P 500					Beta DJIA				
1/12/01 -31/1/02	Max	min	average	(max-min)/ average	max/min	Max	min	average	(max-min)/ average	max/min
MMM	0.79	0.36	0.57	0.76	2.22	1.09	0.69	0.87	0.46	1.58
AA	1.22	0.45	0.78	0.99	2.73	1.49	0.96	1.18	0.45	1.55
AXP	1.76	1.24	1.46	0.36	1.42	1.75	1.33	1.48	0.28	1.32
T	1.02	0.32	0.77	0.91	3.16	0.75	0.15	0.48	1.24	4.94
BA	1.22	0.57	0.95	0.68	2.14	1.60	1.00	1.28	0.47	1.59
CAT	1.04	0.60	0.78	0.58	1.75	1.36	0.96	1.18	0.34	1.41
C	1.96	1.45	1.66	0.31	1.35	1.71	1.49	1.58	0.14	1.15
KO	1.11	0.55	0.80	0.70	2.02	1.18	0.67	0.88	0.58	1.76
DD	0.87	0.54	0.74	0.45	1.62	1.19	0.93	1.07	0.25	1.28
EK	0.70	0.37	0.56	0.60	1.92	0.93	0.61	0.75	0.42	1.51
XOM	0.67	0.24	0.43	1.00	2.81	0.79	0.34	0.54	0.85	2.35
GE	1.39	1.12	1.25	0.22	1.25	1.35	0.94	1.14	0.36	1.43
HPQ	1.69	0.94	1.34	0.56	1.80	1.36	0.86	1.13	0.44	1.58
HD	1.44	0.99	1.24	0.36	1.45	1.23	0.89	1.08	0.31	1.38
HON	1.42	1.03	1.22	0.32	1.39	1.74	1.26	1.52	0.32	1.39
IBM	1.36	0.76	1.18	0.51	1.79	1.45	0.92	1.19	0.45	1.58
@INTC	1.85	1.25	1.55	0.39	1.48	1.53	0.92	1.25	0.49	1.66
IP	1.02	0.49	0.73	0.73	2.09	1.33	0.93	1.10	0.37	1.43
JNJ	0.75	0.33	0.47	0.89	2.26	0.71	0.41	0.52	0.56	1.70
JPM	1.93	1.38	1.59	0.34	1.39	1.76	1.25	1.44	0.36	1.41
MCD	0.81	0.48	0.66	0.50	1.70	0.90	0.53	0.72	0.51	1.68
MRK	0.95	0.24	0.61	1.18	4.04	0.80	0.31	0.56	0.88	2.58
@MSFT	1.91	1.27	1.53	0.42	1.51	1.43	0.82	1.15	0.54	1.76
MO	0.42	-0.15	0.12	4.70		0.59	-0.01	0.31	1.93	
SBC	0.82	0.23	0.48	1.23	3.59	0.52	0.18	0.33	1.04	2.97
UTX	1.39	0.98	1.20	0.35	1.43	1.65	1.09	1.40	0.40	1.52
WMT	1.20	0.77	0.99	0.43	1.55	1.11	0.67	0.90	0.49	1.66
GM	1.20	0.78	1.01	0.41	1.52	1.22	0.88	1.03	0.33	1.38
PG	0.91	-0.04	0.45	2.10		1.07	0.24	0.66	1.26	4.49
DIS	1.37	0.84	1.05	0.51	1.63	1.28	0.81	1.00	0.47	1.58
Average of (+)	1.21	0.68	0.94	0.78	1.96	1.23	0.77	0.99	0.57	1.85

	Beta W 5000					Volatility				
1/12/01 -31/1/02	Max	min	average	(max-min)/ average	max/min	Max	min	average	(max-min)/ average	max/min
MMM	0.69	0.18	0.49	1.06	3.93	0.31	0.25	0.27	0.23	1.25
AA	1.21	0.44	0.77	1.00	2.72	0.42	0.36	0.38	0.17	1.19
AXP	1.62	1.11	1.33	0.38	1.46	0.39	0.29	0.34	0.29	1.34
T	1.03	0.35	0.77	0.87	2.92	0.45	0.36	0.40	0.24	1.26
BA	1.17	0.54	0.91	0.69	2.15	0.44	0.35	0.38	0.23	1.25
CAT	0.85	0.49	0.64	0.56	1.74	0.38	0.31	0.35	0.18	1.20
C	1.79	1.31	1.52	0.32	1.37	0.43	0.34	0.37	0.24	1.26
KO	0.97	0.44	0.68	0.78	2.21	0.34	0.27	0.31	0.22	1.25
DD	0.76	0.38	0.60	0.63	2.00	0.35	0.28	0.31	0.25	1.28
EK	0.70	0.31	0.51	0.77	2.27	0.40	0.31	0.35	0.24	1.28
XOM	0.61	0.22	0.38	1.05	2.82	0.22	0.17	0.19	0.28	1.31
GE	1.25	0.97	1.11	0.25	1.29	0.31	0.26	0.28	0.20	1.22
HPQ	1.74	0.98	1.43	0.53	1.78	0.50	0.41	0.46	0.19	1.21
HD	1.39	0.85	1.15	0.47	1.64	0.41	0.33	0.36	0.24	1.27
HON	1.28	0.79	1.04	0.47	1.62	0.48	0.41	0.44	0.16	1.17
IBM	1.33	0.73	1.12	0.53	1.82	0.44	0.34	0.38	0.25	1.27
@INTC	1.86	1.30	1.55	0.36	1.43	0.55	0.42	0.49	0.26	1.30
IP	0.88	0.40	0.60	0.81	2.23	0.38	0.30	0.34	0.23	1.25
JNJ	0.56	0.18	0.34	1.11	3.17	0.27	0.20	0.23	0.32	1.37
JPM	1.92	1.32	1.54	0.39	1.46	0.45	0.34	0.40	0.27	1.32
MCD	0.68	0.27	0.50	0.83	2.55	0.31	0.27	0.29	0.14	1.15
MRK	0.71	0.09	0.42	1.47	8.10	0.33	0.27	0.30	0.21	1.24
@MSFT	1.78	1.22	1.47	0.38	1.46	0.52	0.39	0.46	0.28	1.32
MO	0.31	-0.25	0.00	474.83		0.39	0.30	0.35	0.24	1.28
SBC	0.75	0.14	0.40	1.54	5.37	0.34	0.24	0.29	0.33	1.40
UTX	1.25	0.84	1.06	0.39	1.49	0.37	0.30	0.34	0.19	1.21
WMT	1.04	0.60	0.82	0.54	1.75	0.36	0.28	0.32	0.26	1.30
GM	1.18	0.75	1.00	0.43	1.58	0.38	0.33	0.35	0.14	1.15
PG	0.61	-0.08	0.33	2.06		0.38	0.28	0.33	0.30	1.35
DIS	1.30	0.80	0.99	0.50	1.61	0.38	0.31	0.34	0.22	1.24
Average of (+)	1.11	0.59	0.85	16.53	2.35	0.39	0.31	0.35	0.23	1.26

Appendix 3

ARE CALCULATED BETAS GOOD FOR ANYTHING?

Statistics of the indexes

Historical volatilities, betas with respect to other indexes and correlations of the S&P 500, the Dow Jones Industrial Average Index and the Wilshire 5000 Index. Each day volatilities are calculated using 5 years of monthly data, i.e. on December 18, 2001, the volatility is the annualized standard deviation of the 60 monthly returns of the company. The returns of each month are calculated on the 18th of each month. For the periods 31/12/1996-31/5/2002, 1/1/2001-31/5/2002, and 1/12/01-31/1/02, the table contains the maximum, the minimum, the average, maximum minus minimum divided by the average, and maximum divided by the minimum.

	31/12/1996-31/5/2002					1/1/2001-31/5/2002					1/12/01-31/1/02				
	Max	min	average	(max-min)/ average	max/min	Max	min	average	(max-min)/ average	max/min	Max	min	average	(max-min)/ average	max/min
Volatility S&P 500	0.20	0.07	0.14	0.89	2.62	0.20	0.14	0.17	0.30	1.37	0.19	0.17	0.18	0.14	1.15
DOW JONES IND.	0.20	0.09	0.14	0.78	2.30	0.20	0.14	0.18	0.32	1.41	0.19	0.17	0.18	0.13	1.14
WILSHIRE 5000	0.20	0.08	0.14	0.88	2.58	0.20	0.15	0.18	0.28	1.34	0.20	0.17	0.19	0.15	1.16
Beta S&P 500															
DOW JONES IND.	1.08	0.79	0.96	0.30	1.37	0.98	0.79	0.91	0.21	1.24	0.97	0.85	0.92	0.13	1.14
WILSHIRE 5000	1.06	0.92	1.00	0.14	1.15	1.06	0.95	1.01	0.11	1.12	1.05	0.98	1.02	0.07	1.07
Beta DJ IND															
S&P 500	0.99	0.72	0.86	0.31	1.37	0.93	0.80	0.88	0.15	1.17	0.92	0.82	0.88	0.11	1.12
WILSHIRE 5000	1.01	0.71	0.84	0.36	1.42	0.95	0.74	0.86	0.25	1.29	0.94	0.78	0.87	0.19	1.21
Beta W 5000															
S&P 500	1.04	0.89	0.95	0.16	1.17	0.98	0.89	0.94	0.10	1.10	0.97	0.91	0.94	0.06	1.07
DOW JONES IND.	1.08	0.70	0.90	0.43	1.55	0.90	0.70	0.82	0.24	1.28	0.89	0.78	0.84	0.13	1.14
correlation															
S&P 500 -DJ IND	0.96	0.82	0.91	0.15	1.17	0.93	0.82	0.89	0.12	1.13	0.93	0.88	0.90	0.06	1.06
S&P 500 -W 5000	0.99	0.95	0.98	0.04	1.04	0.99	0.96	0.97	0.03	1.03	0.99	0.96	0.98	0.02	1.02
DJ IND -W 5000	0.93	0.76	0.87	0.20	1.23	0.89	0.76	0.84	0.15	1.16	0.89	0.81	0.85	0.09	1.09

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