Stress Testing and Bank Lending*

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Abstract

Bank stress tests are a major form of regulatory oversight. Banks respond to the strictness of the tests by changing their lending behavior. As regulators care about bank lending, this affects the design of the tests and creates a feedback loop. We demonstrate that stress tests may be (1) lenient, in order to encourage lending in the future, or (2) tough, in order to reduce the risk of costly bank defaults. There may be multiple equilibria. Regulators may strategically delay stress tests. We also analyze bottom-up stress tests and banking supervision exams.

Keywords: Bank regulation, stress tests, bank lending

JEL Codes: G21, G28

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1 Introduction

Stress tests are a new policy tool for bank regulators that were first used in the recent financial crisis and have become regular exercises subsequent to the crisis. They are assessments of a bank's ability to withstand adverse shocks, and are generally accompanied by requirements intended to boost the capital of those banks who have been found to be at risk.

Naturally, bank behavior reacts to stress testing exercises. Acharya, Berger, and Roman (2018) find that all banks that underwent the U.S. SCAP and CCAR tests reduced their risk by raising loan spreads and decreasing their commercial real estate credit and credit card loans activity.[1]

Regulators should take into account the reaction of banks when conducting the tests. One might posit that if regulators want to boost lending, they might make stress tests more lenient. Indeed, in the case of bank ratings, Agarwal et al. (2014) show that state level banking regulators rate banks more leniently than federal regulators due to concerns over the local economy and this may lead to more bank failures.

In this paper, we study the feedback effect between stress testing and bank lending. Banks may take excess risk or not lend enough to the real economy. Regulators react with either a lenient or tough approach. We demonstrate that this behavior may be self-fulfilling and result in coordination failures. A regulator may prefer to conduct an uninformative stress test or to strategically delay the test. We show that when capital is more available or the bank is more systemic, stress tests will be more informative. We compare stress tests to banking supervision exams, and show that the latter's relative informativeness depends on lending externalities.

In the model, there are two sequential stress testing exercises. For simplicity, there is one bank that is tested in both exercises. Each period, the bank decides whether to make a risky loan or to invest in a risk free asset. The regulator can observe the quality of the risky loan, and may require the bank to raise capital (which we denote as "failing" the stress test)

[1] Connolly (2017) and Calem, Correa, and Lee (2017) have similar findings.

or not. Therefore, stress tests in the model are about gathering information and boosting capital, rather than relaying information to the market. This is in line with the annual exercises during non-crisis times, where runs are an unlikely response to stress test results.

In the model, there is uncertainty about whether a regulator is lenient. A lenient regulator conducts uninformative stress test exercises.[2] A strategic regulator may decide to fail a bank depending on its incentives; it trades off the cost of foregone credit with the benefit of reducing costly default. After the first stress test result, the bank updates its beliefs about the preferences of the regulator, decides whether to make a risky loan, and undergoes a second stress test. Thus, the regulator's first stress test serve two roles: to possibly boost capital for the bank in the first period and to signal the regulator's willingness to force the bank to raise capital in the second period.

Banks may take too much or too little risk from the regulator's point of view. On the one hand, the bank will take too much risk due to its limited downside. On the other hand, the bank may take too little risk when potential capital raising requirements extract rents from the bank's owners.[3] The bank's choice affects the leniency of the stress test and the leniency of the stress test affects the bank's choice.

The regulator faces a natural trade-off in conducting the first stress test:

First, the regulator may want to build a reputation for being lenient, which can increase lending by the bank in the second period. Since lenient regulators prefer not to require banks to raise capital, there is an equilibrium where the regulator builds the perception that it is lenient by passing a bank that should fail. This is reminiscent of EU's 2016 stress test, where the pass/fail grading scheme was eliminated and only one bank was found to be undercapitalized.[4] The desire for lenience may be sufficiently large that a regulator may

[2]In the text, we demonstrate that the results can be qualitatively similar if the uncertainty was about whether the regulator were tough (i.e. it always failed banks).

[3]Thakor (1996) provides evidence that the adoption of risk-based capital requirements under Basel I and the passage of FDICIA in 1991 led to banks substituting risky lending with Treasury investments potentially prolonging the economic downturn.

[4]That bank, Monte dei Paschi di Siena, had already failed the 2014 stress test and was well known by the market to be in distress.

prefer not to conduct stress tests or strategically delay them.

Second, the regulator may want to build a reputation for being tough, which can prevent future excess risk-taking. This leads to an equilibrium where it is tough in its first stress test: it fails banks that should pass. Post-crisis, the U.S. has routinely been criticized for being too strict: imposing very adverse scenarios, not providing the model to banks, accompanying the test with asset quality reviews, and conducting qualitative reviews all feed into a stringent test.

Finally, there is one more type of equilibrium, where the regulator doesn't have reputation concerns and acts in accordance with its information.

Intriguingly, there may be multiple equilibria, leading to a natural coordination failure. This occurs due to a strategic complementarity between the strategic regulator's choice of leniency in the first stress test and the bank's second period risk choice. The less likely the strategic regulator is to pass the bank in the first period, the more risk the bank in the second period takes when it observes a pass (as it believes the regulator is lenient). This prompts the strategic regulator to be even tougher, and leads to a self-fulfilling equilibrium. This implies that the presence of stress tests may introduce fragility in the form of excess default or reduced lending to the real economy.

This naturally raises the issue of how a particular equilibrium may be chosen. One way might be if regulators can commit ex-ante to how to use information provided to them (as in games of Bayesian Persuasion). In practice, this might mean announcing stress test scenarios in advance or allowing banks to develop their own scenarios. The regulator might also take costly actions to commit by auditing bank data (e.g. asset quality reviews).

In stress testing exercises, by examining the banking system, a regulator may uncover information about liquidity and systemic linkages that individual banks may be unaware of. We model this as the private information of the regulator. However, we also analyze the case where the regulator only uncovers information about asset quality that the bank already knows. This is similar to banking supervision exams or "bottom-up" stress tests where the regulator allows the bank to perform the test (as in Europe). We find that results may be more or less informative depending on the value the regulator places on lending vs. costly defaults.

In the model, uncertainty about the regulator's preferences plays a key role. Given that (i) increased lending may come with risk to the economy and (ii) bank distress may have systemic consequences, there is ample motivation to keep this information/intention private. This uncertainty may also arise from the political process. Decisionmaking may be opaque, bureaucratic, or tied up in legislative bargaining.[5] Meanwhile, governments with a mandate to stimulate the economy may respond to lobbying from various interest groups or upcoming elections.[6]

There is little direct evidence of regulators behaving strategically during disclosure exercises, but much indirect evidence. The variance in stress test results to date seem to support the idea of regulatory discretion.[7] Beyond Agarwal et. al. (2014) cited above, Bird et al. (2015) show U.S. stress tests were lenient towards large banks and strict with poorly capitalized banks, affecting bank equity issuance and payout policy. The recent Libor scandal revealed that Paul Tucker, deputy governor of the Bank of England, made a statement to Barclays' CEO that was interpreted as a suggestion that the bank lower its Libor submissions.[8] Hoshi and Kashyap (2010) and Skinner (2008) discuss accounting rule changes that the government of Japan used to improve the appearance of its financial institutions during the country's crisis.[9]

[5]Shapiro and Skeie (2015) provide examples of related uncertainty around bailouts during the financial crisis.

[6]Thakor (2014) discusses the political economy of banking.

[7]The 2009 U.S. SCAP was widely perceived as a success (Goldstein and Sapra, 2014), with subsequent U.S. tests retaining credibility. European stress tests have varied in perceived quality (Schuermann, 2014) with the early versions so unsuccessful that Ireland and Spain hired independent private firms to conduct stress tests on their banks.

[8]The CEO of Barclays wrote notes at the time on his conversation with Tucker, who reportedly said, "It did not always need to be the case that [Barclays] appeared as high as [Barclays has] recently." This quote and a report on what happened appear in the *Financial Times* (B. Masters, G. Parker, and K. Burgess, Diamond Lets Loose Over Libor, *Financial Times*, July 3, 2012).

[9]Nevertheless, stress tests do contain significant information that is valued by markets (Flannery, Hirtle, and Kovner (2017) demonstrate this and survey recent evidence).

Theoretical Literature

There are a few papers on reputation management by a regulator. Morrison and White (2013) argue that a regulator may choose to forbear when it knows that a bank is in distress, because liquidating the bank may lead to a poor reputation about the ability of the regulator to screen and trigger contagion in the banking system.[10] Boot and Thakor (1993) also find that bank closure policy may be inefficient due to reputation management by the regulator, but this is due to the regulator being self-interested rather than being worried about social welfare consequences as in Morrison and White (2013). Shapiro and Skeie (2015) show that a regulator may use bailouts to stave off depositor runs and forbearance to stave off excess risk taking by banks. They define the type of the regulator as the regulator's cost of funding. We also have potential contagion through reputation as in these papers, but model the bank's lending decision and how it interacts with the choice of the regulator to force banks to raise capital. Furthermore, we define the regulator's type as whether it lenient (uninformative) or strategic.

There are several recent theoretical papers on regulatory disclosure and stress tests. Goldstein and Sapra (2014) survey the stress test and disclosure literature to describe the costs and benefits of information provision. Prescott (2008) argues that more information disclosure by a bank regulator decreases the amount of information that the regulator can gather on banks. Bouvard, Chaigneau, and de Motta (2015) show that transparency is better in bad times and opacity is better in good times. Goldstein and Leitner (2018) find a similar result in a very different model where the regulator is concerned about risk sharing (the Hirshleifer effect) between banks. Williams (2017) looks at bank portfolio choice and liquidity in this context. Orlov, Zryumov, and Skrzypacz (2018) show that the optimal stress test will test banks sequentially. Faria-e-Castro, Martinez, and Philippon (2016) demonstrate that stress tests will be more informative when the regulator has a strong fiscal position (to stop runs). In contrast to these papers, in our model *reputational incentives* drive the regulator's

[10]Morrison and White (2005) also model reputation as the ability of the regulator to screen, but do not consider the effect of ex-post learning about the type of the regulator.

choices. In addition, we incorporate capital requirements as a key element of stress testing and focus on banks' endogenous choice of risk. We also don't allow the regulator to commit to a disclosure rule (as all of the papers except for Prescott (2008) and Bouvard, Chaigneau, and de Motta (2015) do).

Our paper identifies the regulator's reputation concern as a source of fragility in the banking sector. In a different context, Ordonez (2013, 2017) show banks' reputation concerns, which provides discipline to keep banks from taking excessive risk, can lead to fragility and a crisis of confidence in the market.

2 The model

We consider a model with three types of risk-neutral agents: the regulator, the bank and a capital provider. The model has two periods $t \in \{1, 2\}$ and the regulator conducts a stress test for the bank in each period. We assume that the regulator has a discount factor $\delta \geq 0$ for the payoffs from the second period bank, where δ may be larger than 1 (as, e.g., in Laffont and Tirole, 1993). The discount factor captures the relative importance of the future of the banking sector for the regulator. For simplicity, we do not allow for discounting within a period.

We now provide a very basic timeline of each period, and then proceed in the following subsections to discuss each aspect in detail. In each period t, where $t = \{1, 2\}$, there are three stages:

1. Bank investment choice;
2. Stress test and (possible) recapitalization;
3. Payoffs realize.

2.1 Banks

At stage 1, the bank raises one unit of fully insured deposits, which matures at stage 3.[11] The bank has access to two possible investments. The first is a safe asset that returns $R_0 > 1$ at stage 3. The second is a risky loan, whose quality q_t can be good (g) or bad (b), where the prior probability that the loan is good is denoted by α. A good loan ($q_t = g$) repays R with probability 1 at stage 3, whereas a bad loan ($q_t = b$) repays R with probability $1 - d$ and 0 otherwise at stage 3. We assume that the expected return of the risky loan is higher than that of the safe investment, representing the risk-return trade-off:

Assumption 1. $[\alpha + (1-\alpha)(1-d)]\, R > R_0$.

In order to focus on the reputation building incentives of the regulator when conducting the stress test in the first period, we make the simplifying assumption that in the first period, the bank has extended the risky loan.[12]

2.2 Stress testing

We assume that the credit quality of the risky loan is only learned by the regulator (through the stress test). In Section 6, we demonstrate that the main results don't change if the bank also knows this information. The regulator could have generated private information from having done a stress test on many banks. In this case, it may have gathered more information on asset values/liquidity and/or understand more about systemic risk (not modelled here). This is an element of the macroprudential role of stress tests.

The regulator conducts the stress test at stage 2 by first observing the quality q_t of the bank's risky loan, and then decides whether to require the bank to raise capital. We will henceforth refer to the regulatory action of requiring the bank to raise capital as "failing",

[11] In Section 7.2, we remove the assumption that deposits are fully insured and allow the bank's liabilities to be priced by the market. All of our qualitative results remain.

[12] Allowing endogenous loan origination effort in the first period does not alter the reputation building incentives we demonstrate in Section 4.

and not requiring the bank to raise capital as "passing".[13] The regulator's objective function is to maximize social welfare (which we detail below).

The stress test in the model therefore is not about conveying information to the market about the health of the banks. The test provides the regulator with information on the bank's health, which the regulator uses by requiring recapitalizations. Nevertheless, the stress test accompanied by the recapitalizations does convey information.[14] This information is about the private information of the regulator (the regulator has preferences over bank risk taking; these are defined below). In the second period, the bank reacts to the information inferred from the first period stress test, forming the basis of the reputation mechanism.

2.3 Recapitalization

If a bank fails the stress test, we assume that the bank is required to raise one unit of capital kept in costless storage with zero net return so that the bank with the risky loan will not default at stage 3 even if its borrower does not repay.[15] There is a capital provider who can fund the bank. We assume that the capital provider's outside option for its funding is an alternative investment that produces a total return of $\rho > 1$, which we call the opportunity cost of capital. The opportunity cost of capital is high ($\rho = \rho_H$) with probability γ, and low ($\rho = \rho_L$) with probability $1 - \gamma$. We assume that the ρ is realized after the stress test when the bank approaches the capital provider for funds, and is publicly observable.

We make the following assumption about the expected return on the risky loan:

Assumption 2. *$R < \rho_H$ and $(1 - d)R \geq \rho_L$.*

This assumption implies that recapitalization is only feasible with probability $1 - \gamma$, when the opportunity cost of capital is low. First, if the opportunity cost of capital is high,

[13]To be precise, a "fail" is an announced requirement for the bank to recapitalize. We will allow for recapitalizations to be attempted but not work out, which we still consider a fail.

[14]The stress test results themselves are cheap talk in the model, but the recapitalizations incur costs (and benefits) for the regulator, making signalling possible.

[15]We assume capital earns zero net return for simplicity. The results do not change if capital is reinvested in the safe investment with a return $R_0 > 1$.

then the expected value of a good loan is lower than than the capital provider's outside option. This also implies that recapitalization is infeasible for the bad loan. Second, if the opportunity cost of capital is low, then the expected value of a bad loan is higher than the capital providers' outside option. This also implies that recapitalization is feasible for the good loan.

We assume that the capital provider has some bargaining power β due to the scarcity of capital, enabling it to capture a fraction of the expected surplus of the bank. Raising capital thus results in a (private) dilution cost for the bank's owners. The banking literature generally views equity capital raising as costly for banks (for a discussion, see Diamond (2017)). We model this cost as dilution due to the bargaining power of a capital provider, which fits our scenario of a public requirement by a regulator, though other mechanisms that impose a cost on the bank when trying to shore up capital would also work.[16] We make the following assumption on the effect of recapitalization:

Assumption 3. $\left[\alpha + \gamma(1-\alpha)(1-d)\right](R-1) < R_0 - 1.$

This assumption implies that the bank's owners may not find it worthwhile to originate the risky loan, because the expected payoff may be lower than that from investing in the safe asset ($R_0 - 1$). This is because, if the bank originates a risky loan, and the loan is good (with probability α), the bank's owners receive at most the residual payoff of $R - 1$ after repaying the debtholders; if the loan is bad, however, the bank's owners may not receive the value of the loan because it may have to raise capital (with probability $1 - \gamma$).

2.4 Regulatory preferences

We now examine the externalities from risky lending that affect social welfare and, hence, the regulator's preferences.

[16]For example, the bank may be forced to sell assets at fire-sale prices. This is a loss in value for the bank. And those who are purchasing the assets are distorting their investment decisions, as in our model. Hanson, Kashyap, and Stein (2011) discuss this effect and review the literature on fire sales.

There are two social costs of risky lending. The first is the cost to society of a bank default at stage 3. Specifically, if a bank operates without being recapitalized and the borrower repays 0 at stage 3, the bank defaults and a social cost to society D is incurred. The cost of bank default may represent cost of financing the deposit insurance payout,[17] the loss of value from future intermediation the bank may perform, the cost to resolve the bank, or the cost of contagion.

The second social cost of risky lending is the capital provider's opportunity cost; the alternative investment that goes unfunded when the capital provider recapitalizes the bank. This is only incurred if $\rho = \rho_L$.

We make the following assumption about the social costs of risky lending.

Assumption 4. $dD > \rho_L - 1 > 0$.

This assumption states that a strategic regulator finds that it is beneficial to recapitalize a bank whose risky loan is known to be bad, but not a bank whose risky loan is known to be good.

Finally, we add one more potential externality, which we call the social benefit of risky lending: if the bank originates a risky loan at stage 1, it generates a positive externality equal to B. Broadly, increased credit is positively associated with economic growth and income for the poor (both across countries and U.S. states, see Demirgüç-Kunt and Levine, 2018).[18]

2.5 Regulator reputation

The regulator can be one of two types, a strategic type or a lenient type. The strategic type trades off the social benefits and costs associated with recapitalization when deciding whether to fail a bank. The lenient type is behavioral and always passes the bank. A lenient type could also be considered an uninformative type, as its test does not screen banks. Agents

[17] The deposit insurance payout would be costly if (i) deposit insurance wasn't fairly priced, or (ii) there is a cost (e.g., political) of using the deposit insurance fund.

[18] Moskowitz & Garmaise (2006) provide causal evidence the social effects of credit allocation such as reduced crime.

may view this type as not conducting "serious" stress test exercises. The behavior of the lenient type regulator can be microfounded by a high social net benefit of risky lending.[19] In subsection 7.1.2, we demonstrate that our qualitative results still hold if we replace the behavioral lenient type with a behavioral tough type who always fails banks and recapitalizes them.

The regulator knows its own type, but during the stress testing of bank t (where $t = \{1, 2\}$), the owners of the bank and the capital provider are uncertain about the regulator's type. These agents have a belief that, with probability $1 - z_t$, the regulator is strategic. With probability z_t, the regulator is believed to be a lenient type. In our model, z_1 is the probability that nature chooses the regulator to be a lenient type. The term z_2 is the updated belief that the regulator is a lenient type after the first period stress test.

2.6 Summary of timing

The regulator conducts stress testing of the bank in first period, and then in the second period if the bank has not defaulted in the first period. If the bank defaults in the first period, the bank is closed down and does not continue into the second period. At the beginning of the second period, the beliefs about the type of the regulator will be updated depending on the result of bank's stress test and the realized payoff of the bank in the first period. The timing is illustrated in Figure 1.

We assume that the probabilities that the risky loan opportunity is good in the second period is independent of whether the risky loan opportunity is good in the first period, and that the type of the regulator is independent from the quality of the banks' risky loans. Furthermore, the regulator's type remains the same in both periods.

We use the equilibrium concept of Perfect Bayesian equilibrium.

[19] Specifically, if the lenient type regulator has a low cost of bank default D' or a large benefit from risky lending B', then passing the bank with certainty is indeed the unique equilibrium strategy.

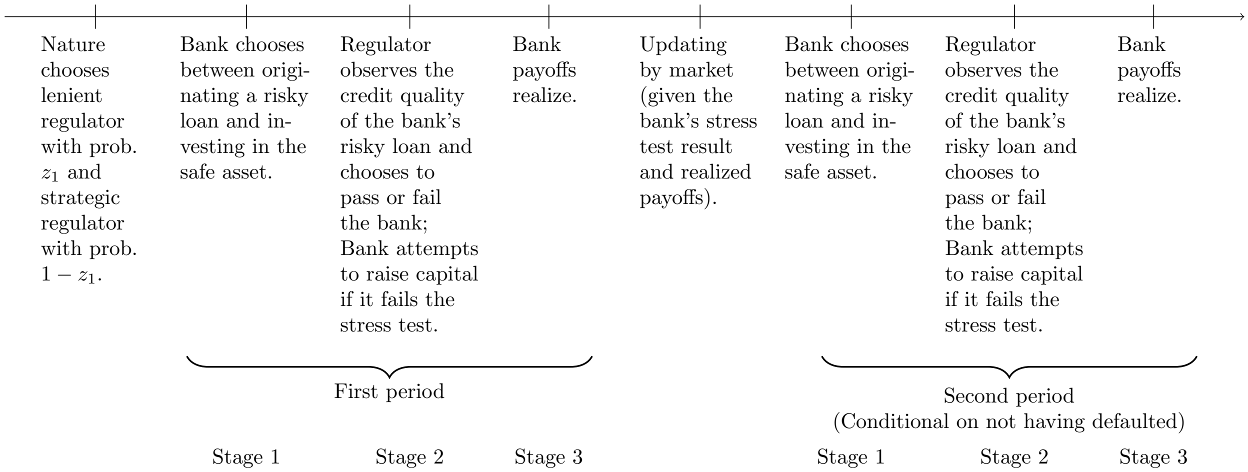

Figure 1: Time line of events

3 Stress testing in the second period

We begin the analysis of the model by using backward induction, and characterize the equilibrium in the second period. We first characterize the strategic regulator's stress test strategy at stage 2, taking as given the bank's investment decision at stage 1.

If the bank invests in the safe asset at stage 1, it is clear that the bank will not default and therefore requires no capital at stage 2. We will focus on describing the equilibrium stress test outcome given that the bank extends a risky loan at stage 1.

Given that the game does not continue after the second period, the regulator has no reputational incentives. The stress test strategy of the strategic regulator at stage 2 depends on the quality of the bank's risky loan $q_2 \in \{g, b\}$. Specifically, the strategic regulator passes the bank if and only if the loan is good, as implied by Assumption 4. Table 1 depicts the regulator's equilibrium stress testing strategy.

At stage 2, given the stress test result, the bank raises one unit of capital if it fails the stress test. Since failing the stress test reveals that the bank's loan is of bad quality, the total value of the bank's equity (including the capital provider's equity) post-recapitalization is $(1 - d)R$, because the one unit of capital raised will all be paid out to the depositors at maturity. The capital provider's outside option is equal to the expected return on the forgone

	Strategic regulator	Lenient regulator
$q_2 = g$	Pass	Pass
$q_2 = b$	Fail	Pass

Table 1: Equilibrium stress testing in the second period.

alternative investment, ρ. Assumption 2 implies that the total surplus is positive if and only if the opportunity cost of capital is low ($\rho = \rho_L$).

If recapitalization is feasible, we define $1 - \phi$ as the fraction of equity that the bank's owners retain. In order to determine this fraction, we now examine how the surplus is split between the capital provider and the bank's owners. When recapitalization is feasible, the capital provider's outside option is ρ_L. We assume the bank's outside option is 0, as the regulator compels the bank to be recapitalized. The total surplus is therefore $(1-d)R - \rho_L$. We use the Nash bargaining solution to define the split of the surplus, where the capital provider gets a fraction of the surplus determined by its bargaining power β. The transfer from the bank's owners to the capital provider is:

$$\rho_L + \beta \left[(1-d)R - \rho_L\right], \tag{1}$$

which is equal to the capital provider's outside option plus the fraction of surplus it obtains through bargaining power.

Therefore the equity given to the capital providers is a fraction ϕ, determined by:

$$\phi(1-d)R = \rho_L + \beta \left[(1-d)R - \rho_L\right] \tag{2}$$

We can now analyze the bank's investment decision at stage 1. At stage 1, the bank anticipates the fraction of equity ϕ it will need to sell to capital providers in exchange for capital. The bank originates a risky loan if and only if:

$$\underbrace{\left[\alpha + (1-\alpha)\left[z_2 + (1-z_2)\gamma\right](1-d)\right](R-1)}_{\text{pass, or fail but recapitalization infeasible}}$$

$$+ \underbrace{(1-\alpha)(1-z_2)(1-\gamma)(1-\phi)(1-d)R}_{\text{fail and recapitalized}} \geq R_0 - 1. \tag{3}$$

The bank originates a risky loan if and only if the expected payoff to the bank's owners is higher when it originates a risky loan (represented by the left hand side of Eq. 3) than when it invests in the safe investment (represented by the right hand side of Eq. 3). Notice that the expected payoff to the bank's owners when it originates a risky loan consists of two terms. First, if the bank does not raise capital and there is no default, it receives the net payoff $R-1$ at stage 4. This is the case if the loan is good, if the loan is bad and the regulator is lenient so that the bank passes the stress test (and the loan doesn't default), or if the loan is bad and the bank fails the stress test but recapitalization is infeasible (and the loan doesn't default). Second, if the bank fails the stress test and is recapitalized, which is the case if the loan is bad and the regulator is strategic, the bank's owners face dilution during recapitalization and thus their payoff is only the retained share ϕ of the bank's equity. The equity is priced after the stress test and reflects the equilibrium choice of the regulator in the second period.

Proposition 1. *In the second period, there exists a unique equilibrium. There exists a unique threshold $z_2^* < 1$, such that in equilibrium the bank originates a risky loan if and only if $z_2 \geq z_2^*$, given by $\Delta(z_2^*) = 0$, where:*

$$\Delta(z_2) \equiv \underbrace{[\alpha + (1-\alpha)(1-d)](R-1) - (R_0 - 1)}_{\textit{profit differential without recapitalization}}$$
$$- \underbrace{(1-\alpha)(1-z_2)(1-\gamma)\left(\rho_L - (1-d) + \beta\left[(1-d)R - \rho_L\right]\right)}_{\textit{dilution cost of recapitalization}}. \tag{4}$$

If the bank extends a risky loan at stage 2, the lenient type regulator passes the bank with certainty and the strategic regulator passes the bank with certainty if and only if the bank's loan is good.

Moreover, there exists $\bar{\beta} < 1$, such that $z_2^* > 0$ if and only if $\beta > \bar{\beta}$.

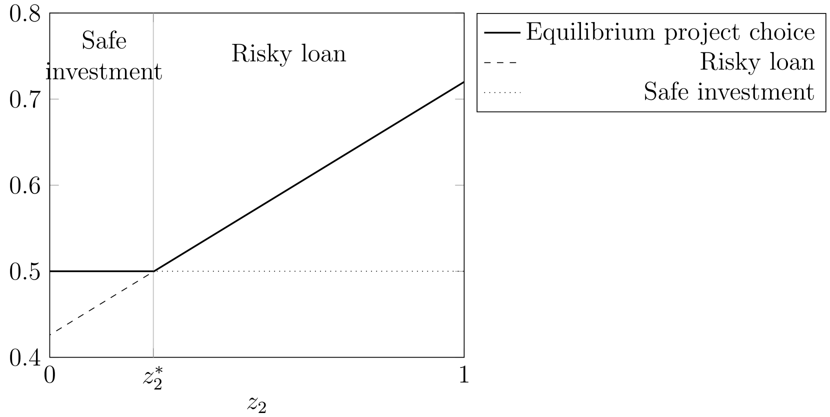

Figure 2: The expected payoff to the bank's owners. The parameters used in this plot are: $R_0 = 1.5$, $R = 2$, $\alpha = 0.3$, $d = 0.4$, $\rho_L = 1.1$, $\gamma = 0.2$, $\beta = 0.25$ and $D = 0.5$. This implies that $z_2^* = 0.25$.

This proposition states that the bank's incentive to originate a risky loan takes into account two factors. On the one hand, the bank benefits from originating the risky loan because it produces a higher expected profit than the safe investment (the profit differential term in $\Delta(z_2)$). On the other hand, the bank faces a dilution cost whenever it is required to raise capital, because the capital provider extracts rents (the dilution cost in $\Delta(z_2)$). When recapitalized, which occurs with probability $(1-\alpha)(1-z_2)(1-\gamma)$, the bank's cost of funding increases from $1-d$ to $\rho_L + \beta\left[(1-d)R - \rho_L\right]$. Here, $1-d$ represents the bank's cost of repaying depositors, taking into account the deposit insurance, and $\rho_L + \beta\left[(1-d)R - \rho_L\right]$ is how much the bank must pay the capital provider. Since the bank only faces the possibility of failing the stress test and thus having to raise capital if it extends a risky loan, the bank only originates the risky loan if the gains from higher NPV outweighs the potential dilution cost of recapitalization.

Importantly, the bank originates a risky loan only if the regulator's reputation of being the lenient type is sufficiently high, i.e. $z_2 \geq z_2^*$, as illustrated in Figure 2. This is because the lenient type regulator does not require the bank to raise capital even if the bank's risky loan is bad (Table 1), reducing the expected dilution cost to the bank.

Since the regulator's reputation z_2 determines the bank's investment decision in equilibrium, we now turn to understanding how the regulator's reputation affects surplus. Let U_R and U_0 denote the strategic regulator's expected surplus from the bank in the second period when the bank originates a risky loan and invests in the safe asset, respectively. We can express the expected surplus as follows:

$$U_R = [\alpha + (1-\alpha)(1-d)]\, R - 1 + X, \tag{5}$$

$$U_0 = R_0 - 1, \tag{6}$$

where X represents the net social costs of risky lending, given by:

$$X \equiv B - (1-\alpha)\left[\gamma d D - (1-\gamma)(\rho_L - 1)\right]. \tag{7}$$

When the bank extends a risky loan, the strategic regulator internalizes the net social benefits of risky lending, consisting of the positive externality of bank lending B as well as the social costs of a potential bank default. Conditional on a bad loan (with probability $1-\alpha$), the expected social costs of a potential bank default include the expected cost of bank default dD if recapitalization is infeasible (with probability γ) and the forgone net return from the capital providers' alternative investment $\rho_L - 1$ when the bank is recapitalized (with probability $1-\gamma$). Notice that X encapsulates all of the externalities from risky lending; the following analysis will only use X rather than the individual components.

It then follows from Proposition 1 that the strategic regulator's expected surplus, for a given reputation z_2, denoted by $U(z_2)$, is given by

$$U(z_2) = \begin{cases} U_R, & \text{if } z_2 > z_2^*, \\ U_0, & \text{if } z_2 < z_2^*, \\ \lambda U_R + (1-\lambda) U_0 \text{ for some } \lambda \in [0,1], & \text{if } z_2 = z_2^*, \end{cases} \tag{8}$$

where we have taken into account that, if $z_2 = z_2^*$, the bank is indifferent between originating

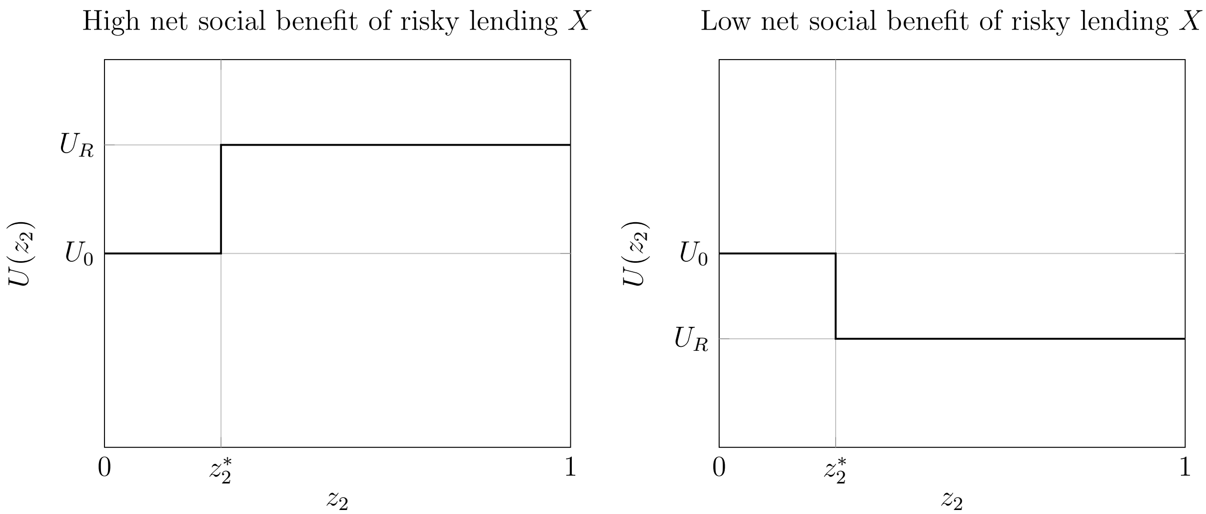

Figure 3: The strategic regulator's expected surplus for a given reputation z_2. The parameters used in this plot are the same as those in Figure 2, implying $U_0 = 0.5$. In addition, in the left panel, $B = 0.2$, implying $X = 0.116$ and $U_R = 0.556$, whereas in the right panel, $B = 0.1$, implying $X = 0.016$ and $U_R = 0.456$.

a risky loan and investing in the safe asset, thus may employ a mixed strategy and randomize between the two investment choices with some probability λ.

The regulator internalizes the social benefits and costs of risky lending, whereas the bank only cares about the private cost of recapitalization. Therefore the bank's investment choice characterized in Proposition 1 generally differs from the socially optimal choice. The more the bank expects the regulator to be the lenient type, the more the bank is willing to originate a risky loan. However, originating a risky loan is indeed socially preferred only if the net social benefits of risky lending X is sufficiently high. This is illustrated in Figure 3. In the following analysis, we demonstrate that the divergence in preferences between the strategic regulator and the bank leads to reputation building incentives for the strategic regulator that depend on the net social benefits of risky lending X.

4 Stress testing in the first period

In this section, we analyze the equilibrium stress testing strategy of the regulator for the bank in the first period, given the equilibrium in the second period. In particular, we consider the incentives of the strategic regulator to pass the bank in the first period at stage 2. These incentives are driven by concerns for the bank in the first period and the reputational consequences of the regulator's observable decision to pass or fail the bank.

At stage 2, given the posterior beliefs held by the bank about the regulator's type after it passes and fails the stress test z_2^R and z_2^f, respectively, let $G_{q_1}(z_2^R, z_2^f)$ denote the net gain of passing the bank relative to failing the bank, given the quality of the bank's risky loan $q_1 \in \{g, b\}$.

$$\begin{aligned} G_g(z_2^R, z_2^f) &= \underbrace{(1-\gamma)(\rho_L - 1)}_{\text{first period bank surplus effect}} + \underbrace{\delta \left[U(z_2^R) - U(z_2^f) \right]}_{\text{reputation effect}}, \\ G_b(z_2^R, z_2^f) &= \underbrace{(1-\gamma)\left[(\rho_L - 1) - dD\right]}_{\text{first period bank surplus effect}} + \underbrace{\delta \left[(1-d)U(z_2^R) - \left[(1-d) + d(1-\gamma)\right] U(z_2^f) \right]}_{\text{reputation effect}} \end{aligned} \tag{9}$$

where in both expressions the first term represents the net gain in terms of the expected surplus in the first period, and the second term represents the reputation concern in terms of the expected surplus in the second period. The first term takes into account that, when failing the bank, recapitalization is only feasible with probability $1-\gamma$. In that case, the first period bank surplus effect of passing the bank relative to failing it is equal to the capital provider's alternative investment (which can now be realized with a pass) less the expected cost of a bank default (which may also realize with a pass) if the quality of the investment is bad. The term z_2^R is the posterior belief about the probability that the regulator is the lenient type, given that the bank passes the stress test in the first period and realizes a payoff of R. If the bank passes the stress test in the first period and realizes a payoff of 0 instead, the bank defaults and does not continue to the second period, producing an expected surplus of 0. The term z_2^f is the posterior belief about the probability that the regulator is the lenient type, given that the bank fails the stress test in the first period.

The first period bank surplus effect is positive if the first bank's risky loan is good as there is no risk of default. This effect is negative if the risky loan is bad, given Assumption 4.

The reputation effect depends on the regulator's posterior reputation after it grades the first period bank and the payoffs are realized. Given that the lenient type regulator passes the bank, if the strategic regulator fails the bank in the first stress test, it is revealed to be strategic ($z_2^f = 0$); the bank will then realize that it will be recapitalized in the second period if its risky investment is of bad quality. In contrast, if the strategic regulator passes the bank in the first stress test, it is pooled with the lenient type regulator who also passes the bank. In equilibrium, the posterior probability that the regulator is the lenient type, given that the bank passes the first stress test and then realizes a payoff of R is given by

$$z_2^R(\pi_g, \pi_b) = \frac{[\alpha + (1-\alpha)(1-d)]z_1}{[\alpha + (1-\alpha)(1-d)]z_1 + [\alpha\pi_g + (1-\alpha)(1-d)\pi_b](1-z_1)} \tag{10}$$

where π_g and π_b denote the strategic regulator's probability of passing the bank in the first stress test, given that the bank's risky loan is good and bad, respectively. As a result, passing or failing the bank in the first period may lead to different investment decisions by the bank in the second period, and hence has a reputation effect.

The following lemma establishes the set of possible equilibrium stress testing strategies, which narrows down our analysis.

Lemma 1. *In any equilibrium, the stress testing strategy of the strategic regulator is either:*

Informative: *it passes the first bank if and only if the bank's risky loan is good;*

Lenient: *it passes the bank with certainty if the bank's risky loan is good, and passes the bank with positive probability* $\pi_b^* > 0$ *if the loan is bad; or*

Tough: *it passes the bank with probability* $\pi_g^* < 1$ *if the bank's risky loan is good, and fails the bank with certainty if the loan if bad.*

This lemma follows from the fact that, in any equilibrium, the strategic regulator faces strictly greater incentives to pass a bank with a good risky loan than a bank with a bad risky loan. This can be seen in Eq. 9. Passing a bank with a bad risky loan results in a possible costly default, while passing a bank with a good risky loan does not have this possibility. A bank default generates two costs. First, it generates a social cost of default D in the first period. Second, it leads to a loss of expected surplus $U(z_2)$ in the second period. Lemma 1 thus represents all possible equilibrium strategies of the strategic regulator in the first period: every possibility where the probability with which the strategic regulator passes a bank with a good risky loan is weakly larger than the probability with which it passes a bank with a bad risky loan.

We now show that for intermediate levels of the net social benefits of risky lending X, there is a unique equilibrium in which the strategic regulator's stress testing strategy in the first period is identical to its strategy in the second period.

Proposition 2. *There exist cutoffs $\overline{X}$ and $\underline{X}$, with $\underline{X} < \overline{X}$, such that for $X \in [\underline{X}, \overline{X}]$, there exists a unique equilibrium in the first period in which the stress testing strategy of the strategic regulator in the first period is identical to that in the second period described in Proposition 1, and is fully informative.*

For intermediate levels of the net social externality of bank lending ($X \in [\underline{X}, \overline{X}]$), the expected surplus for the strategic regulator is not too sensitive to the bank's investment decision in the second period. That is, for intermediate X, the social values of the risky project and the safe asset are close, so the bank's investment choice in the second period does not affect the regulator's surplus much and the regulator can choose its static optimum stress testing strategy in the first period. Proposition 2 shows that, in this case, the equilibrium is unique and is fully informative.

In the following sections, we will show that the other two types of equilibria described in Lemma 1 can arise if the net social benefits of risky lending X is either low or high, and depend on the regulator's reputation building incentives.

4.1 High net social benefits of risky lending $X > \overline{X}$

If the strategic regulator fails the first period bank and recapitalizes it, the regulator reveals the fact that it is strategic. The bank then faces a strong incentive to invest in the safe investment in the second period, in order to avoid failing the stress test. If the strategic regulator passes the bank in the first period, however, it pools with the lenient regulator, increasing the incentive for the bank to engage in risky lending in the second period. If the benefit of lending by the bank in the second period is sufficiently large, the regulator may want to pass the bank in the first period, even when its risky loan is bad, in order to gain a reputation of leniency.

In the following proposition, we demonstrate that for high X, there is still an equilibrium in which the strategic regulator's stress testing strategy in the first period is identical to its strategy in the second period, *but* reputation building incentives to encourage lending by the second period bank can lead to another equilibrium for the bank's stress test.

Proposition 3. *For high net social externalities of bank default $X > \overline{X}$, there exists an equilibrium in the first bank's stress test that is either fully informative or lenient (as described in Lemma 1). There exist $\bar{\delta}_b, \underline{\delta}_b \in \mathbf{R}^+ \cup \{\infty\}$, with $\bar{\delta}_g \geq \underline{\delta}_b$, such that*

- *the fully informative equilibrium exists if and only if $\delta \leq \bar{\delta}_b$; and*
- *a lenient equilibrium exists if and only if $\delta \geq \underline{\delta}_b$.*

Moverover, $\underline{\delta}_b \neq \infty$ if and only if $\beta \geq \bar{\beta}$ and $z_1 > \bar{z}_1$, and $\bar{\delta}_b \neq \infty$ if and only if $\beta > \bar{\beta}$ and $z_1 > \bar{z}_1$, where $\bar{\beta}$ is defined in Proposition 1. In particular, there exists δ_b, such that the equilibrium is unique unless $\delta = \delta_b$ or $\beta = \overline{\beta}$.

Proposition 3 shows that, for certain parameters, the equilibrium stress testing strategy of the regulator in the first period is the same as its strategy in the second period, and is illustrated in Table 1. However, this proposition also shows that the strategic regulator's reputation building incentives to encourage lending by the bank in the second period can

	Strategic regulator	Lenient regulator
$q_1 = g$	Pass	Pass
$q_1 = b$	Pass with probability $\pi_b^* > 0$	Pass

Table 2: Equilibrium stress testing in the first period when the strategic regulator wants to build reputation to incentivize lending by the bank in the second period.

lead to an equilibrium with a lenient stress test in the first period. Table 2 depicts the stress testing in the lenient equilibrium in the first period described in Proposition 3.

Passing the bank with a bad risky loan is costly, as it incurs a higher expected cost of default than the social cost of capital. However, in the lenient equilibrium, by passing the bank, the strategic regulator is able to increase the perception that it is the lenient type, since the lenient type regulator always passes the bank. This is useful to the strategic regulator when this induces the bank to originate a risky loan in the second period. In other words, the regulator enjoys a positive reputation effect from passing the bank with a bad risky loan in the first period.

Proposition 3 identifies the necessary and sufficient conditions for a lenient equilibrium with reputation building to incentivize lending to exist. First, the private cost of capital (β) must be sufficiently high so that the second period bank's investment is responsive to the stress test. Failing the stress test in the first period can then lead the bank to invest in the safe asset in the second period; a "pass" will then influence the bank's behavior. Second, the prior reputation of the regulator being lenient (z_1) must be sufficiently high, so that the posterior reputation of the regulator is sufficiently lenient after passing the bank in the first period to induce the bank in the second period to originate a risky loan. Third, the reputation concern (δ) of the strategic regulator must be sufficiently high, so that the regulator's reputational benefits outweigh the short-term efficiency loss when passing the bank with a bad risky loan.

While the initial European stress tests performed poorly (e.g. passing Irish banks and Dexia), one might argue that during crisis times, the main focus was preventing runs - and without a fiscal backstop it was hard to maintain credibility (Faria-e-Castro, Martinez, and Philippon, 2016). We argue that in normal times, a stress test may be lenient to incentivize banks to lend to the real economy. This may explain the 2016 EU stress test, which eliminated the pass/fail criteria, reduced the number of banks stress tested by about half, used less adverse scenarios than the U.S. and UK, and only singled out one bank as undercapitalized - Monti dei Paschi di Siena, which had failed the previous (2014) stress test and was well known to be in distress.

4.2 Low net social benefits of risky lending $X < \underline{X}$

When the net social benefit of risky lending X is low (e.g. due to a large cost of bank default D), the expected surplus to the strategic regulator from the bank in the second period is higher when the bank invests in the safe asset. If the concerns about excessive risk-taking by the bank in the second period are sufficiently large, the strategic regulator may want to fail the bank with a risky loan in the first period even when it is good, in order to reveal its willingness to fail a bank during the second stress test.

In the following proposition, we demonstrate that the reputation building incentives to reduce excessive risk-taking by the bank leads to another equilibrium for the bank's stress test, in additional to an equilibrium in which the strategic regulator's stress testing strategy in the first period is identical to its strategy in the second period.

Proposition 4. *For low net social externalities of bank lending $X < \underline{X}$, the exists an equilibrium in the first bank's stress test that is either fully informative or tough (as described in Lemma 1). There exists $\bar{\delta}_g, \underline{\delta}_g \in \mathbf{R}^+ \cup \{\infty\}$, with $\bar{\delta}_g \geq \underline{\delta}_g$, such that*

- *the fully informative equilibrium exists if and only if $\delta \leq \bar{\delta}_g$; and*
- *the tough equilibrium exists if and only if $\delta \geq \underline{\delta}_g$.*

Moreover, $\underline{\delta}_g \neq \infty$ if and only if $\beta > \bar{\beta}$, and $\bar{\delta}_g \neq \infty$ if and only if $\beta > \bar{\beta}$ and $z_1 > \bar{z}_1$, where $\bar{\beta}$ is defined in Proposition 1 and $\bar{z}_1$ is defined in Proposition 3.

	Strategic regulator	Lenient regulator
$q_1 = g$	Pass with probability $\pi_g^* < 1$	Pass
$q_1 = b$	Fail	Pass

Table 3: Equilibrium stress testing in the first period when the strategic regulator wants to build reputation to reduce excessive risk-taking by the second bank in the second period.

Proposition 4 shows that, for certain parameters, the equilibrium stress testing strategy of the regulator in the first period is the same as its strategy in the second period, and is illustrated in Table 1. However, this proposition also shows that the strategic regulator's reputation building incentives to reduce excessive risk taking by the bank in the second period can lead to an equilibrium with a tough stress test in the first period. Table 3 depicts the stress testing in the tough equilibrium in the first period described in Proposition 4.

In the fully informative equilibrium, the strategic regulator passes the bank with a good risky loan in the first period, which maximizes the expected surplus from the bank. Failing the bank in this case would possibly result in a costly recapitalization of the bank with no benefit, since the good loan will not default. However, in the tough equilibrium, by failing this bank, the strategic regulator is able to reveal its willingness to recapitalize a bank, and thus reduce the bank's incentive to engage in excessive risk taking in the second period.

Proposition 4 identifies two further necessary and sufficient conditions for an equilibrium with reputation building to reduce excessive risk-taking to exist. First, the private cost of capital (β) must be sufficiently high. This makes it possible to make the bank refrain from taking excessive risk. Second, the reputation concern (δ) of the regulator must be sufficiently high, so that the regulator's reputational benefit outweighs the short-term efficiency loss when recapitalizing the bank with a good risky loan.

Proposition 4 also indicates that the informative and tough equilibria coexist when $\delta \in (\underline{\delta}_g, \bar{\delta}_g)$. This is due to a strategic complementarity between the regulator's first period stress test and the bank's belief updating process in the second period. We discuss this in more detail in the next section.

U.S. stress tests have generally been regarded as much more strict than European ones.

First, the Federal Reserve performs the stress test itself on data provided by the banks (and does not provide the model to the banks), whereas in Europe, it has been the case that the banks themselves perform the test. Second, the U.S. stress tests have regularly been accompanied by Asset Quality Reviews, whereas this has been infrequent for European stress tests. Third, one of the most feared elements of the U.S. stress tests has been the fact that there is a qualitative element that can (and has been) used to fail banks.[20] In line with our results above, the fact that U.S. stress tests have been institutionalized as occurring on a yearly basis implies that reputation concerns are important. Furthermore, a swifter recovery from the crisis means that capital raising for banks was likely to be easier in the U.S.

5 Discussion

Having characterized the equilibria of the model, we discuss the implications of the model in this section. First, we examine the reasons for equilibrium multiplicity. Second, we point to the possibility that stress tests may be strategically delayed. Third, we consider how stress tests may vary with the availability of capital. Finally, we explore the implications of the model for stress test design when banks are systemic.

5.1 Multiplicity

Proposition 4 imply that there exist parameter values for which the fully informative equilibrium coexists with a tough equilibrium. This is because the regulator's reputation concern is self-fulfilling.

Specifically, the strategic regulator's stress testing strategy and the bank's belief updating process are strategic complements when the net social benefits of risky lending are low ($X < \underline{X}$). Here, the bank realizes that the strategic regulator's surplus is lower when the

[20]The qualitative element for domestic banks was removed for domestic banks in March 2019 (see "US financial regulators relax Obama-era rules," by Kiran Stacey and Sam Fleming, Financial Times, March 7, 2019).

bank makes the risky loan and therefore when the strategic regulator is perceived to be the lenient type. If the bank conjectures that the strategic regulator adopts a tougher stress test strategy (lower π_g), the bank infers that the regulator who passes the bank in the first period is more likely to be lenient (higher z_2^R). Consequently, the bank increases its risk-taking in the second period after a pass result in the first period, resulting in *even lower* expected surplus for the strategic regulator. In turn, this further decreases the net gain for the strategic regulator from passing the bank in the first period, justifying a tougher testing strategy. It is indeed this strategic complementarity that leads to equilibrium multiplicity.

By contrast, the regulator's stress testing strategy and the bank's belief updating process are strategic substitutes when the net social benefit of risky lending is high ($X > \overline{X}$). Here, the bank realizes that the strategic regulator's surplus is larger when the bank extends the risky loan and therefore when the strategic regulator is perceived to be the lenient type. If the bank conjectures that the strategic regulator adopts a lenient stress testing strategy (higher π_b), the bank infers that the regulator who passed the bank in the first period is more likely to be the strategic type (lower z_2^R). Consequently, the bank may refrain from originating a risky loan in the second period after a pass result in the first period, resulting in *lower* expected surplus for the strategic regulator. In turn, this reduces the net gain for the strategic regulator from passing the bank in the first period.[21] Because of this strategic substitutability, the type of equilibrium multiplicity does not arise.[22]

[21] While formally, multiplicity arises only when X is high, the driver for this is the assumption that the behavioural type is lenient. The strategic complementarity flips (i.e., there is co-existence of informative and *lenient* equilibria) when we change the behavioral type to being tough. We describe this in Section 7.1.2.

[22] Note that Proposition 3 implies that there exists multiplicity in the knife-edge cases when $\delta = \delta_b$ and when $\beta = \bar{\beta}$, but for different reasons than the strategic complementarity discussed above. In both cases, the fully informative equilibrium coexists with a lenient equilibrium. If $\delta = \delta_b$, then in both types of equilibria, the bank invests in the risky loan in the second period if and only if it passes the stress test in the first period. Such multiplicity stems from the fact that the bank's investment decision in the second period follows a threshold strategy. Therefore a range of stress testing strategies (in terms of the mixing probability π_b) leads to posterior beliefs held by the bank that are consistent with the same investment strategy in the second period, implying the same reputation effect on the regulator's stress testing incentives that justifies the mixed strategies. If $\beta = \bar{\beta}$, then in any equilibrium, after the failure of the first bank, the posterior reputation of the regulator is $z_2^f = 0$, such that the bank is indifferent between investments in the second period. As a result, risky lending by the bank in the second period after failing the stress test in the first period justifies a fully informative equilibrium, while safe investment by the bank in the second period after failing the stress test in the first period justifies a lenient equilibrium.

The fact that we have potentially coexisting equilibria raises the issue of how a particular equilibrium may be chosen. Commitment by the regulator in an ex ante stage would facilitate this. Of course, in a crisis, committing to future actions may not be feasible. The regulator has access to several policy variables that might prove useful as commitment devices. Committing to how signals from banks are used is standard in the Bayesian Persuasion literature, but requires substantial independence from political pressure and processes that are well defined. A more practical alternative is committing to stress test scenarios. Stress test scenarios can be more or less lenient, given the effect desired. Asset quality reviews also commit more resources and reveal more information about bank positions.

5.2 Strategic delay of stress tests

There may exist an equilibrium in which both types of regulator pass the bank in the first period with certainty. This is equivalent to an economy where the regulator does not conduct stress tests for the bank in the first period.

Corollary 1. *For high net social externalities of risky lending ($X > \overline{X}$), the regulator passes the bank in the first period in equilibrium with certainty if and only if $\delta \geq \bar{\delta}_b$ and $z_1 \geq z_2^*$, where z_2^* is defined in Proposition 1.*

The timing of European stress tests has been quite irregular compared with the annual U.S. exercises (they were conducted in 2010, 2011, 2014, 2016, and 2018). Delay in this situation may be a way of choosing lenience.

5.3 Availability of capital

Holding the expected availability of capital in the second period constant, let γ_1 denote the probability that recapitalization is infeasible in the first period. The following corollary assesses how the regulator's stress testing strategy is affected by the availability of capital in the first period.

Corollary 2. • *For a high net social benefit of risky lending $X > \overline{X}$, $\bar{\delta}_b$ and $\underline{\delta}_b$ are decreasing in γ_1.*

• *For a low net social benefit of risky lending $X < \underline{X}$, $\bar{\delta}_g$ and $\underline{\delta}_g$ are decreasing in γ_1.*

The corollary demonstrates that when there is a higher probability that recapitalization is infeasible in the first period, the strategic regulator's reputation building incentives are exacerbated and the stress test becomes less informative. This is because the strategic regulator trades off the cost/benefit of recapitalizing the bank in the first period against the regulator's cost/benefit of affecting the bank's investment decision in the second period. While the reputation effect depends only on the bank's updated belief about the regulator's type, the cost of passing a bad bank or failing a good bank in the first period is smaller if recapitalization is infeasible in the first period with a high probability. This result is related to that of Faria-e-Castro, Martinez, and Philippon (2016) in that as recapitalizing the bank becomes more difficult, the test becomes less informative. Nevertheless, we demonstrate this link through a dynamic reputation model, whereas they have the regulator committing upfront to the informativeness of the stress test.

5.4 Stress tests of systemic banks

Holding the net social benefit of risky lending in the second period constant, let D_1 denote the social cost of a potential bank default in the first period. The following corollary assesses how the regulator's stress testing strategy is affected by the social cost of a potential bank default in the first period.

Corollary 3. • *For a high net social benefit of risky lending $X > \overline{X}$, $\bar{\delta}_b$ and $\underline{\delta}_b$ are increasing in D_1.*

• *For a low net social benefit of risky lending $X < \underline{X}$, $\bar{\delta}_g$ and $\underline{\delta}_g$ are independent of D_1.*

The corollary demonstrates that, when the social cost of a potential bank failure in the first period is higher, the strategic regulator becomes less lenient when facing reputation building incentives to incentivize lending ($X > \overline{X}$). This is because when considering whether to pass a bad bank in the first period, the strategic regulator trades off the cost/benefit of recapitalizing the bank in the first period against the regulator's cost/benefit of affecting the bank's investment decision in the second period. While the reputation effect depends only on the bank's updated belief about the regulator's type, the cost of passing a bad bank in the first period is larger if the cost of a potential bank failure in the first period is larger. The strategic regulator's stress testing strategy when facing reputation building incentives to curb excessive risk taking ($X < \underline{X}$) is unaffected, since in this case the strategic regulator's main focus is whether to pass or fail a good bank, which does not run the risk of default.

In both U.S. and Europe there have been ongoing debates since the inception of stress tests about how large/systemic a bank must be in order to be included in the stress testing exercise. To the extent that larger and more systemic banks have higher expected cost of default, our model predicts that they should be subject to (weakly) more informative tests.

6 The bank and the regulator both learn the asset quality

In this section, we consider a stress test where the signal about the quality q_t of the bank's risky loan observed by the regulator during the stress test is also observed by the bank. This could be the case because:

- The stress test only uncovers the private information the bank already has about its loan quality.[23] This is indeed the case for banking supervision examinations. These exams are conducted on a regular basis by collecting information and assessing the health of banks on multiple dimensions and have real effects.[24] They do not use

[23]For example, in Walther and White (2018) the regulator and the bank both observe the bank's asset value, while creditors do not. They consider the effectiveness of bail-ins in this scenario.

[24]Agarwal et. al. (2014) demonstrate real effects of exams: leniency leads to more bank failures, a

information from the entire banking system to assess the position of each bank (which can be the source of the regulator's private information in the baseline model). In the United States, this has historically been conducted using the CAMEL rating system, though in recent years variations on this rating system have been implemented;[25] or

- The stress test produces/uncovers new information but regulators share that information with the bank. This second case resembles the European stress test exercises, which use a "bottom-up" approach where the regulator provides the model and basic parameters to banks, who perform the test themselves.[26] In contrast, the U.S. uses a "top-down" approach where the regulators does the test themselves and do not provide all of the information about the model or results.[27]

As in the baseline model, the equilibrium in the second period for given belief z_2 held by the bank is as described in Lemma 1.

Unlike in the baseline model, here the bank in the second period forms posterior beliefs $z_2 = z^R_{2,q_1}$ ($z_2 = z^f_{2,q_1}$) about the probability that the regulator is lenient given the bank passes (fails) the stress test in the first period *and* the loan quality in the first period is q_1. Therefore in the first period, taking the bank's posterior beliefs described above as given, the incentives of the strategic regulator to pass the bank is characterized by $G_{q_1}(z^R_{2,q_1}, z^f_{2,q_1})$, where $G_{q_1}(\cdot)$ is defined by Eq. 9.

In equilibrium, as in the baseline model, the posterior belief that the regulator is lenient given that the bank fails the stress test in the first period is $z^f_{2,q_1} = 0$, since only a strategic

higher proportion of banks unable to repay TARP money in the crisis, and a larger discount on assets of banks liquidated by the FDIC. Hirtle, Kovner, and Plosser (2018) demonstrate real effects of more banking supervision effort (measured by hours).

[25]The RFI/C(D) system was recently supplanted by the LFI system for large financial institutions. See (https://www.davispolk.com/files/2018-11-06_federal_reserve_finalizes_new_supervisory_ratings_system_for_large_financial_institutions.pdf).

[26]Note that we do not model the inherent moral hazard problem when a bank is permitted to do its own stress test.

[27]See Baudino et al (2018) and Niepmann and Stebunovs (2018) for a discussion of top-down vs. bottom up approaches. The U.S. has recently made more information available about its test after complaints about opacity by banks (https://uk.reuters.com/article/uk-usa-fed-stresstests/fed-gives-u-s-banks-more-stress-test-information-unveils-2019-scenarios-idUKKCN1PU2GE).

regulator fails a bank. In addition, the posterior belief of the bank that the regulator is lenient given that it had a loan of quality q_1 and passed the first stress test is given by:

$$z^R_{2,q_1}(\pi_{q_1}) = \frac{z_1}{z_1 + (1 - z_1)\pi_{q_1}}. \tag{11}$$

We can now compare the results in this case to those in the baseline model and examine the effect of bank information on the equilibrium stress testing strategy of the regulator.

Proposition 5. *The equilibria when the bank has information about the risky loan's quality q_t are characterized as follows.*

- *For intermediate levels of net social benefits of risky lending $X \in [\underline{X}, \overline{X}]$, there exists a unique informative equilibrium (as described in Proposition 2).*
- *For high net social benefits of risky lending $X > \overline{X}$, there exists an equilibrium for the first bank's stress test, which is either fully informative or lenient. The parameter space in terms of (β, z_1, δ) for which the fully informative equilibrium exists is strictly smaller than in the baseline model, and that for which the lenient equilibrium exists is strictly larger than in the baseline model.*
- *For low net social benefits of risky lending $X < \underline{X}$, there exists an equilibrium for the first bank's stress test, which is either fully informative or tough. The parameter space in terms of (β, z_1, δ) for which the fully informative equilibrium exists is strictly larger than in the baseline model, and that for which the tough equilibrium exists is identical to the baseline model.*

Proposition 5 provides two key insights. First, when the net social benefit of risky lending is high ($X > \overline{X}$), the bank having knowledge of the risky loan's quality exacerbates the strategic regulator's reputation concerns, resulting in a less informative stress test. The reason is as follows. Since the strategic regulator is more likely to pass a bank with a good loan than a bank with a bad loan, after a pass on the first stress test, the bank's posterior belief about the likelihood the regulator's type is lenient is lower if the loan was good quality than if the loan was bad quality. That is, $z_{2,g}^R \leq z_2^R \leq z_{2,b}^R$. As a result, passing the bank with a bad loan in the first period is more likely to induce the bank to originate a risky loan in the second period, exacerbating the strategic regulator's incentives to pass the bank to incentivize lending.

Second, when the net social benefit of risky lending is low ($X < \underline{X}$), the bank having knowledge of the risky loan's quality reduces the strategic regulator's reputation concerns, resulting in a more informative stress test. The reason is as follows. Since $z_{2,g}^R \leq z_2^R$, passing the good bank in the first period is less likely to induce the bank to originate a risky loan in the second period. This reduces the strategic regulator's concern about excessive risk taking, resulting in a more informative stress test in the first period.

Therefore, banking supervision exams (or a bottom-up stress test) will be less informative than top-down stress tests when regulators are concerned about lending. This is in line with the evidence in Agarwal et. al. (2014). On the other hand, when bank defaults are more of a concern, banking supervision exams (or bottom-up stress tests) will be more informative than top-down stress tests.

7 Robustness and extensions

In this section, we show that the our model is robust to two extensions. First, we consider different types for the behavioral regulator. Second, we remove the assumption of deposit insurance and allow fair pricing of capital market debt of the bank.

7.1 Different types for the behavioral regulator

In the model, we assumed that one of the types of regulator was behavioral and always passed the bank in its stress test - the lenient type. In this section, we demonstrate the robustness of our main results by considering two alternative models: one in which the behavioral regulator fails the bank and recapitalizes it if and only if the bank has a bad quality loan (we call this the *informative type*), and one in which the behavioral regulator always fails the bank and recapitalizes it regardless of its quality (we call this the *tough type*). These results show that reputation building incentives for the strategic regulator are present and can lead to multiple equilibria similar to those in our baseline model whenever the behavioral type regulator's stress testing strategy differs from (i.e., is either more lenient or more tough than) that of the strategic regulator when there are no reputation concerns (as in the second period).

7.1.1 An informative type regulator

In this subsection, we consider an alternative model in which the behavioral regulator fails the bank and recapitalizes it if and only if the bank has a loan that is bad. In this case, both the strategic regulator and the behavioral regulator use an identical stress testing strategy in the second period. Anticipating the regulator's stress testing strategy, the bank's lending behavior in the second period does not depend on its belief about the regulator's type. As a result, the strategic regulator faces no reputation building incentives, since it cannot influence the bank's lending strategy through its reputation. The following proposition states that, in this case, the equilibrium is always fully informative, analogous to informative equilibrium of the baseline model.

Proposition 6. *Consider the model with an informative type regulator. There exists a unique equilibrium in which, in each period, if the bank originates a risky loan, the strategic regulator passes the bank with certainty if and only if the bank's loan is good.*

7.1.2 A tough type regulator

In this subsection, we consider an alternative model in which the behavioral regulator instead always fails the bank and recapitalizes it. The following proposition characterizes the equilibria with the tough type regulator, and demonstrates that the types of equilibria when the behavioral regulator is lenient are also the only types of equilibria when the behavioral regulator is tough.

Proposition 7. *Consider the model with a tough type regulator. In the second period, there exists a unique equilibrium in which, if the bank extends a risky loan, the strategic regulator passes the bank with certainty if and only if the bank's loan is good.*

An equilibrium exists in the first period, and the possible equilibria are as described in Lemma 1.

- *For $X \in [\underline{X}, \overline{X}]$, there exists a unique equilibrium in which the stress testing strategy of the strategic regulator in the first period is identical to that in the second period.*

- *For $X > \overline{X}$, the equilibrium is either fully informative or lenient. There can exist a unique equilibrium, or a fully informative equilibrium can coexist with a lenient equilibrium.*

- *For $X < \underline{X}$, the equilibrium is either fully informative or tough. There can exist a unique equilibrium, or a fully informative equilibrium can coexist with a tough equilibrium. In particular, there exists δ_g, such that the equilibrium is unique unless $\delta = \delta_g$ or $\beta = \bar{\beta}$.*

Notice that, in contrast to the baseline model, when the behavioral regulator is a tough type, equilibrium multiplicity that arises due to the regulator's self-fulfilling reputation concern exists only with a lenient equilibrium, but not with a tough equilibrium. This is because, in this case, the strategic regulator's stress testing strategy and the bank's belief updating process are strategic complements only when the net social benefit of risky lending is high ($X > \overline{X}$). Here, the bank realizes that the strategic regulator's surplus is larger when the bank originates the risky loan and the regulator is perceived to be the lenient type. If the bank conjectures that the strategic regulator adopts a more lenient stress test strategy (higher π_b), the bank infers that the regulator who passes the bank in the first period is more likely to be the strategic regulator (who would be relatively lenient). Consequently, the bank is more likely to originate a risky loan in the second period after a pass result in the first period, resulting in higher expected surplus for the strategic regulator. This further increases the net gain for the strategic regulator from passing the bank in the first period, justifying a more lenient testing strategy.[28]

7.2 No deposit insurance

In this section, we no longer assume that the bank is funded by fully insured deposits. Instead, we assume that, at stage 1 of each period, the bank raises one unit of funds from-competitive debtholders maturing at stage 1.5 with an exogenous repayment of 1.[29] Then bank then chooses whether to invest in the safe investment or originate a risky loan. At stage 1.5, the bank must rollover its debt. After observing the bank's investment decision, the debtholders require a promised repayment of $\tilde{R}_t \in [1, R]$ at stage 3. Accordingly, should the bank fail the stress test at stage 2, it is required to raise $\tilde{R}_t$ unit of capital in order to eliminate bank default. Notice that Assumption 1 implies that the bank is always solvent, and thus financing at stage 1 and refinancing at stage 1.5 are always feasible.

In order to account for the potentially higher amount of recapitalization, we modify Assumptions 2 and 4, respectively, as follows:

Assumption 5. *$R < \rho_H$ and $(1-d)R \geq R^{max}\rho_L$, where $R^{max} \equiv \frac{1}{\alpha+(1-\alpha)(1-d)}$.*

Assumption 6. *$dD > R^{max}(\rho_L - 1) > 0$.*

The maximum promised repayment $R^{\max}$ is derived given debtholders' belief that the bank is not going to be recapitalized.

We can now solve the model by backward induction and show that the main results of the baseline model remain valid.

[28]Like in the baseline model, there exists multiplicity when the net social benefit of risky lending is low $X < \underline{X}$ due to the threshold nature of the bank's investment decision in the second period.

[29]While the promised repayment can be greater than 1, the renegotiation-proof repayment is equal to 1 assuming that the bank's liquidation value is equal to 1.

7.2.1 Stress testing in the second period

We begin by characterizing the equilibrium in the second period.

If the bank makes a safe investment at stage 1, it is clear that the bank will not default and therefore requires no capital at stage 2. The bank rolls over its debt at stage 1.5 with a promised repayment of $\tilde{R}_2 = 1$.

If the bank makes a risky investment at stage 1, Assumption 6 ensures that the strategic regulator passes the bank if and only if the loan is good, as depicted in Table 1.

At stage 2, given the stress test result, the bank raises $\tilde{R}_2$ unit of capital if it fails the stress test. Analogous to Eq. 2, the equity given to the capital providers is a fraction $\phi(\tilde{R}_2)$, determined by

$$\phi(\tilde{R}_2)(1-d)R = \tilde{R}_2\rho_L + \beta\left[(1-d)R - \tilde{R}_2\rho_L\right].$$

Anticipating the stress testing strategy of the regulator at stage 2, we can now proceed back to the point when the bank rolls over its maturing debt at stage 1.5 and determine the promised repayment $\tilde{R}_2(z_2)$ to debtholders:

$$[\alpha + (1-\alpha)(1 - [z_2 + (1-z_2)\gamma]\, d)]\, \tilde{R}_2(z_2) = 1.$$

Eq. 13 takes into account that debtholders will be repaid if the loan doesn't default. There is no default if the is of good quality (with probability α). If the loan is of bad quality (with probability $1-\alpha$), defaults with probability d if (i) the regulator is lenient (with probability z_2) or (ii) the regulator is strategic (with probability $1-z_2$) but recapitalization is unfeasible (with probability γ).

We can now analyze the bank's investment decision at stage 1. At stage 1, given the promised repayment $\tilde{R}_2$ to debtholders and the fraction of equity ϕ it will need to sell to capital providers in exchange for capital, the bank originates a risky loan if and only if

$$[\alpha + (1-\alpha)[z_2 + (1-z_2)\gamma](1-d)]\left(R - \tilde{R}_2\right)$$

$$+ (1-\alpha)(1-z_2)(1-\gamma)(1-\phi)(1-d)R \geq R_0 - 1. \tag{12}$$

Notice that the analogous expressions in the baseline model given by Eq. 3 is equivalent to this expression if $\tilde{R}_2 = 1$.

We can now characterize the equilibrium in the second period, analogous to that characterized in Proposition 1:

Lemma 2. *In the second period, there exists a unique equilibrium. There exists a unique threshold $\tilde{z}_2^* < 1$, such that in equilibrium the bank originates a risky loan if and only if $z_2 \geq \tilde{z}_2^*$. If the bank extends a risky loan at stage 1, the lenient type regulator passes the bank with certainty and the strategic regulator passes the bank if and only if the bank's loan is good.*

Moreover, there exists $\tilde{\beta} < 1$, such that $\tilde{z}_2^* > 0$ if and only if $\beta > \tilde{\beta}$.

It then follows that the strategic regulator's expected surplus, for a given reputation z_2, denoted by $\tilde{U}(z_2)$, is given by

$$\tilde{U}(z_2) = \begin{cases} U_R, & \text{if } z_2 > \tilde{z}_2^*, \\ U_0, & \text{if } z_2 < \tilde{z}_2^*, \\ \lambda U_R + (1-\lambda)U_0 \text{ for some } \lambda \in [0,1], & \text{if } z_2 = \tilde{z}_2^*. \end{cases} \tag{13}$$

7.2.2 Stress testing in the first period

We can now analyze the equilibrium stress testing strategy of the regulator for the bank in the first period, given the equilibrium in the second period.

At stage 2, given the promised repayment of the bank's debt $\tilde{R}_1$ and the posterior beliefs held by the bank about the regulator's type after it passes and fails the stress test z_2^R and z_2^f, respectively, let $\tilde{G}_{q_1}(\tilde{R}_1, z_2^R, z_2^f)$ denote the net gain of passing the bank relative to failing the bank given the quality of the bank's risky loan $q_1 \in \{g, b\}$:

$$\tilde{G}_g(\tilde{R}_1, z_2^R, z_2^f) = (1-\gamma)\tilde{R}_1(\rho_L - 1) + \delta\left[\tilde{U}(z_2^R) - \tilde{U}(z_2^f)\right],$$
$$\tilde{G}_g(\tilde{R}_1, z_2^R, z_2^f) = (1-\gamma)\left[\tilde{R}_1(\rho_L - 1) - dD\right] + \delta\left[(1-d)\tilde{U}(z_2^R) - [(1-d) + d(1-\gamma)]\,\tilde{U}(z_2^f)\right]. \tag{14}$$

Notice that the analogous expressions in the baseline model given by Eq. 9 is equivalent to this expression if $\tilde{R}_1 = 1$.

In equilibrium, the posterior beliefs are as in the baseline model, where $z_2^f = 0$ and $z_2^R = z_2^R(\pi_g, \pi_b)$, given by Eq. 10. The equilibrium promised repayment at stage 1 anticipates the regulator's stress testing strategy and is equal to $\tilde{R}_1(\pi_b)$, given by

$$[\alpha + (1-\alpha)(1 - d + (1-z_1)(1-\pi_b)(1-\gamma)d)]\,\tilde{R}_1(\pi_b) = 1. \tag{15}$$

We now show in next proposition that the regulator's reputation building incentives in equilibrium are similar to those in the baseline model.

Proposition 8. *The equilibria without deposit insurance are characterized as follows.*

- *For intermediate levels of the net social benefits of risky lending $X \in [\underline{X}, \overline{X}]$, there exists a unique equilibrium as described in Proposition 2.*

- *For high net social benefits of risky lending $X > \overline{X}$, there exists an equilibrium in the first bank's stress test, which is either fully informative or lenient (as described in Lemma 1).*

- *For low net social benefits of risky lending $X < \underline{X}$, there exists an equilibrium in the first bank's stress test, which is either fully informative or tough (as described in Lemma 1).*

In particular, the self-fulfilling reputation building incentives of the strategic regulator leads to multiple equilibria for low net social benefits of risky lending ($X < \underline{X}$), as in the baseline model. Moreover, another type of equilibrium multiplicity can arise for high net social benefits of risky lending ($X > \overline{X}$) due to endogenous social cost of recapitalizing the bank. Specifically, if the investors conjecture that the strategic regulator adopts a more lenient stress test strategy (higher π_b), they require a higher promised repayment $\tilde{R}_1$ since they expect higher probability of default. Consequently, the regulator would have to require the bank to raise a higher amount of capital (equal to $\tilde{R}_1$) to eliminate a potential bank default, increasing the social cost of recapitalizing the bank. In turn, this raises the net gain for the strategic regulator from passing the bank, justifying a more lenient stress testing strategy.

8 Conclusion

Stress tests have been incorporated recently into the regulatory toolkit. The tests provide assessments of bank risk in adverse scenarios. Regulators respond to negative information by requiring banks to raise capital. However, regulators have incentives to be tough by asking even some safe banks to raise capital or to be lenient by allowing some risky banks to get by without raising capital. These incentives are driven by the weight the regulator places on lending in the economy versus stability. Banks respond to the leniency of the stress test by altering their lending policies. We demonstrate that in equilibrium, regulators may be tough and discourage lending or lenient and encourage lending. These equilibria can be self-fulfilling and the regulator may get trapped in one of them, leading to a loss of surplus. Banking supervision exams will lead to similar results but be less informative.

It would be of great interest to study regulators' reputation and effects on the real economy when stress tests deal with multiple banks in a macroprudential setting.

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9 Proofs

9.1 Proof of Proposition 1

$\Delta(z_2)$ given by Eq. 4 is obtained by substituting Eq. 2 into Eq. 3 to eliminate ϕ and rearranging.

Notice that $\Delta(z_2)$ is strictly increasing in z_2^*. Moreover, $\Delta(1) > 0$ as implied by Assumption 1, and $\Delta(z_2) \to -\infty$ as $z_2 \to -\infty$. Therefore a unique z_2^* as defined by $\Delta(z_2^*) = 0$ exists, where $z_2^* < 1$.

We now derive a condition for $z_2^* > 0$. This is the case if and only if

$$\begin{aligned}\Delta(0) = {} & [\alpha + \gamma(1-\alpha)(1-d)]\,(R-1) - (R_0 - 1) \\ & - (1-\alpha)(1-\gamma)(1-\beta)\,[(1-d)R - \rho_L] < 0. \qquad (16)\end{aligned}$$

Notice the above expression is strictly decreasing in β. Moreover, Assumption 3 implies that $\Delta(0) < 0$ for $\beta = 1$. Therefore there exists a unique $\bar{\beta} < 1$, such that $z_2^* > 0$ if and only if $\beta > \bar{\beta}$, where $\bar{\beta}$ is defined by

$$[\alpha + \gamma(1-\alpha)(1-d)]\,(R-1) - (R_0 - 1) - (1-\alpha)(1-\gamma)(1-\bar{\beta})\,[(1-d)R - \rho_L] = 0. \qquad (17)$$

∎

9.2 Proof of Proposition 2

Let $\underline{X}$ be defined such that $U_R = U_0$, i.e.,

$$[\alpha + (1-\alpha)(1-d)]\,R - 1 + \underline{X} = R_0 - 1. \qquad (18)$$

Let $\overline{X}$ be defined such that $(1-d)U_R = [(1-d)+d(1-\gamma)]\,U_0$, i.e.,

$$(1-d)\left([\alpha+(1-\alpha)(1-d)]\,R-1+\overline{X}\right) = [(1-d)+d(1-\gamma)]\,(R_0-1). \tag{19}$$

It is straightforward to show that $\overline{X} > \underline{X}$.

We now consider the case where $X \in [\underline{X}, \overline{X}]$. Notice that $X \geq \underline{X}$ and the fact that $z_2^R(\pi_g,\pi_b) \geq z_2^f = 0$ for all (π_g,π_b) implies that $U_R \geq U(z_2^R(\pi_g,\pi_b)) \geq U(z_2^f) \geq U_0$. It follows that:

$$\begin{aligned}
G_g(z_2^R(\pi_g,\pi_b),0) &= (1-\gamma)\tilde{R}_1(\pi_b)(\rho_L-1)+\delta\left[U(z_2^R(\pi_g,\pi_b))-U(z_2^f)\right] \\
&\geq (1-\gamma)\tilde{R}_1(\pi_b)(\rho_L-1) > 0.
\end{aligned} \tag{20}$$

Moreover, this also implies that:

$$\begin{aligned}
G_b(z_2^R(\pi_g,\pi_b),z_2^f) &= (1-\gamma)\left[\tilde{R}_1(\pi_b)(\rho_L-1)-dD\right] \\
&\quad +\delta\left[(1-d)U(z_2^R(\pi_g,\pi_b))-[(1-d)+d(1-\gamma)]\,U(z_2^f)\right] \\
&\leq (1-\gamma)\left[\tilde{R}_1(\pi_b)(\rho_L-1)-dD\right] \\
&\quad +\delta\left[(1-d)U_R-[(1-d)+d(1-\gamma)]\,U_0\right] < 0,
\end{aligned} \tag{21}$$

where the second inequality follows because of Assumption 4 and $X \leq \overline{X}$.

Since $G_g(z_2^R(\pi_g,\pi_b),0) > 0 > G_b(z_2^R(\pi_g,\pi_b),z_2^f)$ for all (π_g,π_b), in this case there exists a unique equilibrium in which the strategic regulator passes the bank in the first period if and only if the risky loan is good. ∎

9.3 Proof of Proposition 3

Since $X > \overline{X} > \underline{X}$, we have $G_g(z_2^R(\pi_g,\pi_b),0) > 0$ for all (π_g,π_b) as shown in the proof of Proposition 2. Therefore $\pi_g = 1$ in any equilibrium.

Before we proceed, given $\pi_g = 1$, we establish some properties of $G_b(z_2^R(1, \pi_b), z_2^f)$, where we have $z_2^R(1, \pi_b) > z_2^f = 0$.

- If $\beta = \bar{\beta}$, where $\bar{\beta}$ is defined in Proposition 1, then Eq. 8 implies that $U(z_2^R(1, \pi_b)) = U_R$ for all $\pi_b \in [0, 1]$ and $U(z_2^f)$ takes a continuum of values in $[U_0, U_R]$. In turn, this implies that $G_b(z_2^R(1, \pi_b), z_2^f)$ takes a continuum of values between $(1 - \gamma)[(\rho_L - 1) - dD] < 0$ and $(1 - \gamma)[(\rho_L - 1) - dD] + \delta[(1 - d)U_R - [(1 - d) + d(1 - \gamma)]U_0]$.

- If $\beta \neq \bar{\beta}$, then $z_2^* > 0$ and let us define $\hat{\pi}_b$ such that

$$z_2^R(1, \hat{\pi}_b) = \frac{[\alpha + (1 - \alpha)(1 - d)]\, z_1}{[\alpha + (1 - \alpha)(1 - d)]\, z_1 + [\alpha + (1 - \alpha)(1 - d)\hat{\pi}_b]\,(1 - z_1)} = z_2^*. \tag{22}$$

  Notice that $z_2^R(1, \pi_b)$ is strictly decreasing in π_b, therefore $G_b(z_2^R(1, \pi_b), 0)$ is continuous and decreasing in π_b. Specifically, $G_b(z_2^R(1, \pi_b), 0)$ is equal to $(1 - \gamma)[(\rho_L - 1) - dD] + \delta[(1 - d)U_R - [(1 - d) + d(1 - \gamma)]U_0]$ for all $\pi_b < \hat{\pi}_b$, is equal to $(1-\gamma)[(\rho_L - 1) - dD] < 0$ for all $\pi_b > \hat{\pi}_b$, and takes a continuum of values in between at $\pi_b = \hat{\pi}_b$.

We can now characterize the two possible types of equilibrium: a fully informative equilibrium with $\pi_b = 0$ and a lenient equilibrium with $\pi_b > 0$.

First, an equilibrium with $\pi_b = 0$ exists if and only if $G_b(z_2^R(1, 0), 0) \leq 0$. Notice that $G_b(z_2^R(1, 0), 0) > 0$ implies that $U(z_2^R(1, 0)) > U(z_2^f)$, which in turn implies that $G_b(z_2^R(1, 0), 0)$ is strictly increasing in δ and that $z_2^R(1, 0) \geq z_2^* \geq 0$. Therefore there exists $\bar{\delta}_b(\beta, z_1) \in \mathbf{R}^+ \cup \{\infty\}$, such that an equilibrium with $\pi_b = 0$ exists if and only if $\delta \leq \bar{\delta}_b(\beta, z_1)$, where $\bar{\delta}_g(z_1) \neq \infty$ if and only if $z_2^R(1, 0) > z_2^* > 0$. More specifically, $\bar{\delta}_b(\beta, z_1)$ is defined by

$$\bar{\delta}_b(\beta, z_1) = \begin{cases} \delta_b, & \text{if } \beta > \bar{\beta} \text{ and } z_1 > \bar{z}_1, \\ \infty, & \text{otherwise.} \end{cases} \tag{23}$$

where $\beta > \bar{\beta}$ ensures that $z_2^* > 0$ by Proposition 1 and $\bar{z}_1$ is defined such that $z_2^R(1, 0) = z_2^*$, i.e.,

$$\frac{[\alpha + (1-\alpha)(1-d)]\,\bar{z}_1}{[\alpha + (1-\alpha)(1-d)]\,\bar{z}_1 + \alpha(1-\bar{z}_1)} = z_2^*, \tag{24}$$

and δ_b is defined by

$$(1-\gamma)\left[(\rho_L - 1) - dD\right] + \delta_b\left[(1-d)U_R - [(1-d) + d(1-\gamma)]\,U_0\right] = 0. \tag{25}$$

Next, an equilibrium with $\pi_b > 0$ exists if and only if $G_b(z_2^R(1,\pi_b),0) \geq 0$ for some $\pi_b \in (0,1]$. Since we have shown that $G_b(z_2^R(1,\pi_b),0)$ is decreasing in π_b, an equilibrium with $\pi_b > 0$ exists if and only if $\lim_{\pi_b \to 0} G_b(z_2^R(1,\pi_b),0) \geq 0$. Notice that $\lim_{\pi_b \to 0} G_b(z_2^R(1,\pi_b),0) \geq 0$ implies that $\lim_{\pi_b \to 0} U(z_2^R(1,\pi_b)) > U(z_2^f)$, which in turn implies that $\lim_{\pi_b \to 0} G_b(z_2^R(1,\pi_b),0)$ is strictly increasing in δ and that $z_2^R(1,0) \geq z_2^* \geq 0$. Therefore there exists $\underline{\delta}_b(\beta,z_1) \in \mathbf{R}^+ \cup \{\infty\}$, such that an equilibrium with $\pi_b > 0$ exists if and only if $\delta \geq \underline{\delta}_b(\beta,z_1)$, where $\underline{\delta}_b(\beta,z_1) \neq \infty$ if and only if $z_2^R(1,0) > z_2^* \geq 0$. More specifically, $\underline{\delta}_b(\beta,z_1)$ is defined by

$$\underline{\delta}_b(\beta,z_1) = \begin{cases} \delta_b, & \text{if } \beta \geq \bar{\beta} \text{ and } z_1 > \bar{z}_1, \\ \infty, & \text{otherwise,} \end{cases} \tag{26}$$

where $\beta \geq \bar{\beta}$ ensures that $z_2^* \geq 0$, and $z_1 > \bar{z}_1$ ensures that $z_2^R(1,0) > z_2^*$.

To summarize, an equilibrium with $\pi_b = 0$ exists if and only if $\delta \leq \bar{\delta}_b(\beta,z_1)$, whereas an equilibrium with $\pi_b > 0$ exists if and only if $\delta \geq \bar{\delta}_b(\beta,z_1)$. Finally, it is immediate from Eqs. 25 and 28 that $\bar{\delta}_b(\beta,z_1) \geq \underline{\delta}_b(\beta,z_1)$, with strict inequality if and only if $\beta = \bar{\beta}$. ∎

9.4 Proof of Proposition 4

Notice that $X < \underline{X}$ and the fact that $z_2^R(\pi_g,\pi_b) \geq z_2^f$ implies that $U(z_2^R(\pi_g,\pi_b)) \leq U(z_2^f)$. It follows that

$$G_b(z_2^R(\pi_g,\pi_b), z_2^f) = (1-\gamma)\left[\tilde{R}_1(\rho_L - 1) - dD\right]$$

$$
\begin{aligned}
&+\delta\left[(1-d)U(z_2^R(\pi_g,\pi_b)) - [(1-d)+d(1-\gamma)]\,U(z_2^f)\right] \\
\leq\ & (1-\gamma)\left[\tilde{R}_1(\rho_L-1)-dD\right] \\
&+\delta\left[(1-d)U(z_2^f) - [(1-d)+d(1-\gamma)]\,U(z_2^f)\right] < 0.
\end{aligned}
\tag{27}
$$

Before we proceed, given $\pi_b = 0$, we establish some properties of $G_g(z_2^R(\pi_g, 0), z_2^f)$, where we have $z_2^R(\pi_g, 0) > z_2^f = 0$.

- If $\beta = \bar{\beta}$, where $\bar{\beta}$ is defined in Proposition 1, then Eq. 8 implies that $U(z_2^R(\pi_g, 0)) = U_R$ for all $\pi_g \in [0, 1]$ and $U(z_2^f)$ takes a continuum of values in $[U_0, U_R]$. In turn, this implies that $G_g(z_2^R(\pi_g, 0, z_2^f)$ takes a continuum of values between $(1-\gamma)(\rho_L - 1) > 0$ and $(1-\gamma)(\rho_L - 1) + \delta\,[U_R - U_0]$.

- If $\beta \neq \bar{\beta}$, then $z_2^* > 0$. Notice that $z_2^R(\pi_g, 0)$ is strictly decreasing in π_g, therefore $G_g(z_2^R(\pi_g, 0), 0)$ is continuous and increasing in π_g.

We can now characterize the two possible types of equilibrium: fully informative equilibrium with $\pi_g = 1$ and tough equilibrium with $\pi_g < 1$.

First, an equilibrium with $\pi_g = 1$ exists if and only if $G_g(z_2^R(1, 0), 0) \geq 0$. Notice that $G_g(z_2^R(1, 0), 0) < 0$ implies that $U(z_2^R(1, 0)) < U_0$, which in turn implies that $G_g(z_2^R(1, 0), 0)$ is strictly decreasing in δ and that $z_2^R(1, 0) \geq z_2^* \geq 0$. Therefore there exists $\bar{\delta}_g(\beta, z_1) \in \mathbf{R}^+ \cup \{\infty\}$, such that an equilibrium with $\pi_g = 1$ exists if and only if $\delta \leq \bar{\delta}_g(\beta, z_1)$, where $\bar{\delta}_g(\beta, z_1) \neq \infty$ if and only if $z_2^R(1, 0) > z_2^* > 0$. More specifically, $\bar{\delta}_g(\beta, z_1)$ is defined by

$$
\bar{\delta}_g(\beta, z_1) = \begin{cases} \delta_g, & \text{if } \beta > \bar{\beta} \text{ and } z_1 > \bar{z}_1, \\ \infty, & \text{otherwise}, \end{cases} \tag{28}
$$

where $\beta > \bar{\beta}$ ensures that $z_2^* > 0$ by Proposition 1, $z_1 > \bar{z}_1$ ensures that $z_2^R(1, 0) > z_2^*$ by the definition of $\bar{z}_1$ given by Eq. 26, and δ_g is defined by

$$(1-\gamma)(\rho_L - 1) + \delta_g [U_R - U_0] = 0. \tag{29}$$

Second, an equilibrium with $\pi_g < 1$ exists if and only if $G_g(z_2^R(\pi_g, 0), 0) \leq 0$ at some $\pi_g < 1$. Since we have shown above that $G_g(\tilde{R}_1(0), z_2^R(\pi_g, 0), 0)$ is increasing in π_g, an equilibrium with $\pi_g < 1$ exists if and only if $G_g(\tilde{R}_1(0), z_2^R(0,0), 0) = G_g(\tilde{R}_1(0), 1, 0) \leq 0$. Notice that $G_g(\tilde{R}_1(0), 1, 0) \leq 0$ implies that $G_g(\tilde{R}_1(0), 1, 0)$ is strictly decreasing in δ and that $1 \geq z_2^* \geq 0$. Therefore there exists $\underline{\delta}_g(z_1) \in \mathbf{R}^+ \cup \{\infty\}$, such that an equilibrium with $\pi_g < 1$ exists if and only if $\delta \geq \underline{\delta}_g(\beta, z_1)$, where $\underline{\delta}_g(z_1) \neq \infty$ if and only if $1 \geq z_2^* \geq 0$. More specifically, $\underline{\delta}_g(\beta, z_1)$ is defined by

$$\underline{\delta}_g(\beta, z_1) = \begin{cases} \delta_g, & \text{if } \beta \geq \bar{\beta}, \\ \infty, & \text{otherwise,} \end{cases} \tag{30}$$

where $\beta \geq \bar{\beta}$ ensures that $z_2^* \geq 0$ by Proposition 1, and $\delta_g(z_1)$ is defined by Eq. 31. $1 > z_2^*$ is satisfied by Proposition 1.

To summarize, an equilibrium with $\pi_g = 1$ exists if and only if $\delta \leq \bar{\delta}_g(\beta, z_1)$, whereas an equilibrium with $\pi_g < 1$ exists if and only if $\delta \geq \underline{\delta}_g(\beta, z_1)$. Finally, it is immediate from Eqs. 30 and 32 that $\bar{\delta}_g(\beta, z_1) \geq \underline{\delta}_g(\beta, z_1)$, with strict inequality if and only if $\beta \geq \bar{\beta}$ and $z_1 \leq \bar{z}_1$. ∎

9.5 Proof of Corollary 1

Using similar logic as the proof of Proposition 3, an equilibrium with $\pi_b = 1$ exists if and only if $G_b(z_2^R(1,1), 0) = G_b(z_1, 0) \geq 0$. Notice that $G_b(z_1, 0) \geq 0$ implies that $G_b(z_1, 0)$ is strictly increasing in δ and that $z_1 \geq z_2^* \geq 0$. an equilibrium in which π_b exists if and only if $z_1 \geq z_2^*$ and $\delta \geq \delta_b$. This is condition is equivalent to $z_1 \geq z_2^*$ and $\delta \geq \bar{\delta}_b$, since $z_1 \geq z_2^*$ implies that $z_1 > \bar{z}_1$ and thus $\bar{\delta}_b = \delta_b$. ∎

9.6 Proof of Corollary 2

This corollary follows immediately from Propositions 3–4, the implicit function theorem, and the observation that $G_g(\cdot)$ is decreasing in γ_1, and $G_b(\cdot)$ is increasing in γ_1.

9.7 Proof of Corollary 3

This corollary follows immediately from Propositions 3–4, the implicit function theorem, and the observation that $G_g(\cdot)$ is independent of D_1, and $G_b(\cdot)$ is decreasing in D_1.

9.8 Proof of Proposition 5

We prove this proposition by analyzing the three regions of X separately.

- $X \in [\underline{X}, \overline{X}]$. The proof is identical to the proof of Proposition 2.

- $X > \overline{X}$. The characterization of the equilibrium follows the logic of the proof of Proposition 3. Recall that $z^R_{2,g}(\pi_g) > z^f_{2,g} = 0$ for all $\pi_g \in [0,1]$. We then have that $G_g(z^R_{2,g}(\pi_g), z^f_{2,g}) > 0$ for all π_g and therefore $\pi_g = 1$ in any equilibrium. The equilibrium is thus either fully informative or lenient. A fully informative equilibrium (i.e. one with $\pi_b = 0$) exists if and only if $G_b(z^R_{2,b}(\pi_b), 0) \leq 0$, whereas a lenient equilibrium (i.e. one with $\pi_b > 0$) exists if and only if $G_b(z^R_{2,b}(\pi_b), 0) \geq 0$ for some $\pi_b > 0$.

  We first show that, the parameter space for which the fully informative equilibrium exists is (weakly) smaller than in the baseline model, and that for which the lenient equilibrium exists is (weakly) larger than in the baseline model. This follows because, for any $\pi_b \in [0,1]$, $G_b(z^R_{2,b}(\pi_b), 0) \geq G_b(z^R_2(1, \pi_b), 0)$, since $G_b(z^R_2, 0)$ is increasing in z^R_2 and $z^R_{2,b}(\pi_b) \geq z^R_2(1, \pi_b)$.

  We now show that the above statement holds with strict inequality by demonstrating that there exist parameter values for which the fully informative equilibrium is the unique equilibrium in the baseline model but does not exist in the model in which the bank observes q_t. Specifically, suppose $\beta > \bar{\beta}$, $z_1 = \bar{z}_1 - \epsilon$, where $\epsilon > 0$, and $\delta > \delta_b$, where δ_b is defined by Eq. 27. In this case, $z_1 < \bar{z}_1$ implies that $\bar{\delta}_b(\beta, z_1) = \underline{\delta}_b(\beta, z_1) = \infty$ as shown in the proof of Proposition 3, therefore the unique equilibrium in the baseline model is fully informative. Recall that $\bar{z}_1$ is defined such that $z_2^R(1,0) = z_2^*$. Therefore for ϵ sufficiently small, $z_1 < \bar{z}_1$ implies that $\Delta(0) \leq \Delta(z_2^R(1,0)) < 0 < \Delta(z_{2,b}^R(0))$. This implies that $U(z_{2,b}^R(0)) = U_R$ and $U(z_{2,b}^f) = 0$, and therefore $G(z_{2,b}^R(0), 0) > 0$ as implied by $\delta > \delta_b$. Therefore a fully informative equilibrium does not exist when the bank observes q_t, whereas by continuity of $G_b(z_{2,b}^R(\pi_b), 0)$ in π_b, a lenient equilibrium does exist.

- $X < \underline{X}$. The characterization of the equilibrium follows the logic of the proof of Proposition 4. We have that $G_b(z_{2,b}^R(\pi_b), z_{2,b}^f) < 0$ for all π_b and therefore $\pi_b = 0$ in any equilibrium. The equilibrium is thus either fully informative or tough. A fully informative equilibrium (i.e. one with $\pi_g = 1$) exists if and only if $G_g(z_{2,g}^R(1), 0) \geq 0$, whereas a tough equilibrium (i.e. one with $\pi_g < 1$) exists if and only if $G_g(z_{2,g}^R(\pi_g), 0 \leq 0$ for some $\pi_g < 1$.

  We first show that the parameter space for which the fully informative equilibrium exists is (weakly) larger than in the baseline model, and that for which the tough equilibrium exists is (weakly) smaller than in the baseline model. This follows because, for any $\pi_g \in [0,1]$, $G_g(z_{2,g}^R(\pi_g), 0) \geq G_g(z_2^R(\pi_g, 0), 0)$, since $G_g(z_2^R, 0)$ is decreasing in z_2^R and $z_{2,g}^R(\pi_g) < z_2^R(\pi_g, 0)$.

  Following similar logic as for the case where $X > \overline{X}$, we can show that the parameter space for which the fully informative equilibrium exists is strictly larger than in the baseline model. Specifically, suppose $\beta > \bar{\beta}$, $z_1 = \bar{z}_1 - \epsilon$, where $\epsilon > 0$, and $\delta > \delta_g$, where δ_g is defined by Eq. 31. For ϵ sufficiently small, the unique equilibrium in the baseline model is tough, whereas a fully informative equilibrium exist in the model in which the bank observes q_t.

Finally, we show that the parameter space for which a tough equilibrium exists is identical to the baseline model. Since $G_g(z_2^R, 0)$ is decreasing in z_2^R and $z_{2,g}^R(pi_g)$ is decreasing in π_g, $G_g(z_{2,g}^R(\pi_g), 0)$ is increasing in π_g and therefore a tough equilibrium exists if and only if $G_g(z_{2,g}^R(0)) = G_g(z_1, 0) \leq 0$. Recall that we have shown in the proof of Proposition 4 that a tough equilibrium exists if and only if $G_g(z_2^R(0,0),0) = G_g(z_1, 0) \geq 0$. Therefore the condition and thus the parameter space for which a tough equilibrium exists is identical to the baseline model.

∎

9.9 Proof of Proposition 6

Let z_t denote the market's ex ante belief in period $t \in \{1, 2\}$ that the regulator is the informative type. Following backward induction, we first solve for the equilibrium in the second period. Analogous to Proposition 1, the equilibrium in the second period is characterized in the following lemma.

Lemma 3. *In the second period, there exists a unique equilibrium, in which the bank originates a risky loan if and only if $\beta \leq \bar{\beta}$, where $\bar{\beta}$ is defined in Proposition 1.*

This lemma follows similar logic as the proof of Proposition 1. Given that the behavioral regulator passes the second bank at stage 3 if and only if the loan is good, the bank originates a risky loan at stage 1 if and only if $\Delta(0) \geq 0$, which is the case if and only if $\beta \leq \bar{\beta}$.

Since this lemma implies that the second bank's investment choice does not depend on the regulator's reputation, this proposition then follows. ∎

9.10 Proof of Proposition 7

Let s_t denote the market's ex ante belief in period $t \in \{1, 2\}$ that the regulator is the strategic type. That is, a higher s_t reflects a market belief that the regulator is likely to be more lenient.

Following backward induction, we start by considering the second period. As in the baseline model, we focus on describing the equilibrium stress test outcome given that the bank originates a risky loan at stage 1. The strategic regulator at stage 2 passes the bank if and only if the loan is good, as implied by Assumption 4.

Conditional on failing the stress test, the probability that the bank is good is given by

$$\alpha_2^T = \frac{\alpha(1-s_2)}{\alpha(1-s_2)+(1-\alpha)}.$$

Recapitalization is feasible if and only if the opportunity cost of capital is low ($\rho = \rho_L$), in which case the equity given to the capital providers is a fraction ϕ^T, determined by:

$$\phi^T \left[\alpha_2^T + (1-\alpha_2^T)(1-d)\right] R = \rho_L + \beta \left(\left[\alpha_2^T + (1-\alpha_2^T)(1-d)\right] R - \rho_L\right). \tag{31}$$

At stage 1, the bank originates a risky loan if and only if

$$\begin{aligned}[\alpha\left[1-(1-s_2)(1-\gamma)\right] + (1-\alpha)\gamma(1-d)]\,(R-1)& \\ + (1-\gamma)(1-\phi^T)\left[\alpha(1-s_2)+(1-\alpha)(1-d)\right] R &\geq R_0 - 1. \end{aligned}\tag{32}$$

Analogous to Proposition 1, the equilibrium in the second period is characterized in the following lemma.

Lemma 4. *In the second period, there exists a unique equilibrium. There exists a unique threshold s_2^*, such that in equilibrium the bank originates a risky loan if and only if $s_2 \geq s_2^*$, given by $\Delta^T(s_2^*) = 0$, where*

$$\begin{aligned}\Delta^T(s_2) \equiv [\alpha\left[1-(1-s_2)(1-\gamma)\right] + (1-\alpha)\gamma(1-d)]\,(R-1) - (R_0-1)& \\ + (1-\gamma)(1-\beta)\left(\left[\alpha(1-s_2)+(1-\alpha)(1-d)\right] R - \rho_L\right)&. \end{aligned}\tag{33}$$

Moreover, $s_2^* < 1$ if and only if $\beta < \bar{\beta}$, where $\bar{\beta}$ is defined in Proposition 1. Finally, there *exists $\underline{\beta} < \bar{\beta}$, such that $s_2^* > 0$ if and only if $\beta > \underline{\beta}$.*

Proof. Eq. 35 is obtained by substituting Eq. 33 into Eq. 34 to eliminate ϕ^T. Moreover, notice that $\Delta^T(s_2)$ is increasing in s_2, and $\Delta^T(1) = \Delta(0)$. Therefore $s_2^* < 1$ if and only if $\beta < \bar{\beta}$, since $\bar{\beta}$ is defined such that $\Delta(0) = 0$ at $\beta = \bar{\beta}$. Finally, $s_2^* > 0$ if and only if $\Delta^T(0) < 0$. This is the case if and only if $\beta > \underline{\beta}$, where $\underline{\beta}$ is defined by

$$\begin{aligned}\gamma\left[\alpha + (1-\alpha)(1-d)\right](R-1) - (R_0 - 1)&\\ + (1-\gamma)(1-\underline{\beta})\left(\left[\alpha + (1-\alpha)(1-d)\right]R - \rho_L\right) &= 0. \qquad (34)\end{aligned}$$

We have that $\underline{\beta} < \bar{\beta}$ because $\Delta^T(s_2)$ is increasing in s_2 and decreasing in β. ∎

It then follows that the strategic regulator's expected surplus from the bank in the second period, for a given reputation s_2, denoted by $U^T(s_2)$, is given by

$$U^T(s_2) = \begin{cases} U_R, & \text{if } s_2 > s_2^*, \\ U_0, & \text{if } s_2 < s_2^*, \\ \lambda U_R + (1-\lambda)U_0 \text{ for some } \lambda \in [0,1], & \text{if } s_2 = s_2^*. \end{cases} \qquad (35)$$

We now move to analyzing the equilibrium stress test of the regulator for the bank in the first period, given the equilibrium in the second period. The incentives of the strategic regulator to pass the first period bank is given by

$$\begin{aligned} G_g^T(s_2^p, s_2^R) &= (1-\gamma)(\rho_L - 1) + \delta\left[U^T(s_2^p) - U^T(s_2^R)\right], \\ G_b^T(s_2^p, s_2^R, s_2^0) &= (1-\gamma)\left[(\rho_L - 1) - dD\right] \\ &\quad + \delta\left[(1-d)U^T(s_2^p) - (1-d)U^T(s_2^R) - d(1-\gamma)U^T(s_2^0)\right]. \qquad (36)\end{aligned}$$

Analogous to Eq. 9, the first term in Eq. 38 represents the net gain in terms of the expected surplus from the bank in the first period and the second term represents the reputation concern in terms of the expected surplus from the bank in the second period. In contrast to the baseline setup, passing the first period bank reveals that the regulator is strategic, i.e. $s_2^p = 1$. Subsequently, the bank continues to the second period with probability 1 if its risky loan is good, or with probability $1-d$ if its risky loan is bad. The term s_2^R (s_2^0) is the posterior belief held by the market about the probability that the regulator is strategic, given that the first period bank fails the stress test and that the realized payoff is R (0). Since the bank recapitalizes with probability $1-\gamma$ after failing the stress test, it continues to the second period with probability 1 if its payoff is R and with probability $1-\gamma$ if its payoff is 0. In equilibrium, the posterior probabilities are given by

$$\begin{aligned} s_2^R(\pi_g,\pi_b) &= \frac{[\alpha(1-\pi_g)+(1-\alpha)(1-d)(1-\pi_b)]\, s_1}{[\alpha(1-\pi_g)+(1-\alpha)(1-d)(1-\pi_b)]\, s_1 + [\alpha+(1-\alpha)(1-d)]\,(1-s_1)}, \\ s_2^0(\pi_b) &= \frac{(1-\pi_b)s_1}{(1-\pi_b)s_1+(1-s_1)}. \end{aligned} \tag{37}$$

In particular, if the first bank fails the stress test, the market updates its belief to $s_2^R, s_2^0 \le s_1$ if the bank realizes a payoff of R and 0, respectively, reflecting the fact that the strategic regulator is less likely to fail a bank than the tough regulator. Moreover, we have $s_2^R < s_2^0$, since the strategic regulator is more likely to fail a bad bank than a good bank.

We can now prove this proposition by considering the three regions of X separately.

- $X \in [\underline{X}, \overline{X}]$. Recall that $s_2^p = 1 \ge s_2^0(\pi_b) \ge s_2^R(\pi_g,\pi_b)$ for all (π_g,π_b). Therefore $X \ge \underline{X}$ implies that $U_R \ge U^T(s_2^p) \ge U^T(s_2^0(\pi_b)) \ge U^T(s_2^R(\pi_g,\pi_b)) \ge U_0$ for all (π_g,π_b). It follows that

$$\begin{aligned} G_g^T(s_2^p, s_2^R(\pi_g,\pi_b)) &= (1-\gamma)(\rho_L-1)+\delta\left[U^T(s_2^p)-U^T(s_2^R(\pi_g,\pi_b))\right] \\ &\ge (1-\gamma)(\rho_L-1) > 0. \end{aligned} \tag{38}$$

Moreover, we have

$$
\begin{aligned}
& G_b^T(s_2^p, s_2^R(\pi_g,\pi_b), s_2^0(\pi_b)) \\
&= (1-\gamma)\left[(\rho_L - 1) - dD\right] \\
&\qquad\qquad + \delta\left[(1-d)U^T(s_2^p) - (1-d)U^T(s_2^R(\pi_g,\pi_b)) - d(1-\gamma)U^T(s_2^0(\pi_b))\right] \\
&\leq (1-\gamma)\left[(\rho_L-1) - dD\right] + \delta\left[(1-d)U_R - \left[(1-d) + d(1-\gamma)\right]U_0\right] < 0, \qquad (39)
\end{aligned}
$$

where the last inequality follows from $X \leq \overline{X}$.

Since $G_g^T(s_2^p, s_2^R(\pi_g,\pi_b)) \geq 0 \geq G_b^T(s_2^p, s_2^R(\pi_g,\pi_b), s_2^0(\pi_b))$ for all (π_g,π_b), there exists a unique equilibrium in which the regulator passes the bank in the first period if and only if the risky loan is good.

- $X > \overline{X}$. As shown above, this implies that $G_g^T(\tilde{R}_1^T(\pi_b), s_2^p, s_2^R(\pi_g,\pi_b)) > 0$ for all (π_g,π_b). Therefore $\pi_g = 1$ in any equilibrium.

  Given $\pi_g = 1$, we first establish some properties of $G_b^T(1, s_2^R(1,\pi_b), s_2^0(\pi_b))$. Notice that $s_2^R(1,\pi_b)$ and $s_2^0(\pi_b)$ defined by Eq. 39 are strictly decreasing in π_b, implying that $G_b^T(1, s_2^R(1,\pi_b), s_2^0(\pi_b))$ is increasing in π_b.

  We can now characterize the two possible types of equilibrium: a fully informative equilibrium with $\pi_b = 0$ and a lenient equilibrium with $\pi_b > 0$.

  First, an equilibrium with $\pi_b = 0$ exists if and only if

$$
\begin{aligned}
& G_b^T(1, s_2^R(1,0), s_2^0(0)) \\
&= (1-\gamma)\left[(\rho_L-1) - dD\right] \\
&\qquad\qquad + \delta\left[(1-d)U^T(1) - (1-d)U^T(s_2^R(1,0)) - d(1-\gamma)U^T(s_1)\right] \leq 0 \qquad (40)
\end{aligned}
$$

  Notice that $G_b^T(1, s_2^R(1,0), s_1) \leq 0$ implies that $G_b^T(1, s_2^R(1,0), s_1)$ is strictly increasing in δ and that $1 \geq s_2^* \geq s_2^R(1,0)$. Therefore there exists $\bar{\delta}_b^T \in \mathbf{R}^+ \cup \{\infty\}$, such that a fully informative equilibrium exists if and only if $\delta \leq \bar{\delta}_b^T$, where $\bar{\delta}_b^T \neq \infty$ if and only if

$$(1-d)U^T(1) - (1-d)U^T(s_2^R(1,0)) - d(1-\gamma)U^T(s_1) > 0. \quad (41)$$

When this is the case, $\bar{\delta}_b^T$ is defined by

$$\begin{aligned}&(1-\gamma)\left[(\rho_L - 1) - dD\right]\\&\qquad + \bar{\delta}_b^T \left[(1-d)U^T(1) - (1-d)U^T(s_2^R(1,0)) - d(1-\gamma)U^T(s_1)\right] = 0. \quad (42)\end{aligned}$$

Second, an equilibrium with $\pi_b > 0$ exists if and only if $G_b^T(1, s_2^R(1,\pi_b), s_2^0(\pi_b)) \geq 0$ for some $\pi_b \in (0,1]$. Because $G_b^T(1, s_2^R(1,\pi_b), s_2^0(\pi_b))$ is increasing in π_b, an equilibrium with $\pi_b > 0$ exists if and only if $G_b^T(1, s_2^R(1,1), s_2^0(1)) = G_b^T(1,0,0) \geq 0$. Again, $G_b^T(1,0,0) \geq 0$ implies that $G_b^T(1,0,0)$ is strictly increasing in δ and that $1 \geq s_2^* \geq 0$. Therefore there exists $\underline{\delta}_b^T \in \mathbf{R}^+ \cup \{\infty\}$, such that a lenient equilibrium exists if and only if $\delta \geq \underline{\delta}_b^T$, where $\underline{\delta}_b^T \neq \infty$ if and only if $1 \geq s_2^* \geq 0$. More specifically, $\underline{\delta}_b^T \neq \infty$ if and only if $\beta \in [\underline{\beta}, \bar{\beta}]$, in which case it is defined by

$$(1-\gamma)\left[(\rho_L - 1) - dD\right] + \underline{\delta}_b^T \left[(1-d)U_T - \left[(1-d) + d(1-\gamma)\right]U_0\right] = 0. \quad (43)$$

Finally, $\underline{\delta}_b^T \leq \bar{\delta}_b^T$. This follows because by definition, $\delta > \bar{\delta}_b^T$ and thus $G_b^T(1, s_2^R(1,1), s_2^0(1)) \geq 0$ implies that $G_b^T(1, s_2^R(1,0), s_2^0(0)) \geq 0$ and thus $\delta > \underline{\delta}_b^T$.

- $X < \underline{X}$. The fact that $s_2^p = 1$ is greater than or equal to both $s_2^R(\pi_g, \pi_b)$ and $s_2^0(\pi_b)$ implies that $U^T(s_2^p)$ is less than or equal to both $U^T(s_2^R(\pi_g,\pi_b))$ and $U^T(s_2^0(\pi_b))$ for all (π_g, π_b). It follows that:

$$\begin{aligned}&G_b^T(s_2^p, s_2^R(\pi_g,\pi_b), s_2^0(\pi_b))\\&= (1-\gamma)\left[(\rho_L - 1) - dD\right]\end{aligned}$$

$$+\,\delta\left[(1-d)U^T(s_2^p)-(1-d)U^T(s_2^R(\pi_g,\pi_b))-d(1-\gamma)U^T(s_2^0(\pi_b))\right]$$
$$\leq (1-\gamma)\left[(\rho_L-1)-dD\right]+\delta\left[(1-d)U^T(s_2^p)-\left[(1-d)+d(1-\gamma)\right]U^T(s_2^p)\right]<0. \tag{44}$$

Therefore $\pi_b = 0$ in any equilibrium.

Given $\pi_b = 0$, we first establish some properties of $G_g^T(1, s_2^R(\pi_g, 0))$. Notice that $s_2^R(\pi_g, 0)$ is strictly decreasing in π_g, implying that $G_g^T(1, s_2^R(\pi_g, 0))$ is decreasing in π_g.

We can now characterize the two possible types of equilibrium: a fully informative equilibrium with $\pi_g = 1$ and a tough equilibrium with $\pi_g < 1$.

First, an equilibrium with $\pi_g^* = 1$ exists if and only if $G_g^T(1, s_2^R(1, 0)) \geq 0$. Notice that $G_g^T(1, s_2^R(1, 0)) < 0$ implies that $G_g^T(1, s_2^R(1, 0))$ is strictly decreasing in δ and that $1 \geq s_2^R(1, 0)$. Therefore there exists $\bar{\delta}_g^T \in \mathbf{R}^+ \cup \{\infty\}$, such that the a fully informative equilibrium exists if and only if $\delta \leq \bar{\delta}_g^T$, where $\bar{\delta}_g^T \neq \infty$ if and only if $1 > s_2^* > s_2^R(1, 0)$. More specifically, $\bar{\delta}_g^T$ is defined by

$$\bar{\delta}_g^T = \begin{cases} \delta_g, & \text{if } \beta \in (\underline{\beta}, \bar{\beta}) \text{ and } s_1 < \bar{s}_1, \\ \infty, & \text{otherwise,} \end{cases} \tag{45}$$

where $\beta \in (\underline{\beta}, \bar{\beta})$ ensures that $1 > s_2^* > 0$, $s_1 < \bar{s}_1$ ensures that $s_2^R(1, 0) < s_2^*$, where $\bar{s}_1$ is defined by

$$\frac{(1-\alpha)(1-d)\bar{s}_1}{(1-\alpha)(1-d)\bar{s}_1 + \left[\alpha + (1-\alpha)(1-d)\right](1-\bar{s}_1)} = s_2^*. \tag{46}$$

and δ_g^T is defined by 31.

Second, an equilibrium with $\pi_g < 1$ exists if and only if $G_g^T(1, s_2^R(\pi_g, 0)) \leq 0$ for some $\pi_g \in [0, 1)$. Since we have that $G_g^T(1, s_2^R(\pi_g, 0))$ is decreasing in π_g, an equilibrium with $\pi_g < 1$ exists if and only if $\lim_{\pi_g \to 1} G_g^T(1, s_2^R(\pi_g, 0)) \leq 0$. Notice that $\lim_{\pi_g \to 1} G_g^T(1, s_2^R(\pi_g, 0)) \leq 0$ implies that $U^T(1) < \lim_{\pi_g \to 1} U^T(s_2^R(\pi_g, 0))$, which in turn implies that $\lim_{\pi_g \to 1} G_g^T(1, s_2^R(\pi_g, 0))$ is strictly decreasing in δ and that $1 \geq s_2^* > s_2^R(1, 0)$. Therefore there exists $\underline{\delta}_g^T \in \mathbf{R}^+ \cup \{\infty\}$, such that the an equilibrium with $\pi_g < 1$ exists if and only if $\delta \geq \underline{\delta}_g^T$, where $\bar{\delta}_g^T \neq \infty$ if and only if $1 \geq s_2^* > s_2^R(1, 0)$. More specifically, $\underline{\delta}_g^T$ is defined by

$$\underline{\delta}_g^T = \begin{cases} \delta_g, & \text{if } \beta \in (\underline{\beta}, \bar{\beta}] \text{ and } s_1 < \bar{s}_2, \\ \infty, & \text{otherwise,} \end{cases} \tag{47}$$

where $\beta \in (\underline{\beta}, \bar{\beta}]$ ensures that $1 \geq s_2^* > 0$ and $s < \bar{s}_2$ ensures that $s_2^* > s_2^R(1, 0)$.

∎

9.11 Proof of Lemma 2

After substituting Eq. 12 into Eq. 14 to eliminate ϕ, we have that the bank originates a risky loan if and only if $\tilde{\Delta}(z_2) \geq 0$, where

$$\begin{aligned}\tilde{\Delta}(z_2) = {} & [\alpha + (1-\alpha)(1-d)]\, R - R_0 \\ & - (1-\alpha)(1-z_2)(1-\gamma)\left(\tilde{R}_2(z_2)(\rho_L - 1) + \beta\left[(1-d)R - \tilde{R}_2(z_2)\rho_L\right]\right).\end{aligned} \tag{48}$$

We first show that $\tilde{\Delta}(z_2)$ is strictly increasing in z_2. The derivative with respect to z_2 is:

$$\begin{aligned}\frac{\frac{\partial \tilde{\Delta}(z_2)}{\partial z_2}}{(1-\alpha)(1-\gamma)} = {} & \left(\tilde{R}_2(z_2)(\rho_L - 1) + \beta\left[(1-d)R - \tilde{R}_2(z_2)\rho_L\right]\right) \\ & - (1-z_2)\frac{\partial \tilde{R}_2(z_2)}{\partial z_2}\left[(1-\beta)\rho_L - 1\right],\end{aligned} \tag{49}$$

where:

$$\frac{\partial \tilde{R}_2(z_2)}{\partial z_2} = \left[\tilde{R}_2(z_2)\right]^2 (1-\alpha)(1-\gamma)d > 0. \tag{50}$$

Consider two cases. First, if $(1-\beta)\rho_L - 1 \leq 0$, both terms in Eq. 51 are positive and therefore $\frac{\partial \Delta(z_2)}{\partial z_2} > 0$ for all z_2. Second, if $(1-\beta)\rho_L - 1 > 0$, then using Eq. 52, we can rewrite Eq. 51 as

$$\frac{\frac{\partial \tilde{\Delta}(z_2)}{\partial z_2}}{(1-\alpha)(1-\gamma)} = \beta(1-d)R + [(1-\beta)\rho_L - 1]\,\tilde{R}_2(z_2)\left[1-(1-z_2)(1-\alpha)(1-\gamma)d\tilde{R}_2(z_2)\right].$$

The definition of $\tilde{R}_2(z_2)$ given by Eq. 13 implies that $1-(1-z_2)(1-\alpha)(1-\gamma)d\tilde{R}_2(z_2) > 0$ and thus $\frac{\partial \tilde{\Delta}(z_2)}{\partial z_2} > 0$ for all z_2.

Moreover, $\tilde{\Delta}(1) > 0$ by Assumption 1 and $\tilde{\Delta}(z_1) \to -\infty$ as $z_2 \to -\infty$. Therefore there exists a unique $\tilde{z}_2^* < 1$, defined by $\tilde{\Delta}(\tilde{z}_2^*) = 0$, such that $\tilde{\Delta}(z_2) \geq 0$ and thus the bank originates a risky loan if and only if $z_2 > \tilde{z}_2^*$.

We now derive a condition for $\tilde{z}_2^* > 0$. This is the case if and only if

$$\begin{aligned} \tilde{\Delta}(0) = {} & [\alpha + (1-\alpha)(1-d)]\,R - R_0 \\ & - (1-\alpha)(1-\gamma)\left(\tilde{R}_2(0)(\rho_L - 1) + \beta\left[(1-d)R - \tilde{R}_2(0)\rho_L\right]\right) < 0. \end{aligned} \tag{51}$$

Notice the above expression is strictly decreasing in β. Moreover, it must be the case that $\tilde{\Delta}(0) < 0$ for $\beta = 1$. To see this, notice that, at $\beta = 1$, we have

$$\begin{aligned} & \tilde{\Delta}(0) = [\alpha + (1-\alpha)(1-d)]\,R - R_0 - (1-\alpha)(1-\gamma)\left[(1-d)R - \tilde{R}_2(0)\right] < 0 \\ \Leftrightarrow \quad & [\alpha + (1-\alpha)\gamma(1-d)]\,R - R_0 + (1-\alpha)(1-\gamma)\tilde{R}_2(0) < 0 \\ \Leftrightarrow \quad & [\alpha + (1-\alpha)\gamma(1-d)]\left(R - \tilde{R}_2(0)\right) - (R_0 - 1) < 0, \end{aligned} \tag{52}$$

which is implied by Part (ii) of Assumption 3. Therefore there exists a unique $\tilde{\beta} < 1$, such that $z_2^* > 0$ if and only if $\beta > \tilde{\beta}$, where $\tilde{\beta}$ is defined by

$$[\alpha + (1-\alpha)(1-d)]\,R - R_0 - (1-\alpha)(1-\gamma)\left(\tilde{R}_2(0)(\rho_L - 1) + \tilde{\beta}\left[(1-d)R - \tilde{R}_2(0)\rho_L\right]\right) = 0. \tag{53}$$

$\blacksquare$

9.12 Proof of Proposition

We prove this proposition by analyzing the three regions of X separately.

- $X \in [\underline{X}, \overline{X}]$. The proof is identical to the proof of Proposition 2.
- $X > \overline{X}$. The characterization of the equilibrium follows the logic of the proof of Proposition 3. Recall that $z_2^R(\pi_g, \pi_b) > z_2^f = 0$ for all (π_g, π_b). We then have that $\tilde{G}_g(\tilde{R}_2(\pi_b), z_2^R(\pi_g, \pi_b), z_2^f) > 0$ for all π_g and therefore $\pi_g = 1$ in any equilibrium. The equilibrium is thus either fully informative or lenient.
- $X < \underline{X}$. The characterization of the equilibrium follows the logic of the proof of Proposition 4. We have that $\tilde{G}_b(\tilde{R}_2(\pi_b), z_2^R(\pi_g, \pi_b), z_2^f) < 0$ for all π_b and therefore $\pi_b = 0$ in any equilibrium. The equilibrium is thus either fully informative or tough.

$\blacksquare$