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MEAN-SEMIVARIANCE BEHAVIOR: AN ALTERNATIVE BEHAVIORAL MODEL

Javier Estrada *

* Professor of Financial Management, IESE

IESE Business School - Universidad de Navarra
Avda. Pearson, 21 - 08034 Barcelona. Tel.: (+34) 93 253 42 00 Fax: (+34) 93 253 43 43
Camino del Cerro del Águila, 3 (Ctra. de Castilla, km. 5,180) - 28023 Madrid. Tel.: (+34) 91 357 08 09 Fax: (+34) 91 357 29 13

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Abstract:

The most widely-used measure of an asset's risk, beta, stems from an equilibrium in which investors display mean-variance behavior. This behavioral criterion assumes that portfolio risk is measured by the variance (or standard deviation) of returns, which is a questionable measure of risk. The semivariance of returns is a more plausible measure of risk (as Markowitz himself admits) and is backed by theoretical, empirical, and practical considerations. It can also be used to implement an alternative behavioral criterion, mean-semivariance behavior, that is almost perfectly correlated to both expected utility and the utility of mean compound return.

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I. Introduction

Risk is a slippery concept and its proper definition, critical for academics and practitioners alike, is under continuous evolution. Though not free from controversy, the most widely-accepted definition of an asset's risk in a diversified portfolio is the asset's beta. This definition of risk, in turn, stems from an equilibrium in which investors display mean-variance behavior (MVB); that is, a model in which investors choose their optimal portfolio by maximizing a utility function that depends solely on the mean and variance of the portfolio returns.

Levy and Markowitz (1979) defended MVB as an approximately-correct criterion in the sense that it yields a level of utility highly correlated to an investor's expected utility.[1] However, when receiving his Nobel prize, Markowitz (1991) stated that "... it can further help evaluate the adequacy of mean and variance, *or alternate practical measures*, as criteria." (Emphasis added.) In addition, he stated that "[p]erhaps... some other measure of portfolio risk will serve in a two parameter analysis ... *Semivariance seems more plausible than variance as a measure of risk, since it is concerned only with adverse deviations*." (Emphasis added.)

In this article I follow Markowitz's suggestions and evaluate the plausibility of semivariance as a measure of risk, and of mean-semivariance behavior (MSB) as a behavioral criterion. More precisely, following Levy and Markowitz (1979), I evaluate whether MSB is an approximately-correct criterion in the sense that it yields a level of utility highly correlated to an investor's expected utility. I also outline several additional reasons for which semivariance is a better measure of risk than variance, and analyze the relationship between MSB and an alternative behavioral criterion, namely, the maximization of expected compound return.

The rest of this article is organized as follows. Part II tackles MVB and some extensions, as well as the Levy-Markowitz (1979) defense of this criterion. Part III provides a similar defense of MSB, suggests additional reasons that support both the MSB criterion and

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1 See, also, Pulley (1981, 1985), Kroll, Levy, and Markowitz (1984), and Reid and Tew (1986).

the semideviation as a measure of risk, and discusses the relationship between MSB and the maximization of expected compound return. Finally, part IV contains some concluding remarks. An appendix with exhibits and formulas concludes the article.

II. Mean-Variance Behavior (and Extensions)

It is a well-established result in modern financial theory that MVB is exactly consistent with expected utility maximization (EUM) under either one of two conditions: 1) That the investor's utility function is quadratic; or 2) that the returns of the investor's portfolio are jointly normally distributed. In either case, the optimal portfolio chosen by an investor who maximizes a utility function that depends on only two parameters, the mean and the variance of the portfolio returns, would be the same portfolio chosen by the investor if he maximized directly his expected utility function.

However, the plausibility of a quadratic utility function is questioned by the fact that it implies than an investor's absolute risk aversion is increasing with his wealth, although the opposite would be reasonably expected. Furthermore, the normality of returns is questioned by loads of data that display either skewness or kurtosis (or both).[2]

1) A Mean-Variance Approximation to Expected Utility

Not ready to give up on what would eventually become the standard behavioral criterion in modern financial theory, Levy and Markowitz (1979) moved to defend MVB from a different perspective: They asked whether an investor choosing a portfolio on the basis of mean and variance would *almost* maximize his expected utility. In other words, they did not question the implausibility of the conditions that make MVB *exactly* consistent with EUM; rather, they asked whether the simpler choice based on mean and variance would yield a level of expected utility *almost* equal to that obtained by a much more complicated direct maximization of the expected utility function.

Exhibit 1 in the appendix reproduces Table 1 in Markowitz (1991), which in turn is taken from Levy and Markowitz (1979). The exhibit shows the correlation coefficients between an investor's expected utility (*EU*) given by

$$EU = (1/T)\cdot \sum_{t=1}^{T} U(R_t), \qquad (1)$$

and his approximate expected utility based on MVB (AEU_{MVB}) given by

$$AEU_{MVB} = U(\mu) + (1/2)\cdot\sigma^2\cdot U''(\mu)\ , \qquad (2)$$

where U denotes the investor's utility function, R and T denote returns and the number of returns in the sample, and μ and σ^2 denote the mean and variance of returns.

2 Although monthly stock returns in developed markets do not seem to depart significantly from normality, high-frequency returns in these markets and returns in emerging markets do depart significantly from this assumption; see Estrada (2000 and 2001), Aparicio and Estrada (2001) and references therein.

As Exhibit 1 clearly shows, the correlation between expected utility and approximate expected utility is nearly perfect for three different utility functions and several parameter values. On the basis of this table, Levy and Markowitz (1979) and Markowitz (1991) conclude that MVB is a good approximation to EUM. Academics and practitioners seemed to agree, as they widely use beta as a measure of risk, which follows from an equilibrium in which investors display MVB.

2) Two Further Approximations to Expected Utility

The behavioral criterion proposed by Markowitz uses the first two terms of a Taylor approximation (around mean return) to expected utility. If an additional term is added to (2), we obtain an approximate expected utility based not just on mean and variance but also on skewness (AEU_{Skw}). Such an approximation is given by

$$AEU_{Skw} = U(\mu) + (1/2)\cdot\sigma^2\cdot U''(\mu) + (1/6)\cdot Skw\cdot U'''(\mu), \quad (3)$$

where *Skw* denotes the skewness in the returns of the investor's portfolio. The importance of skewness in the assessment of risk has been stressed by Leland (1999), Harvey and Siddique (2000), and Chen, Hong, and Stein (2001), among others.

Furthermore, if an additional term is added to (3), we then obtain an approximate expected utility based not just on mean, variance, and skewness but also on kurtosis (AEU_{Krt}). Such an approximation is given by

$$AEU_{Krt} = U(\mu) + (1/2)\cdot\sigma^2\cdot U''(\mu) + (1/6)\cdot Skw\cdot U'''(\mu) + (1/24)\cdot Krt\cdot U''''(\mu), \quad (4)$$

where *Krt* denotes the kurtosis in the returns of the investor's portfolio. The importance of both skewness and kurtosis in the assessment of risk has been stressed by Bekaert, Erb, Harvey, and Viskanta (1998) and Aparicio and Estrada (2001), among others.

3) Is MVB a Good Approximation to Expected Utility?

The data used to evaluate the relationship between expected utility and different approximations to expected utility is the entire MSCI database of both developed and emerging markets available at the end of the year 2000. This database contains monthly data on 22 developed markets and 28 emerging markets for varied sample periods, some starting as far back as Jan/1970. Summary statistics for all these markets, together with the earliest month for which data is available for each market, are reported in **Exhibit 2** in the appendix.

The utility functions and parameter values used in the evaluation are the same as those used by Levy and Markowitz (1979). I thus use a logarithmic utility function, $U = \ln(1+R)$; a power utility function, $U = (1+R)^a$, for several values of a; and a negative exponential utility function, $U = -e^{-b(1+R)}$, for several values of b. The details of the computations are provided in the appendix; the results are shown in Table 1 below, which reports correlation coefficients between expected utility and three different approximations to

[3] More recently, De Athayde (2001) proposes a nonparametric approach to derive a mean-semivariance efficient frontier, shows the convexity of this frontier, and derives asset pricing relationships with and without a risk-free rate.

expected utility based on mean-variance, mean-variance-skewness, and mean-variance-skewness-kurtosis.

Table 1: EUM, MVB, and Extensions

Utility Function	All markets AEU_{MVB}	All markets AEU_{Skw}	All markets AEU_{Krt}	Developed Markets AEU_{MVB}	Developed Markets AEU_{Skw}	Developed Markets AEU_{Krt}	Emerging Markets AEU_{MVB}	Emerging Markets AEU_{Skw}	Emerging Markets AEU_{Krt}
$U = \ln(1+R)$	0.996	0.997	0.999	1.000	1.000	1.000	0.996	0.997	0.999
$U = (1+R)^a$									
$a = 0.1$	0.997	0.998	0.999	1.000	1.000	1.000	0.997	0.998	0.999
$a = 0.3$	0.999	0.999	1.000	1.000	1.000	1.000	0.999	0.999	1.000
$a = 0.5$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$a = 0.7$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$a = 0.9$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$U = -e^{-b(1+R)}$									
$b = 0.1$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$b = 0.5$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$b = 1$	0.998	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000
$b = 3$	0.980	0.979	0.994	0.999	0.995	1.000	0.967	0.967	0.991
$b = 5$	0.951	0.910	0.954	0.991	0.966	0.996	0.928	0.869	0.930
$b = 10$	0.759	0.577	0.689	0.907	0.759	0.979	0.731	0.526	0.626
Averages	**0.973**	**0.955**	**0.970**	**0.991**	**0.977**	**0.998**	**0.968**	**0.946**	**0.962**

All numbers in the table show correlation coefficients between expected utility, given by (1), and approximate expected utility, given by (2), (3), and (4).

Focus on the AEU_{MVB} columns first, which indicate the correlation between expected utility and its mean-variance approximation. Consistent with the results reported by Levy and Markowitz (1979), MVB does seem to be a very good approximation to expected utility; all correlation coefficients are well above 0.900, with two exceptions for the highest value of the *b* coefficient of the negative exponential utility function (b=10). These columns also show that AEU_{MVB} seems to approximate expected utility in developed markets slightly better than it does in emerging markets.

The mean-variance approximation is so good that, as the AEU_{Skw} and AEU_{Krt} columns show, there is very little room for improvement. In fact, considering skewness, or both skewness and kurtosis, *worsens* the approximation to expected utility in a few cases. We can therefore conclude, unsurprisingly, that Markowitz's insight was correct: MVB does provide a good approximation to EUM.

III. Mean-Semivariance Behavior

I show in this part that MSB is an approximately-correct criterion in the same sense that Levy and Markowitz (1979) showed MVB to be approximately correct. I also provide some reasons for which the standard deviation is an implausible measure of risk, and some other reasons for which the semideviation is a plausible measure of risk. Finally, I argue that

MSB is also an approximately-correct criterion with respect to an alternative behavioral model, namely, the maximization of expected compound return.

1) The Semideviation

Although the standard deviation of returns is widely used as a measure of risk, several problems limit its usefulness. First, the standard deviation is an appropriate measure of risk only when the underlying distribution of returns is symmetric. Second, it can be applied straightforwardly as a risk measure only when the underlying distribution of returns is Normal. Third, the two previous conditions, symmetry and normality, are seriously questioned by the empirical evidence. And fourth, although widely used, the equilibrium measure of risk that follows from MVB, beta, is also seriously questioned by the empirical evidence.

An alternative measure of risk that has received recent support from both academics and practitioners (see references below), and that Markowitz supported from the start, is the downside standard deviation of returns, or semideviation for short, which for any benchmark return B can be denoted as Σ_B and is given by

$$\Sigma_B = \sqrt{E\{\text{Min}[(R-B)\,,0]^2\}}\ , \tag{5}$$

where R denotes returns. As can be noticed by simple inspection of (5), the semideviation gives a positive weight only to the deviations below the benchmark; that is, returns below B increase Σ_B, but returns above B do not. Essentially, the semideviation defines risk as volatility below the benchmark.[3]

Some reasons that support the plausibility of the semideviation as an appropriate measure of risk are discussed below in section 3). But before turning to them, let us follow the pathbreaking insight of Levy and Markowitz (1979) and ask whether an investor choosing portfolios on the basis of mean and semivariance would almost maximize his expected utility.

2) A Mean-Semivariance Approximation to Expected Utility

An approximation to expected utility based on mean and semivariance can be obtained by replacing the variance of returns in (2) by two times the semivariance of returns. Hence, an investor's approximate expected utility based on MSB (AEU_{MSB}) can be calculated with the expression

$$AEU_{MSB} = U(\mu) + (1/2)\cdot(2\Sigma^2)\cdot U''(\mu) = U(\mu) + \Sigma^2\cdot U''(\mu)\ . \tag{6}$$

The rationale for this approximation is the following. If the underlying distribution of returns is symmetric, then $2\cdot\Sigma^2 = \sigma^2$. In that case, both (2) and (6) would yield exactly the same level of (approximate) expected utility. However, when the underlying distribution is skewed, then $2\cdot\Sigma^2 \neq \sigma^2$, and (2) and (6) would yield different levels of (approximate)

[3] Throughout this article, I will use the arithmetic mean return (μ) of each distribution as the benchmark return B, and for ease of notation from now on I will write Σ_μ simply as Σ.

expected utility. More precisely, in the presence of negative skewness, $2\cdot\Sigma^2 > \sigma^2$ and $AEU_{MSB} < AEU_{MVB}$; and in the presence of positive skewness, $2\cdot\Sigma^2 < \sigma^2$ and $AEU_{MSB} > AEU_{MVB}$.

The details of all the computations are provided in the appendix; the results of the analysis are reported in Table 2 below.

Table 2: EUM and MSB

Utility Function	All Markets	Developed Markets	Emerging Markets
$U = \ln(1+R)$	0.989	0.999	0.991
$U = (1+R)^a$			
$a = 0.1$	0.992	0.999	0.993
$a = 0.3$	0.995	1.000	0.996
$a = 0.5$	0.998	1.000	0.998
$a = 0.7$	0.999	1.000	0.999
$a = 0.9$	1.000	1.000	1.000
$U = -e^{-b(1+R)}$			
$b = 0.1$	1.000	1.000	1.000
$b = 0.5$	0.997	1.000	0.997
$b = 1$	0.988	0.999	0.989
$b = 3$	0.982	0.995	0.972
$b = 5$	0.977	0.988	0.968
$b = 10$	0.825	0.918	0.822
Averages	**0.979**	**0.991**	**0.977**

All numbers in the table show correlation coefficients between expected utility, given by (1), and approximate expected utility, given by (6).

Table 2, similar in aim and scope to Exhibit 1 used by Levy and Markowitz (1979) to argue that MVB is an approximately-correct criterion with respect to expected utility, shows that MSB is also an approximately-correct criterion in the same sense. The three columns show correlation coefficients well above 0.900 in all cases, with two exceptions for the highest value of the *b* coefficient of the negative exponential utility function (b=10). These columns also show, as was the case for AEU_{MVB}, that AEU_{MSB} performs slightly better in developed markets than in emerging markets.

A comparison between Tables 1 and 2 above suggests that MSB provides an approximation to expected utility virtually identical to that provided by MVB for all utility functions and parameter values, in both developed markets and emerging markets. For high values of the *b* coefficient of the negative exponential utility function ($b \geq 3$), however, MSB tends to outperform the MVB. In fact, on average, the mean-semivariance approximation outperforms the mean-variance approximation in emerging markets and in the full sample of all markets. (Both approximations have the same average performance in developed markets.)

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[5] Throughout the article, all hypotheses are tested at the 5% significance level.

Although the correlations between expected utility and AEU_{MVB} are virtually identical to those between expected utility and AEU_{MSB}, an interesting question to ask is whether these two sets of correlations are statistically indistinguishable from each other. The answer to such question, however, is not straightforward for we run into the problem of comparing non-nested hypotheses; that is, when comparing the power of AEU_{MVB} and AEU_{MSB} to explain the variability of expected utility, AEU_{MSB} cannot be considered a special case of AEU_{MVB}.

Although there is no widely accepted test to determine whether two competing non-nested models have a significantly different explanatory power, the *J*-test proposed by Davidson and MacKinnon (1981) provides some evidence in the right direction. Consider the two competing models

$$H_0: \quad EU_i = \alpha_0 + \alpha_1 \cdot AEU_{MVB,i} + u_i \,, \tag{7}$$

$$H_1: \quad EU_i = \beta_0 + \beta_1 \cdot AEU_{MSB,i} + v_i, \tag{8}$$

where EU denotes expected utility, the αs and βs are coefficients to be estimated, and u and v are error terms.

The *J*-test consists of first estimating the αs and the βs; then generating the predicted values from equation (7), $\hat{\alpha}_0 + \hat{\alpha}_1 \cdot AEU_{MVB,i}$, and equation (8), $\hat{\beta}_0 + \hat{\beta}_1 \cdot AEU_{MSB,i}$; then running the regressions

$$EU_i = \alpha_0 + \alpha_1 \cdot AEU_{MVB,i} + \alpha_2 \cdot (\hat{\beta}_0 + \hat{\beta}_1 \cdot AEU_{MSB}) + u_i' \tag{9}$$

$$EU_i = \beta_0 + \beta_1 \cdot AEU_{MSB,i} + \beta_2 \cdot (\hat{\alpha}_0 + \hat{\alpha}_1 \cdot AEU_{MVB}) + v_i' \quad ; \tag{10}$$

and finally testing for the significance of α_2 and β_2. The idea is that if model H_0 is correct, then the fitted values of model H_1 should have no explanatory power in (9), and α_2 should not be significant when evaluated with the standard *t*-test. Similarly, if model H_1 is correct, then the fitted values of model H_0 should have no explanatory power in (10), and β_2 should not be significant when evaluated with the standard *t*-test.

Table 3 below reports the *p*-values for the *t*-tests on the significance of α_2 in (9) and of β_2 in (10) using all the markets in the sample. As can be seen from the table, α_2 is significant in all cases and β_2 is significant in all but one case. Thus, the *J*-tests indicate that neither approximation significantly outperforms the other when explaining the variability of expected utility, and that AEU_{MSB} significantly outperforms AEU_{MVB} in the case of the negative exponential utility function for b=10.

Table 3: *J*-Tests

	Log U	*Power U*					*Exponential U*					
	a=0.1	*a*=0.3	*a*=0.5	*a*=0.7	*a*=0.9	*b*=0.1	*b*=0.5	*b*=1	*b*=3	*b*=5	*b*=10	
α_2	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
β_2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.14

All numbers in the table show the *p*-values of the α_2 and β_2 coefficients from equations (9) and (10).

Essentially, three important points follow from Tables 1-3. First, that an investor who chooses portfolios on the basis of mean and semivariance would in fact almost maximize his expected utility. Second, that the approximation to expected utility provided by MSB is on average at least as good as that provided by MVB. And third, that MSB can be defended along the same lines used by Levy and Markowitz (1979) to defend MVB.

3) More on the Semideviation

Having established that MSB is an approximately-correct criterion in the sense that it yields a level of utility highly correlated to an investor's expected utility, let us now consider some additional reasons that support the plausibility of the semideviation as a measure of risk. Some of these reasons are practical, others are empirical.

From a practical point of view, first, investors obviously do not dislike upside volatility; they only dislike downside volatility. Second, the semideviation is more useful than the standard deviation when the underlying distribution of returns is asymmetric and just as useful when the underlying distribution is symmetric; in other words, the semideviation is at least as useful a measure of risk as the standard deviation. And third, the semideviation combines into one measure the information provided by two statistics, variance and skewness, thus making it possible to use a one-factor model to estimate required returns.

From an empirical point of view, the semideviation has been reported to explain the cross-section of returns of emerging markets (Estrada, 2000, and Harvey, 2000), the cross-section of industries in emerging markets (Estrada, 2001), and the cross-section of Internet stock returns (Estrada, 2002*a*). Additional support for the semideviation as an appropriate measure of risk can be found in Sortino and van der Meer (1991), Clash (1999), and Sortino, van der Meer, and Plantinga (1999), among others.

4) Maximizing Expected Compound Return

All the arguments considered in the previous sections are based on the assumption that maximizing expected utility is the correct behavioral criterion for investors. However, an alternative plausible criterion for investors is to maximize the *expected compound return* of their portfolio, a strategy sometimes associated with long-term investing and discussed at length by Markowitz (1959).[4]

Hakansson (1971), and more recently Booth and Fama (1992) and Wilcox (1997, 1998), also support the maximization of expected compound return, an approach that essentially consists of maximizing the geometric mean return of a portfolio (or a portfolio's "rate of growth," in Markowitz's words).

Let *R* and *r* denote simple (arithmetic) and logarithmic (continuously-compounded) returns, respectively, and let μ and σ^2 be the mean and variance of *R*; then, by definition, $r=\ln(1+R)$. Furthermore, approximating by Taylor the expected value of *r* around μ we obtain

$$E(r) = E\{\ln(1+R)\} = \ln(1+\mu) - \frac{(1/2)\cdot\sigma^2}{(1+\mu)^2} + \frac{(1/3)\cdot Skw}{(1+\mu)^3} - \frac{(1/4)\cdot Krt}{(1+\mu)^4} + \ldots \quad . \tag{11}$$

[4] Markowitz (1959) in fact devotes an entire chapter of his book (chapter VI, "Return in the Long Run") to this issue, plus an additional chapter ("Note on Chapter VI") in his 1991 revision of the same book.

Expression (11) nicely shows why investors like mean return and positive skewness and dislike variance and kurtosis; these last two produce a drag on expected compound return. This expression also shows that the maximization of expected compound return implies a logarithmic utility function for terminal wealth (compare (11) with (A11) in the appendix), which adds to the plausibility of this criterion.

Table 4 below shows the correlation between the utility of mean compound return, computed as $U = U(1+g)$, where g is geometric mean return of R, and both the mean-variance approximation (AEU_{MVB}) and the mean-semivariance approximation (AEU_{MSB}).[5] These correlations show that both MVB and MSB are very good approximations to the utility of mean compound return. These correlations also show, just like those of Tables 1 and 2, that the average performance of MSB is better than the average performance of MVB. And they also show that this average performance is better in *both* developed markets *and* emerging markets.

Table 4: Maximizing Expected Compound return, MVB, and MSB

Utility	All markets		Developed Markets		Emerging Markets	
Function	AEU_{MVB}	AEU_{MSB}	AEU_{MVB}	AEU_{MSB}	AEU_{MVB}	AEU_{MSB}
$U = \ln(1+R)$	0.996	0.989	1.000	0.999	0.996	0.991
$U = (1+R)^a$						
$a = 0.1$	0.995	0.981	0.999	0.998	0.995	0.985
$a = 0.3$	0.977	0.956	0.996	0.993	0.980	0.967
$a = 0.5$	0.940	0.920	0.988	0.985	0.954	0.942
$a = 0.7$	0.892	0.878	0.977	0.975	0.921	0.913
$a = 0.9$	0.837	0.832	0.963	0.963	0.884	0.881
$U = -e^{-b(1+R)}$						
$b = 0.1$	0.837	0.833	0.963	0.963	0.884	0.881
$b = 0.5$	0.943	0.923	0.988	0.986	0.957	0.944
$b = 1$	0.996	0.991	1.000	0.999	0.995	0.992
$b = 3$	0.551	0.694	0.801	0.846	0.462	0.653
$b = 5$	0.352	0.468	0.529	0.590	0.199	0.361
$b = 10$	0.263	0.341	0.258	0.299	0.086	0.198
Averages	**0.798**	**0.817**	**0.872**	**0.883**	**0.776**	**0.809**

All numbers in the table show correlation coefficients between the utility of mean compound return, given by $U = U(1+g)$, and approximate expected utility, given by (2) and (6).

In sum, the whole analysis supports the idea that an investor that maximizes a utility function that depends on mean and semivariance would also maximize both expected utility and the utility of expected compound return. This finding, plus the practical and empirical

[5] Exhibit 4 does not show, but it is the case, that the correlation between expected utility and the utility of mean compound return is very high (above 0.9) for the three utility functions and all parameter values (with the exception of the negative exponential utility function for values of $b \geq 3$). In other words, the criteria of maximizing expected utility and maximizing mean compound return are very similar to each other.

considerations discussed above, make the semideviation an ideal variable for a two-parameter utility function and a behavioral model from which further results and implications could be derived.

IV. Conclusions

The most-widely used measure of an asset's risk, beta, follows from an equilibrium in which investors display MVB. Levy and Markowitz (1979) defended this criterion as approximately correct in the sense that it yields a level of utility almost equal to an investor's expected utility. Markowitz (1991) reaffirmed this view and in fact considered the issue important enough to make it the central topic of his Nobel prize lecture.

This article shows that MSB can be defended with the same arguments that Levy and Markowitz (1979) used to defend MVB. It also provides additional considerations, some practical and some empirical, that support the semideviation as a more plausible measure of risk than the standard deviation. And it reports results showing that MSB is not only consistent with the maximization of expected utility but also with the maximization of the utility of expected compound return. Those considerations and results should round up the reasons for which MSB is a more plausible criterion than MVB. Essentially, this article agrees with Markowitz in that "semivariance seems more plausible than variance as a measure of risk."

A fair question to ask is: If MSB is the correct behavioral model, then what is in this framework the appropriate measure of risk of an asset in a diversified portfolio? In other words, what is the counterpart of beta in a downside risk framework? It turns out that a "downside beta" can be defined and articulated into a one-factor model, similar to the CAPM, that can be used to generate required returns. And it turns out that this downside beta explains the cross-section of stock returns better than beta; see Estrada (2002*b*,*c*).

[6] The 5.11% risk-free rate is based on the yield of 10-year U.S. Treasury Notes at the end of the year 2000. The 5.5% world market risk premium is similar to that used by Stulz (1995).

Appendix

Exhibit 1

Expected Utility and Approximate Expected Utility

Utility Function	Annual Returns 149 Mutual Funds	Annual Returns 97 Stocks	Monthly Returns 97 Stocks	Random Portfolios 5/6 Stocks
$U = \ln(1+R)$	0.997	0.880	0.995	0.998
$U = (1+R)^a$				
$a = 0.1$	0.998	0.895	0.996	0.998
$a = 0.3$	0.999	0.932	0.998	0.999
$a = 0.5$	0.999	0.968	0.999	0.999
$a = 0.7$	0.999	0.991	0.999	0.999
$a = 0.9$	0.999	0.999	0.999	0.999
$U = -e^{-b(1+R)}$				
$b = 0.1$	0.999	0.999	0.999	0.999
$b = 0.5$	0.999	0.961	0.999	0.999
$b = 1$	0.997	0.850	0.997	0.998
$b = 3$	0.949	0.850	0.976	0.958
$b = 5$	0.855	0.863	0.961	0.919
$b = 10$	0.449	0.659	0.899	0.768

ource: Markowitz (1991), Table 1. All numbers in the table show correlation coefficients between expected utility, given by (1), and approximate expected utility, given by (2).

Exhibit 2

Summary Statistics (Monthly Stock Returns)

Developed Markets					*Emerging Markets*				
Market	μ	σ	Σ	Start	Market	μ	σ	Σ	Start
Australia	0.93	7.21	5.26	Jan/70	Argentina	3.16	17.94	10.02	Jan/88
Austria	0.91	6.08	4.00	Jan/70	Brazil	3.22	17.78	11.85	Jan/88
Belgium	1.29	5.48	3.77	Jan/70	Chile	1.87	7.54	5.20	Jan/88
Canada	1.00	5.55	4.05	Jan/70	China	-0.60	12.93	7.88	Jan/93
Denmark	1.26	5.42	3.71	Jan/70	Colombia	-0.14	9.47	6.46	Jan/93
Finland	1.94	8.60	5.70	Jan/88	Czech Rep.	0.26	9.48	6.81	Jan/95
France	1.25	6.60	4.69	Jan/70	Egypt	1.23	8.79	5.14	Jan/95
Germany	1.14	5.90	4.22	Jan/70	Greece	1.87	11.46	6.65	Jan/88
Hong Kong	2.04	11.31	7.66	Jan/70	Hungary	2.04	12.35	8.50	Jan/95
Ireland	1.07	5.73	3.96	Jan/88	India	0.65	8.95	6.04	Jan/93
Italy	0.90	7.57	5.13	Jan/70	Indonesia	1.34	17.31	9.89	Jan/88
Japan	1.22	6.63	4.56	Jan/70	Israel	1.14	7.51	5.31	Jan/93
Netherlands	1.38	5.14	3.73	Jan/70	Jordan	-0.02	4.49	3.12	Jan/88
New Zealand	0.29	7.00	4.73	Jan/88	Korea	0.67	12.41	7.54	Jan/88
Norway	1.21	7.74	5.47	Jan/70	Malaysia	0.97	10.26	6.97	Jan/88
Portugal	0.61	6.74	4.47	Jan/88	Mexico	2.45	10.56	7.79	Jan/88
Singapore	1.15	8.52	5.96	Jan/88	Morocco	1.00	4.78	3.32	Jan/93
Spain	1.03	6.52	4.58	Jan/70	Pakistan	0.16	11.96	8.13	Jan/93
Sweden	1.50	6.55	4.60	Jan/70	Peru	0.89	9.85	6.79	Jan/93
Switzerland	1.24	5.49	3.91	Jan/70	Philippines	0.87	10.45	7.05	Jan/88
UK	1.24	6.87	4.52	Jan/70	Poland	3.19	18.52	10.48	Jan/93
USA	1.08	4.44	3.22	Jan/70	Russia	3.46	23.61	16.14	Jan/95
Average	**1.17**	**6.69**	**4.63**	**N/A**	South Africa	1.03	8.17	5.95	Jan/93
					Sri Lanka	-0.40	9.47	6.65	Jan/93
					Taiwan	1.23	12.34	8.14	Jan/88
					Thailand	0.67	12.64	8.81	Jan/88
					Turkey	2.54	18.39	11.31	Jan/88
					Venezuela	1.52	15.19	10.70	Jan/93
					Average	**1.30**	**11.95**	**7.81**	**N/A**

μ: Mean return; σ: Standard deviation; Σ: Semideviation. All numbers in %. Data through Dec/2000.

Expected Utility and Approximate Expected Utility Calculations

I briefly discuss here the expressions used to evaluate the relationship between EUM, MVB, and MSB. For all three utility functions, an investor's expected utility (*EU*) is defined as in (1); that is,

$$EU = (1/T)\cdot \sum_{t=1}^{T} U(R_t) \text{ ,} \tag{A1}$$

Exhibit 2 (continued)

where U denotes the investor's utility function, and R and T denote returns and the number of returns in the sample, respectively. It thus follows that the expected utility of an investor who has a logarithmic, power, or negative exponential utility function is respectively given by

$$EU = (1/T)\cdot\sum_{t=1}^{T}\ln(1+R_t)\ , \tag{A2}$$

$$EU = (1/T)\cdot\sum_{t=1}^{T}(1+R_t)^a\ , \tag{A3}$$

$$EU = (1/T)\cdot\sum_{t=1}^{T}-e^{-b(1+R_t)}\ . \tag{A4}$$

Let μ, σ^2, and Σ^2 be the mean, variance, and semivariance of any given series of returns, respectively. The approximate expected utility of an investor who displays MVB (AEU_{MVB}) and has a logarithmic, power, or negative exponential utility function is respectively given by

$$AEU_{MVB} = \ln(1+\mu) - \frac{(1/2)\cdot\sigma^2}{(1+\mu)^2}\ , \tag{A5}$$

$$AEU_{MVB} = (1+\mu)^a + (1/2)\cdot\sigma^2\cdot a(a\text{-}1)(1+\mu)^{a-2}\ , \tag{A6}$$

$$AEU_{MVB} = -e^{-b(1+m)} - (1/2)\cdot\sigma^2\cdot b^2\cdot e^{-b(1+m)}\ . \tag{A7}$$

Furthermore, let Skw and Krt be the moments of skewness and kurtosis, respectively, of any given series of returns. Then an approximate expected utility based on mean, variance, and skewness (AEU_{Skw}) of an investor that displays a logarithmic, power, or negative exponential utility function is respectively given by

$$AEU_{Skw} = \ln(1+\mu) - \frac{(1/2)\cdot\sigma^2}{(1+\mu)^2} + \frac{(1/3)\cdot Skw}{(1+\mu)^3}\ , \tag{A8}$$

$$AEU_{Skw} = (1+\mu)^a + (1/2)\cdot\sigma^2\cdot a(a\text{-}1)(1+\mu)^{a-2} + (1/6)\cdot Skw\cdot a(a\text{-}1)(a\text{-}2)(1+\mu)^{a-3}\ , \tag{A9}$$

$$AEU_{Skw} = -e^{-b(1+m)} - (1/2)\cdot\sigma^2\cdot b^2\cdot e^{-b(1+m)} + (1/6)\cdot Skw\cdot b^3\cdot e^{-b(1+\mu)}\ . \tag{A10}$$

An approximate expected utility based on mean, variance, skewness, and kurtosis (AEU_{Krt}) of an investor that displays a logarithmic, power, or negative exponential utility function, on the other hand, is respectively given by

$$AEU_{Krt} = \ln(1+\mu) - \frac{(1/2)\cdot\sigma^2}{(1+\mu)^2} + \frac{(1/3)\cdot Skw}{(1+\mu)^3} - \frac{(1/4)\cdot Krt}{(1+\mu)^4}\ , \tag{A11}$$

Exhibit 2 (continued)

$$AEU_{Krt} = (1+\mu)^{a} + (1/2)\cdot\sigma^{2}\cdot a(a\text{-}1)(1+\mu)^{a-2} + (1/6)\cdot Skw\cdot a(a\text{-}1)(a\text{-}2)(1+\mu)^{a-3} + + (1/24)\cdot Krt\cdot a(a\text{-}1)(a\text{-}2)(a\text{-}3)(1+\mu)^{a-4} , \qquad \text{(A12)}$$

$$AEU_{Krt} = -e^{-b(1+m)} - (1/2)\cdot\sigma^{2}\cdot b^{2}\cdot e^{-b(1+m)} + (1/6)\cdot Skw\cdot b^{3}\cdot e^{-b(1+m)} - (1/24)\cdot Krt\cdot b^{4}\cdot e^{-b(1+m)} . \qquad \text{(A13)}$$

Finally, the approximate expected utility of an investor who displays MSB (AEU_{MSB}) and has a logarithmic, power, or negative exponential utility function is respectively given by

$$AEU_{MSB} = \ln(1+\mu) - \frac{Ó^{2}}{(1+\mu)^{2}} \quad , \qquad \text{(A14)}$$

$$AEU_{MSB} = (1+\mu)^{a} + \Sigma^{2}\cdot a(a\text{-}1)(1+\mu)^{a-2} , \qquad \text{(A15)}$$

$$AEU_{MSB} = -e^{-b(1+m)} - \Sigma^{2}\cdot b^{2}\cdot e^{-b(1+m)} . \qquad \text{(A16)}$$

In order to evaluate the relationship between MVB, MSB, and the maximization of expected compound return, the expressions for AEU_{MVB} and AEU_{MSB} are the same as above. The expressions for the utility of mean compound return for an investor that displays a logarithmic, power, or negative exponential utility function, on the other hand, are respectively given by

$$U = \ln(1+g), \qquad \text{(A17)}$$

$$U = (1+g)^{a}, \qquad \text{(A18)}$$

$$U = -e^{-b(1+g)}, \qquad \text{(A19)}$$

where *g* is the geometric mean of *R*.

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