EFFICIENT PUBLIC-PRIVATE PARTNERSHIPS

Marian Moszoro
EFFICIENT PUBLIC-PRIVATE PARTNERSHIPS

Marian Moszoro

Abstract

This paper presents a model to assess the efficiency of the capital structure in public-private partnerships (PPP). A main argument supporting the PPP approach for investment projects is the transfer of managerial skills and know-how from the private partner to the investment vehicle. The paper shows how different managerial skills and knowledge transfer schemes determine an optimal shareholding structure for the PPP. Under the assumption of lower capital cost of the public partner and lower development outlays when the investment is carried out by a private investor, an optimal capital structure is achieved with both the public and private parties as shareholders, i.e. a mixed public-private capital structure enables internalization of the financial advantage of the public sector and the managerial advantage of the private sector.

JEL Classification: D23; G32; H43; H54; L19

Keywords: Public-Private Partnerships, Joint Ventures, Public investment policy, Knowledge transfer, Hybrid governance structures.

Acknowledgements:

This is an extensively revised version of an earlier paper that circulated as “Optimal Capital Structure of Public-Private Partnerships,” IMF Working Paper WP/08/1. The author particularly thanks Pawel Gasiorowski, Thomas Moser, René Weber, and Andrew Feltenstein from the IMF for their comments and Paul Soto for his research assistance.

1 Lecturer, Finance Department, IESE and Haas School of Business, University of California, Berkeley
EFFICIENT PUBLIC-PRIVATE PARTNERSHIPS

Introduction and motivation of the paper

The provision of public goods provides a strong rationale for public-private hybrids that can efficiently carry out public investment projects. PPPs are gaining momentum, both for governments seeking efficient solutions to public services and for investors as an asset class. The global public-private partnership (PPP) debt market attracted 5% of project lending and amounted to $47.9 billion in 2009, a jump of 33.7% from $35.8 billion in 2006 (Esty and Sesia, 2007, p. 2; 2010, pp. 9-35). With the present financial crisis, the debate on the role of PPPs has attracted new attention. The key justifications for pursuing PPPs as quoted in the literature are lower costs than in solely private investments and higher quality than in the solely public provision of the public good (Vaillancourt-Rosenau, 2000), due to the public partner’s lower cost of capital (LEP, 2007; Grout, 2003), and the transfer of managerial skills (Linder, 1999) and industry-specific know-how (Hennart, 1988) from the private to the public partner.

This paper was motivated by the observation of PPPs with hybrid public-private capital structures in Central and Eastern Europe. That observation inspired the study of PPPs in Western countries, which showed that hybrids are not the exception but rather a common governance practice (Table 1 depicts examples of PPPs with mixed public-private capital structures in the utilities sector in various countries). Weber and Alfen (2010, p. 105) show that out of the ten largest international airports by passengers worldwide, two (Paris CDG and Frankfurt) have a mixed public-private ownership structure. There are other examples (e.g.: European Commission, 2004; PricewaterhouseCoopers, 2006). Referring to Fannie Mae and Freddie Mac, Jaffee and Quigley (2009, p. 3) sustain that “there is little reason to presume that the evolution of these [government-sponsored] enterprises represented the best method of providing services.” However, if, on the contrary, there was a rationale for joint public-private capital structures, the question is whether and under which circumstances PPPs can provide a more efficient structure than entirely public or entirely private enterprises.
### Table 1
Examples of PPPs with joint public-private capital structures

<table>
<thead>
<tr>
<th>Panel A: Water companies</th>
<th>Company and location</th>
<th>Private investors</th>
<th>Private investors’ share ( \theta )</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>AQUA, Bielsko-Biala, Poland</td>
<td>United Utilities Europe</td>
<td>21%, and then 33.18%</td>
<td>acquiring the shares of an existing company; 12 years of contractual commitments</td>
<td></td>
</tr>
<tr>
<td>Dabrowa Gornicza, Poland</td>
<td>RWE Aqua GmbH</td>
<td>34%</td>
<td>acquiring the shares of an existing company; 20 years of contractual commitments</td>
<td></td>
</tr>
<tr>
<td>Glogow, Poland</td>
<td>Gelsen-Wasser AG</td>
<td>46%</td>
<td>acquiring the shares of the existing company</td>
<td></td>
</tr>
<tr>
<td>SAUR Neptun, Gdansk, Poland</td>
<td>SAUR International</td>
<td>51%</td>
<td>30 years of contractual commitments</td>
<td></td>
</tr>
<tr>
<td>Tamrowskie Gory/ Mieściec Solakie, Poland</td>
<td>Veolia Water (Vivendi)</td>
<td>33.85%; after IPO 64%</td>
<td>25 years of contractual commitments</td>
<td></td>
</tr>
<tr>
<td>Stadtentwässerung Schwerte, Germany</td>
<td>RWE Umwelt Aqua</td>
<td>48%</td>
<td>Reported investment costs 12% below the average of German cities</td>
<td></td>
</tr>
<tr>
<td>BerlinWasser, Germany</td>
<td>equally RWE Aqua, Allianz Capital Partners, Veolia Deutschland</td>
<td>49.9%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scottish Water Solutions, United Kingdom</td>
<td>Stirling Water (24.5%), UUGM (24.5)</td>
<td>49%</td>
<td>Stirling Water: Thames Water, KBR, Alfred McAlpine and MJ Gleeson; UUGM: by United Utilities, Galliford Try and Morgan Est</td>
<td></td>
</tr>
<tr>
<td>Apa Nova, Romania</td>
<td>Vivendi</td>
<td>84%</td>
<td>25 years of contractual commitments</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Solid Waste</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Müllheimer Entsorgungsgesellschaft, Germany</td>
<td>Trienekeens</td>
<td>49%</td>
<td></td>
</tr>
<tr>
<td>Szolnok, Hungary</td>
<td>Rethmann</td>
<td>51%</td>
<td>Municipality will retain at least 25% of shares to be considered qualified minority investor under Hungarian law</td>
</tr>
<tr>
<td>Debrecen, Hungary</td>
<td>ASA</td>
<td>51%</td>
<td>Municipality will retain at least 25% of shares to be considered qualified minority investor under Hungarian law</td>
</tr>
<tr>
<td>Varna, Bulgaria</td>
<td>RWE</td>
<td>65%</td>
<td></td>
</tr>
<tr>
<td>Kirklees, United Kingdom</td>
<td>United Waste Services</td>
<td>81%</td>
<td>25 years of contractual commitments</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Airports</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Charles De Gaulle Airport, Paris, France</td>
<td>Free float (29.6%), employees (2%)</td>
<td>31.6%</td>
<td>Schiphol Group owns 8% of Paris CDG, a consortium of: State of Netherlands (69.8%), City of Amsterdam (20%), Aéroports de Paris (8%) and City of Rotterdam (2.2%), classified as a public entity</td>
</tr>
<tr>
<td>International Airport Hamburg, Germany</td>
<td>Hochtief AirPort, Aer Rianta International</td>
<td>initially 36%, then 40%</td>
<td></td>
</tr>
<tr>
<td>Local Airport Kassel-Calden, Germany</td>
<td>IHK, Fraport</td>
<td>50%</td>
<td></td>
</tr>
<tr>
<td>Flughafen Dusseldorf, Germany</td>
<td>HTA (20%), HTAC (10%), Aer Rianta (20%)</td>
<td>50%</td>
<td>acquiring the shares of the existing company</td>
</tr>
<tr>
<td>Frankfurt Airport, Germany</td>
<td>Stadtwerke Frankfurt am Main Holding (20.16%), Julius Bar Holding (10.35%), Deutsche Lufthansa (9.94%), others (27.98%)</td>
<td>68.43%</td>
<td>partial IPO to wide range of investors</td>
</tr>
</tbody>
</table>

Empirical research carried out mostly in the United Kingdom and the United States shows that the private sector is indeed able to build infrastructure cheaper than the public sector (Wright, 1987; Viscusi, Vernon, and Harrington, 2000; HM Treasury, 2003a; European Investment Bank 2004). The potential savings amount to 15-30% and can be attributed to more efficient project management by the private investor, shorter construction time, as well as lower administrative expenses. On the other hand, in developed markets the cost of capital for the private sector is on average 40-260 basis points higher than for the public sector.¹

PPPs are often undertaken by joint venture companies (special purpose vehicles, SPV) with equity contributed by the private and public sectors (European Commission, 2004; Corbacho and Schwartz, 2008; Weber and Alfen, 2010; Schaeffer and Loveridge, 2002). The share in equity of the SPV is usually reflected in the partners’ voting power and management involvement. To be attractive and viable, the shareholding structure of the SPV should secure the interest of both the public and private partner, by providing enough public capital and sufficient private sector managerial skills and know-how. The rationale for hybrids solutions is similar to that developed by Rangan, Samii, and Van Wassenhove (2006, pp. 740-741), but the drivers presented here are easier to operationalize and more management-oriented. Efficient financing of public investments by the private sector requires that the higher financing cost of the private sector be offset by the savings in development outlays due to the transfer of managerial skills and know-how from the private sector. As Hennart (1988, p. 372) points out, some knowledge types are firm-specific assets, i.e. they cannot be acquired separately from the firm. Thus, a full takeover of the private partner holding the know-how will involve substantial management costs if the firm to be acquired is large, if it operates in a different industry or has a different managerial culture – as is the case of public agents and private companies – than the acquirer, or if it is foreign-based.

The paper is organized as follows. Section 2 presents the literature related to mixed public-private partnerships. Section 3 introduces the justification for the model’s key assumptions about the lower cost of capital compared with the private partner and the assumption that the private sector can construct the infrastructure at a lower cost than the public sector. Section 4 outlines the relationships between the level of quality, the required investments in infrastructure, and the cost of public utilities’ services. Section 5 presents a model of managerial skills and knowledge transfer (initially a discrete model, then a continuous one) to describe the optimal public-private capital structure in PPPs for the provision of public utilities infrastructure. It also includes practical examples of existing PPPs in the public utilities sector. Section 6 concludes with advice on the applicability of the model’s solution.

2. Related Literature

A concise historic background of PPP can be found in Grimsey and Lewis (2004, 2005). The first public-private cooperation sets were concessions. Mixed public-private ownership is rather a novel concept. Linder (1999) presents a taxonomy of public-private partnerships. Hybrid governance structures would be defined as “risk-shifting” and “power-sharing” PPPs. Schaeffer and Loveridge (2002) review the literature regarding types of public-private cooperation. They

¹ The Appendix contains data on the spread between United States treasury bonds, municipal bonds, and corporate bonds. See also Homer and Sylla (2005, p. 662) for United States Corporate and Municipal Long-Term Bond Yields spreads in 1990-2004 and Passmore (2005, pp 8-12) for debt advantage of GSEs over private corporations.
conclude that there are “important philosophical and legal differences between private sector and public sector organizations. However, it is exactly because of the differences between the public and the private sector that opportunities for mutually beneficial cooperation arise. The two sectors have complementary powers, and thus, each can help the other accomplish things that may otherwise not be feasible.”

The literature on financing public investments by private capital compares higher outlays on construction for the public sector with the higher payments for a private investor (Irwin, 2008, p. 109). Some researchers (Grout, 1997, 2003; Hart, 2003; Hart, Shleifer, and Vishny, 1997) do not acknowledge a continuum between solely public or solely private shareholding for determining the optimal governance structure. Very few scholars take account of the continuum between these two cost drivers and implications for the governance structure. Beato and Vives (1996) consider “mixed cases share features of the pure [public or pure private] cases,” and therefore “they may be evaluated by using the relative participation of final consumers and the public agency in the private firm’s revenues.” Gerrard (2001) points out mixed public and private ownership in water supply utilities, and Hammami, Ruhashyankiko, and Yehoue (2006) give two examples of ‘semi-private’ ownership of public utilities. Schwartz, Corbacho and Funke (2008) describe the pros and cons of mixed public-private structures. Rangan, Samii, and Van Wassenhove (2006) set the necessary conditions for economic opportunity realization of mixed private-public partnerships based on transaction cost economics and externalities theory. Gazley, Chang, and Bingham (2010) underline the relevancy of a stakeholder outlook on governance to the context of public-private partnerships because of its “systems-centered” perspective on how constituent interests are represented. Rufín and Rivera-Santos (2010) compare PPPs with business-to-business alliances. They predict that PPPs will avoid equity structures and will rely, instead, on less complete and more complex alliance contracts, which contrasts with substantial empirical evidence. The European Commission (2004) provides examples of mixed public and private ownership and risk-sharing. Weber and Alfen (2010) refer to them as ‘horizontal’ or ‘institutional’ PPPs (contrasted to ‘vertical’ or ‘contractual’ PPPs). Hart, Shleifer, and Vishny (1997) develop a dichotomy model addressing when a government should provide a service in-house, and when it should contract out provision, ignoring hybrid solutions. Nonetheless, none of these authors provided formal models of hybrid governance. Based on data from real cases, the models presented below will show that the investment may be optimally financed with the mixed public-private shareholding.

This paper analyzes the conditions for PPPs which will provide for a Pareto-efficient development of public utilities infrastructure. Pareto optimality refers here to achieving the same quality of service at a lower cost or better quality at the same cost for a PPP than for publicly developed or privately developed public investments. It will also evaluate how different forms of managerial skills and know-how transfer determine an optimal capital structure of the SPV. I find that if the cost of capital is lower for public entities and the outlays on building infrastructure are lower when the investment is made by a private investor, it is possible to reach the lowest total cost of construction with both public and private capital as part of the shareholding, i.e. a mixed capital structure enables internalization of both the cost of capital advantage of the public sector and the knowledge advantage of the private sector.

---

2 For the so-called Public Sector Comparator (PSC) and Value for Money (VfM) methodologies, see e.g. Broadbent and Laughlin (2003), and HM Treasury (2004).
3. Public Financial Advantage and Private Managerial Advantage

The paper’s underlying assumption is that it might be optimal for the public partner to become a shareholder in the SPV. In markets with a long and reliable tradition of public projects procurement to the private sector, such an assumption might be counterintuitive. However, the existence of market failures, especially in the case of transition and emerging economies, justifies the government’s involvement in the SPV to correct market failures which result in the inability of the marketplace to provide public goods (Jevčák and Keereman, 2008, pp. 38-39; Tandberg, 2008, p. 66).

One of the market failures relates to the financial market. The domestic financial markets of emerging economies are not deep and efficient enough to provide reasonably priced capital to finance projects carried out for the public. Therefore, governments aiming at infrastructure development have to use savings from the external sector. The current account deficits of the Central and Eastern European countries in the last decade confirm this observation. Therefore, the public sector’s participation in the projects provides a necessary certainty to the project’s financial viability. It also serves as the necessary condition for attracting the participation of foreign capital in financing the project. Of course, public participation does not correct all of the market failures, but it may at least reduce their impact. On the other hand, the conservative approach to risk evaluation by foreign investors leads to an additional premium to cover an excess risk which is difficult to evaluate. The additional premium – an extra reward for uncertainty – may be the reason for the market failure of public investment projects. The public sector’s participation may offset this additional premium and make public investment an interesting business case.

The reason behind a public shareholding in the SPV may also be explained from the point of view of the principal-agent theory. In a PPP, the public and private sector might have conflicting interests regarding the project’s cost and the quality of the public good delivered. Under most legislative systems, the shareholder has much more control over the company’s activities than its contractor. Therefore, the government seeking a high-quality and reasonably-priced public good would have much more control over the project when it is a shareholder of the SPV. Again, the need for better control of quality and prices may be more important in the emerging and transition markets, where public legal regulations are not entrenched strongly enough.

Cost of Public Capital

Some researchers develop a notion of the “social discount rate” that should be used to discount and compare the value achieved for the public sector in public projects. Many prestigious economists (Samuelson, 1964; Vickrey, 1964; Solow, 1965; Baumol, 1968; Arrow and Lind, 1970), analyzing the social discount rate in the 1960’s and 1970’s, claimed that the discount rate for public entities should be lower than for the private sector. Other researchers (Hirshleifer, 1964; Diamond, 1967; Bailey and Jensen, 1972; Drèze, 1974; Kay, 1993; Brealey, Cooper, and Habib, 1997; Klein, 1997, among others) claim that the social discount rate should be higher than the plain public borrowing cost, hence equaling both public and private discount rates.

---

3 This paper is not intended to advocate government intervention in private companies. Nonetheless, recent bailouts by governments worldwide during the period 2008-2010 have made the idea of governments taking a stake in private businesses less outrageous.
They suggest that the public sector’s lower borrowing cost reflects the fact that the public sector does not default, that it can levy taxes to repay the debt, and also that the lower cost of funds does not reflect a more efficient management of risk. These comments may be true in the general case of public borrowing, especially in a closed economy. However, such arguments should not apply to the PPPs, especially those organized in emerging and transition markets, for the following reasons:

- The effective PPP scheme assumes the transfer of risks from the public sector to the private sector (Blöndal, 2005). Therefore, the public sector’s cost of borrowing may not reflect the project’s risks, as these risks are transferred out of the public sector. As the IMF (2004) states, it is difficult to establish whether the transfer of risk out of the public sector is proportional to the difference in funding cost, but the public cost of lending is definitely lower.

- In case of investments which are not very important to the economy (practically: low investments in relation to GDP), the discount rate for the public sector should be lower than for the private sector, because the public sector can better absorb and spread the risks among a greater number of people (Arrow and Lind, 1970; Fisher, 1973).

- Private companies cannot internalize the externalities, and their return on investment comes only from a given project. Flemming and Mayer (1997) showed that private sector investments in the public utilities sector depend on the policy applied in other sectors of the economy and create externalities leading to inefficient piece-meal decision making, when only the perspective of a single sector of the economy is taken into account.

- In the case of incomplete capital markets, capital investors and lenders are not able to protect themselves in the capital market against the risk connected with the securities that are financing the public or private investment and therefore apply various discount rates (Hirshleifer, 1964; Bailey and Jensen, 1972).

- Grout (2003) proved that even in a world without incomplete markets and distorting taxation, it is appropriate to apply a higher discount rate for private entities than for public ones. The argument is based on the differential in the beta (risk) for government payments under a PPP and the beta of government expenditures under the normal public provision of goods.

- Lind (1990) suggests that the government’s long-term borrowing rate is a “good first candidate” for long-run intergenerational problems and that “for most government projects we should compute net benefits (from the project) using the government borrowing rate as the discount rate” (quote after Spackman, 2004).

Moreover, the concept of the social discount rate is based on the social time preference. It would be justified to take into account the social discount rate for projects financed in the domestic market. However, it would be difficult for any government to consider the social discount rate of the external sector, which theoretically should be applied if projects are financed from the savings of the external sector.

---

4 The project’s risk profile remains the same independently of the form of financing. In this paper, I do not address the issue of risk-sharing, which is essentially part of the operating phase.
The two concepts: the cost of capital (i.e. a financial approach) and the social discount rate (social cost of capital, i.e. a general equilibrium economic approach) should not be equaled. I based my assumptions regarding the cost of capital on direct market evidence that the public sector can raise capital cheaper than the private sector5 and, from these funds, can finance different projects. I concentrated on the interest paid on the resources engaged and did not discuss the social cost of capital. Whether public sector projects should be discounted at a lower rate than private sector projects is a highly contentious issue and one that has spawned an abundance of literature.

The amount of literature supporting the lower cost of capital for the public sector (starting from the Arrow-Lind theorem in 1970) is at least as large and strong as that supporting the approach according to which public projects should be discounted at the same rate (see for example: HM Treasury, 2003b; Engel, Fischer, and Galetovic, 2008), irrespective of the source of financing (see discussion in section 2.4 in Grout, 2005). Writing in the 1980s on public sector discount rates and their relation to private sector discount rates, Lind (1982) pointed out that “the profession was no closer to agreement on the theory, on a procedure for computing the discount rate, or on the rate itself than it was in 1966.” The evidence is that the difference between the interest rates for governments and private investors is statistically significant; however, whether public projects should be discounted at the same or a different rate is not a positive, but a normative question.

**Savings on Construction Cost Due To Private Sector's Participation in the SPV**

Empirical research carried out mostly in the United Kingdom and in the United States shows that the private sector can construct the infrastructure cheaper than the public sector (Wright, 1987; Viscusi, Vernon, and Harrington, 2000). Wallace and Junk (1970) even claim that the investment costs of public enterprises are 40% higher than those of private enterprises.

On one hand, PPPs involve sizeable structuring and transaction costs, and force governments to develop expensive internal capacities (lawyers, engineers, and financial advisers; see: Tandberg, 2008, p. 67; Corbacho and Schwartz, 2008, p. 88). However, conversely, savings can be attributed to more efficient project management by the private investor (Linder, 1999, pp. 42-43), shortened construction and development times (Ward and Chapman, 1995), higher cost of innovation efforts under private ownership of assets (Hart, Shleifer, and Vishny, 1997, p. 1143), and lower administrative expenses and less bureaucracy (Starr, 1988, pp. 23-24, 30; Goldsmith, 1997). Several National Audit Office (United Kingdom) reports on PPPs (European Investment Bank, 2004, p. 18; Blöndal, 2005) presented the following statistics confirming the abovementioned sources of savings:

- A report commissioned by the Treasury Taskforce found that the average percentage estimated saving against the Public Sector Comparator in PFI projects was 17%.

- HM Treasury research of 61 Private Finance Initiative (PFI) projects:

---

5 See Appendix. Kosar (2007, p. 6), and Jaffee and Quigley (2009, p. 6) state that GSEs borrow money at significantly lower interest rates than solely private competitors because of the inferred government guarantee. For Emerging Markets, the bond market is not liquid enough to provide continuous data. Most of municipal and corporate bonds are privately placed and have not unveiled yields. Moszoro and Kowalik (2005) listed municipal and corporate bonds with open yields formulas in Poland, supporting the hypothesis of lower interest rates for public entities than for private corporations.
− 89% of projects were delivered on time or early.

− All PFI projects in the HM Treasury sample were delivered within public sector budgets.

− No PFI project was found where the unitary charge had changed following contract signature – other than where user requirements changed.

− 77% of public sector managers stated that their project was meeting their initial expectations.

• 4 Design-Build-Finance-Operate (DBFO) road contracts appear likely to generate net quantifiable financial savings of around £100 million (13%).

• Out of 98 projects surveyed by the NAO in 2001 on public authorities’ perceptions of Value For Money:
  − 81% believed that PFI projects are achieving satisfactory or better Value For Money – only 4% described Value For Money as ‘poor’.
  − 75% of PFI projects were delivered on time or early, and in no case did the public sector bear the cost of construction overruns, a significant improvement on previous non-PFI experience.

• The contract for the Private sector Resource Initiative for the Management of the Estate, a project comprising transfer of the Department of Social Security estate to the private sector, is estimated to deliver savings of £560 million, 22%, over 20 years.

The cost of construction may not always be higher for the public sector (Ahadzi and Bowles, 2004); however, there is enough market evidence to enable it to be higher and to analyze what the consequences for the organization of PPPs are.

The mainstream PPP literature suggests that the key issues involved are the bundling together of the project’s construction and operation and efficient risk allocation between the public and private partners (Rangan, Samii, and Van Wassenhove, 2006), regardless of the capital structure. The bundling of contracts is not in contradiction to my assumption of lower construction costs due to the private sector’s participation in the SPV: it may even reinforce it. What the private sector does when bundling is a “package selling” of: construction, insurance, and financing for a lower price than when contracted independently by the public sector. I sustain that when the private capital’s share is large enough, the advantages of this bundling may be realized because of the economic incentives for the private partner (Hennart, 1988, p. 372).

The private sector, when contracted by the public sector only for building (i.e. with no shares in the SPV), has no incentives to build cheaply but to bid low to win the tender. Where competition is low and the public sector does not have the resources to organize each and every tender, it may happen that:

a) There is price collusion or ex ante agreements (bid rigging) about who should win the tender (Buccicossi, 2008; Laffont and Tirole, 1993, pp. 570-571).

b) After bidding low and winning the tender, the private sector renegotiates the contracts (see asymmetric collusion in Laffont and Tirole, 199, pp. 572-573).
c) When public-private relations are ex ante less flexible and subject to ex post higher accountability (Wang and Bunn, 2004, p. 89), the private contractor charges a higher price than in a private-private relational contract (Spiller, 2008).

d) While the question on the appropriate discount rate is normative, the motivation for why the greater knowledge and expertise of the private sector requires (partial) equity ownership of the project by the private sector is a positive question with different answers depending on the institutional and judicial environment. The corollary is that when the private partner has an equity stake in the project, residual control rights are more properly protected and it has an economic incentive to transfer managerial skills and know-how to innovate in cost and quality (Hart, Shleifer, and Vishny, 1997).

4. The Relationships Within The Model

The amount of capital expenditure on infrastructure is determined by the required quality of the public service supplied by the SPV. Consider a newly formed SPV that has to invest during the development phase and – after the investment phase – operates the infrastructure and provides the public good. Further assume that during the project’s operational phase, the public service is provided by the SPV at a cost that reflects a fixed fee that covers financing of the initial investment outlays and a variable fee that covers the current cost of service. This assumption requires that the capital expenditure $I(q)$ necessary to satisfy the demand at the quality level $q$ should be equal to the present value of fixed fees paid to the SPV $f(q)$ over the life of the project $t$:

$$I(q) = f(q)\frac{1-(1+r)^{-t}}{r}$$

For a sufficiently long life of the project (for $t \to \infty$), this can be expressed as:

$$I(q) = \frac{f(q)}{r}$$

This conclusion can be also derived from market-clearing conditions. In the two-part tariff system, for a given quality of service $q$ and demand function $P(x,q)$, the fee for consumption of $x$ units of the service should include a fixed fee $f(q)$ and a variable fee $p(x,q)$.

$$P(x, q) \cdot x = f(q) + p(x, q) \cdot x$$

From formula (3) the demand function is:

$$P(x, q) = \frac{f(q)}{x} + p(x, q)$$

On the supply side, for each level of service provision $x$, total revenues must include the cost of financing the investment outlays and a unit price at least equal to the marginal cost of service. This leads to the following supply function:
Market clearing conditions require $P(x,q)$ for a given $x$ and $q$ to be equal to $S(x,q)$

$$P(x, q) = S(x, q) \tag{6}$$

Replacing (4) and (5) in (6)

$$\frac{f(q)}{x} + p(x, q) = \frac{I(q) \cdot r}{x} + \text{MC}(x, q) \tag{7}$$

Since, in market clearing, the variable fee $p(x,q)$ should cover marginal cost, then

$$f(q) = I(q) \cdot r \tag{8}$$

5. Determining The Optimal Public-Private Capital Structure

As stated above, it is assumed that the private sector is able to execute the PPP project cheaper than the public sector. Let us denote by $J(q)$ the amount by which development outlays (without financial costs) for a privately executed project are lower than the outlays for a publicly executed one.

Assume also that the transfer of the private partner’s idiosyncratic assets (such as industry-specific know-how in infrastructure engineering, capital allocation, and risk management; see: Rangan, Samii, and Van Wassenhove, 2006, p. 747) to the SPV materializes when the private share in the partnership’s capital achieves a minimum of $e$. Then, as in equation (8), one may determine the level of fixed fees for mixed public-private financing from the following equation:

$$f(q) = \theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot \left( I(q) + (1-\beta) \cdot J(q) \right) \cdot r_{pu} \tag{9}$$

where:

$r_{pr}$ – interest (discount) rate for a private investor

$r_{pu}$ – interest (discount) rate for the public sector, $r_{pr} > r_{pu}$

$\theta$ – share of a private investor in the joint venture, $\theta \in (0,1)$

$\beta$ – discrete variable reflecting the existence of managerial skills and know-how in project execution, so that:

---

6 The condition on a higher interest rate for the private sector than the interest rate for the public sector may hold in the context of a partial equilibrium, when the public sector is characterized by a lower risk. For a general equilibrium context, see e.g. Feltenstein and Ha (1999).
As Hart et al. (1997, pp. 1135-1136) point out, it remains to discuss the extent to which the fruits of the private manager’s efforts devoted to cost and quality innovation are embodied in the private partner’s human capital. In the discrete model framework, I simply assume that if the private partner holds residual control rights greater than or equal to \( e \), where \( e \) denotes the private partner’s ownership in the SPV, then the knowledge and managerial skills transfer to the SPV is realized as described by Eaton and Akbiyikli (2009, pp. 304-306).

Thus, the condition for a PPP to execute the investment at a lower cost than the public partner can be written as:

\[
\theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot I(q) \cdot r_{pu} < (I(q) + (1-\beta) \cdot J(q)) \cdot r_{pu}
\]  
\[
(10)
\]

Sorting and arranging with regard to \( \theta \) one obtains:

\[
\theta \left( \frac{r_{pr}}{r_{pu}} - 1 \right) < \frac{(1-\beta) \cdot J(q)}{I(q)}
\]

or

\[
\theta < \frac{(1-\beta) \cdot J(q)}{I(q)} \left( \frac{r_{pr}}{r_{pu}} - 1 \right)
\]

Condition (11) shows that the project should be fully realized by the public entity if either the private partner does not contribute by allowing savings related to its managerial skills and knowledge, i.e. \( J(q) = 0 \), or the savings are relatively small compared with the difference in financial costs.

Provided there is know-how transfer from the private to the public sector (i.e. \( \beta = 1 \)), a PPP (interior solution of inequality [11]) is more efficient than entirely public or private financing (boundary solution), if the following condition is met:

\[
\theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot I(q) \cdot r_{pu} < \min\left\{ I(q) \cdot r_{pr}, (I(q) + J(q)) \cdot r_{pu} \right\}
\]

\[
(12)
\]

The first part of condition (12), i.e. when \( \theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot I(q) \cdot r_{pu} < I(q) \cdot r_{pr} \), yields:

\[
\theta \cdot r_{pr} + (1-\theta) \cdot r_{pu} < r_{pr}
\]

\[
(13)
\]

\[
(1-\theta)(r_{pu} - r_{pr}) < 0
\]

(14)

Inequality (14) holds for every \( \theta \in [e, 1) \).

The second part of condition (12):
\[ 0 \cdot r_{pu} + (1 - \theta) \cdot r_{pu} - r_{pu} < \frac{J(q)}{I(q)} r_{pu} \]  
\hspace{1cm} (15) 

can be transformed to obtain the condition on \( \theta \):

\[ \theta < \frac{J(q)}{I(q)} \left( \frac{r_{pu}}{r_{pu} - r_{pu}} \right) \]  
\hspace{1cm} (16) 

Condition (16) implies that the private partner’s capital share in the partnership is determined by the percentage saving achieved on the investment as a result of the private sector’s participation in the project and the private sector’s interest rate spread over the rate available to the public sector.

Condition (16) also enables it to be determined when the project should be executed by the private partner only. Setting the right-hand side of the inequality to be larger or equal to unity, we obtain 
\[ \frac{J(q)}{I(q)} \geq \frac{(r_{pu} - r_{pu})}{r_{pu}}. \] Therefore, the private partner should be the sole shareholder if the savings on the development outlays (in relation to \( I(q) \)) are higher than the relative spread between private and public rates.

**Example 1:**

The relationship presented in condition (16) can be used to present the following stylized facts. One can assume 
\[ \frac{J(q)}{I(q)} \] to equal 20%.\(^7\) If private sector interest rates are assumed at 8.50% and an interest rate of long-term commercial loans for related government units at 7%, the PPP would be efficient (in terms of a tradeoff between cost and efficiency) when \( \theta < 0.2(0.07/0.015) \), i.e., when \( \theta < 93\% \) and \( \theta \geq c \). Increasing the spread between the rates to 300 basis points yields a reduced private shareholding in the PPP to a maximum of 47% of the total capital. Therefore, the bigger the difference between the interest rates for the public and private partners, the less room there is for negotiation on capital participation between the parties.

In Apa Nova, Romania, Vivendi holds 84% of the shares (see Table 1). A large participation of the private sector responded to a lower capital advantage of the public sector, i.e. the spread between public and private interest rates was smaller than in other countries, with high expected managerial skills from the private sector.

Figure 1 shows the level of the fixed fee as a function of the public-private capital structure and the interval of efficient public-private financing for the case where \( r_{pr} / r_{pu} > 1 + J(q)/I(q) \).

---

\(^7\) A study conducted by the European Investment Bank (2004, p. 18), quoting a report commissioned by the British Government in January 2000 (original source not provided), found that the savings from executing the investment by a private company equal 17%. If \( J/(I+J) = .17 \), then \( J/I = .17/83 = 20\% \).
As can be seen from Figure 1, the level of the fixed fee \( f(q, \theta) \) starts to rise from the point where the project is realized only by the public sector (\( \theta = 0 \)). The \( f(q, \theta) \) increases as a result of the increased share of the more expensive private capital in the partnership. The rate of increase for (\( 0 < \theta < e \)) equals \( I(q) \cdot (r_{pr} - r_{pu}) - J(q) \cdot r_{pu} \). At \( \theta = e \), knowledge transfer takes place and the \( f(q, \theta) \) drops by \( (1 - e) \cdot J \cdot r_{pu} \). For \( \theta \geq e \), the \( f(q, \theta) \) increases at the rate of \( I(q) \cdot (r_{pr} - r_{pu}) \). At \( \theta = 1 \), with private-only shareholding, the fee equals \( I(q) \cdot r_{pr} \). Therefore, the larger the private share that is needed in the capital for knowledge transfer to take place, the smaller the potential savings from private sector participation.

Know-how transfer can also be described by a continuous function, where \( \beta \) is any continuous, monotonically increasing, and differentiable function of \( \theta \). In the exemplary case of the linear function describing know-how transfer, function (9) can be written as:

\[
f(q, \theta) = 0 \cdot I(q) \cdot r_{pu} + (1 - \theta) \cdot (I(q) + (1 - \theta) \cdot J(q)) \cdot r_{pu}
\]  

(17)

where \( (1 - \theta) \cdot J(q) \) reflects the linear decrease in outlays resulting from know-how transfer, proportional to the private partner’s share.

The first-order conditions for a minimum of function (17) with respect to \( \theta \) are:

\[
\frac{\partial f}{\partial \theta} = I(q) \cdot r_{pr} - (I(q) + 2(1 - \theta) \cdot J(q)) \cdot r_{pu} = 0
\]  

(18)

Therefore, \( f(q, \theta) \) has the minimum at such \( \theta^* \) that:
\[
\left( I(q) + 2(1-\theta^*) \cdot J(q) \right) \cdot r_{pu} = I(q) \cdot r_{pr} \quad (19)
\]
\[
2(1-\theta^*) \cdot J(q) = \frac{I(q) \cdot r_{pr}}{r_{pu}} - I(q) \quad (20)
\]
\[
\theta^* = 1 - \frac{I(q) \left( \frac{r_{pr}}{r_{pu}} - 1 \right)}{2J(q)} \quad (21)
\]

Since the second derivative of (17) with respect to \( \theta \) is positive for each \( J(q) > 0 \), then equation (21) determines the minimum of function (17).

As \( \theta \) ranges from zero to one, the condition for \( \theta^* \) to be an interior minimum \( (0 < \theta < 1) \) exists when \( r_{pr} - r_{pu} > 0 \) \ and \ \frac{r_{pr}}{r_{pu}} - 1 < \frac{2J(q)}{I(q)} \.

Example 2:

Equation (21) can be used to determine the private partner’s optimal shareholding for the same ratio of \( I/J = 5 \) as in Example 1. For the cases in which interest rates applicable to the private sector are on average 25% higher than the rates for the public sector \( (r_{pr}/r_{pu} = 1.25) \), function \( f(q, \theta) \) reaches its minimum at \( \theta^* = 0.375 \). The optimal capital structure would then be a 62.5% capital share owned by the public partner, with 37.5% of the capital owned by the private partner.

PPP with mixed financing will be Pareto-efficient under these assumptions if the following condition is met:

\[
\theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot (I(q) + (1-\theta)J(q)) \cdot r_{pu} < \min \left( I(q) \cdot r_{pr} ; (I(q) + J(q)) \cdot r_{pu} \right) \quad (22)
\]

For \( I(q) \cdot r_{pr} < (I(q) + J(q)) \cdot r_{pu} \) (first part of condition [22]), the private partner’s share \( \theta \) results from the solution of the condition:

\[
\theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot (I(q) + (1-\theta)J(q)) \cdot r_{pu} < I(q) \cdot r_{pr} \quad (23)
\]
\[
I(q) \cdot r_{pr} - (1-\theta) \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot I(q) \cdot r_{pu} + (1-\theta)^2 \cdot J(q) \cdot r_{pu} < I(q) \cdot r_{pr} \quad (24)
\]
\[
(1-\theta) \cdot I(q) \cdot (r_{pu} - r_{pr}) + (1-\theta)^2 \cdot J(q) \cdot r_{pu} < 0 \quad (25)
\]

\[\text{8} \quad \text{It is worth noting that the coefficient of variation (i.e. standard deviation to average) of the corporate to municipal bond yield ratios is lower (.0935) than the standard deviation of the spreads between them (.2841), thus making the ratio a suitable measure of financial advantage over time. See Appendix.\]
This condition is met for

\[ 1 - \frac{I}{J} \left( \frac{r_{pr}}{r_{pu}} - 1 \right) < 0 < 1 \]

For \( I(q) \cdot r_{pr} > (I(q) + J(q) \cdot r_{pu}) \) (second part of condition (22)), the private partner’s share \( \theta \) must meet the following condition:

\[
I(q) \cdot r_{pr} - (1 - \theta)I(q) \cdot r_{pr} + (1 - \theta)^2 J(q) \cdot r_{pu} < (I(q) + J(q)) \cdot r_{pu}
\]  
\[ \quad \text{(26)} \]

\[
-I(q) \cdot (r_{pu} - r_{pr}) - J(q) \cdot r_{pu} + (1 - \theta)J(q) \cdot (r_{pu} - r_{pr}) + (1 - \theta)^2 J(q) \cdot r_{pu} < 0
\]  
\[ \quad \text{(27)} \]

This condition is met for:

\[ 0 < \theta < 2 - \frac{I}{J} \left( \frac{r_{pr}}{r_{pu}} - 1 \right) \]
\[ \quad \text{(28)} \]

From the above analysis, it turns out that for the section:

\[ \max \left[ 0; 1 - \frac{I}{J} \left( \frac{r_{pr}}{r_{pu}} - 1 \right) \right] < \theta < \min \left[ 2 - \frac{I}{J} \left( \frac{r_{pr}}{r_{pu}} - 1 \right), 1 \right] \]
\[ \quad \text{(29)} \]

the PPP will be the efficient form of financing public investments.

Example 3:

The level of expected savings in development outlays due to the private sector's managerial advantage in existing joint venture partnerships can be calculated based on condition (29). In a sample of five public-private water supply and sewage companies in Poland (see Table 1), the level of \( \theta \) ranged between 0.33 and 0.64. For interest rates that on average are 20% higher for the private sector than for the public sector, and assuming that the existing capital structures are at an optimum, we can conclude that the ratio \( I/J \) was expected to be between 3.35 and 6.8, which amounts to savings of development outlays up to 12.8–23.0%. On average, these results are similar to the savings of 17% quoted in the literature for the United Kingdom.

Figure 2 shows the space for Pareto-efficient public-private partnerships, assuming a linear function describing the cost increase due to the lack of know-how in the public sector.
Knowledge transfer begins even with a small private share in the shareholding. An efficient public-private capital structure is achieved in the interval $\theta \in (0, \theta^*)$, where the fixed fee $f(q, \theta)$ is lower than in the case of a solely public or solely private investment. The minimum value of the fixed fee is obtained at $\theta^*$, where all the know-how is transferred from the private partner to the PPP. Further increases of $\theta$ lead to an increasing $f(q, \theta)$ as a result of the higher share of more expensive private capital. As in Figure 1, at $\theta = 1$ the fee equals $I(q) \cdot r_{p\theta}$.

6. Conclusions

The model-based analysis in this paper shows that PPPs may provide public services cheaper than a solely private or solely public entity. Efficiency considerations suggest that ownership of the SPV providing the public service does not have to be exclusively public or private. An optimum investment in public infrastructure requires mixed public and private ownership and governance of the project and knowledge transfer. If the optimum share of private ownership $\theta^*$ lies within the borders as defined by the interest rate spread and the potential savings from private management, a public-private capital structure will be more efficient in terms of lower fixed costs than a solely public or solely private ownership. Moreover, the larger the difference between the interest rates for the public and private partners and the smaller the savings resulting from private sector participation, the less room there is for negotiation of capital participation between the parties.

This model-based conclusion has important policy implications. The public and private partners’ economic motivations differ, and consequently, a PPP’s legal framework must be elaborate. From the public partner’s point of view, the transfer of managerial skills and knowledge that justifies the private partner’s participation in the SPV should be well defined.
and secured in a properly drafted and executable legal document. From the private partner’s point of view, the lower cost derived from the public financing should be secured for the project’s entire lifespan. This might not be problematic if the project’s funding is provided upfront. However, if funding is required over the project’s lifetime, the availability of cheaper financing would imply that the government involved maintains its creditworthiness and, accordingly, follows sound macroeconomic policies.

Therefore, PPPs may be most efficient in countries whose governments follow stability-oriented and predictable macroeconomic policies that are conducive to securing cheaper financing. An equally important advantage is a reliable legal system that provides the instruments to secure the interest of the public partner vis-à-vis the private partner. A lack of confidence between the partners, an insufficient legal framework, and the pursuit of other than stability-oriented macroeconomic policies would undermine the Pareto-efficient solution derived from the model. If any or all of these conditions are violated, the possible savings achieved with the PPP scheme diminish.
### Appendix

Corporate, Municipal, and Treasury 10-year AAA-Bond Yields: October 2006-August 2009

<table>
<thead>
<tr>
<th>Date of Issue</th>
<th>Corporate Bonds (%)</th>
<th>Municipal Bonds (%)</th>
<th>Treasury Bonds (%)</th>
<th>C-M Bond Yields Spread (%)</th>
<th>C/M Bond Yields Ratio</th>
<th>C-T Bond Yields Spread (%)</th>
<th>C/T Bond Yields Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006-10-31</td>
<td>5.01</td>
<td>3.78</td>
<td>4.61</td>
<td>1.23</td>
<td>1.32</td>
<td>0.40</td>
<td>1.09</td>
</tr>
<tr>
<td>2006-11-30</td>
<td>4.91</td>
<td>3.66</td>
<td>4.46</td>
<td>1.25</td>
<td>1.34</td>
<td>0.45</td>
<td>1.10</td>
</tr>
<tr>
<td>2006-12-31</td>
<td>5.10</td>
<td>3.74</td>
<td>4.70</td>
<td>1.36</td>
<td>1.36</td>
<td>0.40</td>
<td>1.09</td>
</tr>
<tr>
<td>2007-01-31</td>
<td>5.15</td>
<td>3.86</td>
<td>4.82</td>
<td>1.29</td>
<td>1.33</td>
<td>0.33</td>
<td>1.07</td>
</tr>
<tr>
<td>2007-02-28</td>
<td>4.80</td>
<td>3.74</td>
<td>4.55</td>
<td>1.06</td>
<td>1.28</td>
<td>0.25</td>
<td>1.05</td>
</tr>
<tr>
<td>2007-03-31</td>
<td>5.02</td>
<td>3.79</td>
<td>4.65</td>
<td>1.23</td>
<td>1.33</td>
<td>0.37</td>
<td>1.08</td>
</tr>
<tr>
<td>2007-04-30</td>
<td>4.95</td>
<td>3.80</td>
<td>4.63</td>
<td>1.15</td>
<td>1.30</td>
<td>0.32</td>
<td>1.07</td>
</tr>
<tr>
<td>2007-05-31</td>
<td>5.21</td>
<td>3.93</td>
<td>4.89</td>
<td>1.28</td>
<td>1.33</td>
<td>0.32</td>
<td>1.07</td>
</tr>
<tr>
<td>2007-06-30</td>
<td>5.43</td>
<td>4.09</td>
<td>5.03</td>
<td>1.34</td>
<td>1.33</td>
<td>0.40</td>
<td>1.08</td>
</tr>
<tr>
<td>2007-07-31</td>
<td>5.53</td>
<td>4.02</td>
<td>4.74</td>
<td>1.51</td>
<td>1.38</td>
<td>0.79</td>
<td>1.17</td>
</tr>
<tr>
<td>2007-08-31</td>
<td>5.45</td>
<td>4.05</td>
<td>4.53</td>
<td>1.40</td>
<td>1.35</td>
<td>0.92</td>
<td>1.20</td>
</tr>
<tr>
<td>2007-09-30</td>
<td>5.40</td>
<td>3.86</td>
<td>4.59</td>
<td>1.54</td>
<td>1.40</td>
<td>0.81</td>
<td>1.18</td>
</tr>
<tr>
<td>2007-10-31</td>
<td>5.27</td>
<td>3.83</td>
<td>4.47</td>
<td>1.44</td>
<td>1.38</td>
<td>0.80</td>
<td>1.18</td>
</tr>
<tr>
<td>2007-11-30</td>
<td>5.11</td>
<td>3.73</td>
<td>3.96</td>
<td>1.38</td>
<td>1.37</td>
<td>1.15</td>
<td>1.29</td>
</tr>
<tr>
<td>2007-12-31</td>
<td>5.04</td>
<td>3.65</td>
<td>4.03</td>
<td>1.39</td>
<td>1.38</td>
<td>1.01</td>
<td>1.25</td>
</tr>
<tr>
<td>2008-01-31</td>
<td>4.70</td>
<td>3.40</td>
<td>3.62</td>
<td>1.30</td>
<td>1.38</td>
<td>1.08</td>
<td>1.30</td>
</tr>
<tr>
<td>2008-02-29</td>
<td>4.62</td>
<td>4.00</td>
<td>3.52</td>
<td>0.62</td>
<td>1.16</td>
<td>1.10</td>
<td>1.31</td>
</tr>
<tr>
<td>2008-03-31</td>
<td>4.74</td>
<td>3.83</td>
<td>3.43</td>
<td>0.91</td>
<td>1.24</td>
<td>1.31</td>
<td>1.38</td>
</tr>
<tr>
<td>2008-04-30</td>
<td>4.93</td>
<td>3.83</td>
<td>3.75</td>
<td>1.10</td>
<td>1.29</td>
<td>1.18</td>
<td>1.31</td>
</tr>
<tr>
<td>2008-05-31</td>
<td>5.25</td>
<td>3.80</td>
<td>4.06</td>
<td>1.45</td>
<td>1.38</td>
<td>1.19</td>
<td>1.29</td>
</tr>
<tr>
<td>2008-06-30</td>
<td>5.21</td>
<td>4.01</td>
<td>3.98</td>
<td>1.20</td>
<td>1.30</td>
<td>1.23</td>
<td>1.31</td>
</tr>
<tr>
<td>2008-07-31</td>
<td>5.08</td>
<td>3.85</td>
<td>3.95</td>
<td>1.23</td>
<td>1.32</td>
<td>1.13</td>
<td>1.29</td>
</tr>
<tr>
<td>2008-08-31</td>
<td>4.99</td>
<td>3.74</td>
<td>3.82</td>
<td>1.25</td>
<td>1.34</td>
<td>1.17</td>
<td>1.31</td>
</tr>
<tr>
<td>2008-09-30</td>
<td>5.32</td>
<td>4.16</td>
<td>3.82</td>
<td>1.16</td>
<td>1.28</td>
<td>1.50</td>
<td>1.39</td>
</tr>
<tr>
<td>2008-10-31</td>
<td>6.82</td>
<td>4.40</td>
<td>3.97</td>
<td>2.42</td>
<td>1.55</td>
<td>2.85</td>
<td>1.72</td>
</tr>
<tr>
<td>2008-11-30</td>
<td>5.72</td>
<td>4.08</td>
<td>2.92</td>
<td>1.64</td>
<td>1.40</td>
<td>2.80</td>
<td>1.96</td>
</tr>
<tr>
<td>2008-12-31</td>
<td>4.77</td>
<td>3.91</td>
<td>2.22</td>
<td>0.86</td>
<td>1.22</td>
<td>2.55</td>
<td>2.15</td>
</tr>
<tr>
<td>2009-01-31</td>
<td>5.25</td>
<td>3.40</td>
<td>2.85</td>
<td>1.85</td>
<td>1.54</td>
<td>2.40</td>
<td>1.84</td>
</tr>
<tr>
<td>2009-02-28</td>
<td>5.52</td>
<td>3.41</td>
<td>3.02</td>
<td>2.11</td>
<td>1.62</td>
<td>2.50</td>
<td>1.83</td>
</tr>
<tr>
<td>2009-03-31</td>
<td>5.20</td>
<td>3.47</td>
<td>2.67</td>
<td>1.73</td>
<td>1.50</td>
<td>2.53</td>
<td>1.95</td>
</tr>
<tr>
<td>2009-04-30</td>
<td>5.40</td>
<td>3.28</td>
<td>3.12</td>
<td>2.12</td>
<td>1.65</td>
<td>2.28</td>
<td>1.73</td>
</tr>
<tr>
<td>2009-05-31</td>
<td>5.60</td>
<td>3.26</td>
<td>3.47</td>
<td>2.34</td>
<td>1.72</td>
<td>2.13</td>
<td>1.61</td>
</tr>
<tr>
<td>2009-06-30</td>
<td>5.54</td>
<td>3.37</td>
<td>3.54</td>
<td>2.17</td>
<td>1.64</td>
<td>2.00</td>
<td>1.57</td>
</tr>
<tr>
<td>2009-07-31</td>
<td>4.83</td>
<td>3.18</td>
<td>3.48</td>
<td>1.65</td>
<td>1.52</td>
<td>1.35</td>
<td>1.39</td>
</tr>
<tr>
<td>2009-08-31</td>
<td>4.44</td>
<td>3.15</td>
<td>3.54</td>
<td>1.29</td>
<td>1.41</td>
<td>0.90</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Min 4.44 3.15 2.22 0.62 1.16 0.25 1.05
Max 6.82 4.40 5.03 2.42 1.72 2.85 2.15
Mean 5.18 3.74 3.93 1.44 1.39 1.25 1.37
Median 5.15 3.80 3.96 1.34 1.36 1.13 1.29
SD 0.41 0.30 0.70 0.41 0.13 0.80 0.30

Source: Reuters and author's calculations.
References


