University of Navarra

CIIF

Working Paper

WP No 579

January, 2005

REPLY TO

"COMMENT ON THE VALUE OF TAX SHIELDS

*IS **NOT** EQUAL TO THE PRESENT VALUE OF TAX SHIELDS"*

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REPLY TO
"COMMENT ON THE VALUE OF TAX SHIELDS IS NOT EQUAL TO THE PRESENT VALUE OF TAX SHIELDS"

Abstract

The Comment is thought provoking and helps a lot in rethinking the value of tax shields. However, the conclusion of Fieten, Kruschwitz, Laitenberger, Löffler, Tham, Vélez-Pareja and Wonder (2005) is not correct because, as will be proven below, the main result of Fernández (2004) is correct for several situations.

Equation (16a) shows that the value of tax shields depends **only** upon the nature of the stochastic process of the net increase of debt.

Classification JEL: G12, G31, G32

Keywords: Value of tax shields, present value of the net increases of debt

REPLY TO

"COMMENT ON THE VALUE OF TAX SHIELDS

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1. Value of tax shields and the stochastic process of net debt increases

For simplicity, Fernández (2004) neglected to use expected value notation. The equations in that paper that are affected by using the expected value notation, where E{·} is the expected value operator, are:

$$ECF_t = PAT_{Lt} - \Delta NFA_t - \Delta WCR_t + \Delta D_t \qquad \textbf{(5a)}$$

Where, $\Delta WCR_t = WCR_t - WCR_{t-1}$ = Increase of Working Capital Requirements in period t.

$\Delta NFA_t = NFA_t - NFA_{t-1}$ = Increase of Net Fixed Assets in period t.

$\Delta D_t = D_t - D_{t-1}$ = Increase of Debt in period t.

$$FCF_t = PATu_t - \Delta NFA_t - \Delta WCR_t \qquad \textbf{(7a)}$$

$$Taxesu_t = [T/(1+T)]\ PATu = [T/(1+T)]\ (FCF_t + \Delta NFA_t + \Delta WCR_t) \qquad \textbf{(9a)}$$

$$TaxesL_t = [T/(1+T)]\ (ECF_t + \Delta NFA_t + \Delta WCR_t - \Delta D_t) \qquad \textbf{(12a)}$$

Below we use the convention of referring to equation numbers in Fernández (2004). For non-growing perpetuities, $E\{\Delta NFA_t\} = E\{\Delta WCR_t\} = E\{\Delta D_t\} = 0$, and equations (5), (7), (9) and (12) in Fernández (2004) are equal to equations (5a), (7a), (9a) and (12a).

For growing perpetuties,

$E\{\Delta NFA_1\} + E\{\Delta WCR_1\} - E\{\Delta D_1\} = g\ (NFA + WCR - D) = g\ Ebv$, and

$E\{\Delta NFA_1\} + E\{\Delta WCR_1\} = g\ (NFA + WCR) = g\ (Ebv + D)$, which make equations (24) and (22) in Fernández (2004) correct.

Define $PV_0[\cdot]$ as the present value operator. The present values at t=0 of equations (9) and (12) are:

$$Gu_0 = [T/(1+T)]\ (Vu_0 + PV_0[\Delta NFA_t + \Delta WCR_t]) \qquad \textbf{(11a)}$$

$$G_{L0} = [T/(1+T)]\ (E_0 + PV_0[\Delta NFA_t + \Delta WCR_t] - PV_0[\Delta D_t]) \qquad \textbf{(14a)}$$

* Published in 2005 in *The Quarterly Review of Economics and Finance*, Volume 45, No 1, pages 188-192.

(11a) is equal to (11) only if $PV_0[\Delta NFA_t + \Delta WCR_t] = 0$. In this situation, equation (10) holds. Analogously, (14a) is equal to (14) only if $PV_0[\Delta NFA_t + \Delta WCR_t - \Delta D_t] = 0$ and equation (13) holds. But there are situations in which, for non-growing perpetuities, $PV_0[\Delta NFA_t + \Delta WCR_t] < 0$.

The value of tax shields comes from the difference between (11a) and (14a):

$$VTS_0 = Gu_0 - G_{L0} = [T/(1+T)]\ (Vu_0 - E_0 + PV_0[\Delta D_t])$$

As, according to equation (1), $Vu_0 - E_0 = D_0 - VTS_0$

$VTS_0 = [T/(1+T)]\ (D_0 - VTS_0 + PV_0[\Delta D_t])$. And the value of tax shields is:

$$\mathbf{VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta D_t]} \qquad \textbf{(16a)}$$

Equation (16a) shows that the value of tax shields depends *only* upon the nature of the stochastic process of the net increase of debt[1]. The problem of equation (16a) is how to calculate $PV_0[\Delta D_t]$, which requires knowing the appropriate discount rate to apply to the increase of debt.

2. Value of tax shields in specific situations

It is illustrative to apply (16a) to specific situations.

2.1. Perpetual debt

If the debt is a constant perpetuity (a consol), $PV_0[\Delta D_t] = 0$, and

$$VTS_0 = T \cdot D_0 \qquad \textbf{(16)}$$

This result is far from being a new idea. Brealey and Myers (2000), Modigliani and Miller (1963), Taggart (1991), Copeland et al. (2000), Fernández (2004) and many others report it. However, the way of reaching this result is new.

2.2. Debt of one-year maturity but perpetually rolled over

As in the previous case, $E\{D_t\} = D_0$, but the debt is expected to be rolled over every year. The appropriate discount rate for the cash flows due to the existing debt is Kd.[2] Define K_{ND} as the appropriate discount rate for the new debt that must be obtained every year, then:

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[1] If the nominal value of debt (N) is not equal to the value of debt (D), because the interest rate (r) is different from the required return to debt flows (Kd), equation (16a) is: $VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta N_t]$.
The relationship between D and N is: $D_0 = PV_0[\Delta N_t] + PV_0[N_t \cdot r_t]$.
If a company has little access to banks or financial markets, these difficulties may be solved by paying a high cost of debt. In these situations, D > N.

[2] We use Kd so as not to complicate the notation. It should be Kd_t, a different rate following the yield curve. Using Kd we may also think of a flat yield curve.

Present value of obtaining the new debt every year[3] = D_0 / K_{ND}

Present value of the principal repayments at the end of every year[4] = $D_0 (1+ K_{ND}) / [(1+Kd) K_{ND}]$

$PV_0[\Delta D_t]$ is the difference of these two expressions. Then:

$$PV_0[\Delta D_t] = - D_0 (K_{ND} - Kd) / [(1+Kd) K_{ND}] \quad \textbf{(50)}$$

If $K_{ND} = Kd$, then $PV_0[\Delta D_t] = 0$

In a constant perpetuity ($E\{FCF_t\} = FCF_0$), it seems reasonable that, if we do not expect credit rationing, $K_{ND} = Kd$, which means that the risk associated with the repayment of the current debt and interest (Kd) is equivalent to the risk associated with obtaining an equivalent amount of debt at the same time (K_{ND}).

2.3. Debt increases are as risky as the free cash flows

Then the correct discount rate for the expected increases of debt is Ku, the required return to the unlevered company. In the case of a constant growing perpetuity

$PV_0[\Delta D_t] = g \cdot D_0 / (Ku-g)$,

And the VTS is:

$$VTS_0 = T \cdot Ku \cdot D_0 / (Ku-g) \quad \textbf{(28)}$$

For g = 0, equations (28) and (16) are equal.

Equation (28) is the main one in Fernández (2004), although the way of deriving it is different.

2.4. The company is expected to repay the current debt without issuing new debt

In this situation, the appropriate discount rate for the negative ΔD_t (principal payments) is Kd, the required return to the debt. In this situation, Myers (1974) applies:

$PV_0[\Delta D_t] = PV_0[E\{\Delta D_t\}; Kd]$

And the VTS is:

$$VTS_0 = D_0 \cdot T + T \cdot PV_0[E\{\Delta D_t\}; Kd] \quad \textbf{(51)}$$

For perpetual debt, equations (51), (28) and (16) are equal.

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[3] Present value of obtaining the new debt every year = $D /(1+K_{ND}) + D /(1+K_{ND})^2 + D /(1+K_{ND})^3 + ...$ because $D = E\{D_t\}$, where D_t is the new debt obtained at the end of year t (beginning of t+1).

[4] The present value of the principal repayment at the end of year 1 is $D /(1+Kd)$.
The present value of the principal repayment at the end of year 2 is $D/[(1+Kd)(1+ K_{ND})]$.
The present value of the principal repayment at the end of year t is $D/[(1+Kd)(1+ K_{ND})^{t-1}]$.
That is because $D = E\{D_t\}$, where D_t is the debt repayment at the end of year t.

For a company that is expected to repay the current debt without issuing new debt, the value of the debt today is: $D_0 = PV_0[E\{D_{t-1}\} \cdot Kd - E\{\Delta D_t\}; Kd]$.

Substituting this expression in (51), we get the Myers (1974) formula:

$VTS_0 = PV_0[T \cdot E\{D_{t-1}\} \cdot Kd; Kd]$

The Comment argues that the Sick (1990) formula is a proven one: $VTS_0 = PV_0[T \cdot E\{D_{t-1}\} \cdot R_F; R_F]$.

Comparing the Sick formula with Myers', it is clear that Sick (1990) is valid only if the debt is risk-free and is not expected to grow. However, the Comment applies it for growing perpetuities by asserting that "$VTS_0 = D_0\ T\ R_F / (R_F\text{-}g)$." A little algebra permits to see that, according to Sick (1990), in a situation where Kd = R_F = 4%, Ku = 9%, and T = 50%, Ke > Ku if g > 2%. This hardly makes any economic sense.

3. Conclusion

The two theories that have some economic sense are Myers (1974) and Fernández (2004). As we have already argued, Myers (1974) should be used when the company is not expected to issue new debt, and Fernández (2004) when the company expects to issue new debt in the future. Both theories provide the same value for non-growing perpetuities. Sick (1990) is valid only if the debt is risk-free and is not expected to grow.

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