University of Navarra

Working Paper

THE VALUE OF TAX SHIELDS IS NOT EQUAL TO THE PRESENT VALUE OF TAX SHIELDS: A CORRECTION

Pablo Fernández*

* Professor of Financial Management, PricewaterhouseCoopers Chair of Finance, IESE

The CIIF, International Center for Financial Research, is an interdisciplinary center with an international outlook and a focus on teaching and research in finance. It was created at the beginning of 1992 to channel the financial research interests of a multidisciplinary group of professors at IESE Business School and has established itself as a nucleus of study within the School's activities.

Ten years on, our chief objectives remain the same:

- Find answers to the questions that confront the owners and managers of finance companies and the financial directors of all kinds of companies in the performance of their duties

- Develop new tools for financial management

- Study in depth the changes that occur in the market and their effects on the financial dimension of business activity

All of these activities are programmed and carried out with the support of our sponsoring companies. Apart from providing vital financial assistance, our sponsors also help to define the Center's research projects, ensuring their practical relevance.

The companies in question, to which we reiterate our thanks, are:
Aena, A.T. Kearney, Caja Madrid, Fundación Ramón Areces, Grupo Endesa, Telefónica and Unión Fenosa.

http://www.iese.edu/ciif/

THE VALUE OF TAX SHIELDS IS NOT EQUAL
TO THE PRESENT VALUE OF TAX SHIELDS:
A CORRECTION

Abstract

I correct some expressions in Fernández (2004) and provide a more general expression for the value of tax shields. This expression is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company. The value of tax shields in a world with no leverage cost is the tax rate times the current debt, plus the tax rate times the present value of the net increases of debt. The value of tax shields depends only on the nature of the stochastic process of the net increase of debt; it does not depend on the nature of the stochastic process of the free cash flow.

THE VALUE OF TAX SHIELDS IS NOT EQUAL
TO THE PRESENT VALUE OF TAX SHIELDS:
A CORRECTION

I provide a more general expression for the value of tax shields than that given in Fernández (2004). The title of Fernández (2004) still applies: The value of tax shields is not equal to the present value of tax shields, but is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company. This correction shows that some of the conclusions of Fernández (2004) are valid only for specific situations. More specifically, formula (28) (VTS = PV[Ku; D·T·Ku]) is valid only under the assumption that the debt increases are as risky as the free cash flows.

1. Correction of formulae

For simplicity, in Fernández (2004) I neglected to include in equations (5) to (14) terms with expected value equal to zero. And I wrongly considered as being zero the present value of a variable with expected value equal to zero. This does not have to be the case in general. Because of that error, Equations (5) to (17), Tables 3 and 4, and Figure 1 of Fernández (2004) are correct only if $PV_0[\Delta NFA_t + \Delta WCR_t] = PV_0[\Delta D_t] = 0$.

More precisely, the key equations of my paper that are affected by using the expected value notation are[1]:

$$ECF_t = PAT_{Lt} - \Delta NFA_t - \Delta WCR_t + \Delta D_t \tag{5a}$$

[1] I will refer with an "a" to the equations of Fernández (2004) that are affected by the mentioned error.

Notation being, ECF = Equity Cash Flow; PAT = Profit after Tax = $\Delta WCR_t = WCR_t -$ WCR_{t-1} = Increase of Working Capital Requirements in period t; $\Delta NFA_t = NFA_t - NFA_{t-1} =$ Increase of Net Fixed Assets in period t; $\Delta D_t = D_t - D_{t-1}$ = Increase of Debt in period t.

$$FCF_t = PATu_t - \Delta NFA_t - \Delta WCR_t \tag{7a}$$

where FCF = Free Cash Flow.

$$Taxes_{Ut} = [T/(1+T)] \ PATu = [T/(1+T)] \ (FCF_t + \Delta NFA_t + \Delta WCR_t) \tag{9a}$$

$$Taxes_{Lt} = [T/(1+T)] \ (ECF_t + \Delta NFA_t + \Delta WCR_t - \Delta D_t) \tag{12a}$$

$Taxes_U$ and $Taxes_L$ are the taxes paid by the unlevered company and those paid by the levered company.

$PV_0[\cdot]$ is the present value operator. The present values at t=0 of equations (9a) and (12a) are:

$$Gu_0 = [T/(1+T)] \ (Vu_0 + PV_0[\Delta NFA_t + \Delta WCR_t]) \tag{11a}$$

$$G_{L0} = [T/(1+T)] \ (E_0 + PV_0[\Delta NFA_t + \Delta WCR_t] - PV_0[\Delta D_t]) \tag{14a}$$

Gu is the present value of the taxes paid by the unlevered company and G_L is the present value of the taxes paid by the levered company.

The value of tax shields (VTS) comes from the difference between (11a) and (14a):

$$VTS_0 = Gu_0 - G_{L0} = [T/(1+T)] \ (Vu_0 - E_0 + PV_0[\Delta D_t]) \tag{15a}$$

As, according to equation (1), $Vu_0 - E_0 = D_0 - VTS_0$, then

$VTS_0 = [T/(1+T)] \ (D_0 - VTS_0 + PV_0[\Delta D_t])$. And the value of tax shields is:

$$\mathbf{VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta D_t]} \tag{16a}$$

Equation (16a) (not in the original paper) is valid for perpetuities and for companies with any pattern of growth. More importantly, this equation shows that the value of tax shields depends **only** upon the nature of the stochastic process of the net increase of debt. The problem of equation (16a) is how to calculate $PV_0[\Delta D_t]$, which requires knowing the appropriate discount rate to apply to the expected increase of debt.[2]

We may not know what are the correct values of Gu and G_L, but we do know the value of the difference, provided we can value $PV_0[\Delta D_t]$, the present value of the net debt increases.

---

[2] If the nominal value of debt (N) is not equal to the value of debt (D), because the interest rate (r) is different from the required return to debt flows (Kd), equation (16a) is: $VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta N_t]$. The relationship between D and N is: $D_0 = PV_0[\Delta N_t] + PV_0[N_t \cdot r_t]$.

2. VTS in specific situations

To develop a better understanding of the result in (16a), we apply it to specific situations and show how this formula is consistent with previous formulae under restrictive scenarios.

2.1. Perpetual debt

If the debt is a constant perpetuity (a consol), $PV_0[\Delta D_t] = 0$, and $VTS_0 = T \cdot D_0$

2.2. Debt of one-year maturity but perpetually rolled over

As in the previous case, $E\{D_t\} = D_0$, but the debt is expected to be rolled over every year. The appropriate discount rate for the cash flows due to the existing debt is Kd^3. Define K_{ND} as the appropriate discount rate for the new debt (the whole amount) that must be obtained every year, then:

Present value of obtaining the new debt every year[4] $= D_0 / K_{ND}$

Present value of the principal repayments at the end of every year[5] $= D_0 (1 + K_{ND}) / [(1+Kd) K_{ND}]$

$PV_0[\Delta D_t]$ is the difference of the last two expressions. Then:

$$PV_0[\Delta D_t] = - D_0 (K_{ND} - Kd) / [(1+Kd) K_{ND}] \tag{14}$$

In a constant perpetuity ($E\{FCF_t\} = FCF_0$), it may be reasonable that, if we do not expect credit rationing, $K_{ND} = Kd$, which means that the risk associated with the repayment of the current debt and interest (Kd) is equivalent to the risk associated with obtaining an equivalent amount of debt at the same time (K_{ND}).

2.3. Debt is proportional to the Equity value

This is the assumption made by Miles and Ezzell (1980) and Arzac and Glosten (2005), who show that if $D_t = L \cdot E_t$, then the value of tax shields for perpetuities growing at a constant rate g is:

$$VTS_0 = \frac{D_0 Kd\, T}{(Ku - g)} \frac{(1 + Ku)}{(1 + Kd)} \tag{50}$$

---

[3] We use Kd so as not to complicate the notation. It should be Kd_t, a different rate following the yield curve. Using Kd we may also think of a flat yield curve.

[4] Present value of obtaining the new debt every year $= D /(1+K_{ND}) + D /(1+K_{ND})^2 + D /(1+K_{ND})^3 + ...$ because $D = E\{D_t\}$, where D_t is the new debt obtained at the end of year t (beginning of t+1).

[5] The present value of the principal repayment at the end of year 1 is $D /(1+Kd)$
The present value of the principal repayment at the end of year 2 is $D/[(1+Kd)(1+ K_{ND})]$
The present value of the principal repayment at the end of year t is $D/[(1+Kd)(1+ K_{ND})^{t-1}]$
Because $D = E\{D_t\}$, where D_t is the debt repayment at the end of year t.

Substituting (50) in (16a), we get:

$$PV_0[\Delta D_t] = D_0 \frac{(Kd - Ku) + g(1 + Kd)}{(Ku - g)(1 + Kd)} \tag{51}$$

For the no growth case (g = 0), equation (51) is:

$PV_0[\Delta D_t] = D (Kd-Ku) / [Ku(1+Kd)] < 0.$

Comparing this expression with equation (14), it is clear that Miles and Ezzell imply that $K_{ND} = Ku$. However, to assume $D_t = L \cdot E_t$ is not a good description of the debt policy of any company because:

1. If the company pays a dividend Div_t, simultaneously the company should reduce debt by an amount $\Delta D_t = - L \cdot Div_t$

2. If the equity value increases, then the company should increase its debt, while if the equity value decreases, then the company should reduce its debt. If the equity value is such that $L \cdot E_t > $ (Assets of the company – Book Value of equity), then the company should hold excess cash only for the sake of complying with the debt policy.

2.4. Debt increases are as risky as the free cash flows

In this situation, the correct discount rate for the expected increases of debt is Ku, the required return to the unlevered company. In the case of a constant growing perpetuity, $PV_0[\Delta D_t] = g \cdot D_0 / (Ku-g)$, and the VTS is equation (28) in Fernández (2004):

$$VTS_0 = T \cdot Ku \cdot D_0 / (Ku-g) \tag{28}$$

2.5. The company is expected to repay the current debt without issuing new debt

In this situation, the appropriate discount rate for the negative ΔD_t (because they are principal payments) is Kd, the required return to the debt. In this situation, Myers (1974) applies: $PV_0[\Delta D_t] = PV_0[E\{\Delta D_t\}; Kd]$, and the VTS is:

$$VTS_0 = D_0 \cdot T + T \cdot PV_0[E\{\Delta D_t\}; Kd] \tag{18}$$

For a company that is expected to repay the current debt without issuing new debt, the value of the debt today is: $D_0 = PV_0[E\{D_{t-1}\} \cdot Kd - E\{\Delta D_t\}; Kd]$.

Substituting this expression in (18), we get the Myers (1974) formula:

$$VTS_0 = PV_0[T \cdot E\{D_{t-1}\} \cdot Kd; Kd]$$

3. Value of net debt increases implied by the alternative theories

Table 1 summarizes the implications of several approaches for the value of tax shields. From equation (16a) the present value of the increases of debt is:

$$PV_0[\Delta D_t]= (VTS_0 - T \cdot D_0) / T$$

Applying this equation to the theories mentioned, we may construct the predictions that each of these theories have for $PV_0[\Delta D_t]$.

As we have already argued, Myers (1974) should be used when the company will not issue new debt; and Fernández (2004) when the company expects to issue new debt in the future and we expect the increases of debt to be as risky as the free cash flow. Modigliani-Miller may be applied only if the debt is risk-free. Miles-Ezzell (1980) may be used only if debt will be a multiple of the equity value $D_t = L \cdot E_t$.

4. Conclusions

I show that the value of tax shields in a world with no leverage cost is the tax rate times the debt, plus the tax rate times the present value of the net increases of debt. This expression is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company. The critical parameter for calculating the value of tax shields is the present value of the net increases of debt. It may vary for different companies, but in some special circumstances it may be calculated.

For perpetual debt, the value of tax shields is equal to the tax rate times the value of debt. When the company is expected to repay the current debt without issuing new debt, Myers (1974) applies, and the value of tax shields is the present value of the interest times the tax rate, discounted at the required return to debt. If the correct discount rate for the increases of debt is the required return to the unlevered company, then formula (28) of Fernández (2004) applies.

Table 1

Comparison of value of tax shields (VTS) in perpetuities

Only three out of the seven approaches correctly compute the value of the tax shield in perpetuities as DT.

The other four theories imply a lower value of the tax shield than DT.

Theories	VTS	$PV_0[\Delta D_t]$ for constant growing perpetuities at a rate g
Correct method	$D \cdot T + T \cdot PV_0[\Delta D_t]$	
Damodaran (1994)	$PV[E\{D \cdot T \cdot Ku - D (Kd - R_F) (1-T)\}; Ku]$	$\dfrac{g \cdot D_0}{Ku - g} - \dfrac{D_0 (Kd - R_F)}{Ku - g} \dfrac{(1-T)}{T}$
Practitioners	$PV[E\{D \cdot T \cdot Kd - D (Kd - R_F)\}; Ku]$	$\dfrac{g \cdot D_0}{Ku - g} - \dfrac{D_0 (Ku - Kd)}{Ku - g} - \dfrac{D_0 (Kd - R_F)}{(Ku - g) T}$
Harris-Pringle (1985), Ruback (1995)	$PV[E\{D \cdot T \cdot Kd\}; Ku]$	$\dfrac{g \cdot D_0}{Ku - g} - \dfrac{D_0 (Ku - Kd)}{Ku - g}$
Myers (1974)	$PV[E\{D \cdot T \cdot Kd\}; Kd]$	$g \cdot D_0 / (Kd - g)$
Miles-Ezzell (1980)	$PV[E\{D \cdot T \cdot Kd\}; Ku] (1+Ku) / (1+Kd)$	$\dfrac{g \cdot D_0}{Ku - g} - \dfrac{D_0 (Ku - Kd)}{(Ku - g)(1 + Kd)}$
Modigliani-Miller (1963)	$PV[E\{D \cdot T \cdot R_F\}; R_F]$	$g \cdot D_0 / (R_F - g)$
Fernández (2004)	$PV[E\{D \cdot T \cdot Ku\}; Ku]$	$g \cdot D_0 / (Ku - g)$

Ku = unlevered cost of equity
Kd = required return to debt
T = corporate tax rate
D = debt value
R_F = risk-free rate
$PV[E\{D \cdot T \cdot Ku\}; Ku]$ = present value of the expected value of $D \cdot T \cdot Ku$ discounted at the rate Ku

References

Arzac, E.R, Glosten, L.R., 2005. A Reconsideration of Tax Shield Valuation. *European Financial Management*, forthcoming.

Damodaran, A., 1994. *Damodaran on Valuation*. John Wiley and Sons, New York.

Fernández, P., 2004. The value of tax shields is NOT equal to the present value of tax shields. *Journal of Financial Economics* 73/1, 145-165.

Harris, R.S., Pringle, J.J., 1985. Risk-adjusted discount rates extensions form the average-risk case. *Journal of Financial Research* 8, 237-244.

Miles, J.A., Ezzell, J.R., 1980. The weighted average cost of capital, perfect capital markets and project life: a clarification. *Journal of Financial and Quantitative Analysis* 15, 719-730.

Modigliani, F., Miller, M., 1963. Corporate income taxes and the cost of capital: a correction. *American Economic Review* 53, 433-443.

Myers, S.C., 1974. Interactions of corporate financing and investment decisions – implications for capital budgeting. *Journal of Finance* 29, 1-25.

Ruback, R., 1995. A note on capital cash flow valuation. Harvard Business School Case No. 9-295-069.

Ruback, R., 2002. Capital cash flows: a simple approach to valuing risky cash flows. *Financial Management* 31, 85-103.