Pricing, Investments and Mergers with Intertemporal Capacity Constraints

Charalambos Christou  Rossitsa Kotseva  Nikolaos Vettas*

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Abstract

We set up a duopoly model with dynamic capacity constraints under demand uncertainty. We endogenize the investment decisions of the firms, examine their intertemporal pricing behavior, their incentives to merge, as well as the welfare implications of the merger. Whereas under certain and constant demand the high capacity firm lets its low capacity rival sell out, this is not always the case under demand uncertainty. The marginal unit of each firm’s available capacity obtains an option value, which depends on both firms’ capacities, the current-period demand and the remaining planning horizon. This option value may be higher when the firms act non-cooperatively compared to the case when they merge, forming a monopoly seller - as a result, trade surplus may be higher when a merger takes place. The prospect of a merger also leads to higher investment levels because of the incentive of each firm to appropriate higher share of the total monopoly surplus. We show that for some levels of the capacity installment cost, a merger that turns the duopoly into a monopoly is welfare improving.

JEL classification: D43, L13, L22

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*Charalambos Christou: Department of Economics, Athens University of Economics and Business, and Center of Planning and Economic Research, Greece; cchris@aueb.gr. Rossitsa Kotseva: Department of Economics, Athens University of Economics and Business, Greece, and Department of Economics, University of Cyprus; kotseva@aueb.gr. Nikolaos Vettas (corresponding author): Department of Economics, Athens University of Economics and Business, Patission 76, Athens 10434, Greece and CEPR, UK.; tel: +30-210-8203344; e-mail: nvettas@aueb.gr.
1 Introduction

Inventories, or dynamic capacity constraints, play a significant role in how firms compete in a market. If a firm’s ability to sell at a given point in time is constrained by past technology or investment choices, market competition will reflect these constraints. A firm that currently holds a small inventory is more restricted in supplying the market in the present and the following periods relative to one with large inventory. Further, a firm that competes more aggressively, by setting a lower current period price, will most likely find itself with a lower inventory in the subsequent periods: succeeding in making a sale increases the current-period profit but decreases the firms’ ability to make a sale in the future. While the issues related to the problem of dynamic pricing with inventories or capacity constraints are both important and interesting, they have not received enough attention in the literature.

In this paper we study aspects of oligopolistic competition related to the dynamic capacity constraints that firms face. We study situations where past investments create constraints on how much a firm can produce and/or sell within a given number of periods. This setting includes situations where there are physical inventories and firms’ sales are constrained to selling a quantity that does not exceed their inventory; it also includes situations where firms’ prior investments in capacity building set a limit on how much a firm can produce within a certain time interval. Thus, we focus on aggregate intertemporal selling constraints across some time interval. Cases where a firm can simply produce to order and without delay at each point and where current sales do not affect the firms’ ability to sell subsequently do not fall into the problem that we analyze here. Our focus is on the strategic aspects of the problem that emerge in an oligopolistic setting where firms face intertemporal selling constraints. Specifically, we set up a simple dynamic duopoly game where each firm can make a sale in each of three periods. The specification of demand and costs is kept as simple as possible. Each firm has a maximum number of units that can be sold over the entire time horizon and seeks to maximize its aggregate profit. The two firms observe the current-period demand before they set prices but are uncertain about future demand. The situation is repeated in the subsequent period, with the inventory of one of the firms possibly reduced as a result of making a sale in the previous period. In this setting, we first examine equilibrium pricing given the capacity constrains. Then, we endogenize the firms’ initial capacity choices.

A number of issues arise as central in specifying a firm’s optimal path of sales and pricing strategies. First, demand considerations: if demand today appears to be low compared to expected demand

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1The problem of pricing under various patterns of intertemporal capacity constraints, though in a single seller context, has been studied in the literature of “yield” (or “revenue”) “management”. For comprehensive reviews of this literature see e.g. Elmaghraby and Keskinocak (2003), McGill and Van Ryzin (1999), and Bitran and Caldentey (2003).
tomorrow, a firm may choose to act more conservatively and save its capacity for sales at a later period of possibly higher demand. Second, strategic considerations: a firm’s expected profit depends on its rival’s price strategy. A firm will tend to postpone making a sale today if it expects its rival will be less aggressive tomorrow possibly because of future strategic reasons. Likewise, anticipating low rival prices or relatively high rival capacity tomorrow implies that the firm has a stronger incentive to make a sale today. In particular, a firm with relatively high capacity will have a high incentive to become a monopolist on the residual demand, thus, being less aggressive in the earlier periods of the pricing subgame. Our set-up does, thus, permit the study of the composite effects of strategic interactions that develop in such dynamic pricing setting. Our study is then advanced to the choice of each firm’s capacity, which, being of more long-run nature than pricing choices, is made at the beginning of the game, before the pricing subgames. Again, in addition to the direct cost of setting capacity levels, there are also strategic effects: a firm’s choice of capacity affects not only its own pricing strategy but also that of its rival. Finally, we examine the incentives of the two firms to merge, thus combining their capacities, and how this possibility influences the optimal investment and pricing decisions and, in turn, welfare. Regarding the timing of the (potential) merger we analyze two possibilities. When it can be realized at the beginning of the pricing subgame only and when it can be realized at the beginning of each period of the pricing subgame.

Given the focus of our analysis, the results we obtain are related, first, to the intertemporal pricing decisions of the firms, second, to their investment decisions and, third, to the impact of mergers. Regarding the first point, we see that the introduction of demand uncertainty over a stochastic demand creates an option value for capacity: a firm may wish to forego a sale in the current period in expectation of higher demand in the future. The option value tends to make firms unwilling to sell in the initial periods of the pricing subgame if there is low demand, thus leading to relatively high prices. The effect of the option value of capacity may be so strong as to lead the competing firms to set equilibrium prices that, on average, exceed the prices that would be chosen by a monopolist. We also find that, if firms begin with different capacity levels, and the low capacity firm cannot satisfy residual demand alone, in equilibrium, it does not always sell out its capacity. On the one hand, the high capacity firm’s non-aggressive pricing strategy reflects its incentive to become a monopolist in the last periods. On the other hand, if demand is initially relatively low, the option value of capacity may be greater for the low capacity firm compared to the high capacity firm, so that the latter is the one to sell first.

Regarding the firms’ investment decisions, the analysis of the capacity setting game shows that, interestingly, capacities are not always “strategic substitutes”, in the sense that the capacity that is the best response of a given firm is not always monotonically decreasing in the capacity of the rival firm. This feature of the problem is indicative of the composite effect of aggregate capacity constraints on the dynamic pricing problem of the firms. Moreover, in the non-cooperative game we
do not obtain investment in excess capacity, that is, total capacity does not exceed total demand. Regarding the case of mergers, in either setting the firms optimally merge at the beginning of the pricing game. However, the impact of mergers on the optimal investment level differs between the two settings. When the possibility of a merger is present in every period, the two firms install more capacity. Also, for a relatively low capacity installment cost, we observe that each firm invests in capacity at a level high enough to satisfy total demand. In general, when we allow for mergers we obtain investment in excess capacity by each firm. This is inefficient from the viewpoint of the merged firm and is due to the unilateral incentive that each firm has to capture a greater share of the profits when a merger occurs.

Finally, we study the welfare implications of the merger. We find that, for a range of the capacity cost levels, a merger may be welfare improving despite the fact that it leads to a monopoly situation and that direct cost efficiencies are absent. The net effect of a merger on welfare depends on the relative magnitude of two opposite effects: higher trade surplus and higher cost of capacity. The positive effect on total surplus from sales is due to the dynamic nature of the problem combined with capacity constraints and future demand uncertainty. These factors create incentives for the firms to refrain from selling in a period with low demand. The unilateral incentives of each firm to forego a sale in order to realize higher expected profit in the future sometimes lead to no trade transactions (i.e. lost surplus) in the duopoly case, while a monopolist in a similar situation sells and so the respective surplus is realized. This is because, *ceteris paribus*, the opportunity cost of a unit of capacity is lower when total capacity is owned by a single firm compared to when it is owned by two competing firms. In some cases, the positive effect on total surplus in the pricing game is outweighed by the higher cost of capacity. However, for certain ranges of the capacity cost, the opposite holds, so the merger is welfare improving.

Our paper is related to the literature on price competition with capacity constraints, which dates back to Edgeworth (1897), whose work brought to notice the problem of non-existence of a pure-strategy equilibrium in a pricing game where firms’ capacities are predetermined. Since then, this problem has become the focus of a variety of analyses, including those that proved the existence of equilibria in mixed-strategy; see e.g. Beckmann (1965), Levitan and Shubik (1972), Osborne and Pitchik (1986), and Dasgupta and Maskin (1986). A distinct strand of the literature endogenizes the choice of capacity prior to firms’ engaging in oligopolistic competition. Kreps and Scheinkman (1983), show that capacity precommitment followed by price competition leads to Cournot outcomes, a result indicative of the importance of capacities for understanding firms’ pricing behavior; Davidson and Deneckere (1986) further explore this issue relaxing the efficient rationing assumption. In the above mentioned papers price competition is studied in one-shot settings. Brock and Scheinkman (1985) started a strand of the literature on price competition in dynamic settings and have studied the role of capacity constraints in collusive equilibria of repeated games; Benoit and Krishna (1987)
have extended the setting by allowing firms to adjust capacity over time. In this literature, capacity constraints affect firms’ pricing strategies only indirectly, by forcing them not to deviate from collusive price schemes. Our approach is also about dynamics but a quite different one: in our model, it is the process of inventory reduction that provides the intertemporal link of the firms’ actions, which in turn forms pricing dynamics. In our model, capacity constraints influence directly firms’ pricing strategies. The paper closer in spirit to our approach is by Dudey (1992). He studies the effect of capacity constraints on firms’ pricing decisions. This work, in part, we generalize in our basic model, by considering a setting where demand is not constant across periods and future demand is unknown. Biglaiser and Vettas (2004) study the problem of dynamic price competition with intertemporal capacity constraints in a setting where the buyers act strategically, choosing which seller to buy from, in their effort to avoid subsequent periods of high prices. In this paper, we do not model buyers as strategic. Our focus is on the sellers’ behavior and we focus on the role of demand uncertainty, the endogenous choice of capacities and the impact of mergers.

The literature on mergers and, in particular, on horizontal mergers has focused on their welfare implications, a topic in the center of interest to both regulators and economic theorists. Williamson (1968) is one of the first to point out the trade-off between increasing market power and cost efficiencies that arise from a merger. The overall effect of a merger may be either positive or negative if the gain from the lower costs is higher or lower than the loss from higher prices. In a quantity competition setting, Farell and Shapiro (1990) and Levin (1990) provide sufficient conditions for a merger to be welfare improving, in terms of a weighted average of firms’ pre-merger market shares. For example, Levin (1990) finds that a profitable merger, involving firms with total pre-merger market share less than 50 percent, is welfare increasing. Polasky and Mason (1998) show that positive long-run effects from mergers may be realized by allowing firms outside the merger to change their capacities in the long-run. Mergers have also been studied in a price-setting framework. For example, Deneckere and Davidson (1985) examine firms’ incentives to merge in a differentiated-product model with simultaneous price competition. They show that post merger prices are higher and all firms in the industry earn higher profits. Assuming firms selling homogeneous products, Baik (1995) allows for capacity adjustment after a merger. Although the focus is not on welfare issues, the paper shows that post merger prices may be lower than pre-merger prices, which potentially implies higher consumer welfare.

In general, regarding horizontal mergers, the prevalent view in the literature so far is that welfare may increase if the merger results in significant cost efficiencies or if there are cost asymmetries between the firms.\(^2\) In the present model we do not consider cost asymmetries between the firms.

\(^2\)An exception is a paper by Huck, Konrad and Mueller (2004) where increased welfare does not stem from cost reductions but from better information flows regarding production levels.
Instead we focus on a source of welfare gains from mergers that has not been pointed out in the literature. This is the opportunity cost of capacity utilization. Given demand uncertainty, a monopolist can potentially manage better a given level of capacity. Our result may be also interpreted in terms of cost efficiency, in a more general sense, where the cost measure includes both the direct capacity cost and the opportunity cost of using capacity, created by demand uncertainty and strategic non-cooperative interaction. Thus, when total capacity is owned by a single firm, the opportunity cost of a certain capacity may by lower compared to when the same capacity is divided between two rival firms. In this sense, a merger may be viewed as leading to a “cost” reduction in a generalized sense.

The paper is structured as follows. The basic model is presented in Section 2. In Section 3 we present the analysis of the basic set-up, where firms do not have the possibility to merge. Sections 4 and 5 examine the setting where mergers are allowed and their welfare implications, respectively. Section 6 concludes. Some proofs are presented in the Appendix.

2 The basic model

Suppose there are two firms, $A$ and $B$, that choose capacities once – at the beginning of the game – and then compete in prices for three periods. In each period, a single buyer arrives, demanding a unit of the product. We denote the valuation of the buyer who arrives in period $t$ by $v_t$, $t = 1, 2, 3$. We further assume that buyers’ valuations are iid draws from a uniform distribution with support $[0, 1]$. The firms produce homogeneous products. Within each period, firms set prices simultaneously, after they have observed the valuation of the current-period (potential) buyer, but without knowledge of the valuations of the future buyers. We denote by $(k^A_t, k^B_t)$ the capacity pair at the beginning of period $t$, by $k^i_t$ the capacity of firm $i$ in period $t$, and by $p^i_t$, the price of firm $i$ in period $t$. We denote by $\pi^i_t \equiv \pi^i_t (p^A_t, p^B_t; v_t, k^A_t, k^B_t)$ the total profit of firm $i$ from period $t$ onwards, as a function of the current-period prices of the firms, the (realized) value of $v_t$ and current capacities and also given that both firms follow equilibrium strategies from period $t + 1$ onwards. The function $\pi^i_t$ computed at the equilibrium current-period prices is denoted by $\Pi^i_t \equiv \Pi^i_t ((k^A_t, k^B_t), v_t)$. The expectation of $\Pi^i_t$ with respect to $v_t$, $E(\Pi^i_t ((k^A_t, k^B_t)))$, represents the expected value of firm $i$’s equilibrium profit at the beginning of period $t$, prior to the realization of $v_t$. Throughout the paper, subscript $t$ refers to the period, $t = 1, 2, 3$, of the pricing subgame and superscripts $i, j$, to the firm $A$ or $B$. We assume that the per unit cost of capacity installment is constant and denote it by $c$. Production costs are assumed to be zero. For the simplicity of the exposition (and without harm in the generality of the analysis) we assume that there is no discounting of profits between periods. We solve the model by backward induction, looking for subgame perfect Nash equilibria of the pricing and capacity-choice game.

After the game described above (our “basic model”) is analyzed, we expand the analysis in Sections
4 and 5 to account for the possibility of mergers between the sellers and to assess their implications.

3 Equilibrium in the basic model

In this Section we analyze equilibrium behavior in our basic model. We begin with the analysis of equilibrium in the pricing subgame, given the firms’ capacities. We first analyze the cases where only one firm holds positive capacity.

3.1 The monopoly case

The monopoly case may arise either because only one firm has invested in positive capacity at the start of the game, or because the rival firm has sold out its capacity in the previous periods. In such cases, there are no strategic considerations at play and the seller sets prices taking into account only its own capacity dynamics and the evolution of demand. As mentioned in the Introduction, these demand considerations have been studied in the “revenue management” literature where the focus is on the pricing behavior of a single firm. In the present model, monopoly situations may arise endogenously depending on the firms’ investment and pricing decisions. The study of these cases allows a clear illustration of how, as a result of capacity constraints and demand uncertainty, the available capacity obtains an option value. We, thus, obtain an important benchmark. Subsequently, in the analysis of the duopoly case we build on the intuition developed here and show how this option value is affected by strategic considerations. Here we abstract away from such strategic interactions and, therefore, the option value depends only on the size of the own capacity relative to the number of the future selling opportunities.

Consider, for instance, the case where one firm holds at least one unit of capacity in the third period. Clearly, this firm would sell to the current-period buyer irrespective of his valuation, since this is the last opportunity for it to sell. In other words, in this case, the available capacity does not have an option value: selling one unit does not have an opportunity cost. The same holds if the monopolist has at least two units in the second period and three units in the first period. For instance, when $k_2^1 = 2$, the seller knows that he could sell only one unit in the third period, consequently, there is no incentive to forego a sale in the second period and keep a unit for a sale in the future, as there are less future opportunities to sell than units of capacity. We summarize in the following.

**Lemma 1** A monopolist $i$ who holds enough capacity $k_i^t$ to satisfy (residual) demand always sells, in equilibrium, and

i) for $k_3^1 \geq 1$, $p_3^1 = v_3$ and $E(\Pi_3^1(k_3^1)) = 1/2$

ii) for $k_2^1 \geq 2$, $p_2^1 = v_2$ and $E(\Pi_2^1(k_2^1)) = 1$
iii) for $k_1^0 = 3$, $p_1^1 = v_1$ and $E \left( \Pi_1^1(k_1^1) \right) = 3/2$.

The above result would not hold if the monopolist holds capacity smaller than (residual) demand. In such cases, the seller may have an incentive not to sell a unit of the product in the current period to a customer with low valuation, if he expects that customers with higher valuations will possibly arrive in the next periods. The option value of one unit of his capacity in period $t$, depends on the expectation of the future profit difference: it is equal to $E(\Pi_{t+1}(k_1^t)) - E(\Pi_{t+1}(k_1^t - 1))$. Suppose, for example, that in the second period the monopolist’s capacity is $k_2^1 = 1$, that is, there is one unit and two opportunities to sell it. The opportunity cost of selling this unit today is the profit the monopolist would gain if he keeps the unit and sells it in the next period which, by Lemma 1, equals $E(\Pi_2^1(1)) - E(\Pi_2^1(0)) = 1/2$. Therefore, if the current period valuation $v_2$ is lower than $1/2$, the monopolist prefers not to sell in the current period, and can achieve this setting any price higher than $v_2$. If, on the other hand, $v_2 > 1/2$, the firm strictly prefers to sell in the present period. Thus, the expected equilibrium profit, before knowing the value $v_2$, can be expressed as

$$E \left( \Pi_2^1(1) \right) = \int_0^{1/2} \frac{1}{2} dv_2 + \int_{1/2}^1 v_2dv_2 = \frac{5}{8}.$$ 

Contrary to the cases described in Lemma 1, where the option value of the available capacity is zero, in the present situation, this option value is positive, equal to $1/2$.

Now, consider the case where $k_1^0 = 1$. The opportunity cost of this unit is equal to $E \left( \Pi_2^1(1) \right) - E(\Pi_2^1(0)) = 5/8$ and the seller has an incentive to sell in period 1 only if $v_1 \geq 5/8$. Finally, suppose $k_1^0 = 2$. Now, the option value of one unit equals $E \left( \Pi_2^1(2) \right) - E(\Pi_2^1(1)) = 1 - 5/8 = 3/8$ and the monopolist sells only if $v_1 \geq 3/8$. The following Lemma summarizes the cases where a monopolist cannot fully satisfy the residual demand, but has to decide in which periods he prefers to sell his available units.

**Lemma 2** Equilibrium monopoly prices and expected profits, with capacity smaller than residual demand, are

i) if $k_1^0 = 1$, $p_1^2 = v_2$, if $v_2 \geq 1/2$ and $p_2^2 > v_2$, if $v_2 < 1/2$. $E \left( \Pi_2^1(1) \right) = 5/8$.

ii) if $k_1^0 = 1$, $p_1^1 = v_1$, if $v_1 \geq 5/8$ and $p_1^1 > v_1$, if $v_1 < 5/8$. $E \left( \Pi_1^1(1) \right) \approx 0.7$.

iii) if $k_1^0 = 2$, $p_1^1 = v_1$, if $v_1 \geq 3/8$ and $p_1^1 > v_1$, if $v_1 < 3/8$. $E \left( \Pi_1^1(2) \right) \approx 1.2$.

Note that the option value of a unit of capacity is an increasing function of the number of remaining periods or, equivalently, selling opportunities, and a decreasing function of current capacity.

Having described the equilibrium pricing behavior of the monopolist and the role of the option value of the available capacity in determining his behavior, we turn to the analysis of the situations...
where both firms hold positive capacity. Therefore we now add a strategic dimension to the problem and examine how it affects the option value of the firms’ capacity which, in turn, shapes the equilibrium pricing incentives.

### 3.2 The duopoly case

We first consider the cases where each firm holds enough capacity to satisfy residual demand alone, that is, where each firm’s capacity is higher or equal to the remaining periods. In all these cases we obtain the standard Bertrand outcome: zero equilibrium prices and expected profits. This is because the option value of capacity is zero for each firm and the products are homogeneous. The following Lemma summarizes equilibrium behavior in these cases.

**Lemma 3** If \( \min\{k^A_t, k^B_t\} \geq 4 - t \), equilibrium prices in period \( t = 1, 2, 3 \), are \( p^A_t = p^B_t = 0 \) and so are the expected profits \( E\{\Pi^A_t(k^A_t, k^B_t)\} = E\{\Pi^B_t(k^A_t, k^B_t)\} = 0 \).

We now turn to the cases where at least one of the firms cannot satisfy residual demand and, therefore, a positive option value now appears, affecting the pricing incentives of both firms. Note that, in contrast to the monopoly situation, in the presence of strategic interaction, the pricing incentives of each firm are affected not only by its own capacity and the level of demand, but also by the capacity of the rival firm. Let us first consider period 2 of the pricing subgame. We have (essentially) two cases: \((1,1)_2\) and \((2,1)_2\).\(^3\) In each of these cases, at least one of the firms has positive option value for its capacity. Regarding the case \((2,1)_2\), if firm \( B \) does not sell in period 2, then both firms will have positive capacity in period 3 and, by Lemma 3 firm \( B \)'s profit will be zero. It follows that the option value of firm \( B \)'s capacity in the second period is zero, that is, firm \( B \) is willing to set as low a price in the current period as it is required to sell. Will \( p^i_2 \) go down to zero? No because, provided that firm \( B \) sells today, firm \( A \) will have expected profit equal to \( 1/2 \). Thus, firm \( A \) attaches a positive option value to its capacity. It follows that, anticipating rational behavior on the part of firm \( B \), firm \( A \) is willing to set a price \( p^A_2 \geq 1/2 \). Note that, if \( v_2 \geq 1/2 \), then both firms have an incentive to sell today, setting \( p^A_2 = p^B_2 = 1/2 \), with firm \( B \) selling in equilibrium (this can be intuitively understood as firm \( B \) setting a price arbitrarily smaller than \( 1/2 \)). Therefore:

**Lemma 4** Second-period equilibrium prices and expected profits with capacities \((2,1)_2\) or \((3,1)_2\), are

\(^3\)As case \((2,1)_2\) is symmetric to \((1,2)_2\) we present only the former. Note, also, that case \((3,1)_2\) is equivalent to \((2,1)_2\) since firm 1 cannot sell more than two units in the remaining two periods. Thus, any capacity greater than 2 has no impact (direct or strategic) on the firms’ decisions and revenues.
\( p_A^2 = p_B^2 = 1/2, \) if \( v_2 \geq 1/2, \) and firm \( B \) sells. If \( v_2 < 1/2, p_A^2 > v_2 \) and \( p_B^2 = v_2. \) Expected profits are

\[
E(\Pi_A^2(2, 1)) = E(\Pi_A^2(3, 1)) = \int_0^1 v_3 dv_3 = \frac{1}{2} \quad \text{and}
\]

\[
E(\Pi_B^2(2, 1)) = E(\Pi_B^2(3, 1)) = \int_0^{1/2} v_2 dv_2 + \int_{1/2}^1 \frac{1}{2} dv_2 = \frac{3}{8}.
\]

Note that, in the duopoly case, the option value of one unit of the capacity of firm \( i \) depends on both firms’ current and expected future capacities, that is, on firm \( j \)’s current price, which depends on the valuation of the current period customer. Thus, if firm \( A \) expects its rival to set a price \( p_B^t \leq v_t, \) firm \( A \)’s option value in period \( t \) is equal to \( E(\Pi_A^t(1, k_t^A, k_t^B - 1)) - E(\Pi_A^t(1, k_t^A - 1, k_t^B)), \) otherwise it is \( E(\Pi_A^t(1, k_t^A, k_t^B)) - E(\Pi_A^t(1, k_t^A - 1, k_t^B)). \)

When firms have asymmetric capacities, as in the case \((2, 1), 2\), the option values they attach to them are also asymmetric, a feature which, in our model, always leads to a pure-strategy equilibrium in prices. This does not necessarily hold when firms have symmetric capacities: In the case \((1, 1), 2\) firm \( A \)’s option value for its capacity is \( E(\Pi_A^t(1, 0)) - E(\Pi_A^t(0, 1)) = 1/2, \) provided that firm \( B \) sells in period \( 2.\) It follows that, if \( v_2 \geq 1/2 \) both firms wish to sell in period 2, at a price at least equal to \( 1/2, \) with Bertrand competition pressing equilibrium prices to \( 1/2. \) However, if \( v_2 < 1/2, \) both firms wish their rival to sell in period 2, anticipating expected profit equal to \( 1/2. \) As a result, in this case no symmetric equilibria in pure strategies exist. The only symmetric price equilibrium is in mixed strategies and we prove, below, that it is unique.\(^5\)

**Lemma 5** Suppose capacities are \((1, 1), 2.\) If \( v_2 \geq \frac{1}{2}, \) prices in the unique symmetric equilibrium are \( p_A^2 = p_B^2 = \frac{1}{2}. \) If \( v_2 < \frac{1}{2}, \) we have a mixed strategy equilibrium: prices are \( p_A^2 = v_2, \) with probability \( \Pr_2(1, v_2) = \frac{4v_2}{1 + 2v_2} \) and \( p_B^2 > v_2, \) with probability \( 1 - \Pr_2(1, v_2) \) for \( i = A, B. \) Expected equilibrium profits are

\[
E(\Pi_A^2(1, 1)) = E(\Pi_B^2(1, 1)) = \int_0^{1/2} \frac{2v_2}{1 + 2v_2} dv_2 + \int_{1/2}^1 \frac{1}{2} dv_2 = \frac{3 - 2 \ln 2}{4} \approx 0.4034.
\]

**Proof.** If \( v_2 \geq 1/2 \) Bertrand competition leads to equilibrium prices \( p_A^2 = p_B^2 = 1/2 \) and each firm sells in period 2 with probability \( 1/2. \) The firm that sells in period 3 has expected profit also equal to \( 1/2. \)

\(^4\)Note that, if both firms do not sell in period 2, their expected profit is zero since both will appear in the last period with positive capacity.

\(^5\)There are also asymmetric pure strategy equilibria, where one of the firms sets a price equal to \( v_2, \) while the other firm charges some higher price. In computing the expected profits at the beginning of the period, we consider only the mixed strategy equilibrium. This is justified by the fact that there is no reasonable way to select between the two pure strategy equilibria since in each of them one of the firms obtains the highest, while the other firm, the lowest possible profits. As a result, each firm would prefer the opposite of what is best for the other.
If \( v_2 < 1/2 \) the unique symmetric mixed-strategy equilibrium is determined as follows:

Suppose that firm \( j \) chooses \( p_2^j \) according to an arbitrary decision rule \( F(p_2^j) \). There are only two sets of pure strategies that are not strictly dominated for firm \( i \). First, all prices lower than \( v_2 \) are strictly dominated (since, if firm \( i \) sells, it would prefer to sell at price \( v_2 \) and would be indifferent otherwise) and, second, firm \( i \) is indifferent for all prices above \( v_2 \) (since it is not going to sell at any of these prices).

It follows that, in equilibrium, each firm randomizes between \( p_2^i = v_2 \) and \( \{p_2^i : p_2^i > v_2\} \).

Firm \( i \)'s expected profit by setting \( p_2^i = v_2 \) equals

\[
\frac{1}{2}\left(\frac{1}{2} + v_2\right) \Pr^j\left(p_2^j = v_2\right) + v_2 \Pr^j\left(p_2^j > v_2\right),
\]

(1)
since, if \( p_2^j = v_2 \) then each firm sells in period 2 or 3 with probability 1/2, with expected profit equal to 1/2 in the latter case.

Firm \( i \)'s expected profit from setting \( p_2^i > v_2 \) equals

\[
\frac{1}{2} \Pr^j\left(p_2^j = v_2\right) + 0 \Pr^j\left(p_2^j > v_2\right).
\]

(2)

Since, in equilibrium firm \( i \) is indifferent among all its pure strategies, we can derive values for \( \Pr^j\left(p_2^j = v_2\right) \) that equate (1) and (2) using the fact that \( \Pr^j\left(p_2^j = v_2\right) = 1 - \Pr^j\left(p_2^j > v_2\right) \). The unique value that equates (1) and (2) is \( \Pr^j\left(p_2^j = v_2\right) = \Pr^j\left(1, v_2\right) = \frac{4v_2}{1+2v_2} \) and, since this probability holds for any of the two firms, it determines the unique mixed-strategy equilibrium, which is also symmetric.\(^6\)

Substituting \( \Pr^j\left(1, v_2\right) \) in either (1) or (2), we obtain the equilibrium profit \( \Pi_2^i = \frac{2v_2}{1+2v_2} \), for \( i = A, B \), and the values for \( E(\Pi_2^A(1, 1)) \) and \( E(\Pi_2^B(1, 1)) \) follow directly.

Thus, when two firms operate non-cooperatively and the demand in the current period is relatively low, they prefer to take the risk of setting relatively high prices (and selling no units with probability \([1 - (1 - \Pr^j(1, v_2))^2]\)) with the view to acquiring higher expected profits in the future. In contrast, if the capacity of the two firms were managed by a monopolist, the price set in period 2 would be equal to \( v_2 \) (irrespective of its value) as each unit would not have an option value. In other words, if \( v_2 \) is quite low, the average price set by the two competing firms is greater than \( v_2 \), the price chosen by a monopolist. Thus, in this setting, the monopolist can better manage his capacity over time. This fact opens a different perspective for the evaluation of firms' performance in different contexts of market concentration and will play an important role in the welfare assessment of the merger we present in a later section.

\(^6\)Throughout the paper \( \Pr_t^i(k, v_t) \) denotes the probability with which firms set price equal to \( v_t \) in case a symmetric mixed strategy is used when firms have symmetric capacities (hence \( k^A = k^B \equiv k \)). In case of asymmetric capacities, \( \Pr_t^i((k_t^A, k_t^B), v_t) \) would denote the probability with which firm \( i \) sets a price equal to \( v_t \) in the mixed strategy equilibrium.
So far we have analyzed second-period equilibrium pricing. The analysis of the first-period pricing subgame follows the same lines as above, with the option value of firms’ capacity determining the intensity with which firms compete for each customer. The following lemma summarizes the price equilibria for the cases where firms’ capacities are positive but do not both exceed residual demand (the proof is in Appendix 1).

**Lemma 6** First-period equilibrium prices are,
i) with \((3, 2)_1\), and if \(v_1 \geq 1/2\), \(p_1^A = p_1^B = 1/2\) and firm B sells. If \(v_1 < 1/2\), \(p_1^A > v_1\) and \(p_1^B = v_1\).

ii) with \((3, 1)_1\), and if \(v_1 \geq 1/2\), \(p_1^A = p_1^B = 1/2\) and firm B sells. If \(v_1 \in [3/8, 1/2)\), \(p_1^A > v_1\) and \(p_1^B = v_1\). If \(v_1 \in [0, 3/8)\), \(p_1^A = v_1\) and \(p_1^B > v_1\).

iii) with \((2, 2)_1\), and if \(v_1 \geq 1/8\), \(p_1^A = p_1^B = 1/8\) and the firms sell with equal probability. If \(v_1 < 1/8\), firms randomize between \(p_1^1 = v_1\) with probability \(Pr_1(2, v_1) = \frac{6+16v_1}{7+8v_1}\), and \(p_1^1 > v_1\) with probability \(1 - Pr_1(2, v_1)\).

iv) with \((2, 1)_1\), and if \(v_1 \geq (2 \ln 2 + 1)/4 \approx 0.6\), \(p_1^A = p_1^B = (2 \ln 2 + 1)/4\) and firm B sells. If \(v_1 \in [(3 - 2 \ln 2)/4, (2 \ln 2 + 1)/4)\), \(p_1^A > v_1\) and \(p_1^B = v_1\). If \(v_1 \in [3/8, (3 - 2 \ln 2)/4)\), firms randomize between \(p_1^1 = v_1\) with probability \(Pr_1^A((2, 1), v_1) = \frac{3-8v_1}{2 \ln 2 - 4v_1}\) and B with probability \(Pr_1^B((2, 1), v_1) = \frac{4+8v_1-4 \ln 2}{2 \ln 2 - 4v_1}\), and \(p_1^1 > v_1\) with respective probabilities \(1 - Pr_1^A((2, 1), v_1)\) and \(1 - Pr_1^B((2, 1), v_1)\). If \(v_1 \in [(2 \ln 2 - 1)/4, 3/8)\), \(p_1^A = v_1\) and \(p_1^B > v_1\). If \(v_1 < (2 \ln 2 - 1)/4\), \(p_1^A > v_1\) and \(p_1^B > v_1\).

v) with \((1, 1)_1\), and if \(v_1 \geq 5/8\), \(p_1^A = p_1^B = 5/8\) and both sell with equal probability. If \(v_1 \in [(3 - 2 \ln 2)/4, 5/8)\), firms randomize between \(p_1^1 = v_1\) with probability \(Pr_1(1, v_1) = \frac{4(3-2 \ln 2-4v_1)}{7 - 8 \ln 2 - 8v_1}\), and \(p_1^1 > v_1\) with probability \(1 - Pr_1(1, v_1)\). If \(v_1 < (3 - 2 \ln 2)/4\), \(p_1^A > v_1\) and \(p_1^B > v_1\).

Note that, when the first-period valuation is low enough, a sale is foregone with probability one, when the sum of first-period capacities does not exceed total demand (cases \((1, 1)_1\) and \((2, 1)_1\)), and with some positive probability, when capacities are symmetric and their sum exceeds total demand (case \((2, 2)_1\)). That some customers may not be served under duopoly causes the inefficiency relative to the monopoly set-up, which we will discuss extensively in the subsequent Sections. Another fact worth noting is that the option values’ dependence on the current period valuation may be so strong as to reverse pricing incentives of firms. This is evident in the case \((3, 1)_1\). The option value for firm B’s capacity is \(E(\Pi_2^B(3, 1)) - E(\Pi_2^B(3, 0)) = E(\Pi_2^B(2, 1)) - E(\Pi_2^B(3, 0)) = 3/8\) (see Lemma 4), which implies that firm B will not set a price \(p_1^B < 3/8\), but will prefer to set any price greater or equal to 3/8. Hence, the marginal option value of the capacity of firm A depends on \(v_1\). Specifically, if \(v_1 < 3/8\), since firm B is not expected to sell in this case, firm A’s marginal option value is \(E(\Pi_2^A(3, 1)) - E(\Pi_2^A(2, 1)) = 0\), whereas if \(v_1 \geq 3/8\) it is equal to \(E(\Pi_2^A(3, 0)) - E(\Pi_2^A(2, 1)) = 1/2\).

It turns out that, if \(v_1 \geq 1/2\), Bertrand competition will press equilibrium prices to 1/2 (firm A’s marginal option value) while firm B will sell at a price arbitrarily smaller than 1/2. If \(v_1 \in [3/8, 1/2)\)
firm A is not willing to sell, hence firm B will sell at a price equal to $v_1$. Finally, if $v_1 < 3/8$ it is firm A that will sell at price $v_1$ since its rival’s option value exceeds $3/8$. We, thus, derive the somewhat counter-intuitive result, the incentive of the high-capacity firm to sell, to be a (discontinuously) \textit{decreasing} function of the current period valuation $v_1$, a fact that is entirely attributed to the nature of the strategic interaction between the two firms. If $v_1$ is relatively high, firm A has an incentive to let its rival sell out its inventory with the view to monopolizing the market thereafter. The same incentives are at work in part (iv) of Lemma 6 with the difference that for low $v_1$ the high capacity firm A is not willing to sell, because retaining that capacity strengthens its competitive position in the next round of the pricing subgame.

The following Proposition summarizes our previous analysis.

\begin{proposition}
If both firms’ initial capacities exceed total residual demand, firms set zero prices. Otherwise, if firms have symmetric capacities and $v_1$ is relatively high, the firms set equal and positive prices, and if $v_1$ is relatively low, they randomize in the unique symmetric mixed-strategy equilibrium. If firms begin with asymmetric capacities, depending on $v_1$, either the high capacity or the low capacity firm or none of the two sells in the first period, and it is possible that the low capacity firm does not sell at all.
\end{proposition}

Our results illustrate the strategic role of both own and rival capacities in determining the intertemporal selling decisions of the firms. Whereas the high capacity firm would have an incentive to let its rival sell out and become a monopolist subsequently, if the first-period valuation is quite low, the high capacity firm may be the one that sells first and this occurs because the low capacity firm has an incentive not to sell in anticipation of higher future demand. Contrary to the (nonstochastic and) constant demand case, analyzed by Dudey (1992), in which the low capacity firm sells out all of its capacity and may do this before any sale by its high capacity rival (when total capacity exceeds total demand), in our model, stochastic and uncertain demand generates a richer set of behavior patterns. Not only the low capacity firm may not sell but it may not sell at all with positive probability.\footnote{In the case $(2,1)_1$ firm B may not sell at all: if $v_1 \in [(2 \ln 2 - 1)/4, 3/8)$, firm A will sell in period 1. The second-period’s capacity pair is $(1,1)_2$ and, if $v_2 < 1/2$, since the firms randomize, with some positive probability no firm sells in the second period. In the third period, with capacities $(1,1)_3$, firm A sells again, with probability $1/2$, at zero price. Hence, firm B ends up selling none of its capacity.}

The analysis thus far has been conducted with capacities being fixed at some (arbitrary) level at the start of the game. We now turn to the capacity investment incentives.

Table 1 presents the payoffs (equilibrium expected profits) of the two firms gross of capacity cost. Regarding expected profits, we observe that the high-capacity firm does not always have higher expected profits. This is the case when considering capacities $(3,2)_1$ and $(2,3)_1$. This finding is in
line with Dudey (1992), where it is shown that the low-capacity firm can make higher profits than
the high-capacity one, as long as the low capacity is higher than one half the total demand and
the high capacity is higher or equal to total demand. To sum up our results so far, we observe
that the positive option value of firms’ capacity reduces their incentives to sell, causing a potential
inefficiency due to lost surplus, which is absent from the monopoly case. The more representative of
such situations are the cases $(1,1)_1$, $(2,1)_1$ and $(2,2)_1$, in which total demand may not be satisfied
and at the same time at least one of the firms may end up with unsold capacity. In contrast, if a
monopolist had capacity less than or equal to total demand, he would always sell out all of his units.\(^8\)

3.3 Capacity choices

Our set-up allows us to consider the issue of strategic choice of capacity when firms anticipate
the (equilibrium) pricing schemes that will emanate from each capacity pair. Depending on the
cost function that we assume for capacity building, we can derive a range of equilibrium capacity
configurations. Not surprisingly, the more costly capacity building becomes, the lower is the incentive
of the two firms to build capacities. We now proceed to analyze capacity choice in the context of a
competing duopoly with the view to incorporating it, subsequently in the paper, in the assessment
of the monopoly situation and the procedure of arriving at it, the merger. The other important
reason we explicitly consider the problem of capacity choice is that it reveals an interesting property
of capacities as strategic variables. Observe in Table 1 that, when capacities are $(2,1)_1$ and firm A
increases its capacity by one unit, the best response of firm B is to increase its capacity by one unit
as well.\(^9\) Thus, capacities can be either strategic complements or strategic substitutes, which is not
observed in static models of price competition (see for example Bulow et al., 1985).

The following Proposition describes the equilibrium capacity configurations when the cost function
exhibits constant returns to scale. We denote the per unit of capacity cost by $c$.

**Proposition 2** Depending on the level of the capacity installment cost, equilibrium capacities are:
$(2,1)$ and $(1,2)$, if $c \in (0.03,0.33]$, $(1,1)$, if $c \in (0.33,0.52]$, $(1,0)$ and $(0,1)$, if $c \in (0.52,0.7]$ and
$(0,0)$, if $c > 0.7$. If $c \leq 0.03$, the firms randomize among all three positive capacity levels.

**Proof.** See Appendix 1. ■

It follows that, when the cost function exhibits (at least) constant returns to scale and the per
unit of capacity cost is relatively not too low (that is, not smaller than 0.03 under constant returns to

\(^8\)For example, in case $(1,1)_1$, if $v_1$ and $v_2$ turn out to be low enough, both firms prefer not to sell in either period,
beginning the last period with $(1,1)_3$ and, in total, only one unit of product is sold.

\(^9\)The above point is made under the assumption of zero capacity cost, but it can be shown to hold also when this
cost is positive.
Table 1: *Equilibrium expected profits*

<table>
<thead>
<tr>
<th>Capacity of firm A</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(0;0)</td>
<td>(0;0.7)</td>
<td>(0;1.2)</td>
<td>(0;1.5)</td>
</tr>
<tr>
<td>1</td>
<td>(0.7;0)</td>
<td>(0.52;0.52)</td>
<td>(0.5;0.85)</td>
<td>(0.45;0.88)</td>
</tr>
<tr>
<td>2</td>
<td>(1.2;0)</td>
<td>(0.85;0.5)</td>
<td>(0.5;0.5)</td>
<td>(0.75;0.5)</td>
</tr>
<tr>
<td>3</td>
<td>(1.5;0)</td>
<td>(0.88;0.45)</td>
<td>(0.5;0.75)</td>
<td>(0.0)</td>
</tr>
</tbody>
</table>

Capacity of firm B

scale), there is no investment in excess capacity, in that total capacity never exceeds the maximum total demand.

4 The case of a merger

Thus far, our analysis indicates that a duopoly may possibly perform worse than a monopoly as a result of the firms’ incentives to restrict their sales in periods with low demand. Within a context of welfare assessment of a monopoly situation as compared to a duopoly, this represents one part of the problem. The other part relates to firms’ incentives to invest. In Proposition 2 we saw that, under constant returns to scale, firms do not invest in excess capacity. What remains to be examined is whether a monopoly over or under-invests. In the context of our basic model, it is obvious that, if we let a monopoly firm decide on its capacity, given the demand specification, it will choose a socially optimal capacity. More important, however, is the same question when viewed in a more dynamic setting, in which the monopoly situation may emerge endogenously, as the result of a merger.

In this Section we examine the investment decisions of the two firms when they expect to possibly merge in the future. Merging allows the sellers to add their capacities together. Provided that the decision to merge has to be made after the choice of capacity, we expect this prospect to alter the firms’ incentives for investment. A critical issue relates to the time when firms may possibly decide to merge. In general, it will matter whether firms *may* merge only at one point in time, before all pricing activity starts or at any period after the choice of capacities. We study each of these cases one at a time, in each of the following subsections to highlight the strategic incentives. Another issue relates to the portion of the total (monopoly) profit each firm will take in case of a merger. There is a wide range of sharing rules one may employ (we can think for example of firms’ capacities as involving bargaining power). We choose to adopt a simple rule: the firms split in half the surplus created by their merger. This appears a reasonable rule as a benchmark at least in cases where each firm has a determining opinion about the merger thus firms have roughly equal bargaining powers.
4.1 Possibility of a merger at the beginning of the pricing subgame

In order to analyze the possibility of a merger, formally, we have to extend our basic model by adding a stage after the capacity choices and before the pricing subgame, in which firms decide whether or not to merge.\(^{10}\) Thus, the prospect of a subsequent merger is open at the stage where firms choose their capacities and this is what influences firms’ investment decisions.

Initially, observe that, since monopoly profit is always higher than total duopoly profit, a merger between the two firms is always profitable and, in equilibrium, will be realized. Thus, the firms’ investment choices will be conditional on an upcoming merger. Supposing that the merger did not occur, firms’ expected payoffs would be given by Table 1. Thus, in the present set-up, their expected profits are increased by an amount that is half the surplus created by the merger.\(^{11}\) The following Proposition summarizes the equilibrium capacities when the capacity cost function exhibits constant returns to scale and unit cost of capacity is equal to \(c\).

**Proposition 3** At the prospect of a potential merger equilibrium capacities are: \((2, 2)\), if \(c \leq 0.18\), \((2, 1)\) and \((1, 2)\), if \(c \in (0.18, 0.33]\), \((1, 1)\), if \(c \in (0.33, 0.6]\), \((1, 0)\) and \((0, 1)\), if \(c \in (0.6, 0.7]\) and \((0, 0)\), if \(c > 0.7\).

**Proof.** See Appendix 2.

Compared to Proposition 2, the above Proposition shows that, when merging is possible after capacities have been chosen, there will be higher investment levels for some ranges of the capacity installment cost.\(^{12}\) This is so because the higher a firm’s capacity is, the greater its expected profit in case no merger occurs, that is, the greater its total expected profit in case of a merger. Moreover, observe that, for relatively low levels of \(c\), firms invest in excessive capacity.

4.2 Possibility of a merger at any stage of the pricing subgame

In this Subsection, we analyze the case in which firms can merge at any time after capacities have been chosen. Formally, we extend our basic model by adding a stage where firms may decide to merge at the beginning of each period of the pricing subgame. In such a set-up not only the investment decisions will be affected but also the pricing behavior of the firms. To give an example, let us

---

\(^{10}\)This is a plausible assumption since, normally, mergers are formed among established firms, that is, firms that already have a positive capacity.

\(^{11}\)For example, if firms merge having capacities \((2, 1)\), firm A’s expected profit will be \(E(\Pi_1^1) + (E(\Pi_2^w) - [E(\Pi_1^1) + E(\Pi_2^w)])/2 \approx 0.925\) (where \(E(\Pi_2^w) = 1.5\), is the profit of a monopoly firm with three units of capacity. The values of \(E(\Pi_1^1)\) and \(E(\Pi_2^w)\) can be seen in Table 1).

\(^{12}\)In a more continuous setting it would be apparent that the sum of the two firms’ capacities exceeds the capacities under no prospect for a future merger, for every \(c\).
consider the case $(2, 1)$. The difference with the basic model stems from the fact that, if the two firms end up in the last period having positive capacities, they will then merge, splitting in half the total monopolistic expected profit of $1/2$. Thus, in the second period, if firm $A$ sells, the third-period expected profits for the two firms are $\pi^A_2 = p^A_2 + 1/4$ and $\pi^B_2 = 1/4$. If firm $B$ sells, it gains $\pi^B_2 = p^B_2$, while firm $A$ becomes a monopolist in the third-period with $\pi^A_2 = 1/2$. If nobody sells, profits are $\pi^A_2 = \pi^B_2 = 1/4$. It follows that the marginal option value for firm $A$ is $1/2$, if $v \geq 1/4$, and zero otherwise, whereas that of firm $B$ is $1/4$. Thus, if $v_2 \geq 1/4$, the equilibrium prices are $p^A_2 = p^B_2 = 1/4$ (and each firm sells with equal probability) and $\Pi^A_2 = 1/4, \Pi^B_2 = 1/2$. If $v_2 < 1/4$, the high-capacity firm sells at price $p^A_2 = v_2$ and $\Pi^A_2 = 1/4$ and $\Pi^B_2 = v_2 + 1/4$. The equilibrium pricing behavior is, thus, substantially different in the two setups (in the basic model, firm $B$ sells for every $v_2$ since the option value of its capacity is zero). However, what remains the same in both setups is the willingness of the two firms in equilibrium to merge in the beginning of every period of the pricing subgame, since the monopoly profit always exceeds the total duopoly profit.

In Table 2 we present the expected profits of the two firms for all possible initial capacities given the merger that will occur at the beginning of the first period of the pricing subgame (details can be found in Appendix 3). A comparison between the first-period expected profits in the two versions of the model that allow for mergers, leads to the following.

**Remark 1** The high-capacity firm is better off, while the low-capacity firm is worse off, if the possibility of a merger is present at the beginning of each period relatively to the possibility of a merger after the initial investment only.

<table>
<thead>
<tr>
<th>Capacity of firm $A$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(0;0)</td>
<td>(0;0.7)</td>
<td>(0;1.2)</td>
<td>(0;1.5)</td>
</tr>
<tr>
<td>1</td>
<td>(0.7;0)</td>
<td>(0.6;0.6)</td>
<td>(0.55;0.95)</td>
<td>(0.48;1.02)</td>
</tr>
<tr>
<td>2</td>
<td>(1.2;0)</td>
<td>(0.95;0.55)</td>
<td>(0.75;0.75)</td>
<td>(0.7;0.8)</td>
</tr>
<tr>
<td>3</td>
<td>(1.5;0)</td>
<td>(1.02;0.48)</td>
<td>(0.8;0.7)</td>
<td>(0.75;0.75)</td>
</tr>
</tbody>
</table>

Table 2: Equilibrium expected profits after a merger in the first period

On the basis of the expected profits presented in Table 2 and assuming constant per unit cost of investment, we obtain the optimal investment levels.

13In contrast, in the basic model, the marginal option values for the case $(2, 1)$ are: for firm $A$, $1/2$ if $v_2 \geq 1/2$ and zero otherwise and for firm $B$ is zero (hence, firm $B$ sells for every $v_2$).
Proposition 4. When firms can merge at the beginning of each period, they do so in the first period. Then, equilibrium capacities are: \((3,3)\), if \(c \leq 0.05\); \((2,2)\), if \(c \in (0.05,0.2]\); \((2,1)\) and 
\((1,2)\) if \(c \in (0.2,0.35]\); \((1,1)\), if \(c \in (0.35,0.6]\); \((1,0)\) and 
\((0,1)\), if \(c \in (0.6,0.7]\), and \((0,0)\), if \(c > 0.7\).

Proof. See Appendix 3.

The possibility of a merger at the beginning of each period leads to higher investment levels compared to the case where this possibility is present only at the beginning of the pricing game. The incentive to overinvest is driven by the expectation of merging in the future. Because the option to merge is present also at later periods, firms must reinforce their competitive position in case their rival denies the proposition for a merger in the first and second period of the pricing subgame. It turns out that each firm’s competitive position is positively related to its capacity, which translates to firms’ choosing relatively higher levels of initial capacity. Note that, from the point of view of the merged firm, this is inefficient, since it can sell at most three units. Note also that, if the capacity installment cost is sufficiently low, there is investment in excess capacity.

5 Welfare implications

In this Section we examine the welfare implications of a merger between the two firms leading to a monopoly situation. First, we compute and compare the total expected duopoly surplus with the total expected monopoly surplus, gross of investment costs. Regarding the case where the two firms merge and form a monopoly (at the beginning of the first period), in our model the monopolist always extracts the whole surplus by setting a price equal to the buyers’ valuation. Therefore, expected total surplus (gross of investment costs), equals the expected first-period monopoly profit. Note that this surplus is the same in the setting where a merger is possible either only before the pricing game or at the beginning of each period. This is because in both settings, in equilibrium, a merger occurs after the initial investment and before firms compete in prices. However, total expected surplus net of investment costs is different in the two settings, because equilibrium capacities (and capacity installment costs) are different.

Regarding the duopoly case, note that, if a unit is sold in period \(t\) the created surplus equals the realized value \(v_t\). The price at which the unit is sold determines the shares of the total surplus that go to the seller and the buyer. The expected total surplus (gross of investment costs) at the
beginning of period \( t \), before firms learn \( v_t \), denoted by \( TS_t(k^A, k^B) \), can be calculated as

\[
TS_t(k^A, k^B) = \int_{v_t \in R_A} (v_t + TS_{t+1}(k^A - 1, k^B)) \, dv_t + \int_{v_t \in R_B} (v_t + TS_{t+1}(k^A, k^B - 1)) \, dv_t + \\
+ \int_{v_t \in R_C} \{Pr_t^A((k^A, k^B), v_t)(1 - Pr_t^B((k^A, k^B), v_t)) (v_t + TS_{t+1}(k^A - 1, k^B)) + \\
(1 - Pr_t^A((k^A, k^B), v_t)) Pr_t^B((k^A, k^B), v_t) (v_t + TS_{t+1}(k^A, k^B - 1)) + \\
+ (1 - Pr_t^A((k^A, k^B), v_t)) (1 - Pr_t^B((k^A, k^B), v_t)) TS_{t+1}(k^A, k^B)\} \, dv_t + \\
+ \int_{v_t \in R_D} TS_{t+1}(k^A, k^B) \, dv_t.
\]

Regions \( R_A, R_B, R_C \) and \( R_D \) designate the subsets of the interval \([0,1]\) where \( v_t \) varies, in which firm \( A \) sells in period \( t \) with certainty \( (R_A) \), firm \( B \) sells with certainty \( (R_B) \), firms play mixed strategies \( (R_C) \), and none of the firms sells \( (R_D) \).\textsuperscript{14} What determines the comparison of trade surpluses of the monopoly and the duopoly case is the measure of \( R_C \) and \( R_D \). Since the option value of the capacity is, in general, greater under duopoly with capacities \((k^A, k^B)\) than under monopoly with capacities \((k^A + k^B)\), regions \( R_C \) and \( R_D \) have, accordingly, greater measure under duopoly. For example, if the duopolistic firms have capacities \((2,1)\), for \( v_1 < (2 \ln 2 - 1)/4 \) (see Lemma 6) none of firms sells whereas the monopolistic firm, optimally, sells for every \( v_t \). In this case \( R_D \) has measure \((2 \ln 2 - 1)/4\). If \( R_C \) has positive measure, with probability \([1 - (1 - Pr_t(k, v_t))^2]\) there is no sale in period \( t \) which implies a welfare cost. On the other hand, since the monopolist appropriates total surplus, his pricing strategy is socially optimal.

**Remark 2** Expected total surplus (gross of investment cost), in the case of duopoly, may be lower than the respective surplus when the two firms merge before they compete in prices.

We have shown thus far, that when faced with demand uncertainty, the merged firm can manage the sale of total capacity over time better than a duopoly. However, we have also established (see Remark 1) that the possibility of a merger may lead to higher capacity. The net welfare (net of investment costs) created by the merger reflects the tension between these two effects: increased trade surplus and increased cost due to (possibly) excess investment. The issue boils down to whether the gains from better managing sales over time are outweighed by a possible increase in capacity installment costs.

First, regarding the case where a merger can be realized only at the beginning of the pricing game, in Table 3 we present the capacity pairs in the merger and the duopoly case for different values of the per unit capacity installment cost that are chosen in the market equilibrium (see Propositions 2

\textsuperscript{14} A more complete notation of region \( R_x \) would be \( R_{x,t}(k^A, k^B) \) which we have avoided for the sake of simplicity.
and 3). The configuration that implies higher welfare between the two settings is marked with an asterisk.

<table>
<thead>
<tr>
<th>Cost</th>
<th>Equilibrium investment levels without a merger</th>
<th>Equilibrium investment levels with a merger</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c \in (0.03, 0.18]$</td>
<td>(2, 1) or (1, 2)</td>
<td>(2, 1) *</td>
</tr>
<tr>
<td>$c \in (0.18, 0.33]$</td>
<td>(2, 1) or (1, 2)</td>
<td>(2, 1) or (1, 2) *</td>
</tr>
<tr>
<td>$c \in (0.33, 0.52]$</td>
<td>(1, 1)</td>
<td>(1, 1) *</td>
</tr>
<tr>
<td>$c \in (0.52, 0.6]$</td>
<td>(0, 1) or (1, 0) *</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>$c \in (0.6, 0.7]$</td>
<td>(0, 1) or (1, 0)</td>
<td>(0, 1) or (1, 0)</td>
</tr>
<tr>
<td>$c &gt; 0.7$</td>
<td>(0, 0)</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

Table 3: Equilibrium capacities without and with a merger, when the latter can be realized only at the beginning of the pricing game.

We observe that, when $c < 0.52$, a merger performs strictly better than the duopoly; although equilibrium capacities may be higher when the prospect of a merger is present and, hence, investment cost may be higher, the merger also achieves higher trade surplus. When $c \in (0.52, 0.6]$ the merger performs worse than the duopoly since the benefit of having one more unit of capacity (in the case of a merger) is outweighed by its cost (details can be found in Appendix 4). If $c > 0.6$ the net surplus is the same in both settings since the market equilibrium outcome is a monopoly.

In Table 4 we present the equilibrium capacity levels when a merger is not possible and when it can be realized at the beginning of each period (the analysis follows the same lines as before and can be found in Appendix 4). The difference with the case where a merger can be realized only at the beginning of the pricing subgame, is that now firms have a greater incentive to invest in capacity (compare Propositions 3 and 4) which results in a greater investment cost more often than before. The merger is now optimal if $c \in (0.03, 0.18] \cup (0.2, 0.33] \cup (0.35, 0.52]$.

We summarize the results of the analysis of this Section in the following Proposition.

**Proposition 5** For certain regions of the capacity installment cost, a merger between the two firms is welfare improving compared to the duopoly case.

Thus, for certain cost levels, welfare increases after a merger, although it leads to a monopoly situation and despite the absence of gains in terms of direct cost reductions and other synergies. The increase of total surplus in our case stems from the opportunity that a merger offers to the two firms to better manage total capacity, given demand uncertainty. Demand uncertainty creates an option value for capacity which is greater when firms act noncooperatively compared to the case they merge
Table 4: Equilibrium capacities without and with a merger, when the latter can be realized at the beginning of each period of the pricing game.

(their capacities). This option value then reduces the incentive of each firms to sell when current demand is relatively low in expectation of higher future demand.

As mentioned previously, one of the possibilities established so far in the literature for a merger to be welfare enhancing, is to lead to significant enough cost savings. In our case, there is no \textit{ex ante} cost asymmetry between the firms and no post-merger reduction of direct production costs. However, if we consider a more “general” cost measure that includes the opportunity costs of capacity, then the effect of a merger, in our model, may also be interpreted in terms of a cost “efficiency” created by the merger.

6 Conclusion

In this paper we have examined the optimal pricing behavior of firms that compete over time facing demand uncertainty and capacity constraints. We have considered a duopolistic setting where, first, firms choose capacity and then they compete in prices over three periods. Buyers arrive sequentially, one in each period, and the firms set prices simultaneously knowing the valuation of the current-period buyer but being uncertain about the valuation of future buyers. Capacities evolve endogenously, according to the sellers’ pricing strategies and the demand realizations. In addition, we have analyzed how mergers affect the intertemporal behavior of the sellers and their investment decisions. Finally, we have studied the impact of mergers on welfare.

The main consequence of the introduction of future demand uncertainty in the dynamic setting is that it creates an option value of the available capacity, as long as the capacity is lower than
residual demand. As a result, depending on the capacity levels and on the current, as well as the expected demand, a firm may have an incentive to avoid selling in a given period. In contrast to the nonstochastic demand case, a low capacity firm not only may have an incentive to forego a sale, if current-period demand is low, but may not sell at all. The characterization of optimal capacity levels for constant per unit cost shows that the firms do not invest in excess capacity. This does not hold, if they expect they can merge in the future. When firms invest with the prospect of a future merger their incentives to overinvest are higher than if no prospect for a merger exists.

The prospect of a merger affects welfare in two ways. For exogenously given capacities, a merger between the two firms may increase total surplus from sales. This happens because the option value of a unit of capacity differs when total capacity is owned by a single firm compared to when it is divided between the two sellers. Therefore, by merging, the two firms can better manage total capacity utilization over time. On the other hand, for some levels of the capacity installment cost, a merger leads to higher investment levels. Regarding the net impact of mergers on welfare, we find that, for certain cost ranges, a merger may be welfare enhancing.

Possible extensions of the present model left for future research include the study of the more general oligopolistic case in which mergers can be realized among one or more subsets of the firms, cases where demand uncertainly also refers to the current period, as well as the study of pricing in richer demand patterns.
References


Appendix 1

- The pricing game

Capacities are \((3, 2)_1\). If firm \(A\) sells today, next-period capacities will be \((2, 2)_2\) and we know, from Lemma 3, that the expected profits of both firms at the beginning of the second period are zero. Therefore the expected profit of firm \(A\), if it sells today, equals \(\pi^A_1 = p_1\) and firm \(B\)’s is \(\pi^B_1 = 0\). If firm \(B\) sells today, next period capacities will be \((3, 1)_1\) and \(\pi^B_1 = p^B_1 + 3/8\), while \(\pi^A_1 = 1/2\) (from Lemma 4). If neither firm sells today, their profits are zero, by Lemma 3. It follows that firm \(B\) will have an incentive to sell irrespectively of \(v_1\), since otherwise it gains nothing (that is, the marginal option value of firm \(B\)’s capacity is zero). Specifically, if \(v_1 \geq 1/2\), both firms are willing to sell but firm \(A\) would never sell at \(p_1 < 1/2\), given that firm \(B\) would choose to sell. Thus, equilibrium prices are \(p^A_1 = p^B_1 = 1/2\), firm \(B\) sells (by setting a marginally lower price) and the respective profits are \(\Pi^A_1 = 1/2, \Pi^B_1 = 7/8\). If \(v_1 < 1/2\), the high-capacity firm lets firm \(B\) sell at a price \(p^B_1 = v_1\), by setting a price \(p^A_1 > v_1\). Equilibrium profits in this case are \(\Pi^A_1 = 1/2\) and \(\Pi^B_1 = v_1 + 3/8\). Consequently, the expected profits at the beginning of the first period are

\[
E(\Pi^A_1(3, 2)) = \int_0^{1/2} \frac{1}{2} dv_1 = \frac{1}{2} \quad \text{and} \quad E(\Pi^B_1(3, 2)) = \int_0^{1/2} (v_1 + \frac{3}{8}) dv_1 + \int_{1/2}^1 (\frac{1}{2} + \frac{3}{8}) dv_1 = \frac{3}{4}.
\]

Capacities are \((3, 1)_1\). If firm \(A\) sells, the profits are \(\pi^A_1 = p^A_1 + 1/2\) and \(\pi^B_1 = 3/8\) (from Lemma 4). If firm \(B\) sells, the profits are \(\pi^B_1 = p^B_1\) and \(\pi^A_1 = 1\) (from Lemma 2). If none of the firms sells, their profits are \(\pi^A_1 = 1/2\) and \(\pi^B_1 = 3/8\) (from Lemma 4). It follows that, if \(v_1 \geq 1/2\), both firms are willing to sell. However, firm \(B\) has an incentive to sell even if the valuation were lower than \(1/2\) and, therefore, equilibrium prices are \(p^A_1 = p^B_1 = 1/2\) and firm \(B\) sells. Equilibrium profits in this case are \(\Pi^A_1 = 1\) and \(\Pi^B_1 = 1/2\). If \(v_1 \in [3/8, 1/2]\), only firm \(B\) has an incentive to sell. Firm \(A\) sets \(p^A_1 > v_1\) gaining profit \(\Pi^A_1 = 1\), while \(p^B_1 = v_1\) and \(\Pi^B_1 = v_1\). If \(v_1 \in [0, 3/8]\), the low-capacity firm is better off when either there is no sale or when the high-capacity firm sells. Therefore, it will never set a price lower or equal to \(v_1\). The high-capacity firm will then prefer to sell itself, since its profit is higher than if there is no sale. Hence, equilibrium prices are \(p^A_1 = v_1\) and \(p^B_1 > v_1\), and equilibrium profits equal \(\Pi^A_1 = v_1 + 1/2\) and \(\Pi^B_1 = 3/8\). If follows that

\[
E(\Pi^A_1(3, 1)) = \int_0^{3/8} (v_1 + \frac{1}{2}) dv_1 + \int_{3/8}^1 dv_1 = \frac{113}{128} \approx 0.883 \quad \text{and} \quad E(\Pi^B_1(3, 1)) = \int_0^{3/8} \frac{3}{8} dv_1 + \int_{3/8}^{1/2} v_1 dv_1 + \int_{1/2}^1 \frac{1}{2} dv_1 = \frac{57}{128} \approx 0.445.
\]

Capacities are \((2, 2)_1\). If firm \(i\) sells in period 1, it gains \(\pi^i_1 = p^i_1 + 3/8\) and its rival, \(\pi^j_1 = 1/2\) (by Lemma 4). If none of the firms makes a sale, they will end up with capacities \((2, 2)_2\) in the
second period, so their expected profits are zero (by Lemma 3). The option value of each firm is $1/2 - 3/8 = 1/8$, that is, a firm would sell only if $v_1 \geq 1/8$. Consequently, depending on $v_1$, there are two possibilities: If $v_1 \geq 1/8$, both firms would like to sell. Equilibrium prices are, therefore, $p^A_1 = p^B_1 = 1/8$ and both firms sell with equal probability, obtaining profits $\Pi^A_1 = \Pi^B_1 = 1/2$. If $v_1 < 1/8$, nobody is willing to sell himself but prefers a sale to no sale. The analysis is analogous to that of capacities $(1, 1)_2$.

Mixed-strategy equilibrium. By the same reasoning as in the case $(1, 1)_2$ (see Lemma 5), the support of the distribution of prices chosen in equilibrium is $[v_1, \infty)$. In equilibrium it must be that the expected profits from charging $p = v_1$ and $p > v_1$ be equal. For firm $i$ we have

$$E\left(\frac{3}{8} + v_1\right)Pr\left(p^i_1 > v_1\right) + \left(\frac{1}{2}\left(\frac{3}{8} + v_1\right) + \frac{1}{4}\right)Pr\left(p^i_1 = v_1\right) = 0Pr\left(p^i_1 > v_1\right) + \frac{1}{2}Pr\left(p^i_1 = v_1\right)$$

Denoting $Pr\left(p^i_1 = v_1\right)$ by $Pr_1(2, v_1)$ we obtain

$$\left(\frac{3}{8} + v_1\right)(1 - Pr_1(2, v_1)) + \left(\frac{1}{2}\left(\frac{3}{8} + v_1\right) + \frac{1}{4}\right)Pr_1(2, v_1) = \frac{1}{2}Pr_1(2, v_1)$$

and solving it we get $Pr_1(2, v_1) = \frac{6 + 16 v_1}{7 + 8 v_1}$. Thus, the equilibrium prices – in the unique mixed-strategy equilibrium – are

$$p^A_1 = p^B_1 = \left\{\begin{array}{ll} v_1 & \text{with prob } \frac{6 + 16 v_1}{7 + 8 v_1}, \\
\text{some } p > v_1 & \text{with prob } 1 - \frac{6 + 16 v_1}{7 + 8 v_1},
\end{array}\right.$$

the expected profits are $(3 + 8 v_1)/(7 + 8 v_1)$ and, it follows that

$$E\left(\Pi^A_1(2, 2)_1\right) = E\left(\Pi^B_1(2, 2)_1\right) = \int_0^{1/8} \frac{3 + 8 v_1}{7 + 8 v_1} dv_1 + \int_{1/8}^{\infty} \frac{1}{2} dv_1 = \frac{9}{16} + \ln \frac{7}{8} \approx 0.4957.$$

Capacities are $(2, 1)_1$. If firm $A$ sells in the current period, $\pi^A_1 = p^A_1 + \frac{3 - 2 \ln 2}{4}$ and $\pi^B_1 = \frac{3 - 2 \ln 2}{4}$ (see Lemma 5). If firm $B$ sells $\pi^B_1 = p^B_1$ and $\pi^A_1 = 1$ (see Lemma 1). If no firm sells $\pi^A_1 = 1/2$ and $\pi^B_1 = 3/8$ (see Lemma 4). It follows that, if $v_1 \geq 1 - \frac{3 - 2 \ln 2}{4} \approx 0.6$, both firms are willing to sell. However, the low-capacity firm has an incentive to sell even for a price lower than (approximately) 0.6. Therefore, in equilibrium, $p^A_1 = p^B_1 \approx 0.6$ and firm $B$ slightly undercutts and sells. Equilibrium profits are $\Pi^A_1 = 1$ and $\Pi^B_1 = 0.6$. If $v_1 \in \left[\frac{3 - 2 \ln 2}{4}, 1 - \frac{3 - 2 \ln 2}{4}\right]$, firm $A$ lets its rival sell and firm $B$ does has an incentive to do so. Equilibrium prices are $p^A_1 > v_1$ and $p^B_1 = v_1$, while $\Pi^A_1 = 1$ and $\Pi^B_1 = v_1$. If $v_1 \in \left[\frac{3}{8}, \frac{3 - 2 \ln 2}{4}\right)$, we obtain a situation similar to the one encountered in case $(2, 2)_1$ above; each firm is better of when a sale takes place, but prefers the rival to sell. The difference from the symmetric-capacity cases is that the mixed-strategy equilibrium is not symmetric. Firms randomize between setting $p = v_1$ and $p > v_1$ but a price equal to the current-period valuation is charged by firm $A$ with probability $Pr^A_1 ((2, 1)_1) = \frac{2 + 8 v_1 - 4 \ln 2}{3 + 4 v_1 - 2 \ln 2}$ and is charged by firm $B$ with
probability \( Pr_1^R((2, 1), v_1) = \frac{3-8v_1}{2\ln 2-4v_1} \). The respective equilibrium profits are \( \Pi_1^A = \frac{5+12v_1-6\ln 2}{6+8v_1-4\ln 2} \) and \( \Pi_1^B = \frac{\ln 64-9-4v_1(\ln 256-9)}{32v_1-16\ln 2} \). If \( v_1 \in \left[ \frac{2\ln 2-1}{4}, \frac{3}{8} \right) \), firm \( B \) is better off when firm \( A \) sells. Moreover, it prefers that there is no sale at all than to be the one to sell. Firm \( A \) is better off when it sells than when not. Equilibrium prices are, therefore, \( p_1^A = v_1 \) and \( p_1^R > v_1 \), and equilibrium profits are \( \Pi_1^A = v_1 + \frac{3-2\ln 2}{4} \) and \( \Pi_1^B = \frac{3-2\ln 2}{4} \). Finally, if \( v_1 < \frac{2\ln 2-1}{4} \), both firms are better off if there is no sale. Therefore, equilibrium prices and profits are \( p_1^A > v_1 \), \( p_1^R > v_1 \) and \( \Pi_1^A = 1/2 \), \( \Pi_1^B = 3/8 \). It follows that

\[
E(\Pi_1^A(2, 1)) = \int_0^{1/2} \frac{1}{2} dv_1 + \int_{1/2}^{1} (v_1 + \frac{3-2\ln 2}{4}) dv_1 + \int_{1/2}^{1/2} \frac{5+12v_1-6\ln 2}{6+8v_1-4\ln 2} dv_1 + \int_{1/2}^{1/2} 1 dv_1 \approx 0.8475
\]

\[
E(\Pi_1^B(2, 1)) = \int_0^{3/2} \frac{3}{2} dv_1 + \int_{3/2}^{1} \frac{3-2\ln 2}{4} dv_1 + \int_{3/2}^{1} \frac{\ln 64-9-4v_1(\ln 256-9)}{32v_1-16\ln 2} dv_1 + \int_{3/2}^{1} v_1 dv_1 + \int_{3/2}^{1} \frac{1-3-2\ln 2}{4} dv_1 \approx 0.4969.
\]

*Capacities are \((1, 1)_1\). Since both firms have only one unit of capacity, if firm \( i \) sells today, its profit equals \( \pi_i^1 = p_i^1 \). The other firm then remains a monopolist and has the option to sell either in the second or in the third period. Thus, its profit is \( \pi_i^1 = 5/8 \) (by Lemma 2). If nobody sells today, the firms carry their capacities to the second period and their profits today are \( \pi_i^A = \pi_i^B = \frac{3-2\ln 2}{4} \approx 0.4034 \) (by Lemma 5). It follows that, if \( v_1 \geq 5/8 \), both firms will be willing to sell. Therefore, equilibrium prices are \( p_1^A = p_1^B = 5/8 \) and both sell with equal probability, gaining \( \Pi_1^A = \Pi_1^B = 5/8 \). If \( v_1 \in \left( \frac{3-2\ln 2}{4}, \frac{5}{8} \right) \), none of the firms is willing to sell itself but prefers a sale to no sale. In this case there is a unique mixed-strategy equilibrium which we derive below. Finally, if \( v_1 < \frac{3-2\ln 2}{4} \), none of the firms is willing to sell. Both firms set prices higher than \( v_1 \) and their equilibrium profits are \( \Pi_1^A = \Pi_1^B = \frac{3-2\ln 2}{4} \).

*Characterization of mixed strategies.* Similarly to the previous symmetric cases, firms mix on \([v_1, \infty)\) and \( p = v_1 \) is a mass point. The expected profit of firm \( i \) from setting \( p^I > v_1 \) are

\[
\left( \frac{3-2\ln 2}{4} \right) Pr(p_i^I > v_1) + \frac{5}{8} Pr(p_i^I = v_1),
\]

and the expected profit from setting \( p^I = v_1 \) are

\[
v_1 Pr(p_i^I > v_1) + \left( \frac{1}{2} v_1 + \frac{5}{2} \right) Pr(p_i^I = v_1).
\]

Equating the expected profits in the two cases and letting \( Pr(p_i^I = v_1) = Pr_1(1, v_1) \) we obtain

\[
\left( \frac{3-2\ln 2}{4} \right) (1 - Pr_1(1, v_1)) + \frac{5}{8} Pr_1(1, v_1) = v_1 (1 - Pr_1(1, v_1)) + \left( \frac{v_1}{2} + \frac{5}{16} \right) Pr_1(1, v_1),
\]

\[27\]
which holds for \( \Pr_1 (1, v_1) = \frac{4(3-2\ln 2-4v_1)}{7-8\ln 2-8v_1} \). Therefore the firms’ equilibrium prices are

\[
p_1^A = p_1^B = \begin{cases} 
  v_1 & \text{with prob } \frac{4(3-2\ln 2-4v_1)}{7-8\ln 2-8v_1}, \\
  \text{some } p > v_1 & \text{with prob } 1 - \frac{4(3-2\ln 2-4v_1)}{7-8\ln 2-8v_1}.
\end{cases}
\]

It follows that the expected profits after the realization of \( v_1 \) are \((16v_1(1+\ln 2)-5(3-\ln 4))/(32v_1 + 32\ln 2 - 28)\) for each firm, and the expected equilibrium profits at the beginning of the period are

\[
E (\Pi_1^A (1, 1) ) = E (\Pi_1^B (1, 1) ) = \frac{3 - 2\ln 2}{4} \int_0^{3 - 2\ln 2} dv_1 + \frac{5/8}{32v_1 + 32\ln 2 - 28} \int_{3 - 2\ln 2}^{5/8} dv_1 + \frac{1}{5/8} \approx 0.517.
\]

- **Best responses**

We determine equilibrium capacity levels assuming that firms choose capacities simultaneously and that the per-unit capacity installment cost, \( c \), is constant. Because of symmetry we will present only firm \( A \)’s best response function to firm \( B \)’s chosen capacity.

- Firm \( A \), as a monopolist (that is, when \( k_1^B = 0 \)):
  - does not invest if \( E (\Pi_1^A (1)) - c < 0 \) or, approximately, \( c > 0.7 \)
  - invests in 1 unit if \( E (\Pi_1^A (1)) - c > E (\Pi_1^A (2)) - 2c \) or, approximately, \( 0.7 - c > 1.2 - 2c \), that is, if \( c \in (0.5, 0.7] \)
  - invests in 2 units if \( E (\Pi_1^A (2)) - 2c > E (\Pi_1^A (3)) - 3c \) or, approximately, \( 1.2 - 2c > 1.5 - 3c \), that is, if \( c \in (0.3, 0.5] \)
  - invests in 3 units if \( E (\Pi_1^A (3)) - 3c \) or, approximately, \( c \leq 0.3 \).

- Firm \( B \), as a monopolist (that is, when \( k_1^B = 0 \)):
  - does not invest if \( E (\Pi_1^B (1, 1)) - c < 0 \) or, approximately, \( c > 0.52 \)
  - invests in 1 unit if \( E (\Pi_1^B (1, 1)) - c > E (\Pi_1^B (2, 1)) - 2c \) or, approximately, \( 0.52 - c > 0.85 - 2c \), that is, if \( c \in (0.33, 0.52] \)
  - invests in 2 units if \( E (\Pi_1^B (2, 1)) - 2c > E (\Pi_1^B (3, 1)) - 3c \) or, approximately, \( 0.85 - 2c > 0.88 - 3c \), that is, if \( c \in (0.03, 0.33] \)
  - invests in 3 units if \( E (\Pi_1^B (3, 1)) - 3c \), that is, if \( c \leq 0.03 \).

- Firm \( B \), as a monopolist (that is, when \( k_1^B = 0 \)):
  - does not invest if \( E (\Pi_1^B (0, 2)) - c < 0 \) or, approximately, \( c > 0.5 \)
  - invests in 1 unit if \( c \leq 0.5 \), since \( E (\Pi_1^B (1, 2)) \approx E (\Pi_2^A (2, 2)) \approx E (\Pi_2^A (3, 2)) \).

- Firm \( B \), as a monopolist (that is, when \( k_1^B = 0 \)):
  - does not invest if \( E (\Pi_1^B (0, 3)) - c < 0 \) or, approximately, \( c > 0.45 \)
  - invests in 1 unit if \( E (\Pi_1^B (1, 3)) - c > E (\Pi_1^B (2, 3)) - 2c \) or, approximately, \( 0.45 - c > 0.75 - 2c \), that is, if \( c \in (0.3, 0.45] \)
- invests in 2 units if $E \left( \Pi^A_1(1, 3) \right) - c < E \left( \Pi^A_1(2, 3) \right) - 2c$ or, $c \leq 0.3$.

- **Equilibrium in capacities**

Finally, we use the best response functions derived above to obtain the following equilibria in investments:

- $c \leq 0.03$ a pure-strategy equilibrium does not exist.
- $c \in (0.03, 0.3]$ equilibrium capacities are $(2, 1)$ and $(1, 2)$.
- $c \in (0.3, 0.33]$ equilibrium capacities are $(2, 1)$ and $(1, 2)$.
- $c \in (0.33, 0.45]$ equilibrium capacities are $(1, 1)$.
- $c \in (0.45, 0.5]$ equilibrium capacities are $(1, 1)$.
- $c \in (0.5, 0.52]$ equilibrium capacities are $(1, 1)$.
- $c \in (0.52, 0.7]$ equilibrium capacities are $(1, 0)$ and $(0, 1)$.
- $c > 0.07$ there is no investment in positive capacity.

**Appendix 2**

- **Expected merger profits.**

We analyze all possible capacity combinations at the beginning of the first period. We compute the expected profit of each firm in case of a merger, assuming that the surplus is shared equally. It is irrelevant which firm makes the offer and, because of symmetry, we present here only the symmetric-capacity cases and the cases where firm A is the high capacity firm.

- **Capacities $(1, 1)_1$**.

Expected duopoly profit of firm $i$ is $E \left( \Pi^A_i(1, 1) \right) \approx 0.52$, $i = 1, 2$. In case of a merger, there will be a monopoly seller with two units of capacity and expected profit $E \left( \Pi^A_i(2) \right) \approx 1.2$. Consequently, the expected profit of each firm in case of a merger equals $E^M \left( \Pi^A_i(1, 1) \right) \approx 0.52 + (1.2 - 0.52 - 0.52)/2 = 0.6$.

- **Capacities $(2, 1)_1$**.

Expected duopoly profits are $E \left( \Pi^A_1(2, 1) \right) \approx 0.85$ and $E \left( \Pi^B_1(2, 1) \right) \approx 0.497$. In case of a merger, there will be a monopoly seller with three units of capacity and expected profit $E \left( \Pi^A_i(3) \right) = 1.5$. Consequently, $E^M \left( \Pi^A_1(2, 1) \right) \approx 0.85 + (1.5 - 0.85 - 0.497)/2 = 0.927$ and $E^M \left( \Pi^B_1(2, 1) \right) \approx 0.497 + (1.5 - 0.85 - 0.497)/2 = 0.574$.

- **Capacities $(2, 2)_1$**.

Expected duopoly profit of firm $i$ is $E \left( \Pi^A_i(2, 2) \right) \approx 0.496$, $i = 1, 2$. In case of a merger, there will be a monopoly seller with four units of capacity. Since at most three units can be sold, one unit
will be left unutilized and the expected profit of the merged firm is \( E(\Pi^I_1(3)) = 1.5 \). Consequently, 
\( E^M(\Pi^I_1(2,2)) \approx 0.496 + (1.5 - 0.496 - 0.496)/2 = 0.75 \).

\( \triangleright \) Capacities \((3,1)_1 \).

Expected duopoly profits are \( E(\Pi^I_1(3,1)) \approx 0.88 \) and \( E(\Pi^B_1(3,1)) \approx 0.445 \). In case of a merger, there will be a monopoly seller with four units of capacity. One unit will be left unutilized and, therefore, expected profit is \( E(\Pi^I_1(3)) = 1.5 \). It follows that, \( E^M(\Pi^I_1(3,1)) \approx 0.88 + (1.5 - 0.88 - 0.445)/2 = 0.966 \) and \( E^M(\Pi^B_1(3,1)) \approx 0.445 + (1.5 - 0.88 - 0.445)/2 = 0.533 \).

\( \triangleright \) Capacities \((3,2)_1 \).

Expected duopoly profits are \( E(\Pi^I_1(3,2)) = 0.5 \) and \( E(\Pi^B_1(3,2)) = 0.75 \). In case of a merger, there will be a monopoly seller with five units of capacity. Two units will be left unutilized and, therefore, expected profit is \( E(\Pi^I_1(3)) = 1.5 \). It follows that, \( E^M(\Pi^I_1(3,2)) \approx 0.5 + (1.5 - 0.5 - 0.75)/2 = 0.625 \) and \( E^M(\Pi^B_1(3,2)) \approx 0.75 + (1.5 - 0.5 - 0.75)/2 = 0.875 \).

\( \triangleright \) Capacities \((3,3)_1 \).

Expected duopoly profit of firm \( i \) is \( E(\Pi^I_i) = 0 \), \( i = 1, 2 \). In case of a merger, there will be a monopoly seller with six units of capacity. Three units will be left unutilized and the expected profit of the merged firm is \( E(\Pi^B_1) = 1.5 \). It follows that, \( E^M(\Pi^I_1(3,3)) = E^M(\Pi^B_1(3,3)) = 1.5/2 = 0.75 \).

- **Best responses**

Here we examine the investment incentives of the two firms assuming constant per unit capacity installment cost and simultaneous choice of capacities. As in the case without merger, due to symmetry, we present only the firm \( A \)'s best response to firm \( B \)'s capacity choice.

The monopoly capacity levels are the same as in the game without merger.

- When one of the firms (firm \( i \)) invests in positive capacity, the other firm’s (firm \( j \)) investment incentives are as follows \( (i, j = A, B \text{ and } i \neq j) \)

  - If firm \( B \) invests in 1 unit of capacity, firm \( A \):
    - does not invest if \( E^M(\Pi^I_1(0,1)) > E^M(\Pi^I_1(1,1)) - c \) or, approximately, if \( c > 0.6 \),
    - invests in 1 unit if \( E^M(\Pi^I_1(1,1)) - c > E^M(\Pi^I_1(2,1)) - 2c \) or, approximately, \( 0.6 - c > 0.93 - 2c \), that is, if \( c \in (0.33, 0.6] \)
    - invests in 2 units if \( E^M(\Pi^I_1(2,1)) - 2c > E^M(\Pi^I_1(3,1)) - 3c \) or, approximately, \( 0.93 - 2c > 0.97 - 3c \), that is, if \( c \in (0.04, 0.33] \)
    - invests in 3 units if \( E^M(\Pi^I_1(2,1)) - 2c < E^M(\Pi^I_1(3,1)) - 3c \), that is, if \( c \leq 0.04 \).

  - If firm \( B \) invests in 2 units of capacity, firm \( A \):
    - does not invest if \( E(\Pi^I_1(0,2)) > E(\Pi^I_1(1,2)) - c \) or, approximately, if \( c > 0.57 \)
    - invests in 1 unit, if \( E(\Pi^I_1(1,2)) - c > E(\Pi^I_1(2,2)) - 2c \), or \( 0.57 - c > 0.75 - 2c \), that is, \( c \in (0.18, 0.57] \), since
    - invests in 2 units if \( c \leq 0.18 \).
If firm $B$ invests in 3 units of capacity, firm $A$:
- does not invest if $E^M(\Pi_t^A(0, 3)) > E^M(\Pi_t^A(1, 3)) - c$ or, approximately, if $c > 0.53$,
- invests in 1 unit if $E^M(\Pi_t^A(1, 3)) - c > E^M(\Pi_t^A(2, 3)) - 2c$ or, approximately, $0.53 - c > 0.88 - 2c$,
that is, if $c \in (0.35, 0.53]$
- invests in 2 units if $c \leq 0.35$ (since $E^M(\Pi_t^A(2, 3)) > E^M(\Pi_t^A(3, 3))$).

Following the same steps as in the analysis of the capacity choice when mergers are not possible (see above), we can determine the best responses of the two firms for different levels of the capacity installment cost. The determination of the equilibrium capacity pairs follows directly.

Appendix 3

- The pricing game

The introduction of the option of merging at the beginning of each stage of the pricing subgame affects the firms’ pricing strategies. We solve the new game by backward induction.

Before we proceed, we have to slightly modify notation. Given that the monopoly profit exceeds the duopoly profit, the firms will find it optimal to merge in the beginning of each period.

Now $\pi_t^i \equiv \pi_t^i(p_t^A, p_t^B, v_t, \{k_s^A, k_s^B\}_{s=t})$ denotes the total profit of firm $i$ from period $t$ on, as a function of the current-period prices of the firms, the (realized) value of $v_t$ and given that firms merge in period $t + 1$. The function $\pi_t^i$ computed at the equilibrium current-period prices is denoted by $\Pi_t^i \equiv \Pi_t^i(k_t^A, k_t^B, v_t)$, and $E(\Pi_t^i(k_t^A, k_t^B))$, denotes the expected value of firm $i$’s equilibrium profit at the beginning of period $t$ (prior to the realization of $v_t$) given that firms merge in period $t + 1$, but before firms decide whether to merge in period $t$. By $E^M(\Pi_t^i(k_t^A, k_t^B))$ we denote firm $i$’s expected profit in the beginning of period $t$ when the firms merge in period $t$.

We proceed now to the analysis of the pricing game

Third period. The optimal third-period prices and expected non-cooperative profits are the same as in the basic model. When both firms hold positive capacities, given that the monopolist’s profit exceeds the sum of the duopolistic profits, the firms choose to merge at the beginning of the third period, sharing equally the monopolistic profit, $1/2$. As a result we obtain that:

At the beginning of the third period, the equilibrium expected profits of the two firms are
- $i)$ $E^M(\Pi_3^A(k_3^A)) = 1/2$ and $E^M(\Pi_3^B(k_3^B)) = 0$, if $k_3^A > 0$ and $k_3^B = 0$, and
- $ii)$ $E^M(\Pi_3^A(k_3^A, k_3^B)) = E^M(\Pi_3^B(k_3^A, k_3^B)) = 1/4$, if $k_3^A, k_3^B > 0$.

Second period. Equilibrium second-period prices are affected by the possibility of a merger at the beginning of the third period. Therefore, we examine the pricing behavior of the two firms
given future expected profits.\textsuperscript{15} The monopoly equilibrium prices and expected profits are given by Lemmas 1 and 2. Regarding the rest of the cases, we have:

- **Case 1:** Both firms have enough capacity to satisfy residual demand, that is, capacities are $(2, 2)$, $(2, 3)$ or $(3, 3)$.
  
  i) if firm A sells, \( \pi_A^1 = p_A^1 + EM(\Pi_A^1(k_A^1, k_B^1)) = p_A^1 + 1/4 \) and \( \pi_B^1 = EM(\Pi_B^1(k_A^1, k_B^1)) = 1/4 \).

  ii) if firm B sells, \( \pi_A^1 = EM(\Pi_A^1(k_A^1, k_B^1)) = 1/4 \) and \( \pi_B^1 = p_B^1 + EM(\Pi_B^1(k_A^1, k_B^1)) = p_B^1 + 1/4 \).

  iii) if nobody sells, \( \pi_A^1 = \pi_B^1 = EM(\Pi_A^1(k_A^1, k_B^1)) = EM(\Pi_B^1(k_A^1, k_B^1)) = 1/4 \).

  Clearly, both firms would like to sell for any positive price and, therefore competition will drive prices down to zero. Thus,

  \[
  E(\Pi_A^1(k_A^1, k_B^1)) = E(\Pi_B^1(k_A^1, k_B^1)) = 0.25.
  \]

- **Case 2:** Firm A cannot satisfy residual demand on its own, that is, capacities are $(1, 2)$ or $(1, 3)$.

  i) if firm A sells, \( \pi_A^2 = p_A^2 + EM(\Pi_A^2(0)) = p_A^2 \) and \( \pi_B^2 = EM(\Pi_B^2(k_B^2)) = 1/2 \).

  ii) if firm B sells, \( \pi_A^2 = EM(\Pi_A^2(1, k_B^2 - 1)) = 1/4 \) and \( \pi_B^2 = p_B^2 + EM(\Pi_B^2(1, k_B^2 - 1)) = p_B^2 + 1/4 \).

  iii) if nobody sells, \( \pi_A^2 = \pi_B^2 = EM(\Pi_A^2(1, k_B^2)) = EM(\Pi_B^2(1, k_B^2)) = 1/4 \).

  o If \( v_2 \geq 1/4 = EM(\Pi_A^2(1, k_B^2 - 1)) - EM(\Pi_A^2(0)) = EM(\Pi_B^2(k_B^2)) - EM(\Pi_B^2(1, k_B^2 - 1)) \), both firms are willing to sell for a price not lower than 1/4. Hence, equilibrium prices and profits are \( p_A^2 = p_B^2 = 1/4 \), each firm sells with probability 1/2, and \( \Pi_A^2 = 1/4 \), \( \Pi_B^2 = 1/2 \).

  o If \( v_2 < 1/4 \), the high-capacity firm always sets \( p_B^2 > v_2 \). Given this, the low-capacity firm sets \( p_A^2 = v_2 \) and sells in equilibrium. Equilibrium profits are \( \Pi_A^2 = 1/4 \) and \( \Pi_B^2 = v_2 + 1/4 \).

  Equilibrium expected profits at the beginning of the second period, with capacities \( k_A^1 = 1 \) and \( k_B^2 \in \{2, 3\} \), are

  \[
  E(\Pi_A^1(1, k_B^2)) = \frac{1}{4} \quad \text{and} \quad E(\Pi_B^1(1, k_B^2)) = \frac{1}{4}(v_2 + \frac{1}{4})dv_2 + \frac{1}{4} \frac{1}{2}dv_2 \approx 0.469
  \]

- **Case 3:** capacities are $(1, 1)$.

In this case, we obtain mixed strategy symmetric equilibrium for certain values of \( v_2 \).

i) if firm A sells, \( \pi_A^2 = p_A^2 + EM(\Pi_A^2(0)) = p_A^2 \) and \( \pi_B^2 = EM(\Pi_B^2(1)) = 1/2 \)

ii) if firm B sells, \( \pi_A^2 = EM(\Pi_A^2(1)) = 1/2 \) and \( \pi_B^2 = p_B^2 + EM(\Pi_B^2(0)) = p_B^2 \).

\textsuperscript{15}Because of symmetry in the payoffs, we present only the analysis of the subgames where firm A is the low-capacity firm as well as those where the firms hold equal capacity.
iii) if nobody sells, \( \pi_2^i = \pi_2^B = E^M (\Pi_3^i (1, 1)) = E^M (\Pi_3^B (1, 1)) = 1/4. \)

Both firms would like to sell for a price higher than \( 1/2 = E^M (\Pi_3^i (1)) - E^M (\Pi_3^A (0)) = E^M (\Pi_3^B (1)) - E^M (\Pi_3^B (0)). \) To the contrary, none of the firms will sell for a price lower than \( 1/4 = E^M (\Pi_3^A (1, 1)) - E^M (\Pi_3^B (0)) = E^M (\Pi_3^B (1, 1)) - E^M (\Pi_3^B (0)). \)

Therefore, equilibrium prices and profits are:

- If \( v_2 \geq 1/2, p_2^A = p_2^B = 1/2 \) (each sells with probability \( 1/2 \)) and \( \Pi_2^A = \Pi_2^B = 1/2, \)
- If \( v_2 < 1/4, p_2^A, p_2^B > v_2, \) since both firms gain \( 1/4 \) in case there is no sale, that is, \( \Pi_2^A = \Pi_2^B = 1/4. \)
- If \( v_2 \in [1/4, 1/2), \) each firm is better-off if its rival sells but would prefer to sell itself, if the other firm does not. There are no pure strategy symmetric equilibria and below we characterize the unique mixed strategy equilibrium.

**Characterization of mixed strategies.** Firms mix on \([v_2, \infty)\) and \( p = v_2 \) is a mass point. The expected profit of firm \( i \) from setting \( p^i > v_2 \) is

\[
\frac{1}{4} \Pr (p_2^i > v_2) + \frac{1}{2} \Pr (p_2^i = v_2),
\]

and the expected profit from setting \( p^i = v_2 \) is

\[
v_2 \Pr (p_2^i > v_2) + \left( \frac{1}{2} v_2 + \frac{1}{4} \right) \Pr (p_2^i = v_2).
\]

Equating the expected profits in the two cases and letting \( \Pr (p_2^i = v_2) = \Pr (1, v_2) \) we obtain

\[
\frac{1}{4} (1 - \Pr (1, v_2)) + \frac{1}{2} \Pr (1, v_2) = v_2 (1 - \Pr (1, v_2)) + \left( \frac{v_2}{2} + \frac{1}{4} \right) \Pr (1, v_2),
\]

which holds for \( \Pr (1, v_2) = \frac{4v_2 - 1}{2v_2}. \) Therefore the firms’ equilibrium prices are

\[
p_2^A = p_2^B = \begin{cases} v_2 \quad \text{with prob} \quad \frac{4v_2 - 1}{2v_2} \\ \text{some } p > v_2 \quad \text{with prob} \quad \frac{1 - 2v_2}{2v_2} \end{cases}
\]

Substituting \( \Pr (1, v_2) = \frac{4v_2 - 1}{2v_2} \) into (4) we obtain the expected profit \( \Pi_2^i = \frac{6v_2 - 1}{8v_2}. \) Expected equilibrium profits at the beginning of the period are

\[
E (\Pi_2^i (1, 1)) = 
\int_{0}^{1/4} \frac{1}{4} dv_2 + \int_{1/4}^{1/2} \frac{6v_2 - 1}{8v_2} dv_2 + \int_{1/2}^{1} \frac{1}{2} dv_2 = \frac{4 - \ln 2}{8} \approx 0.4134.
\]

We summarize the results of second-period pricing subgame as follows

**Second-period equilibrium prices and expected profits are:**

- If capacities \( (2, 2) \), \( (2, 3) \), \( (3, 3) \), \( p_2^A = p_2^B = 0 \) and \( E (\Pi_2^i (k_2^A, k_2^B)) = E (\Pi_2^B (k_2^A, k_2^B)) = 0.25. \)

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ii) With capacities \((1, 2)_2, (1, 3)_2\), \(p^A_2 = p^B_2 = 1/4\) and both sell with equal probability, if \(v_2 \geq 1/4\). If \(v_2 < 1/4, p^A_2 = v_2\) and \(p^B_2 > v_2\). Expected profits are \(E(\Pi^A_2 (1, k^B_2)) = 0.25\) and \(E(\Pi^B_2 (1, k^B_2)) \approx 0.469\).

iii) With capacities \((1, 1)_2\), if \(v_2 \geq 1/2, p^A_2 = p^B_2 = 1/2\) and each firm sells with probability 1/2. If \(v_2 \in [1/4, 1/2), p^i_2 = v_2\) with probability \(Pr (1, v_2) = \frac{4v_2 - 1}{2v_2}\) and \(p^i_2 > v_2\) with probability \(1 - Pr (1, v_2)\), \(i = A, B\). If \(v_2 < 1/4, p^A_2 > v_2\) and \(p^B_2 > v_2\). Expected profits are \(E(\Pi^A_2 (1, 1)_2) = E(\Pi^B_2 (1, 1)_2) = \frac{4 - \ln 2}{8} \approx 0.413\).

**• Incentives to merge.** Given the expected profits at the beginning of the period, we examine the incentives of the two firms to merge. The analysis in this case follows the same line as the analysis of the merger incentives at the beginning of the pricing game. For each capacity combination we compute the expected profits after the merger, assuming that firms share the surplus equally.

**• Capacities \((1, 1)_2\).**

Expected duopoly profits of firm \(i\) is \(E(\Pi^i_2 (1, 1)) = (4 - \ln 2)/8, i = 1, 2\). In case of a merger, there will be a monopoly seller with two units of capacity and expected profit \(E(\Pi^M_2 (2)) = 1\). Consequently, the expected profit of each firm in case of a merger equals \(E^M (\Pi^i_2 (1, 1)) = (4 - \ln 2)/8 + (1 - (4 - \ln 2)/4)/2 = 1/2\).

**• Capacities \((2, 1)_2\) or \((3, 1)_2\).**

Expected duopoly profits are \(E(\Pi^A_2 (k^A_2, 1)) = 15/32\) and \(E(\Pi^B_2 (k^A_2, 1)) = 1/4\). In case of a merger, there will be a monopoly seller with three units of capacity and expected profit is \(E(\Pi^M_2 (3)) = 1.16\). Consequently, the expected profits in case of a merger equal \(E^M (\Pi^A_2 (k^A_2, 1)) = 15/32 + (1 - 15/32 - 1/4)/2 = 39/64 \approx 0.61\) and \(E^M (\Pi^B_2 (k^A_2, 1)) = 1/4 + (1 - 15/32 - 1/4)/2 = 25/64 \approx 0.39\).

**• Capacities \((2, 2)_2, (3, 2)_2\) or \((3, 3)_2\).** These are the cases where the per-period profits equal zero and, therefore, expected duopoly profits are \(E(\Pi^A_2 (k^A_2, k^B_2)) = E(\Pi^B_2 (k^A_2, k^B_2)) = 1/4\). In case of a merger, there will be a monopolist with capacity \(k^M_2 = (k^A_2 + k^B_2) \in \{4, 5, 6\}\). There will be idle capacity, since at most two units can be sold and the expected profit of the merged firm is \(E^M (\Pi^i_2 (k^M_2)) = 1\). Thus, each firm obtains \(E^M (\Pi^i_2 (k^A_2, k^B_2)) = 1/2\).

Table 5 summarizes the above results.

**First period.** Next, we examine the first-period pricing behavior of the two firms given that there will be a merger at the beginning of the second period. Equilibrium prices and expected profits in the monopoly case are given by Lemmas 1 and 2. In addition, with capacities \((3, 3)_1\), first-period prices are driven down to zero, since each firm holds enough capacity to satisfy total demand and expected profits are \(E(\Pi^A_1 (3, 3)) = E(\Pi^B_1 (3, 3)) = 0.5\). Regarding the rest of the cases, we have:

**Capacities are \((2, 2)_1\).**

- i) if firm A sells, \(\pi^A_1 = p^A_1 + E^M (\Pi^A_2 (1, 2)) = p^A_1 + 25/64\) and \(\pi^B_1 = E^M (\Pi^B_2 (1, 2)) = 39/64\),

In the case where capacities are \((3, 1)\), the monopolist will leave one unit unutilized.
Setting the probability with which a firm charges an equilibrium follows the same line as before. Each firm randomizes between the rival firm to sell. There are two asymmetric pure-strategy equilibria, where each firm sells. The characterization of the mixed strategies follows the same line as in the previously analyzed cases where mixed strategy equilibria were obtained. Each firm randomizes between the rival firm to sell. The characterization of the mixed strategies follows the same line as before.

\[ v^1_A = E^M(\Pi^2_1(2, 1)) = 39/64 \text{ and } v^1_B = p^1_B + E^M(\Pi^2_2(2, 1)) = p^1_B + 25/64, \]
\[ v^2_A = E^M(\Pi^2_1(2, 2)) = 1/2. \]

ii) if nobody sells \( v^1_A = v^1_B = E^M(\Pi^3_1(2, 2)) = 1/2. \)

\[ \text{If } v^1 \geq 14/64 = E^M(\Pi^3_2(1, 2)) - E^M(\Pi^3_2(1, 2)), \text{ both firms would sell for a price higher than } 14/64. \text{ Thus, equilibrium prices and profits are } p^1_A = p^1_B = 14/64 \text{ and } \Pi^A_1 = \Pi^B_1 = 39/64. \]

\[ \text{If } v^1 < 7/64 = E^M(\Pi^3_2(2, 2)) - E^M(\Pi^3_2(1, 2)), \text{ both firms are better off if there is no sale today and equilibrium prices and profits are } p^1_A, p^1_B > v^1 \text{ and } \Pi^A_1 = \Pi^B_1 = 1/2. \]

\[ \text{If } v^1 \in [7/64, 14/64), \text{ each firm is better off if there is a sale in the current period but prefers the rival firm to sell. There are two asymmetric pure-strategy equilibria, where } p^i_A = v^1, p^i_B > v^1 \text{ and } \Pi^i_1 = v^1 + 25/64, \Pi^i_2 = 39/64, i, j = 1, 2. \]

The characterization of the symmetric mixed strategy equilibrium follows the same line as before. Each firm randomizes between \( p^i_A = v^1 \text{ and } p^i_B > v^1. \)

Setting the probability with which a firm charges \( p^i_A = v^1 \) equal to \( q(v^1) \) and equating the expected profits from using each strategy, we obtain \( q(v^1) = (64v^1 - 7)/32v^1. \)

Thus, the expected profits equal \( \Pi^A_1 = \Pi^B_1 = (1472v^1 - 49)/2048v^1. \)

Equilibrium expected profits at the beginning of the first period are

\[ E(\Pi^1_2(2, 2)) = \int_0^{1/64} \frac{1}{2} dv^1 + \int_{1/64}^{14/64} \frac{(1472v^1 - 49)}{2048v^1} dv^1 + \int_{14/64}^{7/64} \frac{39}{64} dv^1 = \frac{1248 - 49 \ln 2}{2048} \approx 0.593. \]

Capacities are \((1, 1)_1.\)

i) if firm \( i \) sells, \( \pi^i_A = p^i_A + E^M(\Pi^2_i(0)) = p^i_A \) and \( \pi^i_B = E^M(\Pi^2_i(1))) = 5/8, \)

ii) if nobody sells \( \pi^A_1 = \pi^B_1 = E^M(\Pi^3_1(1))) = 1/2. \)

\[ \text{If } v^1 > 5/8, \text{ both firms would sell for a price higher than } 5/8. \text{ Thus, equilibrium prices and profits are } p^A_1 = p^B_1 = 5/8 \text{ and } \Pi^A_1 = \Pi^B_1 = 5/8. \]

\[ \text{If } v^1 \in [1/2, 5/8), \text{ each firm prefers a sale in the current period but is better off if the rival firm sells. The characterization of the mixed strategies follows the same line as in the previously analyzed cases where mixed strategy equilibria were obtained. Each firm randomizes between } p^i_A = v^1 \text{ and } p^i_B > v^1. \]

Setting \( \Pr(p^i_A = v^1) = \Pr(1, v^1) \) and equating the expected profits from using

<table>
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<th>Capacity of firm A</th>
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<th>2</th>
<th>3</th>
</tr>
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<tr>
<td>(0.0)</td>
<td>(0.063)</td>
<td>(0.5;0.5)</td>
<td>(0.39;0.61)</td>
<td>(0.39;0.61)</td>
</tr>
<tr>
<td>(0.63;0)</td>
<td>(0.5;0.5)</td>
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<td>(0.5;0.5)</td>
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</tr>
<tr>
<td>(1;0)</td>
<td>(0.61;0.39)</td>
<td>(0.5;0.5)</td>
<td>(0.5;0.5)</td>
<td></td>
</tr>
<tr>
<td>(1;0)</td>
<td>(0.61;0.39)</td>
<td>(0.5;0.5)</td>
<td>(0.5;0.5)</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Equilibrium expected profits after a merger in the second period
each strategy, we find that \( \Pr (1, v_1) = \frac{8(2v_1 - 1)}{(8v_1 - 3)} \). Thus, the expected profits equal \( \Pi^A_1 = \Pi^B_1 = \frac{(12v_1 - 5)}{2(8v_1 - 3)} \).

- If \( v_1 < 1/2 \), both firms are better off if there is no sale today and equilibrium prices and profits are \( p^A_1, p^B_1 > v_1 \) and \( \Pi^A_1 = \Pi^B_1 = 1/2 \).

Equilibrium expected profits at the beginning of the first period are

\[
E \left( \Pi^A_1(1, 1) \right) = \int_0^{1/2} \frac{1}{2} \, dv_1 + \int_{1/2}^{5/8} \frac{(12v_1 - 5)}{2(8v_1 - 3)} \, dv_1 + \int_{5/8}^{1} \frac{5}{8} \, dv_1 = \frac{37 - \ln 4}{64} \approx 0.557.
\]

We summarize the results obtained for the symmetric capacities cases. First-period equilibrium prices and expected profits are:

i) for \((1, 1)\): if \( v_1 < 1/2, p^A_1, p^B_1 > v_1 \). If \( v_1 \in [1/2, 5/8], p^A_1 = v_1, \) with probability \( \Pr (1, v_1) = \frac{8(2v_1 - 1)}{(8v_1 - 3)} \) and \( p^B_1 > v_1, \) with probability \( 1 - \Pr (1, v_1) \). If \( v_1 > 5/8, p^A_1 = p^B_1 = 5/8 \) and each firm sells with probability 1/2, \( i = A, B. \) Expected profits equal \( E(\Pi^A_1(1, 1)) = E(\Pi^B_1(1, 1)) = (37 - \ln 4)/64 \approx 0.557. \)

ii) for \((2, 2)\): if \( v_1 < 7/64, p^A_1 > v_1 \) and \( p^B_1 > v_1 \). If \( v_1 \in [7/64, 14/64], p^A_1 = v_1, \) with probability \( \Pr (2, v_1) = \frac{(64v_1 - 7)/32v_1 \text{ and } p^A_1 > v_1, \text{ with probability } (1 - \Pr (1, v_1)), i = A, B. \) If \( v_1 > 14/64, p^A_1 = p^B_1 = 14/64 \) and each firm sells with probability 1/2. Expected profits are \( E(\Pi^A_1(2, 2)) = E(\Pi^B_1(2, 2)) = (1248 - 49 \ln 2)/2048 \approx 0.593. \)

ii) for \((3, 3)\): \( p^A_1 = p^B_1 = 0 \) and \( E(\Pi^A_1) = E(\Pi^B_1) = 0.5. \)

We now turn to the analysis of the subgames initiated by asymmetric capacity levels.\(^{17}\)

**Capacities are \((1, 2)\)**

i) if firm A sells, \( \pi^A_1 = p^A_1 + M(\Pi^A_2(0)) = p^A_1 \) and \( \pi^B_1 = E^M(\Pi^B_2(2)) = 1. \)

ii) if firm B sells, \( \pi^A_1 = E^M(\Pi^A_2(1, 1)) = 1/2 \) and \( \pi^B_1 = p^B_1 + E^M(\Pi^B_2(1, 1)) = p^B_1 + 1/2. \)

iii) if nobody sells \( \pi^A_1 = E^M(\Pi^A_2(1, 2)) = 25/64 \) and \( \pi^B_1 = E^M(\Pi^B_2(1, 2)) = 39/64. \)

- If \( v_1 \geq 1/2 = E^M(\Pi^A_2(1, 1)) = E^M(\Pi^B_2(2)) - E^M(\Pi^B_2(1, 1)), \) both firms would sell for a price higher than 1/2. Thus, equilibrium prices and profits are \( p^A_1 = p^B_1 = 1/2 \) and \( \Pi^A_1 = \Pi^B_1 = 1/2. \)

- If \( v_1 \in [25/64, 1/2], \) each firm is better off if there is a sale in the current period but prefers the rival firm to sell. We obtain a mixed strategy equilibrium, where again each firm randomizes between \( p^A_1 = v_1 \) and \( p^B_1 > v_1. \) However, this is not a symmetric equilibrium as in the equal-capacities cases.\(^{18}\) We set the probability with firm A charges \( p^A_1 = v_1 \) equal to \( g(v_1) \) and that with which firm B charges \( p^B_1 = v_1, \) equal to \( s(v_1) \). Equating each firm’s expected profits from setting a price equal to \( v_1 \) and a price higher than \( v_1, \) we find that \( g(v_1) = (64v_1 - 25)/(32v_1 - 9) \) and \( s(v_1) = \)

\(^{17}\)Again, because of symmetry of the payoffs, we consider only the subgames where firm A is the low-capacity firm.

\(^{18}\)As before, there are also asymmetric pure strategy equilibria. However, to be consistent with the previous analysis, in computing the expected profits at the beginning of the period, we consider only the mixed strategy equilibrium.
would sell for a price higher than
and equilibrium prices and pro...ts are $p^A_1, p^B_1 > v_1$ and $\Pi^A_1 = 25/64, \Pi^B_1 = 39/64$.

If $v_1 < 7/64 = E^M(\Pi^A_2(1,2)) - E^M(\Pi^B_2(1,1))$, both firms are better off if there is no sale today and equilibrium prices and profits are $p^A_1, p^B_1 > v_1$ and $\Pi^A_1 = 25/64, \Pi^B_1 = 39/64$.

If $v_1 \in [7/64, 25/64)$, the low-capacity firm is better off to let the high-capacity firm sell. Since the latter has an incentive to do so, equilibrium prices and profits are $p^A_1 > v_1, p^B_1 = v_1$ and $\Pi^A_1 = 1/2$ and $\Pi^B_1 = v_1 + 1/2$.

Equilibrium expected profits at the beginning of the first period are
\[
E(\Pi^A_1(1,2)) = \int_0^{7/64} \frac{25}{64} dv_1 + \int_{7/64}^{25/64} \frac{1}{2} dv_1 + \int_{25/64}^{1/2} \frac{78v_1 - 25}{128v_1 - 36} dv_1 + \int_{1/2}^{1} \frac{1}{2} dv_1 \approx 0.483
\]
\[
E(\Pi^B_1(1,2)) = \int_0^{7/64} \frac{39}{64} dv_1 + \int_{7/64}^{25/64} \left(v_1 + \frac{1}{2}\right) dv_1 + \int_{25/64}^{1/2} \frac{178v_1 + 11}{128v_1 + 36} dv_1 + \int_{1/2}^{1} 1dv_1 \approx 0.884
\]

Capacities are $(1,3)_1$.

i) if firm A sells, $\pi^A_1 = p^A_1 + E^M(\Pi^A_2(0)) = p^A_1$ and $\pi^B_1 = E^M(\Pi^B_2(3)) = 1$.

ii) if firm B sells, $\pi^A_1 = E^M(\Pi^A_2(1,2)) = 25/64$ and $\pi^B_1 = p^B_1 + E^M(\Pi^B_2(1,2)) = p^B_1 + 39/64$.

iii) if nobody sells $\pi^A_1 = E^M(\Pi^A_2(1,3)) = 25/64$ and $\pi^B_1 = E^M(\Pi^B_2(1,3)) = 39/64$.

If $v_1 \geq 25/64 = E^M(\Pi^A_2(1,2)) - E^M(\Pi^A_2(0)) = E^M(\Pi^B_2(3)) - E^M(\Pi^B_2(1,2))$, both firms would sell for a price higher than 25/64. Thus, equilibrium prices and profits are $p^A_1 = p^B_1 = 25/64$ and $\Pi^A_1 = 25/64, \Pi^B_1 = 1$.

If $v_1 < 25/64$, the low-capacity firm is better off if the high-capacity firm sells today. Therefore, it sets $\Pi^A_1 = 25/64$ and $\Pi^B_1 = 1 + 39/64$.

Equilibrium expected profits at the beginning of the first period are
\[
E(\Pi^A_1(1,3)) = \int_0^{25/64} \frac{25}{64} dv_1 + \int_{25/64}^{1} \frac{25}{64} dv_1 = \frac{25}{64} \approx 0.391
\]
\[
E(\Pi^B_1(1,3)) = \int_0^{25/64} \left(v_1 + \frac{39}{64}\right) dv_1 + \int_{25/64}^{1} 1dv_1 \approx 0.924
\]

Capacities are $(2,3)_1$.

i) if firm A sells, $\pi^A_1 = p^A_1 + E^M(\Pi^A_2(1,3)) = p^A_1 + 25/64$ and $\pi^A_1 = E^M(\Pi^B_2(1,3)) = 39/64$.

ii) if firm B sells, $\pi^A_1 = E^M(\Pi^A_2(2,2)) = 1/2$ and $\pi^A_1 = p^B_1 + E^M(\Pi^B_2(2,2)) = p^B_1 + 1/2$.

iii) if nobody sells $\pi^A_1 = \pi^B_1 = E^M(\Pi^A_2(2,3)) = E^M(\Pi^B_2(2,3)) = 1/2$.

If $v_1 \geq 7/64 = E^M(\Pi^A_2(2,2)) - E^M(\Pi^A_2(1,3)) = E^M(\Pi^B_2(1,3)) - E^M(\Pi^B_2(2,2))$, both firms
would sell for a price higher than 7/64. Thus, equilibrium prices and profits are $p^A_1 = p^B_1 = 7/64$ and $\Pi^A_1 = 1/2, \Pi^B_1 = 39/64$. 

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If \( v_1 < 7/64 \), similarly to the previous case, the low-capacity firm is better off if the high-capacity firm sells today. Therefore, it sets \( p^A_1 > v_1 \) and firm B sells for \( p^B_1 = v_1 \). Equilibrium profits are \( \Pi^A_1 = 1/2 \) and \( \Pi^B_1 = v_1 + 1/2 \).

Equilibrium expected profits at the beginning of the first period are

\[
E (\Pi^A_1(2,3)) = \frac{7}{64} \int_0^1 \frac{1}{2} dv_1 + \int_{7/64}^1 \frac{1}{2} dv_1 = \frac{1}{2}
\]

\[
E (\Pi^B_1(2,3)) = \frac{7}{64} \int_0^1 \left( v_1 + \frac{1}{2} \right) dv_1 + \int_{7/64}^1 \frac{39}{64} dv_1 \approx 0.603
\]

The results for asymmetric first-period capacities can be summarised as follows.

**First-period equilibrium prices and expected profits with asymmetric capacities are:**

1. **for (1, 2)_1**: if \( v_1 < 7/64, p^A_1 > v_1, p^B_1 = v_1 \). If \( v_1 \in [7/64, 25/64) \), \( p^A_1 > v_1, p^B_1 = v_1 \). If \( v_1 \in [25/64, 1/2) \), \( p^A_1 = v_1 \), with probability \( g(v_1) = (64v_1 - 25) / (32v_1 - 9) \) and \( p^A_1 > v_1 \), with probability \( (1 - g(v_1)) \), \( p^B_1 = v_1 \) with probability \( s(v_1) = (64v_1 - 7) / (32v_1 + 9) \) and \( p^B_1 > v_1 \), with probability \( (1 - s(v_1)) \). If \( v_1 \geq 1/2, p^A_1 = p^B_1 = 1/2 \) and both sell with equal probability. Equilibrium expected profits equal \( E(\Pi^A_1(1,2)) \approx 0.483 \) and \( E(\Pi^B_1(1,2)) \approx 0.884 \).

2. **for (1, 3)_1**: if \( v_1 < 25/64, p^A_1 > v_1, p^B_1 = v_1 \). If \( v_1 \geq 25/64, p^A_1 = p^B_1 = 25/64 \) and both sell with equal probability. Equilibrium expected profits are \( E(\Pi^A_1(1,3)) \approx 0.391 \) and \( E(\Pi^B_1(1,3)) \approx 0.924 \).

3. **for (2, 3)_1**: if \( v_1 < 7/64, p^A_1 > v_1, p^B_1 = v_1 \). If \( v_1 \geq 7/64, p^A_1 = p^B_1 = 7/64 \) and both sell with equal probability. Equilibrium expected profits are \( E(\Pi^A_1(2,3)) \approx 0.5 \) and \( E(\Pi^B_1(2,3)) \approx 0.603 \).

- **Incentives to merge.** Since a merger leads to a monopoly situation, it is profitable for both firms irrespective of their capacities. Below, we compute the expected profits after a merger as before, assuming that the surplus is shared equally.

  ▶ **Capacities (1, 1)_1.**
  Each firm obtains \( E^M (\Pi^A_1(1,1)) \approx 0.557 + (1.2 - 0.557 - 0.557)/2 = 0.6 \)

  ▶ **Capacities (1, 2)_1.**
  Firm A obtains \( E^M (\Pi^A_1(1,2)) \approx 0.483 + (1.5 - 0.483 - 0.884)/2 = 0.55 \) and firm B obtains \( E^M (\Pi^B_1(1,2)) \approx 0.884 + (1.5 - 0.483 - 0.884)/2 = 0.95 \).

  ▶ **Capacities (1, 3)_1.**
  Firm A obtains \( E^M (\Pi^A_1(1,3)) \approx 0.391 + (1.5 - 0.391 - 0.924)/2 = 0.484 \) and firm B obtains \( E^M (\Pi^B_1(1,3)) \approx 0.924 + (1.5 - 0.391 - 0.924)/2 = 1.017 \).

  ▶ **Capacities (2, 2)_1.**
  Each firm obtains \( E^M (\Pi^A_1(2,2)) \approx 0.593 + (1.5 - 0.593 - 0.593)/2 = 0.75 \).
\( \triangleright \text{Capacities } (2, 3)_1. \)
Firm A obtains \( E^M (\Pi^A_1(2, 3)) \approx 0.5 + (1.5 - 0.5 - 0.603)/2 = 0.699 \) and firm B obtains \( E^M (\Pi^B_1(2, 3)) \approx 0.603 + (1.5 - 0.5 - 0.603)/2 = 0.802. \)

\( \triangleright \text{Capacities } (3, 3)_1. \)
Each firm obtains \( E^M (\Pi^A_1(3, 3)) = 1.5/2 = 0.75. \)
Table 5 summarizes the above results.

\begin{itemize}
  \item \textbf{Capacity choice.}
\end{itemize}

As before, we examine the investment incentives of the two firms, assuming constant per unit capacity installment cost, \( c. \) The monopoly case is identical to the one in the basic model.

Regarding the rest of the cases, when one of the firms (firm \( i \)) invests in positive capacity, the other firm’s (firm \( j \)) investment incentives are as follows \((i, j = A, B \text{ and } i \neq j)\):

- If firm \( B \) invests in 1 unit of capacity, firm \( A \):
  - does not invest if \( E^M (\Pi^A_1(0, 1)) > E^M (\Pi^A_1(1, 1)) - c \) or, approximately, if \( c > 0.6, \)
  - invests in 1 unit, if \( E^M (\Pi^A_1(1, 1)) - c > E^M (\Pi^A_1(2, 1)) - 2c \) or, approximately, if \( 0.6 - c > 0.95 - 2c \), that is, if \( c \in (0.35, 0.6), \)
  - invests in 2 units, if \( E^M (\Pi^A_1(2, 1)) - 2c > E^M (\Pi^A_1(3, 1)) - 3c \) or, approximately, if \( 0.95 - 2c > 1.02 - 3c \), that is, if \( c \in (0.07, 0.35], \)

- If firm \( B \) invests in 2 units of capacity, firm \( A \):
  - does not invest if \( E^M (\Pi^A_1(0, 2)) > E^M (\Pi^A_1(1, 2)) - c \) or, approximately, if \( c > 0.55, \)
  - invests in 1 unit, if \( E^M (\Pi^A_1(1, 2)) - c > E^M (\Pi^A_1(2, 2)) - 2c \) or, approximately, if \( 0.55 - c > 0.75 - 2c \), that is, if \( c \in (0.2, 0.55], \)
  - invests in 2 units if \( E^M (\Pi^A_1(2, 2)) - 2c > E^M (\Pi^A_1(3, 2)) - 3c \) or, approximately, if \( 0.75 - 2c > 0.8 - 3c \), that is, if \( c \in (0.05, 0.2], \)
  - invests in 3 units if \( E^M (\Pi^A_1(2, 2)) - 2c < E^M (\Pi^A_1(3, 2)) - 3c \), that is, if \( c \leq 0.05. \)

- If firm \( i \) invests in 3 units of capacity, firm \( j \):
  - does not invest if \( E^M (\Pi^A_1(0, 3)) > E^M (\Pi^A_1(1, 3)) - c \) or, approximately, if \( c > 0.48, \)
  - invests in 1 unit, if \( E^M (\Pi^A_1(1, 3)) - c > E^M (\Pi^A_1(2, 3)) - 2c \) or, approximately, if \( 0.48 - c > 0.7 - 2c \), that is, if \( c \in (0.22, 0.48], \)
  - invests in 2 units, if \( E^M (\Pi^A_1(2, 3)) - 2c > E^M (\Pi^A_1(3, 3)) - 3c \) or, approximately, if \( 0.7 - 2c > 0.75 - 3c \), that is, if \( c \in (0.05, 0.22] \)
  - invests in 3 units if \( E^M (\Pi^A_1(2, 3)) - 2c < E^M (\Pi^A_1(3, 3)) - 3c \), that is, if \( c \leq 0.05. \)
Equilibrium capacity pairs follow directly (see Proposition 4).
Appendix 4

We assume, without loss of generality, that $k^A_t \geq k^B_t$.

- Monopoly cases:
  Note that $TS_t (k^A, 0) = E \left( \Pi^*_t (k^A) \right)$ (see Lemmas 1 and 2).

- Duopoly cases
  If $\min \{k^A_t, k^B_t\} \geq 4 - t$, (see Lemma 3) then firms set zero prices in period $t$ onward. Thus, the per period trade surplus is $\int_0^1 v_t dv_t = 1/2$ and $TS_t (k^A, k^B) = (4 - t)/2$.

  If $\min \{k^A_t, k^B_t\} < 4 - t$, the total surplus is calculated using (3).

Example:

$TS_2 (2, 1) = 1 - \frac{2 \ln 2 + 1}{4}$

$TS_2 (1, 1) = \int_0^1 \left( v_2 + TS_3 (1, 0) \right) dv_1 = \int_0^1 \left( v_2 + \frac{1}{2} \right) dv_1 = 1,$

where, by Lemma 4, region $R^t_A = \left( R^t_C = \left( R^t_D = \emptyset \right.$ and region $R^t_B = [3/8, 1]$ (since firm $B$ sells for all $v_2$).

$TS_2 (1, 1) = \int_0^{1/2} \left( \left[ 1 - \left( 1 - Pr_2 (1, v_2) \right)^2 \right] (v_2 + TS_3 (1, 0)) + (1 - Pr_2 (1, v_2))^2 TS_3 (1, 1) \right) dv_2 + \int_{1/2}^1 \left( v_2 + TS_3 (1, 0) \right) dv_2 = \frac{19}{8} - \ln 4 \approx 0.989$,

where $Pr_2 (1, v_2) = \frac{4v_2}{1 + 3v_2}$ (by Lemma 5). If $v_2 \geq 1/2$, firms set price $1/2$ and one of them sells with certainty, that is, $R^t_A = 0, R^t_B = \emptyset$ and $R^t_D = 0, R^t_C = 0, R^t_B = 0, R^t_D = \emptyset$. Note that $TS_3 (1, 0) = TS_3 (0, 1)$.

Finally, $TS_2 (2, 0) = E \left( \Pi^*_2 (2) \right) = 1$ (see Lemma 1).